Computer algebra independent integration tests

6-Hyperbolic-functions/6.5-Hyperbolic-secant/6.5.7-d-hyper-^m-a+b-c-sech-^n-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [220]. This is test number [180].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (220)	% 0. (0)
Mathematica	% 100. (220)	% 0. (0)
Maple	% 81.82 (180)	% 18.18 (40)
Maxima	% 47.73 (105)	% 52.27 (115)
Fricas	% 95.91 (211)	% 4.09 (9)
Sympy	% 4.55 (10)	% 95.45 (210)
Giac	% 66.82 (147)	% 33.18 (73)

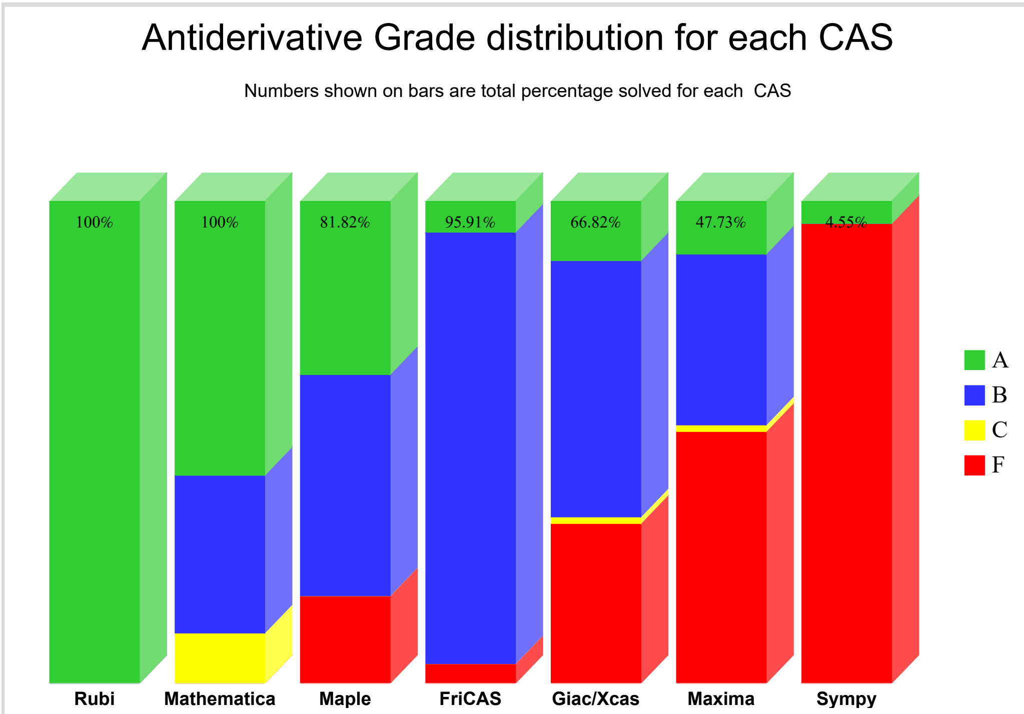
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

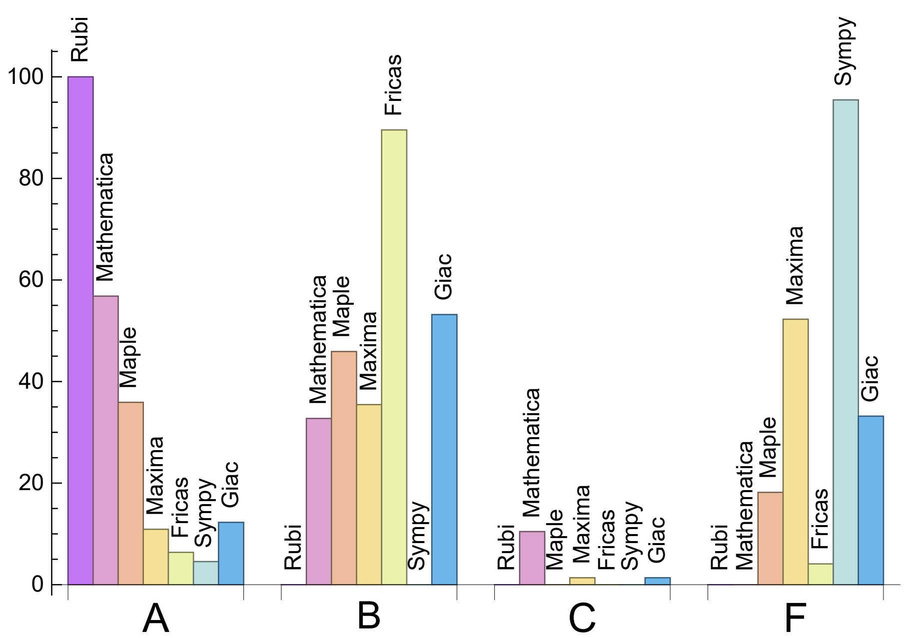
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	56.82	32.73	10.45	0.
Maple	35.91	45.91	0.	18.18
Maxima	10.91	35.45	1.36	52.27
Fricas	6.36	89.55	0.	4.09
Sympy	4.55	0.	0.	95.45
Giac	12.27	53.18	1.36	33.18

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.13	84.68	1.	76.	1.
Mathematica	1.92	261.24	2.56	130.5	1.77
Maple	0.06	344.84	3.27	148.5	2.31
Maxima	1.31	369.8	4.87	252.	3.61
Fricas	2.9	6312.58	66.27	3996.	53.05
Sympy	16.2	87.2	1.84	72.5	1.54
Giac	1.46	269.37	3.21	224.	3.

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {33, 34, 35, 41, 42, 43, 44, 46, 48, 66, 68, 69, 151, 155, 157, 158, 160, 162, 164, 166, 168, 169}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

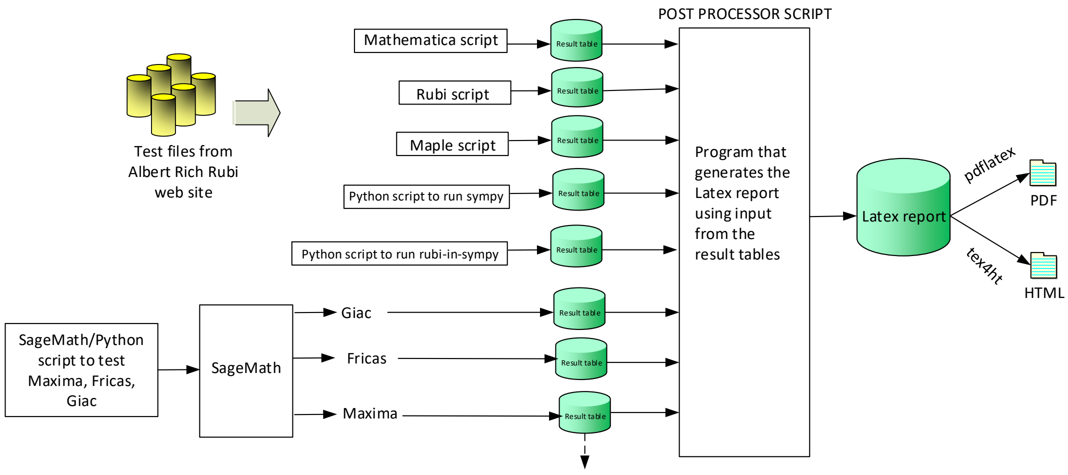
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 6, 8, 9, 10, 11, 12, 15, 18, 20, 21, 23, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 71, 73, 74, 75, 76, 77, 78, 83, 84, 86, 88, 89, 93, 94, 95, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 111, 113, 115, 117, 119, 121, 123, 125, 127, 129, 131, 133, 135, 138, 140, 142, 144, 146, 148, 150, 152, 154, 156, 159, 161, 163, 165, 167, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 182, 183, 184, 185, 186, 187, 188, 190, 192, 193, 194, 195, 197, 201, 202, 203, 204, 208, 209, 210, 211, 212, 214, 216, 217, 219, 220 }

B grade: { 5, 7, 13, 14, 16, 17, 19, 22, 24, 25, 27, 30, 32, 35, 38, 40, 41, 43, 48, 70, 72, 79, 80, 81, 82, 85, 87, 90, 91, 92, 96, 112, 114, 116, 118, 120, 122, 124, 126, 128, 130, 132, 134, 136, 137, 139, 141, 143, 145, 147, 149, 151, 153, 155, 157, 158, 160, 162, 164, 180, 181, 191, 196, 198, 199, 200, 205, 206, 207, 213, 215, 218 }

C grade: { 26, 28, 29, 31, 33, 34, 36, 37, 39, 42, 44, 45, 46, 47, 66, 68, 69, 108, 110, 166, 168, 169, 189 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 28, 29, 31, 36, 44, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 103, 104, 105, 106, 107, 108, 109, 111, 115, 116, 117, 118, 119, 127, 128, 129, 130, 131, 136, 137, 142, 152, 163, 180, 189, 198, 207, 216 }

B grade: { 22, 24, 25, 26, 27, 30, 32, 33, 34, 35, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 110, 112, 113, 114, 120, 121, 122, 123, 124, 125, 126, 132, 133, 134, 135, 138, 139, 140, 141, 143, 144, 145, 146, 147, 148, 149, 150, 151, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175 }

C grade: { }

F grade: { 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220 }

2.1.4 Maxima

A grade: { 3, 4, 6, 12, 20, 49, 51, 52, 57, 59, 60, 65, 103, 104, 105, 106, 115, 118, 127, 138, 140, 170, 171, 172

B grade: { 1, 2, 5, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 50, 53, 54, 55, 56, 58, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 102, 107, 108, 109, 110, 111, 112, 113, 114, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 142, 144, 146, 148, 150, 152, 154, 156, 159, 161, 163, 165, 167 }

C grade: { 173, 174, 175 }

F grade: { 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 139, 141, 143, 145, 147, 149, 151, 153, 155, 157, 158, 160, 162, 164, 166, 168, 169, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220 }

2.1.5 FriCAS

A grade: { 1, 3, 4, 49, 50, 51, 57, 59, 65, 171, 172, 173, 174, 175 }

B grade: { 2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 52, 53, 54, 55, 56, 58, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 170, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 212, 213, 214, 215, 216, 217, 219 }

C grade: { }

F grade: { 47, 48, 168, 169, 186, 193, 211, 218, 220 }

2.1.6 Sympy

A grade: { 51, 103, 105, 113, 115, 125, 127, 142, 207, 216 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 104, 106, 107, 108, 109, 110, 111, 112, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220 }

2.1.7 Giac

A grade: { 4, 6, 8, 12, 16, 17, 20, 30, 32, 33, 35, 52, 56, 57, 78, 80, 82, 89, 106, 107, 108, 118, 140, 143, 145, 151, 153 }

B grade: { 1, 2, 3, 5, 7, 9, 10, 11, 13, 14, 15, 18, 19, 21, 22, 23, 24, 25, 27, 38, 40, 41, 43, 46, 48, 49, 50, 51, 53, 54, 55, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 84, 87, 91, 93, 96, 98, 100, 102, 103, 104, 105, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 141, 142, 144, 146, 147, 148, 149, 150, 152, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172 }

C grade: { 173, 174, 175 }

F grade: { 26, 28, 29, 31, 34, 36, 37, 39, 42, 44, 45, 47, 74, 76, 77, 79, 81, 83, 85, 86, 88, 90, 92, 94, 95, 97, 99, 101, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	54	78	174	300	0	193
normalized size	1	1.	0.77	1.11	2.49	4.29	0.	2.76
time (sec)	N/A	0.081	0.314	0.032	1.016	2.553	0.	1.165

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	53	55	150	220	0	132
normalized size	1	1.	1.2	1.25	3.41	5.	0.	3.
time (sec)	N/A	0.056	0.049	0.029	1.004	2.522	0.	1.186

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	57	45	84	177	0	130
normalized size	1	1.	1.33	1.05	1.95	4.12	0.	3.02
time (sec)	N/A	0.056	0.186	0.028	1.035	2.445	0.	1.174

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	35	26	49	99	0	63
normalized size	1	1.	1.46	1.08	2.04	4.12	0.	2.62
time (sec)	N/A	0.033	0.021	0.011	1.005	2.639	0.	1.144

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	67	36	108	541	0	104
normalized size	1	1.	2.48	1.33	4.	20.04	0.	3.85
time (sec)	N/A	0.045	0.049	0.028	1.076	2.518	0.	1.145

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	37	44	53	246	0	46
normalized size	1	1.	1.37	1.63	1.96	9.11	0.	1.7
time (sec)	N/A	0.046	0.07	0.027	1.036	2.416	0.	1.13

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	131	70	267	2469	0	198
normalized size	1	1.	2.43	1.3	4.94	45.72	0.	3.67
time (sec)	N/A	0.071	0.047	0.036	1.027	2.712	0.	1.163

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	84	73	252	666	0	109
normalized size	1	1.	1.87	1.62	5.6	14.8	0.	2.42
time (sec)	N/A	0.058	0.052	0.036	1.048	2.467	0.	1.159

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	153	109	285	880	0	328
normalized size	1	1.	1.34	0.96	2.5	7.72	0.	2.88
time (sec)	N/A	0.138	1.621	0.04	1.052	2.705	0.	1.182

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	83	108	359	541	0	207
normalized size	1	1.	1.15	1.5	4.99	7.51	0.	2.88
time (sec)	N/A	0.087	0.51	0.045	1.044	2.765	0.	1.179

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	126	90	216	641	0	204
normalized size	1	1.	1.73	1.23	2.96	8.78	0.	2.79
time (sec)	N/A	0.107	0.934	0.036	1.042	2.901	0.	1.18

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	59	43	88	336	0	103
normalized size	1	1.	1.31	0.96	1.96	7.47	0.	2.29
time (sec)	N/A	0.046	0.171	0.018	1.051	2.53	0.	1.202

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	108	72	266	3016	0	193
normalized size	1	1.	2.08	1.38	5.12	58.	0.	3.71
time (sec)	N/A	0.077	0.553	0.036	1.055	2.678	0.	1.2

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	109	91	189	733	0	151
normalized size	1	1.	2.18	1.82	3.78	14.66	0.	3.02
time (sec)	N/A	0.062	1.524	0.037	1.063	2.572	0.	1.222

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	144	126	478	7422	0	317
normalized size	1	1.	1.38	1.21	4.6	71.37	0.	3.05
time (sec)	N/A	0.136	1.539	0.046	1.093	2.978	0.	1.224

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	151	138	385	1054	0	155
normalized size	1	1.	2.01	1.84	5.13	14.05	0.	2.07
time (sec)	N/A	0.083	1.265	0.045	1.05	2.55	0.	1.169

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	651	182	570	1858	0	474
normalized size	1	1.	3.58	1.	3.13	10.21	0.	2.6
time (sec)	N/A	0.232	2.424	0.046	1.079	2.647	0.	1.18

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	119	179	660	1029	0	277
normalized size	1	1.	1.2	1.81	6.67	10.39	0.	2.8
time (sec)	N/A	0.111	1.21	0.043	1.077	2.655	0.	1.2

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	143	480	145	598	1508	0	385
normalized size	1	1.28	4.29	1.29	5.34	13.46	0.	3.44
time (sec)	N/A	0.193	1.82	0.039	1.059	2.552	0.	1.19

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	93	58	127	702	0	138
normalized size	1	1.	1.45	0.91	1.98	10.97	0.	2.16
time (sec)	N/A	0.059	0.247	0.019	1.059	2.522	0.	1.19

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	134	118	483	8645	0	313
normalized size	1	1.	1.61	1.42	5.82	104.16	0.	3.77
time (sec)	N/A	0.1	1.22	0.04	1.1	3.092	0.	1.189

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	380	148	483	1580	0	338
normalized size	1	1.	5.43	2.11	6.9	22.57	0.	4.83
time (sec)	N/A	0.073	2.736	0.043	1.057	2.549	0.	1.195

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	224	192	751	17022	0	470
normalized size	1	1.	1.56	1.33	5.22	118.21	0.	3.26
time (sec)	N/A	0.178	3.734	0.05	1.07	3.63	0.	1.168

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	213	213	896	2480	0	481
normalized size	1	1.	2.05	2.05	8.62	23.85	0.	4.62
time (sec)	N/A	0.105	2.303	0.051	1.096	2.558	0.	1.233

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	294	708	0	4338	0	312
normalized size	1	1.	2.51	6.05	0.	37.08	0.	2.67
time (sec)	N/A	0.196	2.407	0.089	0.	2.969	0.	1.179

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	372	261	0	3343	0	0
normalized size	1	1.	5.24	3.68	0.	47.08	0.	0.
time (sec)	N/A	0.105	2.146	0.061	0.	2.954	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	236	383	0	2132	0	186
normalized size	1	1.	3.15	5.11	0.	28.43	0.	2.48
time (sec)	N/A	0.11	0.904	0.068	0.	2.856	0.	1.216

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	328	44	0	1643	0	0
normalized size	1	1.	6.98	0.94	0.	34.96	0.	0.
time (sec)	N/A	0.048	1.045	0.023	0.	2.783	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	232	67	0	1461	0	0
normalized size	1	1.	4.22	1.22	0.	26.56	0.	0.
time (sec)	N/A	0.082	0.849	0.051	0.	2.889	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	179	147	0	1589	0	107
normalized size	1	1.	3.38	2.77	0.	29.98	0.	2.02
time (sec)	N/A	0.071	0.701	0.061	0.	2.809	0.	1.264

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	338	134	0	5080	0	0
normalized size	1	1.	3.89	1.54	0.	58.39	0.	0.
time (sec)	N/A	0.118	1.954	0.058	0.	3.186	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	216	258	0	4602	0	173
normalized size	1	1.	2.88	3.44	0.	61.36	0.	2.31
time (sec)	N/A	0.099	2.027	0.076	0.	3.003	0.	1.281

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	1330	1025	0	12945	0	454
normalized size	1	1.	6.86	5.28	0.	66.73	0.	2.34
time (sec)	N/A	0.273	14.197	0.109	0.	3.926	0.	1.189

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	114	114	861	561	0	9646	0	0
normalized size	1	1.	7.55	4.92	0.	84.61	0.	0.
time (sec)	N/A	0.144	4.678	0.082	0.	3.99	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	131	131	791	537	0	7262	0	325
normalized size	1	1.	6.04	4.1	0.	55.44	0.	2.48
time (sec)	N/A	0.196	11.18	0.094	0.	2.886	0.	1.183

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	479	74	0	4732	0	0
normalized size	1	1.	5.7	0.88	0.	56.33	0.	0.
time (sec)	N/A	0.065	2.702	0.031	0.	2.88	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	377	431	0	5986	0	0
normalized size	1	1.	3.81	4.35	0.	60.46	0.	0.
time (sec)	N/A	0.13	1.121	0.07	0.	3.336	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	220	310	0	5806	0	328
normalized size	1	1.	2.39	3.37	0.	63.11	0.	3.57
time (sec)	N/A	0.084	2.461	0.085	0.	2.47	0.	1.365

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	462	496	0	16629	0	0
normalized size	1	1.	3.14	3.37	0.	113.12	0.	0.
time (sec)	N/A	0.206	2.154	0.083	0.	3.486	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	295	577	0	14893	0	358
normalized size	1	1.	2.4	4.69	0.	121.08	0.	2.91
time (sec)	N/A	0.195	5.93	0.102	0.	2.992	0.	1.374

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	242	242	3080	1677	0	30538	0	714
normalized size	1	1.	12.73	6.93	0.	126.19	0.	2.95
time (sec)	N/A	0.399	26.712	0.125	0.	3.981	0.	1.226

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	154	154	1217	1296	0	21811	0	0
normalized size	1	1.	7.9	8.42	0.	141.63	0.	0.
time (sec)	N/A	0.218	10.78	0.098	0.	3.479	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	187	187	2544	1320	0	23448	0	517
normalized size	1	1.	13.6	7.06	0.	125.39	0.	2.76
time (sec)	N/A	0.292	18.788	0.112	0.	4.003	0.	1.188

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	116	116	453	107	0	12176	0	0
normalized size	1	1.	3.91	0.92	0.	104.97	0.	0.
time (sec)	N/A	0.084	9.168	0.035	0.	2.96	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	440	1476	0	20882	0	0
normalized size	1	1.	2.86	9.58	0.	135.6	0.	0.
time (sec)	N/A	0.223	2.365	0.082	0.	4.276	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	126	126	981	819	0	17042	0	485
normalized size	1	1.	7.79	6.5	0.	135.25	0.	3.85
time (sec)	N/A	0.1	6.802	0.102	0.	3.703	0.	1.417

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	524	1555	0	0	0	0
normalized size	1	1.	2.46	7.3	0.	0.	0.	0.
time (sec)	N/A	0.342	3.737	0.101	0.	0.	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	165	165	985	1443	0	0	0	568
normalized size	1	1.	5.97	8.75	0.	0.	0.	3.44
time (sec)	N/A	0.273	5.361	0.121	0.	0.	0.	1.523

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	45	66	131	163	0	157
normalized size	1	1.	0.74	1.08	2.15	2.67	0.	2.57
time (sec)	N/A	0.046	0.092	0.04	1.136	2.066	0.	1.218

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	50	34	115	105	0	97
normalized size	1	1.	1.67	1.13	3.83	3.5	0.	3.23
time (sec)	N/A	0.05	0.017	0.037	1.149	2.099	0.	1.167

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	33	37	51	74	60	89
normalized size	1	1.	1.06	1.19	1.65	2.39	1.94	2.87
time (sec)	N/A	0.031	0.031	0.035	1.16	1.972	47.101	1.211

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	35	26	38	266	0	55
normalized size	1	1.	1.46	1.08	1.58	11.08	0.	2.29
time (sec)	N/A	0.03	0.017	0.031	1.6	2.15	0.	1.194

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	48	45	109	882	0	115
normalized size	1	1.	1.2	1.12	2.72	22.05	0.	2.88
time (sec)	N/A	0.027	0.021	0.02	1.723	2.094	0.	1.27

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	43	39	34	151	427	0	82
normalized size	1	1.43	1.3	1.13	5.03	14.23	0.	2.73
time (sec)	N/A	0.04	0.012	0.019	1.043	1.924	0.	1.361

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	60	83	248	2931	0	213
normalized size	1	1.	0.86	1.19	3.54	41.87	0.	3.04
time (sec)	N/A	0.05	0.104	0.021	1.685	2.155	0.	1.201

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	65	71	56	405	938	0	115
normalized size	1	1.3	1.42	1.12	8.1	18.76	0.	2.3
time (sec)	N/A	0.046	0.013	0.02	1.058	2.063	0.	1.145

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	58	79	142	193	0	204
normalized size	1	1.	0.71	0.96	1.73	2.35	0.	2.49
time (sec)	N/A	0.093	0.129	0.041	1.042	2.204	0.	1.158

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	72	66	142	1057	0	149
normalized size	1	1.	1.47	1.35	2.9	21.57	0.	3.04
time (sec)	N/A	0.059	0.026	0.041	1.739	2.211	0.	1.202

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	52	51	85	196	0	180
normalized size	1	1.	1.11	1.09	1.81	4.17	0.	3.83
time (sec)	N/A	0.078	0.142	0.036	1.137	2.149	0.	1.163

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	80	63	136	1678	0	157
normalized size	1	1.	1.43	1.12	2.43	29.96	0.	2.8
time (sec)	N/A	0.068	0.04	0.041	1.6	2.23	0.	1.175

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	71	106	271	3445	0	232
normalized size	1	1.	0.79	1.18	3.01	38.28	0.	2.58
time (sec)	N/A	0.084	0.131	0.027	1.735	2.218	0.	1.155

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	93	70	437	1054	0	211
normalized size	1	1.	1.75	1.32	8.25	19.89	0.	3.98
time (sec)	N/A	0.066	0.03	0.023	1.12	2.074	0.	1.166

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	104	169	470	7609	0	397
normalized size	1	1.	0.81	1.32	3.67	59.45	0.	3.1
time (sec)	N/A	0.149	0.238	0.026	1.7	2.491	0.	1.136

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	144	102	906	1828	0	266
normalized size	1	1.	1.8	1.27	11.32	22.85	0.	3.32
time (sec)	N/A	0.076	0.03	0.026	1.154	2.055	0.	1.159

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	70	93	176	370	0	257
normalized size	1	1.	0.83	1.11	2.1	4.4	0.	3.06
time (sec)	N/A	0.111	0.409	0.038	1.106	2.087	0.	1.192

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	81	81	483	103	242	3555	0	240
normalized size	1	1.	5.96	1.27	2.99	43.89	0.	2.96
time (sec)	N/A	0.093	6.836	0.045	1.701	2.289	0.	1.192

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	64	77	216	670	0	215
normalized size	1	1.	0.89	1.07	3.	9.31	0.	2.99
time (sec)	N/A	0.092	0.479	0.044	1.138	2.301	0.	1.19

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	575	125	298	4986	0	274
normalized size	1	1.	6.18	1.34	3.2	53.61	0.	2.95
time (sec)	N/A	0.103	7.902	0.048	1.687	2.675	0.	1.183

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	1430	193	493	8805	0	420
normalized size	1	1.	9.73	1.31	3.35	59.9	0.	2.86
time (sec)	N/A	0.143	9.735	0.03	1.716	2.49	0.	1.165

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	319	116	938	2103	0	408
normalized size	1	1.	4.31	1.57	12.68	28.42	0.	5.51
time (sec)	N/A	0.08	1.415	0.026	1.233	2.055	0.	1.168

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	297	280	751	16578	0	656
normalized size	1	1.	1.52	1.43	3.83	84.58	0.	3.35
time (sec)	N/A	0.232	9.324	0.03	1.704	2.733	0.	1.17

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	348	158	1681	3222	0	486
normalized size	1	1.	3.22	1.46	15.56	29.83	0.	4.5
time (sec)	N/A	0.094	1.709	0.029	1.126	2.391	0.	1.178

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	95	493	0	4313	0	296
normalized size	1	1.	0.81	4.21	0.	36.86	0.	2.53
time (sec)	N/A	0.189	0.522	0.084	0.	2.752	0.	1.185

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	79	256	0	3991	0	0
normalized size	1	1.	1.04	3.37	0.	52.51	0.	0.
time (sec)	N/A	0.09	0.287	0.083	0.	2.792	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	67	278	0	2188	0	178
normalized size	1	1.	0.89	3.71	0.	29.17	0.	2.37
time (sec)	N/A	0.116	0.228	0.079	0.	2.423	0.	1.186

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	128	0	1891	0	0
normalized size	1	1.	1.	2.46	0.	36.37	0.	0.
time (sec)	N/A	0.061	0.121	0.069	0.	2.375	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	82	0	1291	0	0
normalized size	1	1.	1.	2.28	0.	35.86	0.	0.
time (sec)	N/A	0.042	0.065	0.047	0.	2.313	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	107	0	1065	0	63
normalized size	1	1.	1.	2.97	0.	29.58	0.	1.75
time (sec)	N/A	0.061	0.079	0.043	0.	2.187	0.	1.151

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	194	107	0	1431	0	0
normalized size	1	1.	3.53	1.95	0.	26.02	0.	0.
time (sec)	N/A	0.069	0.682	0.048	0.	2.302	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	182	141	0	1661	0	100
normalized size	1	1.	3.5	2.71	0.	31.94	0.	1.92
time (sec)	N/A	0.07	0.68	0.045	0.	2.271	0.	1.167

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	213	189	0	3997	0	0
normalized size	1	1.	2.48	2.2	0.	46.48	0.	0.
time (sec)	N/A	0.103	1.865	0.047	0.	2.524	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	214	313	0	4709	0	159
normalized size	1	1.	2.78	4.06	0.	61.16	0.	2.06
time (sec)	N/A	0.089	2.233	0.051	0.	2.382	0.	1.251

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	113	517	0	13824	0	0
normalized size	1	1.	0.9	4.14	0.	110.59	0.	0.
time (sec)	N/A	0.154	0.863	0.101	0.	2.905	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	103	551	0	8946	0	446
normalized size	1	1.	0.72	3.83	0.	62.12	0.	3.1
time (sec)	N/A	0.235	1.318	0.102	0.	2.783	0.	1.158

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	234	385	0	7525	0	0
normalized size	1	1.	2.34	3.85	0.	75.25	0.	0.
time (sec)	N/A	0.132	1.832	0.091	0.	2.621	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	124	332	0	4533	0	0
normalized size	1	1.	1.51	4.05	0.	55.28	0.	0.
time (sec)	N/A	0.066	0.302	0.068	0.	2.395	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	187	260	0	3563	0	180
normalized size	1	1.	2.53	3.51	0.	48.15	0.	2.43
time (sec)	N/A	0.074	1.001	0.059	0.	2.345	0.	1.267

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	108	241	0	3996	0	0
normalized size	1	1.	1.48	3.3	0.	54.74	0.	0.
time (sec)	N/A	0.071	0.217	0.059	0.	2.425	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	88	380	0	3765	0	189
normalized size	1	1.	1.06	4.58	0.	45.36	0.	2.28
time (sec)	N/A	0.09	0.207	0.058	0.	2.322	0.	1.174

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	282	361	0	5195	0	0
normalized size	1	1.	2.79	3.57	0.	51.44	0.	0.
time (sec)	N/A	0.117	2.47	0.064	0.	2.675	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	229	413	0	6961	0	306
normalized size	1	1.	2.27	4.09	0.	68.92	0.	3.03
time (sec)	N/A	0.142	3.79	0.063	0.	2.498	0.	1.274

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	489	448	0	15336	0	0
normalized size	1	1.	3.2	2.93	0.	100.24	0.	0.
time (sec)	N/A	0.2	5.048	0.067	0.	3.426	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	156	1435	0	27738	0	552
normalized size	1	1.	0.76	7.03	0.	135.97	0.	2.71
time (sec)	N/A	0.377	4.004	0.128	0.	3.766	0.	1.213

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	292	1238	0	23050	0	0
normalized size	1	1.	1.9	8.04	0.	149.68	0.	0.
time (sec)	N/A	0.185	3.652	0.118	0.	3.495	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	214	1174	0	15863	0	0
normalized size	1	1.	1.51	8.27	0.	111.71	0.	0.
time (sec)	N/A	0.137	2.857	0.082	0.	3.137	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	258	612	0	11784	0	386
normalized size	1	1.	2.39	5.67	0.	109.11	0.	3.57
time (sec)	N/A	0.09	2.078	0.073	0.	2.794	0.	1.353

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	159	1038	0	14071	0	0
normalized size	1	1.	1.29	8.44	0.	114.4	0.	0.
time (sec)	N/A	0.108	0.812	0.079	0.	2.931	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	250	1084	0	12354	0	374
normalized size	1	1.	2.	8.67	0.	98.83	0.	2.99
time (sec)	N/A	0.107	3.421	0.092	0.	2.874	0.	1.355

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	125	592	0	12078	0	0
normalized size	1	1.	1.18	5.58	0.	113.94	0.	0.
time (sec)	N/A	0.092	0.282	0.077	0.	2.767	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	125	1236	0	13508	0	413
normalized size	1	1.	0.87	8.58	0.	93.81	0.	2.87
time (sec)	N/A	0.142	0.961	0.078	0.	2.868	0.	1.341

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	247	1202	0	18826	0	0
normalized size	1	1.	1.61	7.86	0.	123.05	0.	0.
time (sec)	N/A	0.205	4.601	0.084	0.	3.8	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	57	98	124	876	0	142
normalized size	1	1.	1.19	2.04	2.58	18.25	0.	2.96
time (sec)	N/A	0.064	0.024	0.034	1.194	2.077	0.	1.317

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	45	72	105	2871	80	157
normalized size	1	1.	0.92	1.47	2.14	58.59	1.63	3.2
time (sec)	N/A	0.056	0.023	0.035	1.708	2.395	2.732	1.251

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	41	60	57	396	0	93
normalized size	1	1.	1.28	1.88	1.78	12.38	0.	2.91
time (sec)	N/A	0.057	0.016	0.028	1.172	2.23	0.	1.249

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	29	36	973	42	108
normalized size	1	1.	1.	1.	1.24	33.55	1.45	3.72
time (sec)	N/A	0.028	0.022	0.013	1.15	2.085	0.726	1.195

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	31	88	0	31
normalized size	1	1.	1.	1.07	2.07	5.87	0.	2.07
time (sec)	N/A	0.013	0.003	0.007	1.153	1.998	0.	1.146

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	44	26	88	178	0	76
normalized size	1	1.	1.57	0.93	3.14	6.36	0.	2.71
time (sec)	N/A	0.051	0.038	0.033	1.733	2.097	0.	1.177

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	41	30	63	103	0	36
normalized size	1	1.	2.28	1.67	3.5	5.72	0.	2.
time (sec)	N/A	0.059	0.031	0.032	1.159	2.025	0.	1.192

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	52	50	146	990	0	109
normalized size	1	1.	1.68	1.61	4.71	31.94	0.	3.52
time (sec)	N/A	0.057	0.168	0.04	1.184	2.149	0.	1.227

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	49	70	230	370	0	90
normalized size	1	1.	1.44	2.06	6.76	10.88	0.	2.65
time (sec)	N/A	0.063	0.023	0.04	1.155	2.044	0.	1.242

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	62	86	339	2898	0	158
normalized size	1	1.	1.22	1.69	6.65	56.82	0.	3.1
time (sec)	N/A	0.078	0.262	0.041	1.208	2.242	0.	1.28

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	395	181	876	1868	0	371
normalized size	1	1.	5.13	2.35	11.38	24.26	0.	4.82
time (sec)	N/A	0.105	1.166	0.046	1.221	2.085	0.	1.322

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	107	150	450	6669	129	325
normalized size	1	1.	1.39	1.95	5.84	86.61	1.68	4.22
time (sec)	N/A	0.095	0.307	0.043	2.359	2.435	8.809	1.283

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	281	115	439	1099	0	261
normalized size	1	1.	4.76	1.95	7.44	18.63	0.	4.42
time (sec)	N/A	0.098	0.9	0.037	1.188	2.076	0.	1.239

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	81	48	74	3012	63	215
normalized size	1	1.	1.69	1.	1.54	62.75	1.31	4.48
time (sec)	N/A	0.05	0.132	0.011	1.116	2.184	2.919	1.162

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	106	47	162	447	0	107
normalized size	1	1.	2.65	1.18	4.05	11.18	0.	2.68
time (sec)	N/A	0.034	0.42	0.022	1.111	2.007	0.	1.139

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	84	60	217	1709	0	231
normalized size	1	1.	1.58	1.13	4.09	32.25	0.	4.36
time (sec)	N/A	0.08	0.243	0.041	1.862	2.213	0.	1.184

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	82	64	96	265	0	88
normalized size	1	1.	2.28	1.78	2.67	7.36	0.	2.44
time (sec)	N/A	0.085	0.7	0.033	1.202	2.11	0.	1.261

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	82	86	278	1553	0	217
normalized size	1	1.	1.49	1.56	5.05	28.24	0.	3.95
time (sec)	N/A	0.089	0.209	0.044	1.79	2.279	0.	1.318

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	160	96	362	478	0	131
normalized size	1	1.	3.48	2.09	7.87	10.39	0.	2.85
time (sec)	N/A	0.092	0.818	0.04	1.161	2.067	0.	1.324

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	77	142	381	3093	0	198
normalized size	1	1.	1.48	2.73	7.33	59.48	0.	3.81
time (sec)	N/A	0.094	0.232	0.046	1.152	2.232	0.	1.436

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	256	163	828	1050	0	225
normalized size	1	1.	4.	2.55	12.94	16.41	0.	3.52
time (sec)	N/A	0.097	1.101	0.049	1.193	2.165	0.	1.458

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	107	228	940	6593	0	292
normalized size	1	1.	1.24	2.65	10.93	76.66	0.	3.4
time (sec)	N/A	0.125	0.508	0.053	1.218	2.467	0.	1.653

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	301	274	1962	3414	0	637
normalized size	1	1.	2.74	2.49	17.84	31.04	0.	5.79
time (sec)	N/A	0.118	6.144	0.049	1.182	2.219	0.	1.586

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	128	253	880	12142	178	518
normalized size	1	1.	1.24	2.46	8.54	117.88	1.73	5.03
time (sec)	N/A	0.098	0.886	0.051	1.568	2.699	22.626	1.355

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	479	180	1064	2249	0	481
normalized size	1	1.	5.21	1.96	11.57	24.45	0.	5.23
time (sec)	N/A	0.11	1.803	0.042	1.24	2.181	0.	1.309

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	100	67	115	6511	87	362
normalized size	1	1.	1.41	0.94	1.62	91.7	1.23	5.1
time (sec)	N/A	0.058	0.292	0.013	1.127	2.754	8.921	1.238

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	268	83	448	1173	0	246
normalized size	1	1.	3.67	1.14	6.14	16.07	0.	3.37
time (sec)	N/A	0.047	0.914	0.026	1.034	2.497	0.	1.163

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	114	111	405	5819	0	439
normalized size	1	1.	1.36	1.32	4.82	69.27	0.	5.23
time (sec)	N/A	0.107	0.62	0.048	1.627	2.722	0.	1.209

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	126	111	232	863	0	178
normalized size	1	1.	2.07	1.82	3.8	14.15	0.	2.92
time (sec)	N/A	0.103	1.76	0.043	1.172	2.456	0.	1.267

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	110	145	424	4127	0	394
normalized size	1	1.	1.36	1.79	5.23	50.95	0.	4.86
time (sec)	N/A	0.116	1.226	0.053	1.787	2.722	0.	1.402

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	343	149	494	837	0	209
normalized size	1	1.	5.72	2.48	8.23	13.95	0.	3.48
time (sec)	N/A	0.101	1.707	0.043	1.228	2.374	0.	1.423

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	101	194	570	4574	0	335
normalized size	1	1.	1.25	2.4	7.04	56.47	0.	4.14
time (sec)	N/A	0.121	0.858	0.047	1.701	2.749	0.	1.581

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	303	199	1115	1249	0	284
normalized size	1	1.	4.39	2.88	16.16	18.1	0.	4.12
time (sec)	N/A	0.109	1.098	0.046	1.254	2.371	0.	1.607

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	98	310	981	6665	0	323
normalized size	1	1.	1.27	4.03	12.74	86.56	0.	4.19
time (sec)	N/A	0.121	0.773	0.05	1.269	2.478	0.	1.734

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	455	129	949	2410	0	451
normalized size	1	1.	4.1	1.16	8.55	21.71	0.	4.06
time (sec)	N/A	0.07	1.599	0.032	1.461	2.17	0.	1.159

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	724	185	1724	4396	0	725
normalized size	1	1.	4.44	1.13	10.58	26.97	0.	4.45
time (sec)	N/A	0.095	6.542	0.039	1.212	2.155	0.	1.189

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	98	331	177	1895	0	240
normalized size	1	1.	1.4	4.73	2.53	27.07	0.	3.43
time (sec)	N/A	0.112	0.29	0.062	1.767	2.665	0.	2.243

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	196	386	0	1854	0	147
normalized size	1	1.	3.32	6.54	0.	31.42	0.	2.49
time (sec)	N/A	0.178	1.107	0.067	0.	2.316	0.	1.915

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	41	196	104	294	0	107
normalized size	1	1.	0.91	4.36	2.31	6.53	0.	2.38
time (sec)	N/A	0.088	0.106	0.054	1.752	2.33	0.	1.791

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	174	257	0	1087	0	104
normalized size	1	1.	3.78	5.59	0.	23.63	0.	2.26
time (sec)	N/A	0.14	0.299	0.059	0.	2.187	0.	1.49

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	26	38	69	198	124	76
normalized size	1	1.	1.13	1.65	3.	8.61	5.39	3.3
time (sec)	N/A	0.035	0.175	0.017	1.133	2.189	9.338	1.398

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	172	152	0	1122	0	88
normalized size	1	1.	3.74	3.3	0.	24.39	0.	1.91
time (sec)	N/A	0.046	0.247	0.052	0.	2.308	0.	1.171

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	42	133	135	305	0	131
normalized size	1	1.	0.91	2.89	2.93	6.63	0.	2.85
time (sec)	N/A	0.087	0.089	0.059	1.043	2.79	0.	1.47

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	193	189	0	1980	0	120
normalized size	1	1.	3.11	3.05	0.	31.94	0.	1.94
time (sec)	N/A	0.18	1.112	0.072	0.	2.662	0.	1.613

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	100	199	252	2144	0	288
normalized size	1	1.	1.37	2.73	3.45	29.37	0.	3.95
time (sec)	N/A	0.122	0.222	0.07	1.21	3.021	0.	2.076

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	380	301	0	6730	0	217
normalized size	1	1.	4.37	3.46	0.	77.36	0.	2.49
time (sec)	N/A	0.286	3.269	0.085	0.	2.621	0.	2.265

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	109	351	208	2044	0	281
normalized size	1	1.	1.43	4.62	2.74	26.89	0.	3.7
time (sec)	N/A	0.117	0.444	0.083	1.634	2.703	0.	2.997

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	228	667	0	3748	0	252
normalized size	1	1.	2.51	7.33	0.	41.19	0.	2.77
time (sec)	N/A	0.194	2.108	0.086	0.	2.389	0.	2.521

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	81	186	146	1260	0	163
normalized size	1	1.	1.59	3.65	2.86	24.71	0.	3.2
time (sec)	N/A	0.09	0.711	0.076	1.23	2.205	0.	2.136

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	326	417	0	4431	0	198
normalized size	1	1.	3.84	4.91	0.	52.13	0.	2.33
time (sec)	N/A	0.169	4.489	0.081	0.	2.506	0.	1.923

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	79	60	143	1243	0	163
normalized size	1	1.	1.61	1.22	2.92	25.37	0.	3.33
time (sec)	N/A	0.062	0.572	0.025	1.269	2.284	0.	1.591

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	221	423	0	4199	0	224
normalized size	1	1.	2.38	4.55	0.	45.15	0.	2.41
time (sec)	N/A	0.096	2.029	0.073	0.	2.539	0.	1.143

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	115	292	282	2457	0	332
normalized size	1	1.	1.39	3.52	3.4	29.6	0.	4.
time (sec)	N/A	0.129	0.288	0.084	1.374	3.305	0.	1.694

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	121	121	268	481	0	8483	0	381
normalized size	1	1.	2.21	3.98	0.	70.11	0.	3.15
time (sec)	N/A	0.284	2.847	0.099	0.	2.823	0.	2.168

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	130	367	518	8284	0	517
normalized size	1	1.	1.18	3.34	4.71	75.31	0.	4.7
time (sec)	N/A	0.175	1.295	0.096	1.268	4.779	0.	2.692

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	161	161	350	634	0	22652	0	408
normalized size	1	1.	2.17	3.94	0.	140.7	0.	2.53
time (sec)	N/A	0.411	5.227	0.109	0.	3.605	0.	3.034

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	148	148	515	1713	0	12740	0	501
normalized size	1	1.	3.48	11.57	0.	86.08	0.	3.39
time (sec)	N/A	0.32	6.055	0.106	0.	2.895	0.	4.319

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	136	579	278	4238	0	252
normalized size	1	1.	1.77	7.52	3.61	55.04	0.	3.27
time (sec)	N/A	0.12	2.074	0.093	1.221	2.398	0.	3.532

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	139	1317	1306	0	14841	0	398
normalized size	1	1.	9.47	9.4	0.	106.77	0.	2.86
time (sec)	N/A	0.294	14.278	0.104	0.	3.121	0.	3.189

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	131	672	282	4297	0	236
normalized size	1	1.	1.62	8.3	3.48	53.05	0.	2.91
time (sec)	N/A	0.126	1.39	0.093	1.299	2.499	0.	2.753

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	139	1317	1173	0	16496	0	417
normalized size	1	1.	9.47	8.44	0.	118.68	0.	3.
time (sec)	N/A	0.246	14.552	0.104	0.	3.311	0.	2.405

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	129	82	261	4103	0	252
normalized size	1	1.	1.77	1.12	3.58	56.21	0.	3.45
time (sec)	N/A	0.082	1.896	0.027	1.214	2.466	0.	1.818

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	146	146	301	1292	0	15505	0	450
normalized size	1	1.	2.06	8.85	0.	106.2	0.	3.08
time (sec)	N/A	0.181	6.113	0.093	0.	3.329	0.	1.177

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	155	1046	566	9400	0	641
normalized size	1	1.	1.19	8.05	4.35	72.31	0.	4.93
time (sec)	N/A	0.186	1.073	0.101	1.375	5.792	0.	1.932

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	182	182	2083	1442	0	27173	0	543
normalized size	1	1.	11.45	7.92	0.	149.3	0.	2.98
time (sec)	N/A	0.403	7.031	0.12	0.	3.812	0.	2.58

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	172	1128	934	23458	0	1034
normalized size	1	1.	1.13	7.42	6.14	154.33	0.	6.8
time (sec)	N/A	0.243	1.937	0.123	1.468	9.424	0.	3.535

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	232	232	3334	1610	0	0	0	651
normalized size	1	1.	14.37	6.94	0.	0.	0.	2.81
time (sec)	N/A	0.514	7.528	0.134	0.	0.	0.	3.926

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	207	207	1405	2892	0	0	0	811
normalized size	1	1.	6.79	13.97	0.	0.	0.	3.92
time (sec)	N/A	0.346	6.859	0.119	0.	0.	0.	1.181

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	25	120	45	620	0	97
normalized size	1	1.	0.86	4.14	1.55	21.38	0.	3.34
time (sec)	N/A	0.031	0.023	0.108	1.65	1.953	0.	1.127

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	79	18	55	0	35
normalized size	1	1.	1.	5.64	1.29	3.93	0.	2.5
time (sec)	N/A	0.022	0.006	0.116	1.676	1.876	0.	1.126

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	79	30	55	0	42
normalized size	1	1.	1.	5.64	2.14	3.93	0.	3.
time (sec)	N/A	0.024	0.011	0.099	1.682	1.858	0.	1.137

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	27	123	45	4	0	112
normalized size	1	1.	0.79	3.62	1.32	0.12	0.	3.29
time (sec)	N/A	0.029	0.015	0.102	1.66	2.054	0.	1.137

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	81	18	4	0	42
normalized size	1	1.	1.	5.06	1.12	0.25	0.	2.62
time (sec)	N/A	0.02	0.006	0.122	1.708	1.976	0.	1.123

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	81	30	4	0	50
normalized size	1	1.	1.	5.06	1.88	0.25	0.	3.12
time (sec)	N/A	0.022	0.009	0.101	1.704	1.97	0.	1.14

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	114	0	0	12714	0	0
normalized size	1	1.	1.37	0.	0.	153.18	0.	0.
time (sec)	N/A	0.147	0.708	0.156	0.	5.889	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	192	0	0	24571	0	0
normalized size	1	1.	1.54	0.	0.	196.57	0.	0.
time (sec)	N/A	0.313	0.44	0.125	0.	5.808	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	90	0	0	7024	0	0
normalized size	1	1.	1.53	0.	0.	119.05	0.	0.
time (sec)	N/A	0.107	0.373	0.112	0.	3.266	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	150	0	0	12953	0	0
normalized size	1	1.	1.72	0.	0.	148.89	0.	0.
time (sec)	N/A	0.218	0.265	0.109	0.	3.548	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	90	43	0	4658	0	0
normalized size	1	1.	2.25	1.08	0.	116.45	0.	0.
time (sec)	N/A	0.063	0.257	0.034	0.	2.288	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	134	0	0	8756	0	0
normalized size	1	1.	2.27	0.	0.	148.41	0.	0.
time (sec)	N/A	0.045	0.679	0.102	0.	2.77	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	111	0	0	10376	0	0
normalized size	1	1.	1.98	0.	0.	185.29	0.	0.
time (sec)	N/A	0.108	0.121	0.116	0.	2.914	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	75	0	0	3892	0	0
normalized size	1	1.	1.56	0.	0.	81.08	0.	0.
time (sec)	N/A	0.178	0.525	0.119	0.	2.529	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	156	0	0	15598	0	0
normalized size	1	1.	1.88	0.	0.	187.93	0.	0.
time (sec)	N/A	0.143	0.514	0.122	0.	4.281	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	149	0	0	7036	0	0
normalized size	1	1.	1.77	0.	0.	83.76	0.	0.
time (sec)	N/A	0.248	0.587	0.126	0.	3.424	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	191	0	0	0	0	0
normalized size	1	1.	1.53	0.	0.	0.	0.	0.
time (sec)	N/A	0.221	0.91	0.128	0.	0.	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	129	0	0	12290	0	0
normalized size	1	1.	1.7	0.	0.	161.71	0.	0.
time (sec)	N/A	0.128	0.971	0.095	0.	6.048	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	197	0	0	23956	0	0
normalized size	1	1.	1.58	0.	0.	191.65	0.	0.
time (sec)	N/A	0.361	0.808	0.09	0.	5.828	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	65	56	0	6842	0	0
normalized size	1	1.	1.14	0.98	0.	120.04	0.	0.
time (sec)	N/A	0.084	0.114	0.014	0.	3.284	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	152	0	0	12542	0	0
normalized size	1	1.	1.73	0.	0.	142.52	0.	0.
time (sec)	N/A	0.091	0.26	0.082	0.	3.591	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	159	0	0	12166	0	0
normalized size	1	1.	2.27	0.	0.	173.8	0.	0.
time (sec)	N/A	0.132	0.491	0.095	0.	3.584	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	144	0	0	10029	0	0
normalized size	1	1.	1.78	0.	0.	123.81	0.	0.
time (sec)	N/A	0.232	0.327	0.102	0.	3.452	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	280	0	0	0	0	0
normalized size	1	1.	1.65	0.	0.	0.	0.	0.
time (sec)	N/A	0.194	9.017	0.193	0.	0.	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	109	0	0	7483	0	0
normalized size	1	1.	1.65	0.	0.	113.38	0.	0.
time (sec)	N/A	0.131	0.491	0.125	0.	3.496	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	169	0	0	13245	0	0
normalized size	1	1.	1.88	0.	0.	147.17	0.	0.
time (sec)	N/A	0.228	0.495	0.125	0.	3.841	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	105	0	0	4733	0	0
normalized size	1	1.	2.5	0.	0.	112.69	0.	0.
time (sec)	N/A	0.093	0.212	0.121	0.	2.62	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	107	0	0	8377	0	0
normalized size	1	1.	1.78	0.	0.	139.62	0.	0.
time (sec)	N/A	0.193	0.168	0.115	0.	3.041	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	70	30	0	4004	0	0
normalized size	1	1.	2.8	1.2	0.	160.16	0.	0.
time (sec)	N/A	0.053	0.081	0.035	0.	2.179	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	62	0	0	3023	0	0
normalized size	1	1.	2.14	0.	0.	104.24	0.	0.
time (sec)	N/A	0.025	0.04	0.149	0.	2.165	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	124	0	0	10525	0	0
normalized size	1	1.	2.21	0.	0.	187.95	0.	0.
time (sec)	N/A	0.103	0.259	0.12	0.	3.028	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	94	0	0	3951	0	0
normalized size	1	1.	1.77	0.	0.	74.55	0.	0.
time (sec)	N/A	0.188	0.114	0.124	0.	2.484	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	159	0	0	17920	0	0
normalized size	1	1.	1.77	0.	0.	199.11	0.	0.
time (sec)	N/A	0.161	0.826	0.13	0.	4.514	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	129	0	0	8805	0	0
normalized size	1	1.	1.9	0.	0.	129.49	0.	0.
time (sec)	N/A	0.147	0.577	0.104	0.	4.185	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	169	0	0	14456	0	0
normalized size	1	1.	1.97	0.	0.	168.09	0.	0.
time (sec)	N/A	0.252	0.381	0.099	0.	4.313	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	103	0	0	6137	0	0
normalized size	1	1.	2.1	0.	0.	125.24	0.	0.
time (sec)	N/A	0.115	0.229	0.098	0.	2.748	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	128	0	0	5075	0	0
normalized size	1	1.	2.51	0.	0.	99.51	0.	0.
time (sec)	N/A	0.218	0.713	0.095	0.	2.648	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	98	46	0	5840	44	0
normalized size	1	1.	2.28	1.07	0.	135.81	1.02	0.
time (sec)	N/A	0.073	0.396	0.015	0.	2.617	6.611	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	107	0	0	5858	0	0
normalized size	1	1.	1.88	0.	0.	102.77	0.	0.
time (sec)	N/A	0.045	0.81	0.085	0.	2.815	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	155	0	0	18668	0	0
normalized size	1	1.	1.96	0.	0.	236.3	0.	0.
time (sec)	N/A	0.14	0.651	0.101	0.	4.71	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	120	0	0	10016	0	0
normalized size	1	1.	1.36	0.	0.	113.82	0.	0.
time (sec)	N/A	0.274	0.42	0.103	0.	4.428	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	178	0	0	0	0	0
normalized size	1	1.	1.51	0.	0.	0.	0.	0.
time (sec)	N/A	0.339	0.692	0.106	0.	0.	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	126	0	0	12878	0	0
normalized size	1	1.	1.66	0.	0.	169.45	0.	0.
time (sec)	N/A	0.158	0.521	0.108	0.	4.855	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	290	0	0	9694	0	0
normalized size	1	1.	3.22	0.	0.	107.71	0.	0.
time (sec)	N/A	0.262	2.016	0.102	0.	4.469	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	124	0	0	12041	0	0
normalized size	1	1.	1.82	0.	0.	177.07	0.	0.
time (sec)	N/A	0.124	0.915	0.098	0.	4.456	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	290	0	0	12787	0	0
normalized size	1	1.	3.3	0.	0.	145.31	0.	0.
time (sec)	N/A	0.248	1.158	0.093	0.	4.722	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	112	61	0	10838	65	0
normalized size	1	1.	1.81	0.98	0.	174.81	1.05	0.
time (sec)	N/A	0.087	0.773	0.014	0.	4.478	52.206	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	130	0	0	15431	0	0
normalized size	1	1.	1.37	0.	0.	162.43	0.	0.
time (sec)	N/A	0.09	0.423	0.086	0.	5.245	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	242	0	0	0	0	0
normalized size	1	1.	2.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.2	1.042	0.1	0.	0.	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	155	0	0	26619	0	0
normalized size	1	1.	1.17	0.	0.	200.14	0.	0.
time (sec)	N/A	0.376	0.782	0.103	0.	10.539	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	330	0	0	0	0	0
normalized size	1	1.	1.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.191	7.458	0.152	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [190] had the largest ratio of [0.5833]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	5	1.	21	0.238
2	A	3	2	1.	21	0.095
3	A	4	4	1.	21	0.19
4	A	3	2	1.	19	0.105
5	A	3	3	1.	19	0.158
6	A	3	2	1.	21	0.095
7	A	4	4	1.	21	0.19
8	A	3	2	1.	21	0.095
9	A	6	5	1.	23	0.217
10	A	3	2	1.	23	0.087
11	A	5	5	1.	23	0.217
12	A	3	2	1.	21	0.095
13	A	4	3	1.	21	0.143
14	A	3	2	1.	23	0.087
15	A	5	5	1.	23	0.217
16	A	3	2	1.	23	0.087
17	A	6	5	1.	23	0.217
18	A	3	2	1.	23	0.087
19	A	6	5	1.28	23	0.217
20	A	3	2	1.	21	0.095
21	A	4	3	1.	21	0.143
22	A	3	2	1.	23	0.087
23	A	5	4	1.	23	0.174
24	A	3	2	1.	23	0.087
25	A	6	6	1.	23	0.261
26	A	4	4	1.	23	0.174
27	A	5	5	1.	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
28	A	3	3	1.	21	0.143
29	A	4	4	1.	21	0.19
30	A	3	3	1.	23	0.13
31	A	5	5	1.	23	0.217
32	A	4	4	1.	23	0.174
33	A	7	6	1.	23	0.261
34	A	5	4	1.	23	0.174
35	A	6	6	1.	23	0.261
36	A	4	4	1.	21	0.19
37	A	5	5	1.	21	0.238
38	A	4	4	1.	23	0.174
39	A	6	6	1.	23	0.261
40	A	5	4	1.	23	0.174
41	A	8	6	1.	23	0.261
42	A	6	5	1.	23	0.217
43	A	7	6	1.	23	0.261
44	A	5	4	1.	21	0.19
45	A	6	6	1.	21	0.286
46	A	5	4	1.	23	0.174
47	A	7	7	1.	23	0.304
48	A	6	5	1.	23	0.217
49	A	3	3	1.	21	0.143
50	A	3	2	1.	21	0.095
51	A	2	2	1.	21	0.095
52	A	2	2	1.	19	0.105
53	A	2	2	1.	19	0.105
54	A	3	3	1.43	21	0.143
55	A	3	3	1.	21	0.143
56	A	3	2	1.3	21	0.095
57	A	4	4	1.	23	0.174
58	A	4	3	1.	23	0.13
59	A	5	4	1.	23	0.174
60	A	5	4	1.	21	0.19
61	A	4	4	1.	21	0.19
62	A	3	2	1.	23	0.087
63	A	5	5	1.	23	0.217
64	A	3	2	1.	23	0.087
65	A	6	5	1.	23	0.217
66	A	5	4	1.	23	0.174
67	A	5	4	1.	23	0.174
68	A	6	5	1.	21	0.238
69	A	5	5	1.	21	0.238
70	A	3	2	1.	23	0.087
71	A	6	6	1.	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
72	A	3	2	1.	23	0.087
73	A	6	6	1.	23	0.261
74	A	4	3	1.	23	0.13
75	A	5	5	1.	23	0.217
76	A	3	3	1.	21	0.143
77	A	2	2	1.	21	0.095
78	A	2	2	1.	23	0.087
79	A	4	4	1.	23	0.174
80	A	3	3	1.	23	0.13
81	A	5	5	1.	23	0.217
82	A	4	3	1.	23	0.13
83	A	5	4	1.	23	0.174
84	A	6	6	1.	23	0.261
85	A	5	4	1.	21	0.19
86	A	3	3	1.	21	0.143
87	A	3	3	1.	23	0.13
88	A	3	3	1.	23	0.13
89	A	3	3	1.	23	0.13
90	A	5	5	1.	23	0.217
91	A	5	4	1.	23	0.174
92	A	6	6	1.	23	0.261
93	A	7	6	1.	23	0.261
94	A	6	5	1.	21	0.238
95	A	4	4	1.	21	0.19
96	A	4	3	1.	23	0.13
97	A	4	4	1.	23	0.174
98	A	4	4	1.	23	0.174
99	A	4	3	1.	23	0.13
100	A	4	4	1.	23	0.174
101	A	6	6	1.	23	0.261
102	A	4	3	1.	21	0.143
103	A	4	3	1.	21	0.143
104	A	4	3	1.	21	0.143
105	A	3	2	1.	19	0.105
106	A	3	2	1.	12	0.167
107	A	4	3	1.	19	0.158
108	A	4	3	1.	21	0.143
109	A	4	3	1.	21	0.143
110	A	4	3	1.	21	0.143
111	A	4	3	1.	21	0.143
112	A	4	3	1.	23	0.13
113	A	4	3	1.	23	0.13
114	A	4	3	1.	23	0.13
115	A	4	3	1.	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
116	A	4	3	1.	14	0.214
117	A	4	3	1.	21	0.143
118	A	4	3	1.	23	0.13
119	A	4	3	1.	23	0.13
120	A	4	3	1.	23	0.13
121	A	4	3	1.	23	0.13
122	A	4	3	1.	23	0.13
123	A	5	4	1.	23	0.174
124	A	4	3	1.	23	0.13
125	A	5	4	1.	23	0.174
126	A	4	3	1.	23	0.13
127	A	4	3	1.	21	0.143
128	A	4	3	1.	14	0.214
129	A	4	3	1.	21	0.143
130	A	4	3	1.	23	0.13
131	A	4	3	1.	23	0.13
132	A	4	3	1.	23	0.13
133	A	4	3	1.	23	0.13
134	A	4	3	1.	23	0.13
135	A	4	3	1.	23	0.13
136	A	4	3	1.	14	0.214
137	A	4	3	1.	14	0.214
138	A	4	3	1.	23	0.13
139	A	6	6	1.	23	0.261
140	A	4	3	1.	23	0.13
141	A	5	5	1.	23	0.217
142	A	2	2	1.	21	0.095
143	A	3	3	1.	14	0.214
144	A	4	3	1.	21	0.143
145	A	6	6	1.	23	0.261
146	A	4	3	1.	23	0.13
147	A	7	7	1.	23	0.304
148	A	4	3	1.	23	0.13
149	A	6	6	1.	23	0.261
150	A	4	3	1.	23	0.13
151	A	6	6	1.	23	0.261
152	A	4	3	1.	21	0.143
153	A	5	5	1.	14	0.357
154	A	4	3	1.	21	0.143
155	A	7	7	1.	23	0.304
156	A	4	3	1.	23	0.13
157	A	8	7	1.	23	0.304
158	A	7	7	1.	23	0.304
159	A	4	3	1.	23	0.13

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
160	A	7	7	1.	23	0.304
161	A	4	3	1.	23	0.13
162	A	7	7	1.	23	0.304
163	A	4	3	1.	21	0.143
164	A	6	6	1.	14	0.429
165	A	4	3	1.	21	0.143
166	A	8	8	1.	23	0.348
167	A	4	3	1.	23	0.13
168	A	9	8	1.	23	0.348
169	A	7	6	1.	14	0.429
170	A	4	4	1.	12	0.333
171	A	3	3	1.	12	0.25
172	A	3	3	1.	12	0.25
173	A	4	4	1.	10	0.4
174	A	3	3	1.	10	0.3
175	A	3	3	1.	10	0.3
176	A	7	6	1.	17	0.353
177	A	9	9	1.	17	0.529
178	A	6	6	1.	17	0.353
179	A	8	8	1.	17	0.471
180	A	5	5	1.	15	0.333
181	A	6	6	1.	12	0.5
182	A	7	5	1.	15	0.333
183	A	6	6	1.	17	0.353
184	A	8	6	1.	17	0.353
185	A	7	7	1.	17	0.412
186	A	9	7	1.	17	0.412
187	A	7	6	1.	17	0.353
188	A	9	9	1.	17	0.529
189	A	6	5	1.	15	0.333
190	A	7	7	1.	12	0.583
191	A	8	6	1.	15	0.4
192	A	8	8	1.	17	0.471
193	A	8	8	1.	16	0.5
194	A	6	5	1.	17	0.294
195	A	8	8	1.	17	0.471
196	A	5	5	1.	17	0.294
197	A	7	7	1.	17	0.412
198	A	4	4	1.	15	0.267
199	A	3	3	1.	12	0.25
200	A	7	5	1.	15	0.333
201	A	6	6	1.	17	0.353
202	A	8	6	1.	17	0.353
203	A	6	5	1.	17	0.294

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
204	A	8	8	1.	17	0.471
205	A	5	5	1.	17	0.294
206	A	5	5	1.	17	0.294
207	A	5	5	1.	15	0.333
208	A	4	4	1.	12	0.333
209	A	8	6	1.	15	0.4
210	A	7	7	1.	17	0.412
211	A	9	9	1.	17	0.529
212	A	6	5	1.	17	0.294
213	A	7	7	1.	17	0.412
214	A	6	6	1.	17	0.353
215	A	7	7	1.	17	0.412
216	A	6	5	1.	15	0.333
217	A	6	6	1.	12	0.5
218	A	9	7	1.	15	0.467
219	A	8	8	1.	17	0.471
220	A	7	6	1.	16	0.375

Chapter 3

Listing of integrals

3.1 $\int \left(a+b\,\mathrm{sech}^2(c+dx)\right)\sinh^4(c+dx)\,dx$

Optimal. Leaf size=70

$$-\frac{(5a-4b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{3}{8}x(a-4b)+\frac{a\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{b\tanh(c+dx)}{d}$$

```
[Out] (3*(a - 4*b)*x)/8 - ((5*a - 4*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (a*Co
sh[c + d*x]^3*Sinh[c + d*x])/(4*d) + (b*Tanh[c + d*x])/d
```

Rubi [A] time = 0.0808843, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4132, 455, 1157, 388, 206}

$$-\frac{(5a-4b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{3}{8}x(a-4b)+\frac{a\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x]^4,x]
```

```
[Out] (3*(a - 4*b)*x)/8 - ((5*a - 4*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (a*Co
sh[c + d*x]^3*Sinh[c + d*x])/(4*d) + (b*Tanh[c + d*x])/d
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)\sinh^4(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+b-bx^2\right)}{\left(1-x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{a\cosh^3(c+dx)\sinh(c+dx)}{4d} + \frac{\text{Subst}\left(\int \frac{-a-4ax^2+4bx^4}{\left(1-x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{4d}$$

$$= -\frac{(5a-4b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{a\cosh^3(c+dx)\sinh(c+dx)}{4d} - \frac{\text{Subs}}{}$$

$$= -\frac{(5a-4b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{a\cosh^3(c+dx)\sinh(c+dx)}{4d} + \frac{b\tan}{}$$

$$= \frac{3}{8}(a-4b)x - \frac{(5a-4b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{a\cosh^3(c+dx)\sinh(c+}{4d}$$

Mathematica [A] time = 0.313628, size = 54, normalized size = 0.77

$$\frac{12(a-4b)(c+dx)-8(a-b)\sinh(2(c+dx))+a\sinh(4(c+dx))+32b\tanh(c+dx)}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x]^4,x]
```

```
[Out] (12*(a - 4*b)*(c + d*x) - 8*(a - b)*Sinh[2*(c + d*x)] + a*Sinh[4*(c + d*x)]
 + 32*b*Tanh[c + d*x])/(32*d)
```

Maple [A] time = 0.032, size = 78, normalized size = 1.1

$$\frac{1}{d}\left(a\left(\left(\frac{(\sinh(dx+c))^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(\frac{(\sinh(dx+c))^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)*sinh(d*x+c)^4,x)
```

```
[Out] 1/d*(a*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+b*(1
/2*sinh(d*x+c)^3/cosh(d*x+c)-3/2*d*x-3/2*c+3/2*tanh(d*x+c)))
```

Maxima [B] time = 1.01564, size = 174, normalized size = 2.49

$$\frac{1}{64}a\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)-\frac{1}{8}b\left(\frac{12(dx+c)}{d}+\frac{e^{(-2dx-2c)}}{d}-\frac{17e^{(-2dx-2c)}+1}{d\left(e^{(-2dx-2c)}+e^{(-4dx-4c)}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/64*a*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) - 1/8*b*(12*(d*x + c)/d + e^(-2*d*x - 2*c)/d - (17
*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c))))
```

Fricas [A] time = 2.55299, size = 300, normalized size = 4.29

$$\frac{a\sinh(dx+c)^5+\left(10a\cosh(dx+c)^2-7a+8b\right)\sinh(dx+c)^3+8(3(a-4b)dx-8b)\cosh(dx+c)+\left(5a\cosh(dx+c)^4-3(7a-8b)\cosh(dx+c)^2-8a+72b\right)\sinh(dx+c)}{64d\cosh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/64*(a*sinh(d*x + c)^5 + (10*a*cosh(d*x + c)^2 - 7*a + 8*b)*sinh(d*x + c)^
3 + 8*(3*(a - 4*b)*d*x - 8*b)*cosh(d*x + c) + (5*a*cosh(d*x + c)^4 - 3*(7*a
 - 8*b)*cosh(d*x + c)^2 - 8*a + 72*b)*sinh(d*x + c))/(d*cosh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)*sinh(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16455, size = 193, normalized size = 2.76

$$\frac{3(dx+c)(a-4b)}{8d}-\frac{\left(18ae^{(4dx+4c)}-72be^{(4dx+4c)}-8ae^{(2dx+2c)}+8be^{(2dx+2c)}+a\right)e^{(-4dx-4c)}}{64d}+\frac{ade^{(4dx+4c)}-8ade^{(2dx \ldots}}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 3/8*(d*x + c)*(a - 4*b)/d - 1/64*(18*a*e^(4*d*x + 4*c) - 72*b*e^(4*d*x + 4*
c) - 8*a*e^(2*d*x + 2*c) + 8*b*e^(2*d*x + 2*c) + a)*e^(-4*d*x - 4*c)/d + 1/
64*(a*d*e^(4*d*x + 4*c) - 8*a*d*e^(2*d*x + 2*c) + 8*b*d*e^(2*d*x + 2*c))/d^
2 - 2*b/(d*(e^(2*d*x + 2*c) + 1))
```

3.2 $\int \left(a+b\,\text{sech}^2(c+dx)\right)\sinh^3(c+dx)\,dx$

Optimal. Leaf size=44

$$-\frac{(a-b)\cosh(c+dx)}{d}+\frac{a\cosh^3(c+dx)}{3d}+\frac{b\,\text{sech}(c+dx)}{d}$$

```
[Out] -(((a - b)*Cosh[c + d*x])/d) + (a*Cosh[c + d*x]^3)/(3*d) + (b*Sech[c + d*x]
)/d
```

Rubi [A] time = 0.0560561, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4133, 448}

$$-\frac{(a-b)\cosh(c+dx)}{d}+\frac{a\cosh^3(c+dx)}{3d}+\frac{b\,\text{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x]^3,x]
```

```
[Out] -(((a - b)*Cosh[c + d*x])/d) + (a*Cosh[c + d*x]^3)/(3*d) + (b*Sech[c + d*x]
)/d
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\,\text{sech}^2(c+dx)\right)\sinh^3(c+dx)\,dx &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)(b+ax^2)}{x^2}\,dx,x,\cosh(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left(a\left(1-\frac{b}{a}\right)+\frac{b}{x^2}-ax^2\right)dx,x,\cosh(c+dx)\right)}{d}\\ &= -\frac{(a-b)\cosh(c+dx)}{d}+\frac{a\cosh^3(c+dx)}{3d}+\frac{b\,\text{sech}(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0488596, size = 53, normalized size = 1.2

$$-\frac{3a\cosh(c+dx)}{4d}+\frac{a\cosh(3(c+dx))}{12d}+\frac{b\cosh(c+dx)}{d}+\frac{b\,\text{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x]^3,x]
```

```
[Out] (-3*a*Cosh[c + d*x])/(4*d) + (b*Cosh[c + d*x])/d + (a*Cosh[3*(c + d*x)])/(1
2*d) + (b*Sech[c + d*x])/d
```

Maple [A] time = 0.029, size = 55, normalized size = 1.3

$$\frac{1}{d}\left(a\left(-\frac{2}{3}+\frac{(\sinh(dx+c))^2}{3}\right)\cosh(dx+c)+b\left(-\frac{(\sinh(dx+c))^2}{\cosh(dx+c)}+2\cosh(dx+c)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)*sinh(d*x+c)^3,x)
```

```
[Out] 1/d*(a*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+b*(-sinh(d*x+c)^2/cosh(d*x+c)+2
*cosh(d*x+c)))
```

Maxima [B] time = 1.00415, size = 150, normalized size = 3.41

$$\frac{1}{24}a\left(\frac{e^{(3dx+3c)}}{d}-\frac{9e^{(dx+c)}}{d}-\frac{9e^{(-dx-c)}}{d}+\frac{e^{(-3dx-3c)}}{d}\right)+\frac{1}{2}b\left(\frac{e^{(-dx-c)}}{d}+\frac{5e^{(-2dx-2c)}+1}{d\left(e^{(-dx-c)}+e^{(-3dx-3c)}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/24*a*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x
- 3*c)/d) + 1/2*b*(e^(-d*x - c)/d + (5*e^(-2*d*x - 2*c) + 1)/(d*(e^(-d*x -
c) + e^(-3*d*x - 3*c))))
```

Fricas [B] time = 2.52248, size = 220, normalized size = 5.

$$\frac{a\cosh(dx+c)^4+a\sinh(dx+c)^4-4(2a-3b)\cosh(dx+c)^2+2\left(3a\cosh(dx+c)^2-4a+6b\right)\sinh(dx+c)^2-9a+}{24d\cosh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 - 4*(2*a - 3*b)*cosh(d*x + c)^2
 + 2*(3*a*cosh(d*x + c)^2 - 4*a + 6*b)*sinh(d*x + c)^2 - 9*a + 36*b)/(d*cos
h(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)*sinh(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18559, size = 132, normalized size = 3.

$$\frac{2\,b}{d\left(e^{(dx+c)} + e^{(-dx-c)}\right)} + \frac{ad^2\left(e^{(dx+c)} + e^{(-dx-c)}\right)^3 - 12\,ad^2\left(e^{(dx+c)} + e^{(-dx-c)}\right) + 12\,bd^2\left(e^{(dx+c)} + e^{(-dx-c)}\right)}{24\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 2*b/(d*(e^(d*x + c) + e^(-d*x - c))) + 1/24*(a*d^2*(e^(d*x + c) + e^(-d*x -
 c))^3 - 12*a*d^2*(e^(d*x + c) + e^(-d*x - c)) + 12*b*d^2*(e^(d*x + c) + e^
(-d*x - c)))/d^3
```

3.3 $\int \left(a+b\,\text{sech}^2(c+dx)\right)\sinh^2(c+dx)\,dx$

Optimal. Leaf size=43

$$-\frac{1}{2}x(a-2b)+\frac{a\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{b\tanh(c+dx)}{d}$$

```
[Out] -((a - 2*b)*x)/2 + (a*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) - (b*Tanh[c + d*x]
)/d
```

Rubi [A] time = 0.0561302, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4132, 455, 388, 206}

$$-\frac{1}{2}x(a-2b)+\frac{a\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x]^2,x]
```

```
[Out] -((a - 2*b)*x)/2 + (a*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) - (b*Tanh[c + d*x]
)/d
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)\sinh^2(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+b-bx^2)}{(1-x^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{a\cosh(c+dx)\sinh(c+dx)}{2d} - \frac{\text{Subst}\left(\int \frac{a-2bx^2}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{2d}$$

$$= \frac{a\cosh(c+dx)\sinh(c+dx)}{2d} - \frac{b\tanh(c+dx)}{d} - \frac{(a-2b)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx\right.}{2d}$$

$$= -\frac{1}{2}(a-2b)x + \frac{a\cosh(c+dx)\sinh(c+dx)}{2d} - \frac{b\tanh(c+dx)}{d}$$

Mathematica [A] time = 0.18568, size = 57, normalized size = 1.33

$$\frac{a(-c-dx)}{2d} + \frac{a\sinh(2(c+dx))}{4d} + \frac{b\tanh^{-1}(\tanh(c+dx))}{d} - \frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x]^2,x]
```

```
[Out] (a*(-c - d*x))/(2*d) + (b*ArcTanh[Tanh[c + d*x]])/d + (a*Sinh[2*(c + d*x)])
/(4*d) - (b*Tanh[c + d*x])/d
```

Maple [A] time = 0.028, size = 45, normalized size = 1.1

$$\frac{1}{d}\left(a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2} - \frac{dx}{2} - \frac{c}{2}\right) + b\left(dx+c-\tanh(dx+c)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)*sinh(d*x+c)^2,x)
```

```
[Out] 1/d*(a*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+b*(d*x+c-tanh(d*x+c)))
```

Maxima [A] time = 1.03543, size = 84, normalized size = 1.95

$$-\frac{1}{8}a\left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d}\right) + b\left(x + \frac{c}{d} - \frac{2}{d\left(e^{(-2dx-2c)}+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/8*a*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + b*(x + c/d - 2/(d*(
e^(-2*d*x - 2*c) + 1)))
```

Fricas [A] time = 2.44495, size = 177, normalized size = 4.12

$$\frac{a\sinh(dx+c)^3 - 4\left((a-2b)dx - 2b\right)\cosh(dx+c) + \left(3a\cosh(dx+c)^2 + a - 8b\right)\sinh(dx+c)}{8d\cosh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(a*sinh(d*x + c)^3 - 4*((a - 2*b)*d*x - 2*b)*cosh(d*x + c) + (3*a*cosh(
d*x + c)^2 + a - 8*b)*sinh(d*x + c))/(d*cosh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\operatorname{sech}^2(c + dx)\right)\sinh^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)*sinh(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*sinh(c + d*x)**2, x)
```

Giac [B] time = 1.1741, size = 130, normalized size = 3.02

$$-\frac{(dx + c)(a - 2\,b)}{2\,d} + \frac{ae^{(2\,dx+2\,c)}}{8\,d} + \frac{ae^{(4\,dx+4\,c)} - 2\,be^{(4\,dx+4\,c)} + 14\,be^{(2\,dx+2\,c)} - a}{8\,d\left(e^{(4\,dx+4\,c)} + e^{(2\,dx+2\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(d*x + c)*(a - 2*b)/d + 1/8*a*e^(2*d*x + 2*c)/d + 1/8*(a*e^(4*d*x + 4*
c) - 2*b*e^(4*d*x + 4*c) + 14*b*e^(2*d*x + 2*c) - a)/(d*(e^(4*d*x + 4*c) +
e^(2*d*x + 2*c)))
```

3.4 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)\sinh(c+dx)\,dx$

Optimal. Leaf size=24

$$\frac{a\cosh(c+dx)}{d}-\frac{b\mathrm{sech}(c+dx)}{d}$$

```
[Out] (a*Cosh[c + d*x])/d - (b*Sech[c + d*x])/d
```

Rubi [A] time = 0.0326643, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4133, 14}

$$\frac{a\cosh(c+dx)}{d}-\frac{b\mathrm{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x],x]
```

```
[Out] (a*Cosh[c + d*x])/d - (b*Sech[c + d*x])/d
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\mathrm{sech}^2(c+dx)\right)\sinh(c+dx)\,dx &= \frac{\mathrm{Subst}\left(\int \frac{b+ax^2}{x^2}\,dx,x,\cosh(c+dx)\right)}{d}\\
&= \frac{\mathrm{Subst}\left(\int \left(a+\frac{b}{x^2}\right)dx,x,\cosh(c+dx)\right)}{d}\\
&= \frac{a\cosh(c+dx)}{d}-\frac{b\mathrm{sech}(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.021398, size = 35, normalized size = 1.46

$$\frac{a\sinh(c)\sinh(dx)}{d}+\frac{a\cosh(c)\cosh(dx)}{d}-\frac{b\mathrm{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x],x]
```

```
[Out] (a*Cosh[c]*Cosh[d*x])/d - (b*Sech[c + d*x])/d + (a*Sinh[c]*Sinh[d*x])/d
```

Maple [A] time = 0.011, size = 26, normalized size = 1.1

$$-\frac{1}{d}\left(\mathrm{bsech}\left(dx+c\right)-\frac{a}{\mathrm{sech}\left(dx+c\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)*sinh(d*x+c),x)
```

```
[Out] -1/d*(b*sech(d*x+c)-1/sech(d*x+c)*a)
```

Maxima [A] time = 1.00482, size = 49, normalized size = 2.04

$$\frac{a\cosh\left(dx+c\right)}{d}-\frac{2\,b}{d\left(e^{(dx+c)}+e^{(-dx-c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c),x, algorithm="maxima")
```

```
[Out] a*cosh(d*x + c)/d - 2*b/(d*(e^(d*x + c) + e^(-d*x - c)))
```

Fricas [A] time = 2.63862, size = 99, normalized size = 4.12

$$\frac{a\cosh\left(dx+c\right)^2+a\sinh\left(dx+c\right)^2+a-2\,b}{2\,d\cosh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a - 2*b)/(d*cosh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\,\mathrm{sech}^2\left(c+dx\right)\right)\sinh\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)*sinh(d*x+c),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*sinh(c + d*x), x)
```

Giac [A] time = 1.14407, size = 63, normalized size = 2.62

$$\frac{a\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{2\,d}-\frac{2\,b}{d\left(e^{(dx+c)}+e^{(-dx-c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*a*(e^(d*x + c) + e^(-d*x - c))/d - 2*b/(d*(e^(d*x + c) + e^(-d*x - c)))
```

3.5 $\int \mathbf{csch}(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)dx$

Optimal. Leaf size=27

$$\frac{b\operatorname{sech}(c+dx)}{d}-\frac{(a+b)\tanh^{-1}(\cosh(c+dx))}{d}$$

```
[Out] -(((a + b)*ArcTanh[Cosh[c + d*x]])/d) + (b*Sech[c + d*x])/d
```

Rubi [A] time = 0.044651, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4133, 453, 206}

$$\frac{b\operatorname{sech}(c+dx)}{d}-\frac{(a+b)\tanh^{-1}(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -(((a + b)*ArcTanh[Cosh[c + d*x]])/d) + (b*Sech[c + d*x])/d
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)dx &= -\frac{\operatorname{Subst}\left(\int \frac{b+ax^2}{x^2(1-x^2)}\,dx,x,\cosh(c+dx)\right)}{d}\\
&= \frac{b\operatorname{sech}(c+dx)}{d}-\frac{(a+b)\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\cosh(c+dx)\right)}{d}\\
&= -\frac{(a+b)\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b\operatorname{sech}(c+dx)}{d}
\end{aligned}$$

Mathematica [B] time = 0.049223, size = 67, normalized size = 2.48

$$\frac{a\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{a\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{b\text{sech}(c+dx)}{d}+\frac{b\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((a*Log[Cosh[c/2 + (d*x)/2]])/d) + (a*Log[Sinh[c/2 + (d*x)/2]])/d + (b*Log
[Tanh[(c + d*x)/2]])/d + (b*Sech[c + d*x])/d
```

Maple [A] time = 0.028, size = 36, normalized size = 1.3

$$\frac{-2\,a\text{Artanh}\left(e^{dx+c}\right)+b\left((\cosh(dx+c))^{-1}-2\,\text{Artanh}\left(e^{dx+c}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/d*(-2*a*arctanh(exp(d*x+c))+b*(1/cosh(d*x+c)-2*arctanh(exp(d*x+c))))
```

Maxima [B] time = 1.07553, size = 108, normalized size = 4.

$$-b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2\,e^{(-dx-c)}}{d\left(e^{(-2\,dx-2\,c)}+1\right)}\right)+\frac{a\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d - 2*e^(-d*x - c)/(d*(
e^(-2*d*x - 2*c) + 1))) + a*log(tanh(1/2*d*x + 1/2*c))/d
```

Fricas [B] time = 2.51786, size = 541, normalized size = 20.04

$$\frac{2\,b\cosh(dx+c)-\left((a+b)\cosh(dx+c)^2+2\,(a+b)\cosh(dx+c)\sinh(dx+c)+(a+b)\sinh(dx+c)^2+a+b\right)\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] (2*b*cosh(d*x + c) - ((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sin
h(d*x + c) + (a + b)*sinh(d*x + c)^2 + a + b)*log(cosh(d*x + c) + sinh(d*x
+ c) + 1) + ((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c
) + (a + b)*sinh(d*x + c)^2 + a + b)*log(cosh(d*x + c) + sinh(d*x + c) - 1)
 + 2*b*sinh(d*x + c))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c)
```

```
+ d*sinh(d*x + c)^2 + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\,\mathrm{sech}^2\,(c + dx)\right) \mathrm{csch}\,(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*csch(c + d*x), x)
```

Giac [B] time = 1.14546, size = 104, normalized size = 3.85

$$-\frac{(a+b)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)}{2\,d}+\frac{(a+b)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)}{2\,d}+\frac{2\,b}{d\left(e^{(dx+c)}+e^{(-dx-c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*(a + b)*log(e^(d*x + c) + e^(-d*x - c) + 2)/d + 1/2*(a + b)*log(e^(d*x
 + c) + e^(-d*x - c) - 2)/d + 2*b/(d*(e^(d*x + c) + e^(-d*x - c)))
```

3.6 $\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)dx$

Optimal. Leaf size=27

$$-\frac{(a+b)\coth(c+dx)}{d}-\frac{b\tanh(c+dx)}{d}$$

```
[Out] -(((a + b)*Coth[c + d*x])/d) - (b*Tanh[c + d*x])/d
```

Rubi [A] time = 0.0460427, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4132, 14}

$$-\frac{(a+b)\coth(c+dx)}{d}-\frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -(((a + b)*Coth[c + d*x])/d) - (b*Tanh[c + d*x])/d
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)dx&=\frac{\operatorname{Subst}\left(\int \frac{a+b-bx^2}{x^2}\,dx,x,\tanh(c+dx)\right)}{d}\\&=\frac{\operatorname{Subst}\left(\int \left(-b+\frac{a+b}{x^2}\right)dx,x,\tanh(c+dx)\right)}{d}\\&=-\frac{(a+b)\coth(c+dx)}{d}-\frac{b\tanh(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0696755, size = 37, normalized size = 1.37

$$-\frac{a\coth(c+dx)}{d}-\frac{b\tanh(c+dx)}{d}-\frac{b\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((a*Coth[c + d*x])/d) - (b*Coth[c + d*x])/d - (b*Tanh[c + d*x])/d
```

Maple [A] time = 0.027, size = 44, normalized size = 1.6

$$\frac{1}{d}\left(-\coth\left(dx+c\right)a+b\left(-\frac{1}{\cosh\left(dx+c\right)\sinh\left(dx+c\right)}-2\tanh\left(dx+c\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/d*(-coth(d*x+c)*a+b*(-1/sinh(d*x+c)/cosh(d*x+c)-2*tanh(d*x+c)))
```

Maxima [A] time = 1.03625, size = 53, normalized size = 1.96

$$\frac{2\,a}{d\left(e^{(-2\,dx-2\,c)}-1\right)}+\frac{4\,b}{d\left(e^{(-4\,dx-4\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 2*a/(d*(e^(-2*d*x - 2*c) - 1)) + 4*b/(d*(e^(-4*d*x - 4*c) - 1))
```

Fricas [B] time = 2.41616, size = 246, normalized size = 9.11

$$-\frac{4\left(\left(a+b\right)\cosh\left(dx+c\right)-b\sinh\left(dx+c\right)\right)}{d\cosh\left(dx+c\right)^{3}+3\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}+d\sinh\left(dx+c\right)^{3}-d\cosh\left(dx+c\right)+\left(3\,d\cosh\left(dx+c\right)^{2}+d\right)\sinh\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -4*((a + b)*cosh(d*x + c) - b*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(
d*x + c)*sinh(d*x + c)^2 + d*sinh(d*x + c)^3 - d*cosh(d*x + c) + (3*d*cosh(
d*x + c)^2 + d)*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\operatorname{sech}^{2}\left(c+dx\right)\right)\operatorname{csch}^{2}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*csch(c + d*x)**2, x)
```

Giac [A] time = 1.12958, size = 46, normalized size = 1.7

$$-\frac{2\left(ae^{(2\,dx+2\,c)}+a+2\,b\right)}{d\left(e^{(4\,dx+4\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -2*(a*e^(2*d*x + 2*c) + a + 2*b)/(d*(e^(4*d*x + 4*c) - 1))
```

3.7 $\int \mathbf{csch}^3(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)dx$

Optimal. Leaf size=54

$$\frac{(a+3b)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{(a+b)\coth(c+dx)\operatorname{csch}(c+dx)}{2d}-\frac{b\operatorname{sech}(c+dx)}{d}$$

```
[Out] ((a + 3*b)*ArcTanh[Cosh[c + d*x]])/(2*d) - ((a + b)*Coth[c + d*x]*Csch[c +
d*x])/(2*d) - (b*Sech[c + d*x])/d
```

Rubi [A] time = 0.0713326, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4133, 456, 453, 206}

$$\frac{(a+3b)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{(a+b)\coth(c+dx)\operatorname{csch}(c+dx)}{2d}-\frac{b\operatorname{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 3*b)*ArcTanh[Cosh[c + d*x]])/(2*d) - ((a + b)*Coth[c + d*x]*Csch[c +
d*x])/(2*d) - (b*Sech[c + d*x])/d
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)dx = \frac{\operatorname{Subst}\left(\int \frac{b+ax^2}{x^2(1-x^2)^2}dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{(a+b)\coth(c+dx)\operatorname{csch}(c+dx)}{2d} - \frac{\operatorname{Subst}\left(\int \frac{-2b-(a+b)x^2}{x^2(1-x^2)}dx, x, \cosh(c+\right.}{2d}$$

$$= -\frac{(a+b)\coth(c+dx)\operatorname{csch}(c+dx)}{2d} - \frac{b\operatorname{sech}(c+dx)}{d} + \frac{(a+3b)\operatorname{Subst}\left(\int\right.}{}$$

$$= \frac{(a+3b)\tanh^{-1}(\cosh(c+dx))}{2d} - \frac{(a+b)\coth(c+dx)\operatorname{csch}(c+dx)}{2d} - \frac{b\operatorname{se}}{}$$

Mathematica [B] time = 0.0468217, size = 131, normalized size = 2.43

$$-\frac{a\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{b\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{b\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d} -$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -(a*Csch[(c + d*x)/2]^2)/(8*d) - (b*Csch[(c + d*x)/2]^2)/(8*d) - (a*Log[Tan
h[(c + d*x)/2]])/(2*d) - (3*b*Log[Tanh[(c + d*x)/2]])/(2*d) - (a*Sech[(c +
d*x)/2]^2)/(8*d) - (b*Sech[(c + d*x)/2]^2)/(8*d) - (b*Sech[c + d*x])/d
```

Maple [A] time = 0.036, size = 70, normalized size = 1.3

$$\frac{1}{d}\left(a\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2} + \operatorname{Artanh}\left(e^{dx+c}\right)\right) + b\left(-\frac{1}{2(\sinh(dx+c))^2\cosh(dx+c)} - \frac{3}{2\cosh(dx+c)} + 3A\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/d*(a*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+b*(-1/2/sinh(d*x+
c)^2/cosh(d*x+c)-3/2/cosh(d*x+c)+3*arctanh(exp(d*x+c))))
```

Maxima [B] time = 1.02697, size = 267, normalized size = 4.94

$$\frac{1}{2}b\left(\frac{3\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{3\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(3e^{(-dx-c)}-2e^{(-3dx-3c)}+3e^{(-5dx-5c)}\right)}{d\left(e^{(-2dx-2c)}+e^{(-4dx-4c)}-e^{(-6dx-6c)}-1\right)}\right) + \frac{1}{2}a\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*b*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d + 2*(3*e^(-d*x
 - c) - 2*e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c))/(d*(e^(-2*d*x - 2*c) + e^(
```

```
-4*d*x - 4*c) - e^(-6*d*x - 6*c) - 1))) + 1/2*a*(log(e^(-d*x - c) + 1)/d -
log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d
*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

Fricas [B] time = 2.71236, size = 2469, normalized size = 45.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a + 3*b)*cosh(d*x + c)^5 + 10*(a + 3*b)*cosh(d*x + c)*sinh(d*x + c
)^4 + 2*(a + 3*b)*sinh(d*x + c)^5 + 4*(a - b)*cosh(d*x + c)^3 + 4*(5*(a + 3
*b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^3 + 4*(5*(a + 3*b)*cosh(d*x + c)
^3 + 3*(a - b)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*(a + 3*b)*cosh(d*x + c) -
 ((a + 3*b)*cosh(d*x + c)^6 + 6*(a + 3*b)*cosh(d*x + c)*sinh(d*x + c)^5 + (
a + 3*b)*sinh(d*x + c)^6 - (a + 3*b)*cosh(d*x + c)^4 + (15*(a + 3*b)*cosh(d
*x + c)^2 - a - 3*b)*sinh(d*x + c)^4 + 4*(5*(a + 3*b)*cosh(d*x + c)^3 - (a
+ 3*b)*cosh(d*x + c))*sinh(d*x + c)^3 - (a + 3*b)*cosh(d*x + c)^2 + (15*(a
+ 3*b)*cosh(d*x + c)^4 - 6*(a + 3*b)*cosh(d*x + c)^2 - a - 3*b)*sinh(d*x +
c)^2 + 2*(3*(a + 3*b)*cosh(d*x + c)^5 - 2*(a + 3*b)*cosh(d*x + c)^3 - (a +
3*b)*cosh(d*x + c))*sinh(d*x + c) + a + 3*b)*log(cosh(d*x + c) + sinh(d*x +
 c) + 1) + ((a + 3*b)*cosh(d*x + c)^6 + 6*(a + 3*b)*cosh(d*x + c)*sinh(d*x
+ c)^5 + (a + 3*b)*sinh(d*x + c)^6 - (a + 3*b)*cosh(d*x + c)^4 + (15*(a + 3
*b)*cosh(d*x + c)^2 - a - 3*b)*sinh(d*x + c)^4 + 4*(5*(a + 3*b)*cosh(d*x +
c)^3 - (a + 3*b)*cosh(d*x + c))*sinh(d*x + c)^3 - (a + 3*b)*cosh(d*x + c)^2
 + (15*(a + 3*b)*cosh(d*x + c)^4 - 6*(a + 3*b)*cosh(d*x + c)^2 - a - 3*b)*s
inh(d*x + c)^2 + 2*(3*(a + 3*b)*cosh(d*x + c)^5 - 2*(a + 3*b)*cosh(d*x + c)
^3 - (a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + a + 3*b)*log(cosh(d*x + c) +
sinh(d*x + c) - 1) + 2*(5*(a + 3*b)*cosh(d*x + c)^4 + 6*(a - b)*cosh(d*x +
c)^2 + a + 3*b)*sinh(d*x + c))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(
d*x + c)^5 + d*sinh(d*x + c)^6 - d*cosh(d*x + c)^4 + (15*d*cosh(d*x + c)^2
- d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x +
 c)^3 - d*cosh(d*x + c)^2 + (15*d*cosh(d*x + c)^4 - 6*d*cosh(d*x + c)^2 - d
)*sinh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^5 - 2*d*cosh(d*x + c)^3 - d*cosh(d
*x + c))*sinh(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\operatorname{sech}^2(c + dx)\right)\operatorname{csch}^3(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*csch(c + d*x)**3, x)
```

Giac [B] time = 1.16261, size = 198, normalized size = 3.67

$$\frac{(a+3b)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)}{4d}-\frac{(a+3b)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)}{4d}-\frac{a\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2+3b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2}{\left(\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-4e^{(dx+c)}-4e^{(-dx-c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/4*(a + 3*b)*log(e^(d*x + c) + e^(-d*x - c) + 2)/d - 1/4*(a + 3*b)*log(e^(
d*x + c) + e^(-d*x - c) - 2)/d - (a*(e^(d*x + c) + e^(-d*x - c))^2 + 3*b*(e
^(d*x + c) + e^(-d*x - c))^2 - 8*b)/(((e^(d*x + c) + e^(-d*x - c))^3 - 4*e^
(d*x + c) - 4*e^(-d*x - c))*d)
```

3.8 $\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)dx$

Optimal. Leaf size=45

$$-\frac{(a+b)\coth^3(c+dx)}{3d}+\frac{(a+2b)\coth(c+dx)}{d}+\frac{b\tanh(c+dx)}{d}$$

```
[Out] ((a + 2*b)*Coth[c + d*x])/d - ((a + b)*Coth[c + d*x]^3)/(3*d) + (b*Tanh[c +
 d*x])/d
```

Rubi [A] time = 0.0579728, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4132, 448}

$$-\frac{(a+b)\coth^3(c+dx)}{3d}+\frac{(a+2b)\coth(c+dx)}{d}+\frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b)*Coth[c + d*x])/d - ((a + b)*Coth[c + d*x]^3)/(3*d) + (b*Tanh[c +
 d*x])/d
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)(a+b-bx^2)}{x^4}\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(b+\frac{a+b}{x^4}+\frac{-a-2b}{x^2}\right)dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{(a+2b)\coth(c+dx)}{d}-\frac{(a+b)\coth^3(c+dx)}{3d}+\frac{b\tanh(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.05242, size = 84, normalized size = 1.87

$$\frac{2a\coth(c+dx)}{3d}-\frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}+\frac{b\tanh(c+dx)}{d}+\frac{5b\coth(c+dx)}{3d}-\frac{b\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (2*a*Coth[c + d*x])/(3*d) + (5*b*Coth[c + d*x])/(3*d) - (a*Coth[c + d*x]*Cs
ch[c + d*x]^2)/(3*d) - (b*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d) + (b*Tanh[c
+ d*x])/d
```

Maple [A] time = 0.036, size = 73, normalized size = 1.6

$$\frac{1}{d}\left(a\left(\frac{2}{3}-\frac{(\operatorname{csch}(dx+c))^2}{3}\right)\coth(dx+c)+b\left(-\frac{1}{3(\sinh(dx+c))^3\cosh(dx+c)}+\frac{4}{3\cosh(dx+c)\sinh(dx+c)}+\frac{8}{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/d*(a*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+b*(-1/3/sinh(d*x+c)^3/cosh(d*x+c
)+4/3/sinh(d*x+c)/cosh(d*x+c)+8/3*tanh(d*x+c)))
```

Maxima [B] time = 1.04763, size = 252, normalized size = 5.6

$$\frac{4}{3}a\left(\frac{3e^{(-2dx-2c)}}{d\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1\right)}-\frac{1}{d\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1\right)}\right)+\frac{16}{3}b\left(\frac{\cdots}{d\left(2e^{(-2dx-2\cdots}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6
*d*x - 6*c) - 1))) + 16/3*b*(2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - 2*
e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) - 1)) - 1/(d*(2*e^(-2*d*x - 2*c) - 2*e^
(-6*d*x - 6*c) + e^(-8*d*x - 8*c) - 1)))
```

Fricas [B] time = 2.46678, size = 666, normalized size = 14.8

$$-\frac{\cdots}{3\left(d\cosh(dx+c)^6+6d\cosh(dx+c)\sinh(dx+c)^5+d\sinh(dx+c)^6-2d\cosh(dx+c)^4+\left(15d\cosh(dx+c)^2-\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -8/3*((a - 2*b)*cosh(d*x + c)^2 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (
a - 2*b)*sinh(d*x + c)^2 + a + 4*b)/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*
sinh(d*x + c)^5 + d*sinh(d*x + c)^6 - 2*d*cosh(d*x + c)^4 + (15*d*cosh(d*x
+ c)^2 - 2*d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 2*d*cosh(d*x + c))
*sinh(d*x + c)^3 - d*cosh(d*x + c)^2 + (15*d*cosh(d*x + c)^4 - 12*d*cosh(d*
x + c)^2 - d)*sinh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^5 - 4*d*cosh(d*x + c)^
```

```
3 + d*cosh(d*x + c))*sinh(d*x + c) + 2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\operatorname{sech}^2(c + dx)\right)\operatorname{csch}^4(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*csch(c + d*x)**4, x)
```

Giac [A] time = 1.15938, size = 109, normalized size = 2.42

$$-\frac{2\,b}{d\left(e^{(2\,dx+2\,c)}+1\right)}+\frac{2\left(3\,be^{(4\,dx+4\,c)}-6\,ae^{(2\,dx+2\,c)}-12\,be^{(2\,dx+2\,c)}+2\,a+5\,b\right)}{3\,d\left(e^{(2\,dx+2\,c)}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -2*b/(d*(e^(2*d*x + 2*c) + 1)) + 2/3*(3*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x +
2*c) - 12*b*e^(2*d*x + 2*c) + 2*a + 5*b)/(d*(e^(2*d*x + 2*c) - 1)^3)
```

3.9 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)^2 \sinh^4(c+dx)\,dx$

Optimal. Leaf size=114

$$-\frac{\left(a^2-8ab+4b^2\right)\tanh(c+dx)}{4d}+\frac{1}{8}x\left(3a^2-24ab+8b^2\right)+\frac{a^2\sinh^4(c+dx)\tanh(c+dx)}{4d}-\frac{a(a-8b)\sinh(c+dx)\text{c}}{8d}$$

```
[Out] ((3*a^2 - 24*a*b + 8*b^2)*x)/8 - (a*(a - 8*b)*Cosh[c + d*x]*Sinh[c + d*x])/
(8*d) - ((a^2 - 8*a*b + 4*b^2)*Tanh[c + d*x])/(4*d) + (a^2*Sinh[c + d*x]^4*
Tanh[c + d*x])/(4*d) - (b^2*Tanh[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.138374, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4132, 463, 455, 1153, 206}

$$-\frac{\left(a^2-8ab+4b^2\right)\tanh(c+dx)}{4d}+\frac{1}{8}x\left(3a^2-24ab+8b^2\right)+\frac{a^2\sinh^4(c+dx)\tanh(c+dx)}{4d}-\frac{a(a-8b)\sinh(c+dx)\text{c}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x]^4,x]
```

```
[Out] ((3*a^2 - 24*a*b + 8*b^2)*x)/8 - (a*(a - 8*b)*Cosh[c + d*x]*Sinh[c + d*x])/
(8*d) - ((a^2 - 8*a*b + 4*b^2)*Tanh[c + d*x])/(4*d) + (a^2*Sinh[c + d*x]^4*
Tanh[c + d*x])/(4*d) - (b^2*Tanh[c + d*x]^3)/(3*d)
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^2 \sinh^4(c+dx)\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^4\left(a+b-bx^2\right)^2}{\left(1-x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{a^2\sinh^4(c+dx)\tanh(c+dx)}{4d} - \frac{\operatorname{Subst}\left(\int \frac{x^4\left(5a^2-4(a+b)^2+4b^2x^2\right)}{\left(1-x^2\right)^2}\,dx, x, \tanh(c\right.}{4d}$$

$$= -\frac{a(a-8b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{a^2\sinh^4(c+dx)\tanh(c+dx)}{4d} - \frac{\text{Su}}{}$$

$$= -\frac{a(a-8b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{a^2\sinh^4(c+dx)\tanh(c+dx)}{4d} - \frac{\text{Su}}{}$$

$$= -\frac{a(a-8b)\cosh(c+dx)\sinh(c+dx)}{8d} - \frac{\left(a^2-8ab+4b^2\right)\tanh(c+dx)}{4d} + \frac{a^2}{}$$

$$= \frac{1}{8}\left(3a^2-24ab+8b^2\right)x - \frac{a(a-8b)\cosh(c+dx)\sinh(c+dx)}{8d} - \frac{\left(a^2-8ab+\right.}{}$$

Mathematica [A] time = 1.6206, size = 153, normalized size = 1.34

$$\frac{\operatorname{sech}^3(c+dx)\left(a\cosh^2(c+dx)+b\right)^2\left(3\cosh^3(c+dx)\left(4dx\left(3a^2-24ab+8b^2\right)+a^2\sinh(4(c+dx))-8a(a-2b)\sinh(2(\right.\right.}{24d(a\cosh(2(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x]^4,x]
```

```
[Out] ((b + a*Cosh[c + d*x]^2)^2*Sech[c + d*x]^3*(32*b^2*Sech[c]*Sinh[d*x] + 64*(
3*a - 2*b)*b*Cosh[c + d*x]^2*Sech[c]*Sinh[d*x] + 3*Cosh[c + d*x]^3*(4*(3*a^
2 - 24*a*b + 8*b^2)*d*x - 8*a*(a - 2*b)*Sinh[2*(c + d*x)] + a^2*Sinh[4*(c +
 d*x)]) + 32*b^2*Cosh[c + d*x]*Tanh[c]))/(24*d*(a + 2*b + a*Cosh[2*(c + d*x
)])^2)
```

Maple [A] time = 0.04, size = 109, normalized size = 1.

$$\frac{1}{d}\left(a^2\left(\left(\frac{(\sinh(dx+c))^3}{4} - \frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c) + \frac{3dx}{8} + \frac{3c}{8}\right) + 2ab\left(1/2\,\frac{(\sinh(dx+c))^3}{\cosh(dx+c)} - 3/2\,dx - 3/2\,c + 3/\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^4,x)
```

```
[Out] 1/d*(a^2*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+2*
a*b*(1/2*sinh(d*x+c)^3/cosh(d*x+c)-3/2*d*x-3/2*c+3/2*tanh(d*x+c))+b^2*(d*x+
c-tanh(d*x+c)-1/3*tanh(d*x+c)^3))
```

Maxima [B] time = 1.05235, size = 285, normalized size = 2.5

$$\frac{1}{64}a^2\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)+\frac{1}{3}b^2\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}\right.}{d\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/64*a^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 1/3*b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) +
 3*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-
6*d*x - 6*c) + 1))) - 1/4*a*b*(12*(d*x + c)/d + e^(-2*d*x - 2*c)/d - (17*e^
(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c))))
```

Fricas [B] time = 2.70513, size = 880, normalized size = 7.72

$$\frac{3a^2\sinh(dx+c)^7+3\left(21a^2\cosh(dx+c)^2-5a^2+16ab\right)\sinh(dx+c)^5+8\left(3\left(3a^2-24ab+8b^2\right)dx-48ab+32b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/192*(3*a^2*sinh(d*x + c)^7 + 3*(21*a^2*cosh(d*x + c)^2 - 5*a^2 + 16*a*b)*
sinh(d*x + c)^5 + 8*(3*(3*a^2 - 24*a*b + 8*b^2)*d*x - 48*a*b + 32*b^2)*cosh
(d*x + c)^3 + 24*(3*(3*a^2 - 24*a*b + 8*b^2)*d*x - 48*a*b + 32*b^2)*cosh(d*
x + c)*sinh(d*x + c)^2 + (105*a^2*cosh(d*x + c)^4 - 30*(5*a^2 - 16*a*b)*cos
h(d*x + c)^2 - 63*a^2 + 528*a*b - 256*b^2)*sinh(d*x + c)^3 + 24*(3*(3*a^2 -
 24*a*b + 8*b^2)*d*x - 48*a*b + 32*b^2)*cosh(d*x + c) + 3*(7*a^2*cosh(d*x +
 c)^6 - 5*(5*a^2 - 16*a*b)*cosh(d*x + c)^4 - (63*a^2 - 528*a*b + 256*b^2)*c
osh(d*x + c)^2 - 15*a^2 + 160*a*b)*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*
cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**2*sinh(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.182, size = 328, normalized size = 2.88

$$\frac{\left(3\,a^{2}-24\,ab+8\,b^{2}\right)(dx+c)}{8\,d}-\frac{\left(18\,a^{2}e^{(4\,dx+4\,c)}-144\,abe^{(4\,dx+4\,c)}+48\,b^{2}e^{(4\,dx+4\,c)}-8\,a^{2}e^{(2\,dx+2\,c)}+16\,abe^{(2\,dx+2\,c)}+a^{2}\right)e^{(-4}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/8*(3*a^2 - 24*a*b + 8*b^2)*(d*x + c)/d - 1/64*(18*a^2*e^(4*d*x + 4*c) - 1
44*a*b*e^(4*d*x + 4*c) + 48*b^2*e^(4*d*x + 4*c) - 8*a^2*e^(2*d*x + 2*c) + 1
6*a*b*e^(2*d*x + 2*c) + a^2)*e^(-4*d*x - 4*c)/d + 1/64*(a^2*d*e^(4*d*x + 4*
c) - 8*a^2*d*e^(2*d*x + 2*c) + 16*a*b*d*e^(2*d*x + 2*c))/d^2 - 4/3*(3*a*b*e
^(4*d*x + 4*c) - 3*b^2*e^(4*d*x + 4*c) + 6*a*b*e^(2*d*x + 2*c) - 3*b^2*e^(2
*d*x + 2*c) + 3*a*b - 2*b^2)/(d*(e^(2*d*x + 2*c) + 1)^3)
```

3.10 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)^2 \sinh^3(c+dx)\,dx$

Optimal. Leaf size=72

$$\frac{a^2\cosh^3(c+dx)}{3d}-\frac{a(a-2b)\cosh(c+dx)}{d}+\frac{b(2a-b)\text{sech}(c+dx)}{d}+\frac{b^2\text{sech}^3(c+dx)}{3d}$$

```
[Out] -((a*(a - 2*b)*Cosh[c + d*x])/d) + (a^2*Cosh[c + d*x]^3)/(3*d) + ((2*a - b)
*b*Sech[c + d*x])/d + (b^2*Sech[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0865373, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4133, 448}

$$\frac{a^2\cosh^3(c+dx)}{3d}-\frac{a(a-2b)\cosh(c+dx)}{d}+\frac{b(2a-b)\text{sech}(c+dx)}{d}+\frac{b^2\text{sech}^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x]^3,x]
```

```
[Out] -((a*(a - 2*b)*Cosh[c + d*x])/d) + (a^2*Cosh[c + d*x]^3)/(3*d) + ((2*a - b)
*b*Sech[c + d*x])/d + (b^2*Sech[c + d*x]^3)/(3*d)
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\text{sech}^2(c+dx)\right)^2\sinh^3(c+dx)\,dx &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)(b+ax^2)^2}{x^4}\,dx, x, \cosh(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left(a(a-2b)+\frac{b^2}{x^4}+\frac{(2a-b)b}{x^2}-a^2x^2\right)dx, x, \cosh(c+dx)\right)}{d}\\ &= -\frac{a(a-2b)\cosh(c+dx)}{d}+\frac{a^2\cosh^3(c+dx)}{3d}+\frac{(2a-b)b\text{sech}(c+dx)}{d}+\end{aligned}$$

Mathematica [A] time = 0.509638, size = 83, normalized size = 1.15

$$\frac{\text{sech}^3(c+dx)\left(-3\left(11a^2-64ab+16b^2\right)\cosh(2(c+dx))+a^2\cosh(6(c+dx))-26a^2-6a(a-4b)\cosh(4(c+dx))+1\right.}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x]^3,x]
```

```
[Out] ((-26*a^2 + 168*a*b - 16*b^2 - 3*(11*a^2 - 64*a*b + 16*b^2)*Cosh[2*(c + d*x
)] - 6*a*(a - 4*b)*Cosh[4*(c + d*x)] + a^2*Cosh[6*(c + d*x)])*Sech[c + d*x]
^3)/(96*d)
```

Maple [A] time = 0.045, size = 108, normalized size = 1.5

$$\frac{1}{d}\left(a^2\left(-\frac{2}{3}+\frac{(\sinh(dx+c))^2}{3}\right)\cosh(dx+c)+2ab\left(-\frac{(\sinh(dx+c))^2}{\cosh(dx+c)}+2\cosh(dx+c)\right)+b^2\left(-\frac{(\sinh(dx+c))^2}{3(\cosh(dx+c))^3}+\frac{2}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^3,x)
```

```
[Out] 1/d*(a^2*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+2*a*b*(-sinh(d*x+c)^2/cosh(d*
x+c)+2*cosh(d*x+c))+b^2*(-1/3*sinh(d*x+c)^2/cosh(d*x+c)^3+2/3*sinh(d*x+c)^2
/cosh(d*x+c)-2/3*cosh(d*x+c)))
```

Maxima [B] time = 1.04399, size = 359, normalized size = 4.99

$$\frac{1}{24}a^2\left(\frac{e^{(3dx+3c)}}{d}-\frac{9e^{(dx+c)}}{d}-\frac{9e^{(-dx-c)}}{d}+\frac{e^{(-3dx-3c)}}{d}\right)+ab\left(\frac{e^{(-dx-c)}}{d}+\frac{5e^{(-2dx-2c)}+1}{d\left(e^{(-dx-c)}+e^{(-3dx-3c)}\right)}\right)-\frac{2}{3}b^2\left(\frac{3}{d\left(3e^{(-2dx-2c)}+3e^{(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/24*a^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*
x - 3*c)/d) + a*b*(e^(-d*x - c)/d + (5*e^(-2*d*x - 2*c) + 1)/(d*(e^(-d*x -
c) + e^(-3*d*x - 3*c)))) - 2/3*b^2*(3*e^(-d*x - c)/(d*(3*e^(-2*d*x - 2*c) +
 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 2*e^(-3*d*x - 3*c)/(d*(3*e^(
-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 3*e^(-5*d*x -
 5*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

Fricas [B] time = 2.76468, size = 541, normalized size = 7.51

$$\frac{a^2\cosh(dx+c)^6+a^2\sinh(dx+c)^6-6\left(a^2-4ab\right)\cosh(dx+c)^4+3\left(5a^2\cosh(dx+c)^2-2a^2+8ab\right)\sinh(dx+c)^4-}{24\left(d\cosh(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(a^2*cosh(d*x + c)^6 + a^2*sinh(d*x + c)^6 - 6*(a^2 - 4*a*b)*cosh(d*x
+ c)^4 + 3*(5*a^2*cosh(d*x + c)^2 - 2*a^2 + 8*a*b)*sinh(d*x + c)^4 - 3*(11*
a^2 - 64*a*b + 16*b^2)*cosh(d*x + c)^2 + 3*(5*a^2*cosh(d*x + c)^4 - 12*(a^2
 - 4*a*b)*cosh(d*x + c)^2 - 11*a^2 + 64*a*b - 16*b^2)*sinh(d*x + c)^2 - 26*
```

```
a^2 + 168*a*b - 16*b^2)/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c
)^2 + 3*d*cosh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**2*sinh(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17876, size = 207, normalized size = 2.88

$$\frac{a^2d^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-12\,a^2d^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)+24\,abd^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{24\,d^3}+\frac{2\left(6\,ab\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-3\,b^2\left(e^{(\cdots}\right.\right.}{3\,d\left(e^{(dx+c)}+e^{(-d\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*d^2*(e^(d*x + c) + e^(-d*x - c))^3 - 12*a^2*d^2*(e^(d*x + c) + e^
(-d*x - c)) + 24*a*b*d^2*(e^(d*x + c) + e^(-d*x - c)))/d^3 + 2/3*(6*a*b*(e^
(d*x + c) + e^(-d*x - c))^2 - 3*b^2*(e^(d*x + c) + e^(-d*x - c))^2 + 4*b^2)
/(d*(e^(d*x + c) + e^(-d*x - c))^3)
```

3.11 $\int \left(a + b\,\mathbf{sech}^2(c+dx)\right)^2 \sinh^2(c+dx)\,dx$

Optimal. Leaf size=73

$$\frac{a^2\sinh^2(c+dx)\tanh(c+dx)}{2d}+\frac{a(a-4b)\tanh(c+dx)}{2d}-\frac{1}{2}ax(a-4b)+\frac{b^2\tanh^3(c+dx)}{3d}$$

```
[Out] -(a*(a - 4*b)*x)/2 + (a*(a - 4*b)*Tanh[c + d*x])/(2*d) + (a^2*Sinh[c + d*x]
^2*Tanh[c + d*x])/(2*d) + (b^2*Tanh[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.107428, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4132, 463, 459, 321, 206}

$$\frac{a^2\sinh^2(c+dx)\tanh(c+dx)}{2d}+\frac{a(a-4b)\tanh(c+dx)}{2d}-\frac{1}{2}ax(a-4b)+\frac{b^2\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x]^2,x]
```

```
[Out] -(a*(a - 4*b)*x)/2 + (a*(a - 4*b)*Tanh[c + d*x])/(2*d) + (a^2*Sinh[c + d*x]
^2*Tanh[c + d*x])/(2*d) + (b^2*Tanh[c + d*x]^3)/(3*d)
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^2 \sinh^2(c+dx)\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^2\left(a+b-bx^2\right)^2}{\left(1-x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{a^2\sinh^2(c+dx)\tanh(c+dx)}{2d} - \frac{\operatorname{Subst}\left(\int \frac{x^2\left(3a^2-2(a+b)^2+2b^2x^2\right)}{1-x^2}\,dx, x, \tanh\right.}{2d}$$

$$= \frac{a^2\sinh^2(c+dx)\tanh(c+dx)}{2d} + \frac{b^2\tanh^3(c+dx)}{3d} - \frac{(a(a-4b))\operatorname{Subst}\left(\right.}{}$$

$$= \frac{a(a-4b)\tanh(c+dx)}{2d} + \frac{a^2\sinh^2(c+dx)\tanh(c+dx)}{2d} + \frac{b^2\tanh^3(c+}{3d}$$

$$= -\frac{1}{2}a(a-4b)x + \frac{a(a-4b)\tanh(c+dx)}{2d} + \frac{a^2\sinh^2(c+dx)\tanh(c+dx)}{2d}$$

Mathematica [A] time = 0.93376, size = 126, normalized size = 1.73

$$\frac{\operatorname{sech}^3(c+dx)\left(a\cosh^2(c+dx)+b\right)^2\left(3a\cosh^3(c+dx)(a\sinh(2(c+dx))-2dx(a-4b))-4b(6a-b)\operatorname{sech}(c)\sinh(dx)\right.}{3d(a\cosh(2(c+dx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x]^2,x]
```

```
[Out] ((b + a*Cosh[c + d*x]^2)^2*Sech[c + d*x]^3*(-4*b^2*Sech[c]*Sinh[d*x] - 4*(6
*a - b)*b*Cosh[c + d*x]^2*Sech[c]*Sinh[d*x] + 3*a*Cosh[c + d*x]^3*(-2*(a -
4*b)*d*x + a*Sinh[2*(c + d*x)]) - 4*b^2*Cosh[c + d*x]*Tanh[c]))/(3*d*(a + 2
*b + a*Cosh[2*(c + d*x)])^2)
```

Maple [A] time = 0.036, size = 90, normalized size = 1.2

$$\frac{1}{d}\left(a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2} - \frac{dx}{2} - \frac{c}{2}\right) + 2\,ab\,(dx+c-\tanh(dx+c)) + b^2\left(-\frac{\sinh(dx+c)}{2\,(\cosh(dx+c))^3} + \frac{\tanh(dx+}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^2,x)
```

```
[Out] 1/d*(a^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+2*a*b*(d*x+c-tanh(d*x+
c))+b^2*(-1/2*sinh(d*x+c)/cosh(d*x+c)^3+1/2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*
x+c)))
```

Maxima [B] time = 1.04187, size = 216, normalized size = 2.96

$$-\frac{1}{8}a^2\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)+2ab\left(x+\frac{c}{d}-\frac{2}{d\left(e^{(-2dx-2c)}+1\right)}\right)+\frac{2}{3}b^2\left(\frac{3e^{(-4dx-4c)}}{d\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/8*a^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 2*a*b*(x + c/d -
2/(d*(e^(-2*d*x - 2*c) + 1))) + 2/3*b^2*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*x
 - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x -
 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

Fricas [B] time = 2.90141, size = 641, normalized size = 8.78

$$\frac{3a^2\sinh(dx+c)^5-4\left(3\left(a^2-4ab\right)dx-12ab+2b^2\right)\cosh(dx+c)^3-12\left(3\left(a^2-4ab\right)dx-12ab+2b^2\right)\cosh(dx+c)\,\mathrm{s}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*a^2*sinh(d*x + c)^5 - 4*(3*(a^2 - 4*a*b)*d*x - 12*a*b + 2*b^2)*cosh
(d*x + c)^3 - 12*(3*(a^2 - 4*a*b)*d*x - 12*a*b + 2*b^2)*cosh(d*x + c)*sinh(
d*x + c)^2 + (30*a^2*cosh(d*x + c)^2 + 9*a^2 - 48*a*b + 8*b^2)*sinh(d*x + c
)^3 - 12*(3*(a^2 - 4*a*b)*d*x - 12*a*b + 2*b^2)*cosh(d*x + c) + 3*(5*a^2*co
sh(d*x + c)^4 + (9*a^2 - 48*a*b + 8*b^2)*cosh(d*x + c)^2 + 2*a^2 - 16*a*b -
 8*b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)
^2 + 3*d*cosh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**2*sinh(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18011, size = 204, normalized size = 2.79

$$\frac{a^2e^{(2dx+2c)}}{8d}-\frac{\left(a^2-4ab\right)(dx+c)}{2d}+\frac{\left(2a^2e^{(2dx+2c)}-8abe^{(2dx+2c)}-a^2\right)e^{(-2dx-2c)}}{8d}+\frac{2\left(6abe^{(4dx+4c)}-3b^2e^{(4dx+4c)}+12ab\cdots\right.}{3d\left(e^{(2dx+2c)}+1\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/8*a^2*e^(2*d*x + 2*c)/d - 1/2*(a^2 - 4*a*b)*(d*x + c)/d + 1/8*(2*a^2*e^(2
*d*x + 2*c) - 8*a*b*e^(2*d*x + 2*c) - a^2)*e^(-2*d*x - 2*c)/d + 2/3*(6*a*b*
e^(4*d*x + 4*c) - 3*b^2*e^(4*d*x + 4*c) + 12*a*b*e^(2*d*x + 2*c) + 6*a*b -
b^2)/(d*(e^(2*d*x + 2*c) + 1)^3)
```

3.12 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)^2 \sinh(c+dx)\,dx$

Optimal. Leaf size=45

$$\frac{a^2\cosh(c+dx)}{d}-\frac{2ab\operatorname{sech}(c+dx)}{d}-\frac{b^2\operatorname{sech}^3(c+dx)}{3d}$$

```
[Out] (a^2*Cosh[c + d*x])/d - (2*a*b*Sech[c + d*x])/d - (b^2*Sech[c + d*x]^3)/(3*
d)
```

Rubi [A] time = 0.0457674, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4133, 270}

$$\frac{a^2\cosh(c+dx)}{d}-\frac{2ab\operatorname{sech}(c+dx)}{d}-\frac{b^2\operatorname{sech}^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x],x]
```

```
[Out] (a^2*Cosh[c + d*x])/d - (2*a*b*Sech[c + d*x])/d - (b^2*Sech[c + d*x]^3)/(3*
d)
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\operatorname{sech}^2(c+dx)\right)^2 \sinh(c+dx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\left(b+ax^2\right)^2}{x^4}\,dx,x,\cosh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(a^2+\frac{b^2}{x^4}+\frac{2ab}{x^2}\right)dx,x,\cosh(c+dx)\right)}{d}\\ &= \frac{a^2\cosh(c+dx)}{d}-\frac{2ab\operatorname{sech}(c+dx)}{d}-\frac{b^2\operatorname{sech}^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.170857, size = 59, normalized size = 1.31

$$\frac{\operatorname{sech}^3(c+dx)\left(3a^2\cosh(4(c+dx))+9a^2+12a(a-2b)\cosh(2(c+dx))-24ab-8b^2\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x],x]
```

```
[Out] ((9*a^2 - 24*a*b - 8*b^2 + 12*a*(a - 2*b)*Cosh[2*(c + d*x)] + 3*a^2*Cosh[4*
(c + d*x)])*Sech[c + d*x]^3)/(24*d)
```

Maple [A] time = 0.018, size = 43, normalized size = 1.

$$-\frac{1}{d}\left(\frac{b^2 (\operatorname{sech}(dx+c))^3}{3}+2\,ab\operatorname{sech}(dx+c)-\frac{a^2}{\operatorname{sech}(dx+c)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^2*sinh(d*x+c),x)
```

```
[Out] -1/d*(1/3*b^2*sech(d*x+c)^3+2*a*b*sech(d*x+c)-a^2/sech(d*x+c))
```

Maxima [A] time = 1.05052, size = 88, normalized size = 1.96

$$\frac{a^2\cosh(dx+c)}{d}-\frac{4\,ab}{d\left(e^{(dx+c)}+e^{(-dx-c)}\right)}-\frac{8\,b^2}{3\,d\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c),x, algorithm="maxima")
```

```
[Out] a^2*cosh(d*x + c)/d - 4*a*b/(d*(e^(d*x + c) + e^(-d*x - c))) - 8/3*b^2/(d*(
e^(d*x + c) + e^(-d*x - c))^3)
```

Fricas [B] time = 2.53017, size = 336, normalized size = 7.47

$$\frac{3\,a^2\cosh(dx+c)^4+3\,a^2\sinh(dx+c)^4+12\left(a^2-2\,ab\right)\cosh(dx+c)^2+6\left(3\,a^2\cosh(dx+c)^2+2\,a^2-4\,ab\right)\sinh(dx}{6\left(d\cosh(dx+c)^3+3\,d\cosh(dx+c)\sinh(dx+c)^2+3\,d\cosh(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^2*cosh(d*x + c)^4 + 3*a^2*sinh(d*x + c)^4 + 12*(a^2 - 2*a*b)*cosh(
d*x + c)^2 + 6*(3*a^2*cosh(d*x + c)^2 + 2*a^2 - 4*a*b)*sinh(d*x + c)^2 + 9*
a^2 - 24*a*b - 8*b^2)/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^
2 + 3*d*cosh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**2*sinh(d*x+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20174, size = 103, normalized size = 2.29

$$\frac{a^2\left(e^{(dx+c)} + e^{(-dx-c)}\right)}{2\,d} - \frac{4\left(3\,ab\left(e^{(dx+c)} + e^{(-dx-c)}\right)^2 + 2\,b^2\right)}{3\,d\left(e^{(dx+c)} + e^{(-dx-c)}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*a^2*(e^(d*x + c) + e^(-d*x - c))/d - 4/3*(3*a*b*(e^(d*x + c) + e^(-d*x
- c))^2 + 2*b^2)/(d*(e^(d*x + c) + e^(-d*x - c))^3)
```

3.13 $\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=52

$$\frac{b(2a+b)\operatorname{sech}(c+dx)}{d}-\frac{(a+b)^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b^2\operatorname{sech}^3(c+dx)}{3d}$$

```
[Out] -(((a + b)^2*ArcTanh[Cosh[c + d*x]])/d) + (b*(2*a + b)*Sech[c + d*x])/d + (
b^2*Sech[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0771389, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4133, 461, 207}

$$\frac{b(2a+b)\operatorname{sech}(c+dx)}{d}-\frac{(a+b)^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b^2\operatorname{sech}^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -(((a + b)^2*ArcTanh[Cosh[c + d*x]])/d) + (b*(2*a + b)*Sech[c + d*x])/d + (
b^2*Sech[c + d*x]^3)/(3*d)
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2 dx = -\frac{\text{Subst}\left(\int \frac{\left(b+ax^2\right)^2}{x^4(1-x^2)}dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{b^2}{x^4}+\frac{b(2a+b)}{x^2}-\frac{(a+b)^2}{-1+x^2}\right)dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{b(2a+b)\text{sech}(c+dx)}{d}+\frac{b^2\text{sech}^3(c+dx)}{3d}+\frac{(a+b)^2\,\text{Subst}\left(\int \frac{1}{-1+x^2}dx, x, c\right.}{d}$$
$$= -\frac{(a+b)^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b(2a+b)\text{sech}(c+dx)}{d}+\frac{b^2\text{sech}^3(c+dx)}{3d}$$

Mathematica [B] time = 0.553278, size = 108, normalized size = 2.08

$$-\frac{4\text{sech}^3(c+dx)\left(a\cosh^2(c+dx)+b\right)^2\left(-3b(2a+b)\cosh^2(c+dx)+3(a+b)^2\cosh^3(c+dx)\left(\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)-\right.\right.}{3d(a\cosh(2(c+dx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (-4*(b + a*Cosh[c + d*x]^2)^2*(-b^2 - 3*b*(2*a + b)*Cosh[c + d*x]^2 + 3*(a
+ b)^2*Cosh[c + d*x]^3*(Log[Cosh[(c + d*x)/2]] - Log[Sinh[(c + d*x)/2]]))*S
ech[c + d*x]^3)/(3*d*(a + 2*b + a*Cosh[2*(c + d*x)])^2)
```

Maple [A] time = 0.036, size = 72, normalized size = 1.4

$$\frac{1}{d}\left(-2a^2\text{Artanh}\left(e^{dx+c}\right)+2ab\left((\cosh(dx+c))^{-1}-2\,\text{Artanh}\left(e^{dx+c}\right)\right)+b^2\left(\frac{1}{3(\cosh(dx+c))^3}+(\cosh(dx+c))^{-1}-2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(-2*a^2*arctanh(exp(d*x+c))+2*a*b*(1/cosh(d*x+c)-2*arctanh(exp(d*x+c)))
+b^2*(1/3/cosh(d*x+c)^3+1/cosh(d*x+c)-2*arctanh(exp(d*x+c))))
```

Maxima [B] time = 1.05542, size = 266, normalized size = 5.12

$$-\frac{1}{3}b^2\left(\frac{3\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{3\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2\left(3e^{(-dx-c)}+10e^{(-3dx-3c)}+3e^{(-5dx-5c)}\right)}{d\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1\right)}\right)-2ab\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/3*b^2*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d - 2*(3*e^(-
d*x - c) + 10*e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c))/(d*(3*e^(-2*d*x - 2*c)
 + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) - 2*a*b*(log(e^(-d*x - c) +
 1)/d - log(e^(-d*x - c) - 1)/d - 2*e^(-d*x - c)/(d*(e^(-2*d*x - 2*c) + 1))
```

```
) + a^2*log(tanh(1/2*d*x + 1/2*c))/d
```

Fricas [B] time = 2.67833, size = 3016, normalized size = 58.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(6*(2*a*b + b^2)*cosh(d*x + c)^5 + 30*(2*a*b + b^2)*cosh(d*x + c)*sinh(
d*x + c)^4 + 6*(2*a*b + b^2)*sinh(d*x + c)^5 + 4*(6*a*b + 5*b^2)*cosh(d*x +
 c)^3 + 4*(15*(2*a*b + b^2)*cosh(d*x + c)^2 + 6*a*b + 5*b^2)*sinh(d*x + c)^
3 + 12*(5*(2*a*b + b^2)*cosh(d*x + c)^3 + (6*a*b + 5*b^2)*cosh(d*x + c))*si
nh(d*x + c)^2 + 6*(2*a*b + b^2)*cosh(d*x + c) - 3*((a^2 + 2*a*b + b^2)*cosh
(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^2 +
2*a*b + b^2)*sinh(d*x + c)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 3*(5
*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 +
 4*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x
+ c))*sinh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 3*(5*(a^2 +
 2*a*b + b^2)*cosh(d*x + c)^4 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2
 + 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 6*((a^2 + 2*a*b + b^2
)*cosh(d*x + c)^5 + 2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b +
b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) +
 3*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x +
c)*sinh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^6 + 3*(a^2 + 2*a*b +
 b^2)*cosh(d*x + c)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*
a*b + b^2)*sinh(d*x + c)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(
a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^2 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 6*(a^2 + 2*a*b
+ b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b +
 b^2 + 6*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 2*(a^2 + 2*a*b + b^2)*cosh(
d*x + c)^3 + (a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x
 + c) + sinh(d*x + c) - 1) + 6*(5*(2*a*b + b^2)*cosh(d*x + c)^4 + 2*(6*a*b
+ 5*b^2)*cosh(d*x + c)^2 + 2*a*b + b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^6 +
 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 + 3*d*cosh(d*x + c)^
4 + 3*(5*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 +
3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x
+ c)^4 + 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 6*(d*cosh(d*x + c)^5 +
2*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\operatorname{sech}^2(c + dx)\right)^2 \operatorname{csch}(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*csch(c + d*x), x)
```

Giac [B] time = 1.19978, size = 193, normalized size = 3.71

$$-\frac{\left(a^2+2\,ab+b^2\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)}{2\,d}+\frac{\left(a^2+2\,ab+b^2\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)}{2\,d}+\frac{2\left(6\,ab\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2+\right.}{3\,d\left(e^{(dx+c)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(a^2 + 2*a*b + b^2)*log(e^(d*x + c) + e^(-d*x - c) + 2)/d + 1/2*(a^2 +
 2*a*b + b^2)*log(e^(d*x + c) + e^(-d*x - c) - 2)/d + 2/3*(6*a*b*(e^(d*x +
c) + e^(-d*x - c))^2 + 3*b^2*(e^(d*x + c) + e^(-d*x - c))^2 + 4*b^2)/(d*(e^
(d*x + c) + e^(-d*x - c))^3)
```

3.14 $\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=50

$$-\frac{2b(a+b)\tanh(c+dx)}{d}-\frac{(a+b)^2\coth(c+dx)}{d}+\frac{b^2\tanh^3(c+dx)}{3d}$$

```
[Out] -(((a + b)^2*Coth[c + d*x])/d) - (2*b*(a + b)*Tanh[c + d*x])/d + (b^2*Tanh[
c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0622623, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4132, 270}

$$-\frac{2b(a+b)\tanh(c+dx)}{d}-\frac{(a+b)^2\coth(c+dx)}{d}+\frac{b^2\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -(((a + b)^2*Coth[c + d*x])/d) - (2*b*(a + b)*Tanh[c + d*x])/d + (b^2*Tanh[
c + d*x]^3)/(3*d)
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\left(a+b-bx^2\right)^2}{x^2}\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(-2b(a+b)+\frac{(a+b)^2}{x^2}+b^2x^2\right)dx,x,\tanh(c+dx)\right)}{d}\\ &= -\frac{(a+b)^2\coth(c+dx)}{d}-\frac{2b(a+b)\tanh(c+dx)}{d}+\frac{b^2\tanh^3(c+dx)}{3d}\end{aligned}$$

Mathematica [B] time = 1.52381, size = 109, normalized size = 2.18

$$-\frac{4\operatorname{sech}^3(c+dx)\left(a\cosh^2(c+dx)+b\right)^2\left(\sinh(dx)\cosh^2(c+dx)\left(b(6a+5b)\operatorname{sech}(c)-3(a+b)^2\operatorname{csch}(c)\coth(c+dx)\right)\right.}{3d(a\cosh(2(c+dx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (-4*(b + a*Cosh[c + d*x]^2)^2*Sech[c + d*x]^3*(b^2*Sech[c]*Sinh[d*x] + Cosh
[c + d*x]^2*(-3*(a + b)^2*Coth[c + d*x]*Csch[c] + b*(6*a + 5*b)*Sech[c])*Si
nh[d*x] + b^2*Cosh[c + d*x]*Tanh[c]))/(3*d*(a + 2*b + a*Cosh[2*(c + d*x)])^
2)
```

Maple [A] time = 0.037, size = 91, normalized size = 1.8

$$\frac{1}{d}\left(-a^2\coth(dx+c)+2\,ab\left(-\frac{1}{\cosh(dx+c)\sinh(dx+c)}-2\,\tanh(dx+c)\right)+b^2\left(-\frac{1}{\sinh(dx+c)\left(\cosh(dx+c)\right)^3}-4\,(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(-a^2*coth(d*x+c)+2*a*b*(-1/sinh(d*x+c)/cosh(d*x+c)-2*tanh(d*x+c))+b^2*
(-1/sinh(d*x+c)/cosh(d*x+c)^3-4*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)))
```

Maxima [B] time = 1.06326, size = 189, normalized size = 3.78

$$-\frac{16}{3}\,b^2\left(\frac{2\,e^{(-2\,dx-2\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)}-2\,e^{(-6\,dx-6\,c)}-e^{(-8\,dx-8\,c)}+1\right)}+\frac{1}{d\left(2\,e^{(-2\,dx-2\,c)}-2\,e^{(-6\,dx-6\,c)}-e^{(-8\,dx-8\,c)}+1\right)}\right)+\frac{2\,a^2}{d\left(e^{(-2\,dx-2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -16/3*b^2*(2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-6*d*x - 6*c) -
 e^(-8*d*x - 8*c) + 1)) + 1/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-6*d*x - 6*c) - e
^(-8*d*x - 8*c) + 1))) + 2*a^2/(d*(e^(-2*d*x - 2*c) - 1)) + 8*a*b/(d*(e^(-4
*d*x - 4*c) - 1))
```

Fricas [B] time = 2.57163, size = 733, normalized size = 14.66

$$-\frac{4\left(\left(3\,a^2+6\,ab+4\,b^2\right)\cosh(dx+c)^3+3\left(3\,a^2+6\,ab+4\,b^2\right)\cosh(dx+c)\sinh(dx+c)^2-2\right.}{3\left(d\cosh(dx+c)^5+5\,d\cosh(dx+c)\sinh(dx+c)^4+d\sinh(dx+c)^5+d\cosh(dx+c)^3+\left(10\,d\cosh(dx+c)^2+3\,d\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -4/3*((3*a^2 + 6*a*b + 4*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 + 6*a*b + 4*b^2)*c
osh(d*x + c)*sinh(d*x + c)^2 - 2*(3*a*b + 2*b^2)*sinh(d*x + c)^3 + (9*a^2 +
 18*a*b + 8*b^2)*cosh(d*x + c) - 2*(3*(3*a*b + 2*b^2)*cosh(d*x + c)^2 + 3*a
*b + 4*b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x
+ c)^4 + d*sinh(d*x + c)^5 + d*cosh(d*x + c)^3 + (10*d*cosh(d*x + c)^2 + 3*
d)*sinh(d*x + c)^3 + (10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x +
```

```
c)^2 - 2*d*cosh(d*x + c) + (5*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 + 2*d
)*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\,\mathrm{sech}^2\left(c+dx\right)\right)^2 \mathrm{csch}^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*csch(c + d*x)**2, x)
```

Giac [B] time = 1.22173, size = 151, normalized size = 3.02

$$-\frac{2\left(a^2+2\,ab+b^2\right)}{d\left(e^{(2\,dx+2\,c)}-1\right)}+\frac{2\left(6\,abe^{(4\,dx+4\,c)}+3\,b^2e^{(4\,dx+4\,c)}+12\,abe^{(2\,dx+2\,c)}+12\,b^2e^{(2\,dx+2\,c)}+6\,ab+5\,b^2\right)}{3\,d\left(e^{(2\,dx+2\,c)}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -2*(a^2 + 2*a*b + b^2)/(d*(e^(2*d*x + 2*c) - 1)) + 2/3*(6*a*b*e^(4*d*x + 4*
c) + 3*b^2*e^(4*d*x + 4*c) + 12*a*b*e^(2*d*x + 2*c) + 12*b^2*e^(2*d*x + 2*c
) + 6*a*b + 5*b^2)/(d*(e^(2*d*x + 2*c) + 1)^3)
```

3.15 $\int \mathrm{csch}^3(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=104

$$-\frac{(3a^2+6ab+5b^2)\coth(c+dx)\mathrm{csch}(c+dx)}{6d}-\frac{b(6a+5b)\mathrm{sech}(c+dx)}{3d}+\frac{(a+b)(a+5b)\tanh^{-1}(\cosh(c+dx))}{2d}+\frac{b^2\mathrm{cs}}{}$$

```
[Out] ((a + b)*(a + 5*b)*ArcTanh[Cosh[c + d*x]])/(2*d) - ((3*a^2 + 6*a*b + 5*b^2)
*Coth[c + d*x]*Csch[c + d*x])/(6*d) - (b*(6*a + 5*b)*Sech[c + d*x])/(3*d) +
 (b^2*Csch[c + d*x]^2*Sech[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.135715, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4133, 462, 456, 453, 206}

$$-\frac{(3a^2+6ab+5b^2)\coth(c+dx)\mathrm{csch}(c+dx)}{6d}-\frac{b(6a+5b)\mathrm{sech}(c+dx)}{3d}+\frac{(a+b)(a+5b)\tanh^{-1}(\cosh(c+dx))}{2d}+\frac{b^2\mathrm{cs}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + b)*(a + 5*b)*ArcTanh[Cosh[c + d*x]])/(2*d) - ((3*a^2 + 6*a*b + 5*b^2)
*Coth[c + d*x]*Csch[c + d*x])/(6*d) - (b*(6*a + 5*b)*Sech[c + d*x])/(3*d) +
 (b^2*Csch[c + d*x]^2*Sech[c + d*x]^3)/(3*d)
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
```

```
x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(b+ax^2\right)^2}{x^4\left(1-x^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{b^2\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{3d} + \frac{\operatorname{Subst}\left(\int \frac{b(6a+5b)+3a^2x^2}{x^2\left(1-x^2\right)^2}\,dx, x, \cosh(c+\right.}{3d}$$

$$= -\frac{\left(3a^2+6ab+5b^2\right)\coth(c+dx)\operatorname{csch}(c+dx)}{6d} + \frac{b^2\operatorname{csch}^2(c+dx)\operatorname{sech}^3(}{3d}$$

$$= -\frac{\left(3a^2+6ab+5b^2\right)\coth(c+dx)\operatorname{csch}(c+dx)}{6d} - \frac{b(6a+5b)\operatorname{sech}(c+dx)}{3d}$$

$$= \frac{(a+b)(a+5b)\tanh^{-1}(\cosh(c+dx))}{2d} - \frac{\left(3a^2+6ab+5b^2\right)\coth(c+dx)}{6d}$$

Mathematica [A] time = 1.53897, size = 144, normalized size = 1.38

$$-\frac{\operatorname{sech}^3(c+dx)\left(a\cosh^2(c+dx)+b\right)^2\left(48b(a+b)\cosh^2(c+dx)+3(a+b)\cosh^3(c+dx)\left((a+b)\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)+\right.\right.}{6d(a\cosh(2(c+dx))+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -((b + a*Cosh[c + d*x]^2)^2*(8*b^2 + 48*b*(a + b)*Cosh[c + d*x]^2 + 3*(a +
b)*Cosh[c + d*x]^3*((a + b)*Csch[(c + d*x)/2]^2 - 4*(a + 5*b)*(Log[Cosh[(c
+ d*x)/2]] - Log[Sinh[(c + d*x)/2]]) + (a + b)*Sech[(c + d*x)/2]^2))*Sech[c
 + d*x]^3)/(6*d*(a + 2*b + a*Cosh[2*(c + d*x)])^2)
```

Maple [A] time = 0.046, size = 126, normalized size = 1.2

$$\frac{1}{d}\left(a^2\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{Artanh}\left(e^{dx+c}\right)\right)+2\,ab\left(-1/2\,\frac{1}{(\sinh(dx+c))^2\cosh(dx+c)}-3/2\,(\cosh(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+2*a*b*(-1/2/sin
h(d*x+c)^2/cosh(d*x+c)-3/2/cosh(d*x+c)+3*arctanh(exp(d*x+c)))+b^2*(-1/2/sin
```

```
h(d*x+c)^2/cosh(d*x+c)^3-5/6/cosh(d*x+c)^3-5/2/cosh(d*x+c)+5*arctanh(exp(d*
x+c))))
```

Maxima [B] time = 1.09309, size = 478, normalized size = 4.6

$$\frac{1}{6}b^2\left(\frac{15\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{15\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2\left(15e^{(-dx-c)}+20e^{(-3dx-3c)}-22e^{(-5dx-5c)}+20e^{(-7dx-7c)}+15e^{(-9dx-}\right.}{d\left(e^{(-2dx-2c)}-2e^{(-4dx-4c)}-2e^{(-6dx-6c)}+e^{(-8dx-8c)}+e^{(-10dx-10c)}+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*(15*log(e^(-d*x - c) + 1)/d - 15*log(e^(-d*x - c) - 1)/d - 2*(15*e^
(-d*x - c) + 20*e^(-3*d*x - 3*c) - 22*e^(-5*d*x - 5*c) + 20*e^(-7*d*x - 7*c
) + 15*e^(-9*d*x - 9*c))/(d*(e^(-2*d*x - 2*c) - 2*e^(-4*d*x - 4*c) - 2*e^(-
6*d*x - 6*c) + e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + a*b*(3*log(e^
(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d + 2*(3*e^(-d*x - c) - 2*e^(-3
*d*x - 3*c) + 3*e^(-5*d*x - 5*c))/(d*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) -
 e^(-6*d*x - 6*c) - 1))) + 1/2*a^2*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x -
 c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e
^(-4*d*x - 4*c) - 1)))
```

Fricas [B] time = 2.97845, size = 7422, normalized size = 71.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^9 + 54*(a^2 + 6*a*b + 5*b^2)*co
sh(d*x + c)*sinh(d*x + c)^8 + 6*(a^2 + 6*a*b + 5*b^2)*sinh(d*x + c)^9 + 8*(
3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^7 + 8*(27*(a^2 + 6*a*b + 5*b^2)*cosh(d
*x + c)^2 + 3*a^2 + 6*a*b + 5*b^2)*sinh(d*x + c)^7 + 56*(9*(a^2 + 6*a*b + 5
*b^2)*cosh(d*x + c)^3 + (3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c
)^6 + 4*(9*a^2 + 6*a*b - 11*b^2)*cosh(d*x + c)^5 + 4*(189*(a^2 + 6*a*b + 5*
b^2)*cosh(d*x + c)^4 + 42*(3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^2 + 9*a^2 +
 6*a*b - 11*b^2)*sinh(d*x + c)^5 + 4*(189*(a^2 + 6*a*b + 5*b^2)*cosh(d*x +
c)^5 + 70*(3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^3 + 5*(9*a^2 + 6*a*b - 11*b
^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 8*(3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c
)^3 + 8*(63*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^6 + 35*(3*a^2 + 6*a*b + 5*b
^2)*cosh(d*x + c)^4 + 5*(9*a^2 + 6*a*b - 11*b^2)*cosh(d*x + c)^2 + 3*a^2 +
6*a*b + 5*b^2)*sinh(d*x + c)^3 + 8*(27*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^
7 + 21*(3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^5 + 5*(9*a^2 + 6*a*b - 11*b^2)
*cosh(d*x + c)^3 + 3*(3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^2
 + 6*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c) - 3*((a^2 + 6*a*b + 5*b^2)*cosh(d*
x + c)^10 + 10*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^2 +
 6*a*b + 5*b^2)*sinh(d*x + c)^10 + (a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^8 +
(45*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^2 + a^2 + 6*a*b + 5*b^2)*sinh(d*x +
 c)^8 + 8*(15*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^3 + (a^2 + 6*a*b + 5*b^2)
*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^6 +
 2*(105*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^4 + 14*(a^2 + 6*a*b + 5*b^2)*co
sh(d*x + c)^2 - a^2 - 6*a*b - 5*b^2)*sinh(d*x + c)^6 + 4*(63*(a^2 + 6*a*b +
 5*b^2)*cosh(d*x + c)^5 + 14*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^3 - 3*(a^2
```

```
 + 6*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(a^2 + 6*a*b + 5*b^2)*
cosh(d*x + c)^4 + 2*(105*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^6 + 35*(a^2 +
6*a*b + 5*b^2)*cosh(d*x + c)^4 - 15*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^2 -
 a^2 - 6*a*b - 5*b^2)*sinh(d*x + c)^4 + 8*(15*(a^2 + 6*a*b + 5*b^2)*cosh(d*
x + c)^7 + 7*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^5 - 5*(a^2 + 6*a*b + 5*b^2
)*cosh(d*x + c)^3 - (a^2 + 6*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 +
(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^2 + (45*(a^2 + 6*a*b + 5*b^2)*cosh(d*x
+ c)^8 + 28*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^6 - 30*(a^2 + 6*a*b + 5*b^2
)*cosh(d*x + c)^4 - 12*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^2 + a^2 + 6*a*b
+ 5*b^2)*sinh(d*x + c)^2 + a^2 + 6*a*b + 5*b^2 + 2*(5*(a^2 + 6*a*b + 5*b^2)
*cosh(d*x + c)^9 + 4*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^7 - 6*(a^2 + 6*a*b
 + 5*b^2)*cosh(d*x + c)^5 - 4*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^3 + (a^2
+ 6*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x
 + c) + 1) + 3*((a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^10 + 10*(a^2 + 6*a*b +
5*b^2)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^2 + 6*a*b + 5*b^2)*sinh(d*x + c)^
10 + (a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^8 + (45*(a^2 + 6*a*b + 5*b^2)*cosh
(d*x + c)^2 + a^2 + 6*a*b + 5*b^2)*sinh(d*x + c)^8 + 8*(15*(a^2 + 6*a*b + 5
*b^2)*cosh(d*x + c)^3 + (a^2 + 6*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^
7 - 2*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^6 + 2*(105*(a^2 + 6*a*b + 5*b^2)*
cosh(d*x + c)^4 + 14*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^2 - a^2 - 6*a*b -
5*b^2)*sinh(d*x + c)^6 + 4*(63*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^5 + 14*(
a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^3 - 3*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c
))*sinh(d*x + c)^5 - 2*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^4 + 2*(105*(a^2
+ 6*a*b + 5*b^2)*cosh(d*x + c)^6 + 35*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^4
 - 15*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^2 - a^2 - 6*a*b - 5*b^2)*sinh(d*x
 + c)^4 + 8*(15*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^7 + 7*(a^2 + 6*a*b + 5*
b^2)*cosh(d*x + c)^5 - 5*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^3 - (a^2 + 6*a
*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + (a^2 + 6*a*b + 5*b^2)*cosh(d*x
 + c)^2 + (45*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^8 + 28*(a^2 + 6*a*b + 5*b
^2)*cosh(d*x + c)^6 - 30*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^4 - 12*(a^2 +
6*a*b + 5*b^2)*cosh(d*x + c)^2 + a^2 + 6*a*b + 5*b^2)*sinh(d*x + c)^2 + a^2
 + 6*a*b + 5*b^2 + 2*(5*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^9 + 4*(a^2 + 6*
a*b + 5*b^2)*cosh(d*x + c)^7 - 6*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^5 - 4*
(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^3 + (a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)
)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(27*(a^2 + 6*a*
b + 5*b^2)*cosh(d*x + c)^8 + 28*(3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^6 + 1
0*(9*a^2 + 6*a*b - 11*b^2)*cosh(d*x + c)^4 + 12*(3*a^2 + 6*a*b + 5*b^2)*cos
h(d*x + c)^2 + 3*a^2 + 18*a*b + 15*b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^10
+ 10*d*cosh(d*x + c)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 + d*cosh(d*x + c)
^8 + (45*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 8*(15*d*cosh(d*x + c)^3 +
 d*cosh(d*x + c))*sinh(d*x + c)^7 - 2*d*cosh(d*x + c)^6 + 2*(105*d*cosh(d*x
 + c)^4 + 14*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^6 + 4*(63*d*cosh(d*x + c)
^5 + 14*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^5 - 2*d*cosh(d
*x + c)^4 + 2*(105*d*cosh(d*x + c)^6 + 35*d*cosh(d*x + c)^4 - 15*d*cosh(d*x
 + c)^2 - d)*sinh(d*x + c)^4 + 8*(15*d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)^
5 - 5*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c)^3 + d*cosh(d*x + c
)^2 + (45*d*cosh(d*x + c)^8 + 28*d*cosh(d*x + c)^6 - 30*d*cosh(d*x + c)^4 -
 12*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 2*(5*d*cosh(d*x + c)^9 + 4*d*c
osh(d*x + c)^7 - 6*d*cosh(d*x + c)^5 - 4*d*cosh(d*x + c)^3 + d*cosh(d*x + c
))*sinh(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\operatorname{sech}^2(c + dx)\right)^2 \operatorname{csch}^3(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*csch(c + d*x)**3, x)
```

Giac [B] time = 1.22353, size = 317, normalized size = 3.05

$$\frac{\left(a^{2}+6\,ab+5\,b^{2}\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)}{4\,d}-\frac{\left(a^{2}+6\,ab+5\,b^{2}\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)}{4\,d}-\frac{a^{2}\left(e^{(dx+c)}+e^{(-dx-c)}\right)+2\,ab\cdots}{\left(\left(e^{(dx+\cdots}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*(a^2 + 6*a*b + 5*b^2)*log(e^(d*x + c) + e^(-d*x - c) + 2)/d - 1/4*(a^2
+ 6*a*b + 5*b^2)*log(e^(d*x + c) + e^(-d*x - c) - 2)/d - (a^2*(e^(d*x + c)
+ e^(-d*x - c)) + 2*a*b*(e^(d*x + c) + e^(-d*x - c)) + b^2*(e^(d*x + c) + e
^(-d*x - c)))/(((e^(d*x + c) + e^(-d*x - c))^2 - 4)*d) - 4/3*(3*a*b*(e^(d*x
 + c) + e^(-d*x - c))^2 + 3*b^2*(e^(d*x + c) + e^(-d*x - c))^2 + 2*b^2)/(d*
(e^(d*x + c) + e^(-d*x - c))^3)
```

3.16 $\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=75

$$\frac{b(2a+3b)\tanh(c+dx)}{d}-\frac{(a+b)^2\coth^3(c+dx)}{3d}+\frac{(a+b)(a+3b)\coth(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}$$

```
[Out] ((a + b)*(a + 3*b)*Coth[c + d*x])/d - ((a + b)^2*Coth[c + d*x]^3)/(3*d) + (
b*(2*a + 3*b)*Tanh[c + d*x])/d - (b^2*Tanh[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.083351, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4132, 448}

$$\frac{b(2a+3b)\tanh(c+dx)}{d}-\frac{(a+b)^2\coth^3(c+dx)}{3d}+\frac{(a+b)(a+3b)\coth(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + b)*(a + 3*b)*Coth[c + d*x])/d - ((a + b)^2*Coth[c + d*x]^3)/(3*d) + (
b*(2*a + 3*b)*Tanh[c + d*x])/d - (b^2*Tanh[c + d*x]^3)/(3*d)
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)(a+b-bx^2)^2}{x^4}\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(b(2a+3b)+\frac{(a+b)^2}{x^4}+\frac{(-a-3b)(a+b)}{x^2}-b^2x^2\right)dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{(a+b)(a+3b)\coth(c+dx)}{d}-\frac{(a+b)^2\coth^3(c+dx)}{3d}+\frac{b(2a+3b)\tanh}{d}\end{aligned}$$

Mathematica [B] time = 1.2647, size = 151, normalized size = 2.01

$$-\frac{\operatorname{csch}(2c)\operatorname{csch}^3(2(c+dx))\left(-3a^2\sinh(2(c+dx))+a^2\sinh(6(c+dx))+3a^2\sinh(4c+2dx)+a^2\sinh(4c+6dx)-6a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -(Csch[2*c]*Csch[2*(c + d*x)]^3*(8*a*(a + 2*b)*Sinh[2*c] - 6*(a + 2*b)^2*Si
nh[2*d*x] - 3*a^2*Sinh[2*(c + d*x)] - 6*a*b*Sinh[2*(c + d*x)] + a^2*Sinh[6*
(c + d*x)] + 2*a*b*Sinh[6*(c + d*x)] + 3*a^2*Sinh[4*c + 2*d*x] + a^2*Sinh[4
*c + 6*d*x] + 8*a*b*Sinh[4*c + 6*d*x] + 8*b^2*Sinh[4*c + 6*d*x]))/(6*d)
```

Maple [A] time = 0.045, size = 138, normalized size = 1.8

$$\frac{1}{d}\left(a^2\left(\frac{2}{3}-\frac{(\operatorname{csch}(dx+c))^2}{3}\right)\coth(dx+c)+2\,ab\left(-1/3\,\frac{1}{(\sinh(dx+c))^3\cosh(dx+c)}+4/3\,\frac{1}{\cosh(dx+c)\sinh(dx+c)}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+2*a*b*(-1/3/sinh(d*x+c)^3/cosh
(d*x+c)+4/3/sinh(d*x+c)/cosh(d*x+c)+8/3*tanh(d*x+c))+b^2*(-1/3/sinh(d*x+c)^
3/cosh(d*x+c)^3+2/sinh(d*x+c)/cosh(d*x+c)^3+8*(2/3+1/3*sech(d*x+c)^2)*tanh(
d*x+c)))
```

Maxima [B] time = 1.0496, size = 385, normalized size = 5.13

$$\frac{4}{3}a^2\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1\right)}-\frac{1}{d\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1\right)}\right)+\frac{32}{3}ab\left(\frac{}{d\left(2\,e^{(-2\,dx-2\,c}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 4/3*a^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e
^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) - 1))) + 32/3*a*b*(2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c)
- 2*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) - 1)) - 1/(d*(2*e^(-2*d*x - 2*c) -
2*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) - 1))) + 32/3*b^2*(3*e^(-4*d*x - 4*c)
/(d*(3*e^(-4*d*x - 4*c) - 3*e^(-8*d*x - 8*c) + e^(-12*d*x - 12*c) - 1)) - 1
/(d*(3*e^(-4*d*x - 4*c) - 3*e^(-8*d*x - 8*c) + e^(-12*d*x - 12*c) - 1)))
```

Fricas [B] time = 2.55032, size = 1054, normalized size = 14.05

$$-\frac{8\left(\left(a^2-4\,ab-4\,b^2\right)\cosh(dx+c)^4+8\left(a^2+2\,ab+2\,b^2\right)\cosh(dx+c)\sinh(dx+c)^3+\right.}{3\left(d\cosh(dx+c)^8+56\,d\cosh(dx+c)^3\sinh(dx+c)^5+28\,d\cosh(dx+c)^2\sinh(dx+c)^6+8\,d\cosh(dx+c)\sinh(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -8/3*((a^2 - 4*a*b - 4*b^2)*cosh(d*x + c)^4 + 8*(a^2 + 2*a*b + 2*b^2)*cosh(
d*x + c)*sinh(d*x + c)^3 + (a^2 - 4*a*b - 4*b^2)*sinh(d*x + c)^4 + 4*(a^2 +
```

```
2*a*b)*cosh(d*x + c)^2 + 2*(3*(a^2 - 4*a*b - 4*b^2)*cosh(d*x + c)^2 + 2*a^
2 + 4*a*b)*sinh(d*x + c)^2 + 3*a^2 + 12*a*b + 12*b^2 + 8*((a^2 + 2*a*b + 2*
b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(
d*x + c)^8 + 56*d*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*d*cosh(d*x + c)^2*si
nh(d*x + c)^6 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 4*d
*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 - 2*d)*sinh(d*x + c)^4 + 8*(7*d*
cosh(d*x + c)^5 - d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*d*cosh(d*x + c)^6
 - 6*d*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 - d*cosh(d*x
 + c)^3)*sinh(d*x + c) + 3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b \operatorname{sech}^2{(c + dx)}\right)^2 \operatorname{csch}^4{(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*csch(c + d*x)**4, x)
```

Giac [A] time = 1.16943, size = 155, normalized size = 2.07

$$-\frac{4\left(3\,a^2 e^{(8\,dx+8\,c)} + 8\,a^2 e^{(6\,dx+6\,c)} + 16\,abe^{(6\,dx+6\,c)} + 6\,a^2 e^{(4\,dx+4\,c)} + 24\,abe^{(4\,dx+4\,c)} + 24\,b^2 e^{(4\,dx+4\,c)} - a^2 - 8\,ab - 8\,b^2\right)}{3\,d\left(e^{(4\,dx+4\,c)} - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -4/3*(3*a^2*e^(8*d*x + 8*c) + 8*a^2*e^(6*d*x + 6*c) + 16*a*b*e^(6*d*x + 6*c
) + 6*a^2*e^(4*d*x + 4*c) + 24*a*b*e^(4*d*x + 4*c) + 24*b^2*e^(4*d*x + 4*c)
 - a^2 - 8*a*b - 8*b^2)/(d*(e^(4*d*x + 4*c) - 1)^3)
```

3.17 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)^3 \sinh^4(c+dx)\,dx$

Optimal. Leaf size=182

$$\frac{b\left(6a^2-23ab-8b^2\right)\tanh^3(c+dx)}{8d}-\frac{3a\left(a^2-12ab+8b^2\right)\tanh(c+dx)}{8d}+\frac{3}{8}ax\left(a^2-12ab+8b^2\right)-\frac{3b^2(5a-16b)\tanh^5}{40d}$$

```
[Out] (3*a*(a^2 - 12*a*b + 8*b^2)*x)/8 - (3*a*(a^2 - 12*a*b + 8*b^2)*Tanh[c + d*x
])/(8*d) + (b*(6*a^2 - 23*a*b - 8*b^2)*Tanh[c + d*x]^3)/(8*d) - (3*(5*a - 1
6*b)*b^2*Tanh[c + d*x]^5)/(40*d) - (3*(a - 2*b)*Sinh[c + d*x]^2*Tanh[c + d*
x]*(a + b - b*Tanh[c + d*x]^2)^2)/(8*d) + (Cosh[c + d*x]*Sinh[c + d*x]^3*(a
 + b - b*Tanh[c + d*x]^2)^3)/(4*d)
```

Rubi [A] time = 0.231556, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4132, 467, 577, 570, 206}

$$\frac{b\left(6a^2-23ab-8b^2\right)\tanh^3(c+dx)}{8d}-\frac{3a\left(a^2-12ab+8b^2\right)\tanh(c+dx)}{8d}+\frac{3}{8}ax\left(a^2-12ab+8b^2\right)-\frac{3b^2(5a-16b)\tanh^5}{40d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x]^4,x]
```

```
[Out] (3*a*(a^2 - 12*a*b + 8*b^2)*x)/8 - (3*a*(a^2 - 12*a*b + 8*b^2)*Tanh[c + d*x
])/(8*d) + (b*(6*a^2 - 23*a*b - 8*b^2)*Tanh[c + d*x]^3)/(8*d) - (3*(5*a - 1
6*b)*b^2*Tanh[c + d*x]^5)/(40*d) - (3*(a - 2*b)*Sinh[c + d*x]^2*Tanh[c + d*
x]*(a + b - b*Tanh[c + d*x]^2)^2)/(8*d) + (Cosh[c + d*x]*Sinh[c + d*x]^3*(a
 + b - b*Tanh[c + d*x]^2)^3)/(4*d)
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 577

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*g*n*(p + 1)), x] + Dist[1/(a*
b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b
*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m + n
*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 0
```

```
] && LtQ[p, -1] && GtQ[q, 0] && !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a
*f])
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3 \sinh^4(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+b-bx^2\right)^3}{\left(1-x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh^3(c+dx)\left(a+b-b\tanh^2(c+dx)\right)^3}{4d} - \frac{\text{Subst}\left(\int \frac{x^2(3(a}{}\right.}{}$$

$$= -\frac{3(a-2b)\sinh^2(c+dx)\tanh(c+dx)\left(a+b-b\tanh^2(c+dx)\right)^2}{8d} + \frac{\cosh}{}$$

$$= -\frac{3(a-2b)\sinh^2(c+dx)\tanh(c+dx)\left(a+b-b\tanh^2(c+dx)\right)^2}{8d} + \frac{\cosh}{}$$

$$= -\frac{3a\left(a^2-12ab+8b^2\right)\tanh(c+dx)}{8d} + \frac{b\left(6a^2-23ab-8b^2\right)\tanh^3(c+d}{8d}$$

$$= \frac{3}{8}a\left(a^2-12ab+8b^2\right)x - \frac{3a\left(a^2-12ab+8b^2\right)\tanh(c+dx)}{8d} + \frac{b\left(6a^2-2\right.}{}$$

Mathematica [B] time = 2.42398, size = 651, normalized size = 3.58

$$\text{sech}(c)\text{sech}^5(c+dx)\left(a\cosh^2(c+dx)+b\right)^3\left(1200adx\left(a^2-12ab+8b^2\right)\cosh(2c+dx)+1200adx\left(a^2-12ab+8b^2\right)c\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x]^4,x]
```

```
[Out] ((b + a*Cosh[c + d*x]^2)^3*Sech[c]*Sech[c + d*x]^5*(1200*a*(a^2 - 12*a*b +
8*b^2)*d*x*Cosh[d*x] + 1200*a*(a^2 - 12*a*b + 8*b^2)*d*x*Cosh[2*c + d*x] +
600*a^3*d*x*Cosh[2*c + 3*d*x] - 7200*a^2*b*d*x*Cosh[2*c + 3*d*x] + 4800*a*b
^2*d*x*Cosh[2*c + 3*d*x] + 600*a^3*d*x*Cosh[4*c + 3*d*x] - 7200*a^2*b*d*x*C
osh[4*c + 3*d*x] + 4800*a*b^2*d*x*Cosh[4*c + 3*d*x] + 120*a^3*d*x*Cosh[4*c
+ 5*d*x] - 1440*a^2*b*d*x*Cosh[4*c + 5*d*x] + 960*a*b^2*d*x*Cosh[4*c + 5*d*
x] + 120*a^3*d*x*Cosh[6*c + 5*d*x] - 1440*a^2*b*d*x*Cosh[6*c + 5*d*x] + 960
*a*b^2*d*x*Cosh[6*c + 5*d*x] - 180*a^3*Sinh[d*x] + 12120*a^2*b*Sinh[d*x] -
14080*a*b^2*Sinh[d*x] + 1280*b^3*Sinh[d*x] - 180*a^3*Sinh[2*c + d*x] - 7080
*a^2*b*Sinh[2*c + d*x] + 11520*a*b^2*Sinh[2*c + d*x] - 310*a^3*Sinh[2*c + 3
```

```
*d*x] + 8760*a^2*b*Sinh[2*c + 3*d*x] - 8960*a*b^2*Sinh[2*c + 3*d*x] - 310*a
^3*Sinh[4*c + 3*d*x] - 840*a^2*b*Sinh[4*c + 3*d*x] + 3840*a*b^2*Sinh[4*c +
3*d*x] - 640*b^3*Sinh[4*c + 3*d*x] - 150*a^3*Sinh[4*c + 5*d*x] + 2520*a^2*b
*Sinh[4*c + 5*d*x] - 2560*a*b^2*Sinh[4*c + 5*d*x] + 128*b^3*Sinh[4*c + 5*d*
x] - 150*a^3*Sinh[6*c + 5*d*x] + 600*a^2*b*Sinh[6*c + 5*d*x] - 15*a^3*Sinh[
6*c + 7*d*x] + 120*a^2*b*Sinh[6*c + 7*d*x] - 15*a^3*Sinh[8*c + 7*d*x] + 120
*a^2*b*Sinh[8*c + 7*d*x] + 5*a^3*Sinh[8*c + 9*d*x] + 5*a^3*Sinh[10*c + 9*d*
x]))/(1280*d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)
```

Maple [A] time = 0.046, size = 182, normalized size = 1.

$$\frac{1}{d}\left(a^3\left(\left(\frac{(\sinh(dx+c))^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+3a^2b\left(1/2\,\frac{(\sinh(dx+c))^3}{\cosh(dx+c)}-3/2\,dx-3/2\,c+3/\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^4,x)
```

```
[Out] 1/d*(a^3*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+3*
a^2*b*(1/2*sinh(d*x+c)^3/cosh(d*x+c)-3/2*d*x-3/2*c+3/2*tanh(d*x+c))+3*a*b^2
*(d*x+c-tanh(d*x+c)-1/3*tanh(d*x+c)^3)+b^3*(-1/2*sinh(d*x+c)^3/cosh(d*x+c)^
5-3/8*sinh(d*x+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c
)^2)*tanh(d*x+c)))
```

Maxima [B] time = 1.07936, size = 570, normalized size = 3.13

$$\frac{1}{64}\,a^3\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)+ab^2\left(3\,x+\frac{3\,c}{d}-\frac{4\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+2\right.}{d\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/64*a^3*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + a*b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3
*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*
d*x - 6*c) + 1))) - 3/8*a^2*b*(12*(d*x + c)/d + e^(-2*d*x - 2*c)/d - (17*e^
(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c)))) + 2/5*b^3*(1
0*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*
d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 5*e^(-8*d*x -
8*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5
*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 1
0*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x
- 10*c) + 1)))
```

Fricas [B] time = 2.64733, size = 1858, normalized size = 10.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/320*(5*a^3*sinh(d*x + c)^9 + 15*(12*a^3*cosh(d*x + c)^2 - a^3 + 8*a^2*b)*
sinh(d*x + c)^7 - 8*(120*a^2*b - 160*a*b^2 + 8*b^3 - 15*(a^3 - 12*a^2*b + 8
*a*b^2)*d*x)*cosh(d*x + c)^5 - 40*(120*a^2*b - 160*a*b^2 + 8*b^3 - 15*(a^3
- 12*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)*sinh(d*x + c)^4 + (630*a^3*cosh(d*
x + c)^4 - 150*a^3 + 1560*a^2*b - 1280*a*b^2 + 64*b^3 - 315*(a^3 - 8*a^2*b)
*cosh(d*x + c)^2)*sinh(d*x + c)^5 - 40*(120*a^2*b - 160*a*b^2 + 8*b^3 - 15*
(a^3 - 12*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^3 + 5*(84*a^3*cosh(d*x + c)^6
 - 105*(a^3 - 8*a^2*b)*cosh(d*x + c)^4 - 62*a^3 + 792*a^2*b - 512*a*b^2 - 6
4*b^3 - 4*(75*a^3 - 780*a^2*b + 640*a*b^2 - 32*b^3)*cosh(d*x + c)^2)*sinh(d
*x + c)^3 - 40*(2*(120*a^2*b - 160*a*b^2 + 8*b^3 - 15*(a^3 - 12*a^2*b + 8*a
*b^2)*d*x)*cosh(d*x + c)^3 + 3*(120*a^2*b - 160*a*b^2 + 8*b^3 - 15*(a^3 - 1
2*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 - 80*(120*a^2*b - 16
0*a*b^2 + 8*b^3 - 15*(a^3 - 12*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c) + 5*(9*a
^3*cosh(d*x + c)^8 - 21*(a^3 - 8*a^2*b)*cosh(d*x + c)^6 - 2*(75*a^3 - 780*a
^2*b + 640*a*b^2 - 32*b^3)*cosh(d*x + c)^4 - 36*a^3 + 504*a^2*b - 256*a*b^2
 + 128*b^3 - 6*(31*a^3 - 396*a^2*b + 256*a*b^2 + 32*b^3)*cosh(d*x + c)^2)*s
inh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*
cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)
^2 + 10*d*cosh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**3*sinh(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18007, size = 474, normalized size = 2.6

$$\frac{3\left(a^3-12\,a^2b+8\,ab^2\right)(dx+c)}{8\,d}-\frac{\left(18\,a^3e^{(4\,dx+4\,c)}-216\,a^2be^{(4\,dx+4\,c)}+144\,ab^2e^{(4\,dx+4\,c)}-8\,a^3e^{(2\,dx+2\,c)}+24\,a^2be^{(2\,dx+2\,c)}\right.}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 3/8*(a^3 - 12*a^2*b + 8*a*b^2)*(d*x + c)/d - 1/64*(18*a^3*e^(4*d*x + 4*c) -
 216*a^2*b*e^(4*d*x + 4*c) + 144*a*b^2*e^(4*d*x + 4*c) - 8*a^3*e^(2*d*x + 2
*c) + 24*a^2*b*e^(2*d*x + 2*c) + a^3)*e^(-4*d*x - 4*c)/d + 1/64*(a^3*d*e^(4
*d*x + 4*c) - 8*a^3*d*e^(2*d*x + 2*c) + 24*a^2*b*d*e^(2*d*x + 2*c))/d^2 - 2
/5*(15*a^2*b*e^(8*d*x + 8*c) - 30*a*b^2*e^(8*d*x + 8*c) + 5*b^3*e^(8*d*x +
8*c) + 60*a^2*b*e^(6*d*x + 6*c) - 90*a*b^2*e^(6*d*x + 6*c) + 90*a^2*b*e^(4*
d*x + 4*c) - 110*a*b^2*e^(4*d*x + 4*c) + 10*b^3*e^(4*d*x + 4*c) + 60*a^2*b*
e^(2*d*x + 2*c) - 70*a*b^2*e^(2*d*x + 2*c) + 15*a^2*b - 20*a*b^2 + b^3)/(d*
(e^(2*d*x + 2*c) + 1)^5)
```

3.18 $\int \left(a+b\,\mathrm{sech}^2(c+dx)\right)^3 \sinh^3(c+dx)\,dx$

Optimal. Leaf size=99

$$-\frac{a^2(a-3b)\cosh(c+dx)}{d}+\frac{a^3\cosh^3(c+dx)}{3d}+\frac{b^2(3a-b)\mathrm{sech}^3(c+dx)}{3d}+\frac{3ab(a-b)\mathrm{sech}(c+dx)}{d}+\frac{b^3\mathrm{sech}^5(c+dx)}{5d}$$

```
[Out] -((a^2*(a - 3*b)*Cosh[c + d*x])/d) + (a^3*Cosh[c + d*x]^3)/(3*d) + (3*a*(a
- b)*b*Sech[c + d*x])/d + ((3*a - b)*b^2*Sech[c + d*x]^3)/(3*d) + (b^3*Sech
[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.110706, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4133, 448}

$$-\frac{a^2(a-3b)\cosh(c+dx)}{d}+\frac{a^3\cosh^3(c+dx)}{3d}+\frac{b^2(3a-b)\mathrm{sech}^3(c+dx)}{3d}+\frac{3ab(a-b)\mathrm{sech}(c+dx)}{d}+\frac{b^3\mathrm{sech}^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x]^3,x]
```

```
[Out] -((a^2*(a - 3*b)*Cosh[c + d*x])/d) + (a^3*Cosh[c + d*x]^3)/(3*d) + (3*a*(a
- b)*b*Sech[c + d*x])/d + ((3*a - b)*b^2*Sech[c + d*x]^3)/(3*d) + (b^3*Sech
[c + d*x]^5)/(5*d)
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\,\mathrm{sech}^2(c+dx)\right)^3\sinh^3(c+dx)\,dx &= -\frac{\mathrm{Subst}\left(\int \frac{(1-x^2)(b+ax^2)^3}{x^6}\,dx,x,\cosh(c+dx)\right)}{d}\\ &= -\frac{\mathrm{Subst}\left(\int \left(a^2(a-3b)+\frac{b^3}{x^6}+\frac{(3a-b)b^2}{x^4}+\frac{3a(a-b)b}{x^2}-a^3x^2\right)dx,x,\cosh(c+dx)\right)}{d}\\ &= -\frac{a^2(a-3b)\cosh(c+dx)}{d}+\frac{a^3\cosh^3(c+dx)}{3d}+\frac{3a(a-b)b\,\mathrm{sech}(c+dx)}{d}+\end{aligned}$$

Mathematica [A] time = 1.21036, size = 119, normalized size = 1.2

$$\frac{4\mathrm{sech}^5(c+dx)\left(a\cosh^2(c+dx)+b\right)^3\left(5a^2\cosh^6(c+dx)(a\cosh(2(c+dx))-5a+18b)+10b^2(3a-b)\cosh^2(c+dx)+\right.}{15d(a\cosh(2(c+dx))+a+2b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x]^3,x]
```

```
[Out] (4*(b + a*Cosh[c + d*x]^2)^3*(6*b^3 + 10*(3*a - b)*b^2*Cosh[c + d*x]^2 + 90
*a*(a - b)*b*Cosh[c + d*x]^4 + 5*a^2*Cosh[c + d*x]^6*(-5*a + 18*b + a*Cosh[
2*(c + d*x)]))*Sech[c + d*x]^5)/(15*d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)
```

Maple [A] time = 0.043, size = 179, normalized size = 1.8

$$\frac{1}{d}\left(a^3\left(-\frac{2}{3}+\frac{(\sinh(dx+c))^2}{3}\right)\cosh(dx+c)+3a^2b\left(-\frac{(\sinh(dx+c))^2}{\cosh(dx+c)}+2\cosh(dx+c)\right)+3ab^2\left(-1/3\frac{(\sinh(dx+c}{(\cosh(dx+c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^3,x)
```

```
[Out] 1/d*(a^3*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+3*a^2*b*(-sinh(d*x+c)^2/cosh(
d*x+c)+2*cosh(d*x+c))+3*a*b^2*(-1/3*sinh(d*x+c)^2/cosh(d*x+c)^3+2/3*sinh(d*
x+c)^2/cosh(d*x+c)-2/3*cosh(d*x+c))+b^3*(-1/5*sinh(d*x+c)^2/cosh(d*x+c)^5+2
/15*sinh(d*x+c)^2/cosh(d*x+c)^3+2/15*sinh(d*x+c)^2/cosh(d*x+c)-2/15*cosh(d*
x+c)))
```

Maxima [B] time = 1.07732, size = 660, normalized size = 6.67

$$\frac{1}{24}a^3\left(\frac{e^{(3dx+3c)}}{d}-\frac{9e^{(dx+c)}}{d}-\frac{9e^{(-dx-c)}}{d}+\frac{e^{(-3dx-3c)}}{d}\right)+\frac{3}{2}a^2b\left(\frac{e^{(-dx-c)}}{d}+\frac{5e^{(-2dx-2c)}+1}{d\left(e^{(-dx-c)}+e^{(-3dx-3c)}\right)}\right)-2ab^2\left(\frac{}{d\left(3e^{(-2dx-2c}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/24*a^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*
x - 3*c)/d) + 3/2*a^2*b*(e^(-d*x - c)/d + (5*e^(-2*d*x - 2*c) + 1)/(d*(e^(-
d*x - c) + e^(-3*d*x - 3*c)))) - 2*a*b^2*(3*e^(-d*x - c)/(d*(3*e^(-2*d*x -
2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 2*e^(-3*d*x - 3*c)/(d*
(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 3*e^(-5
*d*x - 5*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c)
+ 1))) - 8/15*b^3*(5*e^(-3*d*x - 3*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x
 - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1
)) - 2*e^(-5*d*x - 5*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e
^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 5*e^(-7*d
*x - 7*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c
) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)))
```

Fricas [B] time = 2.65544, size = 1029, normalized size = 10.39

$$\frac{5a^3\cosh(dx+c)^8+5a^3\sinh(dx+c)^8-20\left(a^3-9a^2b\right)\cosh(dx+c)^6+20\left(7a^3\cosh(dx+c)^2-a^3+9a^2b\right)\sinh(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/120*(5*a^3*cosh(d*x + c)^8 + 5*a^3*sinh(d*x + c)^8 - 20*(a^3 - 9*a^2*b)*c
osh(d*x + c)^6 + 20*(7*a^3*cosh(d*x + c)^2 - a^3 + 9*a^2*b)*sinh(d*x + c)^6
 - 20*(11*a^3 - 90*a^2*b + 36*a*b^2)*cosh(d*x + c)^4 + 10*(35*a^3*cosh(d*x
+ c)^4 - 22*a^3 + 180*a^2*b - 72*a*b^2 - 30*(a^3 - 9*a^2*b)*cosh(d*x + c)^2
)*sinh(d*x + c)^4 - 425*a^3 + 3960*a^2*b - 1200*a*b^2 + 64*b^3 - 20*(31*a^3
 - 279*a^2*b + 96*a*b^2 + 16*b^3)*cosh(d*x + c)^2 + 20*(7*a^3*cosh(d*x + c)
^6 - 15*(a^3 - 9*a^2*b)*cosh(d*x + c)^4 - 31*a^3 + 279*a^2*b - 96*a*b^2 - 1
6*b^3 - 6*(11*a^3 - 90*a^2*b + 36*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2)/
(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^
3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh
(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**3*sinh(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20028, size = 277, normalized size = 2.8

$$\frac{a^3d^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-12\,a^3d^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)+36\,a^2bd^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{24\,d^3}+\frac{2\left(45\,a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^4-45\,ab^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/24*(a^3*d^2*(e^(d*x + c) + e^(-d*x - c))^3 - 12*a^3*d^2*(e^(d*x + c) + e^
(-d*x - c)) + 36*a^2*b*d^2*(e^(d*x + c) + e^(-d*x - c)))/d^3 + 2/15*(45*a^2
*b*(e^(d*x + c) + e^(-d*x - c))^4 - 45*a*b^2*(e^(d*x + c) + e^(-d*x - c))^4
 + 60*a*b^2*(e^(d*x + c) + e^(-d*x - c))^2 - 20*b^3*(e^(d*x + c) + e^(-d*x
- c))^2 + 48*b^3)/(d*(e^(d*x + c) + e^(-d*x - c))^5)
```

3.19 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)^3 \sinh^2(c+dx)\,dx$

Optimal. Leaf size=112

$$-\frac{3a^2b\tanh(c+dx)}{d}-\frac{1}{2}a^2x(a-6b)+\frac{a^3}{4d(1-\tanh(c+dx))}-\frac{a^3}{4d(\tanh(c+dx)+1)}+\frac{b^2(3a+b)\tanh^3(c+dx)}{3d}-\frac{b^3}{}$$

```
[Out] -(a^2*(a - 6*b)*x)/2 + a^3/(4*d*(1 - Tanh[c + d*x])) - (3*a^2*b*Tanh[c + d*
x])/d + (b^2*(3*a + b)*Tanh[c + d*x]^3)/(3*d) - (b^3*Tanh[c + d*x]^5)/(5*d)
 - a^3/(4*d*(1 + Tanh[c + d*x]))
```

Rubi [A] time = 0.192742, antiderivative size = 143, normalized size of antiderivative = 1.28, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4132, 467, 528, 388, 206}

$$-\frac{b\left(81a^2-28ab-4b^2\right)\tanh(c+dx)}{30d}-\frac{1}{2}a^2x(a-6b)-\frac{7b\tanh(c+dx)\left(a-b\tanh^2(c+dx)+b\right)^2}{10d}-\frac{b(33a-2b)\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x]^2,x]
```

```
[Out] -(a^2*(a - 6*b)*x)/2 - (b*(81*a^2 - 28*a*b - 4*b^2)*Tanh[c + d*x])/(30*d) -
 ((33*a - 2*b)*b*Tanh[c + d*x]*(a + b - b*Tanh[c + d*x]^2))/(30*d) - (7*b*T
anh[c + d*x]*(a + b - b*Tanh[c + d*x]^2)^2)/(10*d) + (Cosh[c + d*x]*Sinh[c
+ d*x]*(a + b - b*Tanh[c + d*x]^2)^3)/(2*d)
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^3 \sinh^2(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+b-bx^2\right)^3}{\left(1-x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)\left(a+b-b\tanh^2(c+dx)\right)^3}{2d} - \frac{\text{Subst}\left(\int \frac{\left(a+b-7bx^2\right)(}{1-}\right.}{}$$

$$= -\frac{7b\tanh(c+dx)\left(a+b-b\tanh^2(c+dx)\right)^2}{10d} + \frac{\cosh(c+dx)\sinh(c+dx)\left(\right.}{2d}$$

$$= -\frac{(33a-2b)b\tanh(c+dx)\left(a+b-b\tanh^2(c+dx)\right)}{30d} - \frac{7b\tanh(c+dx)\left(a+\right.}{}$$

$$= -\frac{b\left(81a^2-28ab-4b^2\right)\tanh(c+dx)}{30d} - \frac{(33a-2b)b\tanh(c+dx)\left(a+b-b\right.}{30d}$$

$$= -\frac{1}{2}a^2(a-6b)x - \frac{b\left(81a^2-28ab-4b^2\right)\tanh(c+dx)}{30d} - \frac{(33a-2b)b\tanh(c}{}$$

Mathematica [B] time = 1.81971, size = 480, normalized size = 4.29

$$\frac{\operatorname{sech}(c)\operatorname{sech}^5(c+dx)\left(2880a^2b\sinh(2c+dx) - 2880a^2b\sinh(2c+3dx) + 720a^2b\sinh(4c+3dx) - 720a^2b\sinh(4c+5d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x]^2,x]
```

```
[Out] (Sech[c]*Sech[c + d*x]^5*(-600*a^2*(a - 6*b)*d*x*Cosh[d*x] - 600*a^2*(a - 6
*b)*d*x*Cosh[2*c + d*x] - 300*a^3*d*x*Cosh[2*c + 3*d*x] + 1800*a^2*b*d*x*Co
sh[2*c + 3*d*x] - 300*a^3*d*x*Cosh[4*c + 3*d*x] + 1800*a^2*b*d*x*Cosh[4*c +
 3*d*x] - 60*a^3*d*x*Cosh[4*c + 5*d*x] + 360*a^2*b*d*x*Cosh[4*c + 5*d*x] -
60*a^3*d*x*Cosh[6*c + 5*d*x] + 360*a^2*b*d*x*Cosh[6*c + 5*d*x] + 75*a^3*Sin
h[d*x] - 4320*a^2*b*Sinh[d*x] + 960*a*b^2*Sinh[d*x] - 160*b^3*Sinh[d*x] + 7
5*a^3*Sinh[2*c + d*x] + 2880*a^2*b*Sinh[2*c + d*x] - 1440*a*b^2*Sinh[2*c +
d*x] - 480*b^3*Sinh[2*c + d*x] + 135*a^3*Sinh[2*c + 3*d*x] - 2880*a^2*b*Sin
h[2*c + 3*d*x] + 480*a*b^2*Sinh[2*c + 3*d*x] + 160*b^3*Sinh[2*c + 3*d*x] +
135*a^3*Sinh[4*c + 3*d*x] + 720*a^2*b*Sinh[4*c + 3*d*x] - 720*a*b^2*Sinh[4*
c + 3*d*x] + 75*a^3*Sinh[4*c + 5*d*x] - 720*a^2*b*Sinh[4*c + 5*d*x] + 240*a
*b^2*Sinh[4*c + 5*d*x] + 32*b^3*Sinh[4*c + 5*d*x] + 75*a^3*Sinh[6*c + 5*d*x
] + 15*a^3*Sinh[6*c + 7*d*x] + 15*a^3*Sinh[8*c + 7*d*x]))/(3840*d)
```

Maple [A] time = 0.039, size = 145, normalized size = 1.3

$$\frac{1}{d}\left(a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3a^2b(dx+c-\tanh(dx+c))+3ab^2\left(-1/2\frac{\sinh(dx+c)}{(\cosh(dx+c))^3}+1/2\left(2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^2,x)
```

```
[Out] 1/d*(a^3*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+3*a^2*b*(d*x+c-tanh(d*
x+c))+3*a*b^2*(-1/2*sinh(d*x+c)/cosh(d*x+c)^3+1/2*(2/3+1/3*sech(d*x+c)^2)*t
anh(d*x+c))+b^3*(-1/4*sinh(d*x+c)/cosh(d*x+c)^5+1/4*(8/15+1/5*sech(d*x+c)^4
+4/15*sech(d*x+c)^2)*tanh(d*x+c)))
```

Maxima [B] time = 1.05875, size = 598, normalized size = 5.34

$$-\frac{1}{8}a^3\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)+3a^2b\left(x+\frac{c}{d}-\frac{2}{d\left(e^{(-2dx-2c)}+1\right)}\right)+\frac{4}{15}b^3\left(\frac{5e^{(-2}}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/8*a^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 3*a^2*b*(x + c/d
- 2/(d*(e^(-2*d*x - 2*c) + 1))) + 4/15*b^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*
d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c)
 + e^(-10*d*x - 10*c) + 1)) - 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 1
0*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x
- 10*c) + 1)) + 15*e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x -
 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))
 + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5
*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 2*a*b^2*(3*e^(-4*d*x - 4*c)
/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(
d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

Fricas [B] time = 2.55223, size = 1508, normalized size = 13.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/120*(15*a^3*sinh(d*x + c)^7 + 4*(90*a^2*b - 30*a*b^2 - 4*b^3 - 15*(a^3 -
6*a^2*b)*d*x)*cosh(d*x + c)^5 + 20*(90*a^2*b - 30*a*b^2 - 4*b^3 - 15*(a^3 -
 6*a^2*b)*d*x)*cosh(d*x + c)*sinh(d*x + c)^4 + (315*a^3*cosh(d*x + c)^2 + 7
5*a^3 - 360*a^2*b + 120*a*b^2 + 16*b^3)*sinh(d*x + c)^5 + 20*(90*a^2*b - 30
*a*b^2 - 4*b^3 - 15*(a^3 - 6*a^2*b)*d*x)*cosh(d*x + c)^3 + 5*(105*a^3*cosh(
d*x + c)^4 + 27*a^3 - 216*a^2*b - 24*a*b^2 + 16*b^3 + 2*(75*a^3 - 360*a^2*b
 + 120*a*b^2 + 16*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 20*(2*(90*a^2*b -
 30*a*b^2 - 4*b^3 - 15*(a^3 - 6*a^2*b)*d*x)*cosh(d*x + c)^3 + 3*(90*a^2*b -
 30*a*b^2 - 4*b^3 - 15*(a^3 - 6*a^2*b)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2
+ 40*(90*a^2*b - 30*a*b^2 - 4*b^3 - 15*(a^3 - 6*a^2*b)*d*x)*cosh(d*x + c) +
```

```
 5*(21*a^3*cosh(d*x + c)^6 + (75*a^3 - 360*a^2*b + 120*a*b^2 + 16*b^3)*cosh
(d*x + c)^4 + 15*a^3 - 144*a^2*b - 48*a*b^2 - 64*b^3 + 3*(27*a^3 - 216*a^2*
b - 24*a*b^2 + 16*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^5 +
 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x
+ c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**3*sinh(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19006, size = 385, normalized size = 3.44

$$\frac{a^3e^{(2\,dx+2\,c)}}{8\,d} - \frac{\left(a^3-6\,a^2b\right)(dx+c)}{2\,d} + \frac{\left(2\,a^3e^{(2\,dx+2\,c)}-12\,a^2be^{(2\,dx+2\,c)}-a^3\right)e^{(-2\,dx-2\,c)}}{8\,d} + \frac{2\left(45\,a^2be^{(8\,dx+8\,c)}-45\,ab^2e^{(8\,dx+8\,c)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/8*a^3*e^(2*d*x + 2*c)/d - 1/2*(a^3 - 6*a^2*b)*(d*x + c)/d + 1/8*(2*a^3*e^
(2*d*x + 2*c) - 12*a^2*b*e^(2*d*x + 2*c) - a^3)*e^(-2*d*x - 2*c)/d + 2/15*(
45*a^2*b*e^(8*d*x + 8*c) - 45*a*b^2*e^(8*d*x + 8*c) + 180*a^2*b*e^(6*d*x +
6*c) - 90*a*b^2*e^(6*d*x + 6*c) - 30*b^3*e^(6*d*x + 6*c) + 270*a^2*b*e^(4*d
*x + 4*c) - 60*a*b^2*e^(4*d*x + 4*c) + 10*b^3*e^(4*d*x + 4*c) + 180*a^2*b*e
^(2*d*x + 2*c) - 30*a*b^2*e^(2*d*x + 2*c) - 10*b^3*e^(2*d*x + 2*c) + 45*a^2
*b - 15*a*b^2 - 2*b^3)/(d*(e^(2*d*x + 2*c) + 1)^5)
```

3.20 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)^3 \sinh(c+dx)\,dx$

Optimal. Leaf size=64

$$-\frac{3a^2b\,\text{sech}(c+dx)}{d}+\frac{a^3\cosh(c+dx)}{d}-\frac{ab^2\,\text{sech}^3(c+dx)}{d}-\frac{b^3\,\text{sech}^5(c+dx)}{5d}$$

```
[Out] (a^3*Cosh[c + d*x])/d - (3*a^2*b*Sech[c + d*x])/d - (a*b^2*Sech[c + d*x]^3)
/d - (b^3*Sech[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0592691, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4133, 270}

$$-\frac{3a^2b\,\text{sech}(c+dx)}{d}+\frac{a^3\cosh(c+dx)}{d}-\frac{ab^2\,\text{sech}^3(c+dx)}{d}-\frac{b^3\,\text{sech}^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x],x]
```

```
[Out] (a^3*Cosh[c + d*x])/d - (3*a^2*b*Sech[c + d*x])/d - (a*b^2*Sech[c + d*x]^3)
/d - (b^3*Sech[c + d*x]^5)/(5*d)
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\,\text{sech}^2(c+dx)\right)^3\sinh(c+dx)\,dx&=\frac{\text{Subst}\left(\int\frac{\left(b+ax^2\right)^3}{x^6}\,dx,x,\cosh(c+dx)\right)}{d}\\&=\frac{\text{Subst}\left(\int\left(a^3+\frac{b^3}{x^6}+\frac{3ab^2}{x^4}+\frac{3a^2b}{x^2}\right)dx,x,\cosh(c+dx)\right)}{d}\\&=\frac{a^3\cosh(c+dx)}{d}-\frac{3a^2b\,\text{sech}(c+dx)}{d}-\frac{ab^2\,\text{sech}^3(c+dx)}{d}-\frac{b^3\,\text{sech}^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.247026, size = 93, normalized size = 1.45

$$\frac{8\,\text{sech}^5(c+dx)\left(a\cosh^2(c+dx)+b\right)^3\left(-15a^2b\cosh^4(c+dx)+5a^3\cosh^6(c+dx)-5ab^2\cosh^2(c+dx)-b^3\right)}{5d(a\cosh(2(c+dx))+a+2b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x],x]
```

```
[Out] (8*(b + a*Cosh[c + d*x]^2)^3*(-b^3 - 5*a*b^2*Cosh[c + d*x]^2 - 15*a^2*b*Cos
h[c + d*x]^4 + 5*a^3*Cosh[c + d*x]^6)*Sech[c + d*x]^5)/(5*d*(a + 2*b + a*Co
sh[2*(c + d*x)])^3)
```

Maple [A] time = 0.019, size = 58, normalized size = 0.9

$$-\frac{1}{d}\left(\frac{b^3\left(\operatorname{sech}(dx+c)\right)^5}{5}+ab^2\left(\operatorname{sech}(dx+c)\right)^3+3a^2b\operatorname{sech}(dx+c)-\frac{a^3}{\operatorname{sech}(dx+c)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^3*sinh(d*x+c),x)
```

```
[Out] -1/d*(1/5*b^3*sech(d*x+c)^5+a*b^2*sech(d*x+c)^3+3*a^2*b*sech(d*x+c)-a^3/sec
h(d*x+c))
```

Maxima [A] time = 1.05862, size = 127, normalized size = 1.98

$$\frac{a^3\cosh(dx+c)}{d}-\frac{6a^2b}{d\left(e^{(dx+c)}+e^{(-dx-c)}\right)}-\frac{8ab^2}{d\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3}-\frac{32b^3}{5d\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c),x, algorithm="maxima")
```

```
[Out] a^3*cosh(d*x + c)/d - 6*a^2*b/(d*(e^(d*x + c) + e^(-d*x - c))) - 8*a*b^2/(d
*(e^(d*x + c) + e^(-d*x - c))^3) - 32/5*b^3/(d*(e^(d*x + c) + e^(-d*x - c))
^5)
```

Fricas [B] time = 2.52159, size = 702, normalized size = 10.97

$$\frac{5a^3\cosh(dx+c)^6+5a^3\sinh(dx+c)^6+30\left(a^3-2a^2b\right)\cosh(dx+c)^4+15\left(5a^3\cosh(dx+c)^2+2a^3-4a^2b\right)\sinh(dx}{10\left(d\cosh(dx+c)^5+5d\cosh(dx+c)\sinh(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/10*(5*a^3*cosh(d*x + c)^6 + 5*a^3*sinh(d*x + c)^6 + 30*(a^3 - 2*a^2*b)*co
sh(d*x + c)^4 + 15*(5*a^3*cosh(d*x + c)^2 + 2*a^3 - 4*a^2*b)*sinh(d*x + c)^
4 + 50*a^3 - 180*a^2*b - 80*a*b^2 - 32*b^3 + 5*(15*a^3 - 48*a^2*b - 16*a*b^
2)*cosh(d*x + c)^2 + 5*(15*a^3*cosh(d*x + c)^4 + 15*a^3 - 48*a^2*b - 16*a*b
^2 + 36*(a^3 - 2*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^2)/(d*cosh(d*x + c)^
5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d
*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**3*sinh(d*x+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19049, size = 138, normalized size = 2.16

$$\frac{a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{2\,d}-\frac{2\left(15\,a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^4+20\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2+16\,b^3\right)}{5\,d\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*a^3*(e^(d*x + c) + e^(-d*x - c))/d - 2/5*(15*a^2*b*(e^(d*x + c) + e^(-d
*x - c))^4 + 20*a*b^2*(e^(d*x + c) + e^(-d*x - c))^2 + 16*b^3)/(d*(e^(d*x +
 c) + e^(-d*x - c))^5)
```

3.21 $\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=83

$$\frac{b\left(3a^2+3ab+b^2\right)\operatorname{sech}(c+dx)}{d}+\frac{b^2(3a+b)\operatorname{sech}^3(c+dx)}{3d}-\frac{(a+b)^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b^3\operatorname{sech}^5(c+dx)}{5d}$$

```
[Out] -(((a + b)^3*ArcTanh[Cosh[c + d*x]])/d) + (b*(3*a^2 + 3*a*b + b^2)*Sech[c +
 d*x])/d + (b^2*(3*a + b)*Sech[c + d*x]^3)/(3*d) + (b^3*Sech[c + d*x]^5)/(5
*d)
```

Rubi [A] time = 0.10033, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4133, 461, 207}

$$\frac{b\left(3a^2+3ab+b^2\right)\operatorname{sech}(c+dx)}{d}+\frac{b^2(3a+b)\operatorname{sech}^3(c+dx)}{3d}-\frac{(a+b)^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b^3\operatorname{sech}^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -(((a + b)^3*ArcTanh[Cosh[c + d*x]])/d) + (b*(3*a^2 + 3*a*b + b^2)*Sech[c +
 d*x])/d + (b^2*(3*a + b)*Sech[c + d*x]^3)/(3*d) + (b^3*Sech[c + d*x]^5)/(5
*d)
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \mathrm{csch}(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3\,dx = -\frac{\mathrm{Subst}\left(\int \frac{(b+ax^2)^3}{x^6(1-x^2)}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\mathrm{Subst}\left(\int \left(\frac{b^3}{x^6}+\frac{b^2(3a+b)}{x^4}+\frac{b(3a^2+3ab+b^2)}{x^2}-\frac{(a+b)^3}{-1+x^2}\right)dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{b\left(3a^2+3ab+b^2\right)\mathrm{sech}(c+dx)}{d}+\frac{b^2(3a+b)\mathrm{sech}^3(c+dx)}{3d}+\frac{b^3\mathrm{sech}^5(c$$
$$= -\frac{(a+b)^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b\left(3a^2+3ab+b^2\right)\mathrm{sech}(c+dx)}{d}+\frac{b^2(}{}$$

Mathematica [A] time = 1.21971, size = 134, normalized size = 1.61

$$-\frac{8\mathrm{sech}^5(c+dx)\left(a\cosh^2(c+dx)+b\right)^3\left(-15b\left(3a^2+3ab+b^2\right)\cosh^4(c+dx)-5b^2(3a+b)\cosh^2(c+dx)+15(a+b\right.}{15d(a\cosh(2(c+dx))+a+2b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (-8*(b + a*Cosh[c + d*x]^2)^3*(-3*b^3 - 5*b^2*(3*a + b)*Cosh[c + d*x]^2 - 1
5*b*(3*a^2 + 3*a*b + b^2)*Cosh[c + d*x]^4 + 15*(a + b)^3*Cosh[c + d*x]^5*(L
og[Cosh[(c + d*x)/2]] - Log[Sinh[(c + d*x)/2]]))*Sech[c + d*x]^5)/(15*d*(a
+ 2*b + a*Cosh[2*(c + d*x)])^3)
```

Maple [A] time = 0.04, size = 118, normalized size = 1.4

$$\frac{1}{d}\left(-2\,a^3\mathrm{Artanh}\left(e^{dx+c}\right)+3\,a^2b\left((\cosh(dx+c))^{-1}-2\,\mathrm{Artanh}\left(e^{dx+c}\right)\right)+3\,ab^2\left(1/3\,(\cosh(dx+c))^{-3}+(\cosh(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(-2*a^3*arctanh(exp(d*x+c))+3*a^2*b*(1/cosh(d*x+c)-2*arctanh(exp(d*x+c)
))+3*a*b^2*(1/3/cosh(d*x+c)^3+1/cosh(d*x+c)-2*arctanh(exp(d*x+c)))+b^3*(1/5
/cosh(d*x+c)^5+1/3/cosh(d*x+c)^3+1/cosh(d*x+c)-2*arctanh(exp(d*x+c))))
```

Maxima [B] time = 1.10028, size = 483, normalized size = 5.82

$$-\frac{1}{15}b^3\left(\frac{15\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{15\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2\left(15\,e^{(-dx-c)}+80\,e^{(-3\,dx-3\,c)}+178\,e^{(-5\,dx-5\,c)}+80\,e^{(-7\,dx-7\,c)}+1\right.}{d\left(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-1}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/15*b^3*(15*log(e^(-d*x - c) + 1)/d - 15*log(e^(-d*x - c) - 1)/d - 2*(15*
e^(-d*x - c) + 80*e^(-3*d*x - 3*c) + 178*e^(-5*d*x - 5*c) + 80*e^(-7*d*x -
```

```
7*c) + 15*e^(-9*d*x - 9*c))/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) +
10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) - a*b^
2*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d - 2*(3*e^(-d*x - c
) + 10*e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c))/(d*(3*e^(-2*d*x - 2*c) + 3*e^
(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) - 3*a^2*b*(log(e^(-d*x - c) + 1)/d
 - log(e^(-d*x - c) - 1)/d - 2*e^(-d*x - c)/(d*(e^(-2*d*x - 2*c) + 1))) + a
^3*log(tanh(1/2*d*x + 1/2*c))/d
```

Fricas [B] time = 3.09183, size = 8645, normalized size = 104.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(30*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 + 270*(3*a^2*b + 3*a*b^2
 + b^3)*cosh(d*x + c)*sinh(d*x + c)^8 + 30*(3*a^2*b + 3*a*b^2 + b^3)*sinh(d
*x + c)^9 + 40*(9*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^7 + 40*(9*a^2*b +
 12*a*b^2 + 4*b^3 + 27*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x
+ c)^7 + 280*(9*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (9*a^2*b + 12*a
*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 4*(135*a^2*b + 195*a*b^2 + 8
9*b^3)*cosh(d*x + c)^5 + 4*(945*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 +
 135*a^2*b + 195*a*b^2 + 89*b^3 + 210*(9*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x
 + c)^2)*sinh(d*x + c)^5 + 20*(189*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^
5 + 70*(9*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^3 + (135*a^2*b + 195*a*b^
2 + 89*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 40*(9*a^2*b + 12*a*b^2 + 4*b^3
)*cosh(d*x + c)^3 + 40*(63*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 35*(
9*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^4 + 9*a^2*b + 12*a*b^2 + 4*b^3 +
(135*a^2*b + 195*a*b^2 + 89*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 40*(27*
(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 21*(9*a^2*b + 12*a*b^2 + 4*b^3)
*cosh(d*x + c)^5 + (135*a^2*b + 195*a*b^2 + 89*b^3)*cosh(d*x + c)^3 + 3*(9*
a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 30*(3*a^2*b + 3*
a*b^2 + b^3)*cosh(d*x + c) - 15*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x +
 c)^10 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^9 +
 (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^10 + 5*(a^3 + 3*a^2*b + 3*a*
b^2 + b^3)*cosh(d*x + c)^8 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 9*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 40*(3*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh
(d*x + c))*sinh(d*x + c)^7 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x +
c)^6 + 10*(21*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + a^3 + 3*a^2
*b + 3*a*b^2 + b^3 + 14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*si
nh(d*x + c)^6 + 4*(63*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 70*
(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + 15*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^
3)*cosh(d*x + c)^4 + 10*(21*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6
 + 35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a
*b^2 + b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x +
 c)^4 + 40*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 7*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*
cosh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x +
 c)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c
osh(d*x + c)^2 + 5*(9*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 28*
(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 30*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 12*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 10*((a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*cosh(d*x + c)^9 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*
```

```
x + c)^7 + 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 4*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos
h(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 15*((a^
3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^10 + 10*(a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sin
h(d*x + c)^10 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 5*(a^3
+ 3*a^2*b + 3*a*b^2 + b^3 + 9*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)
^2)*sinh(d*x + c)^8 + 40*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3
 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 10*(21*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 14*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 70*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh
(d*x + c)^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x +
c)^5 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 10*(21*(a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3
)*cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 15*(a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 40*(3*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*cosh(d*x + c)^7 + 7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)
^5 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^
3 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2 + 5*(9*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 28*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh
(d*x + c)^6 + 30*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + a^3 + 3*
a^2*b + 3*a*b^2 + b^3 + 12*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^2 + 10*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 + 4*
(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 6*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*cosh(d*x + c)^5 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^
3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(
d*x + c) + sinh(d*x + c) - 1) + 10*(27*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x +
 c)^8 + 28*(9*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^6 + 2*(135*a^2*b + 19
5*a*b^2 + 89*b^3)*cosh(d*x + c)^4 + 9*a^2*b + 9*a*b^2 + 3*b^3 + 12*(9*a^2*b
 + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^10 +
10*d*cosh(d*x + c)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 + 5*d*cosh(d*x + c)
^8 + 5*(9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 40*(3*d*cosh(d*x + c)^3
+ d*cosh(d*x + c))*sinh(d*x + c)^7 + 10*d*cosh(d*x + c)^6 + 10*(21*d*cosh(d
*x + c)^4 + 14*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 4*(63*d*cosh(d*x +
c)^5 + 70*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*sinh(d*x + c)^5 + 10*d*co
sh(d*x + c)^4 + 10*(21*d*cosh(d*x + c)^6 + 35*d*cosh(d*x + c)^4 + 15*d*cosh
(d*x + c)^2 + d)*sinh(d*x + c)^4 + 40*(3*d*cosh(d*x + c)^7 + 7*d*cosh(d*x +
 c)^5 + 5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^3 + 5*d*cosh(d
*x + c)^2 + 5*(9*d*cosh(d*x + c)^8 + 28*d*cosh(d*x + c)^6 + 30*d*cosh(d*x +
 c)^4 + 12*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 10*(d*cosh(d*x + c)^9 +
 4*d*cosh(d*x + c)^7 + 6*d*cosh(d*x + c)^5 + 4*d*cosh(d*x + c)^3 + d*cosh(d
*x + c))*sinh(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\operatorname{sech}^2(c + dx)\right)^3 \operatorname{csch}(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*csch(c + d*x), x)
```

Giac [B] time = 1.18934, size = 313, normalized size = 3.77

$$-\frac{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)}{2\,d}+\frac{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)}{2\,d}+\frac{2\left(45\,a^2b\left(e\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(e^(d*x + c) + e^(-d*x - c) + 2)/d
+ 1/2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(e^(d*x + c) + e^(-d*x - c) - 2)/d
 + 2/15*(45*a^2*b*(e^(d*x + c) + e^(-d*x - c))^4 + 45*a*b^2*(e^(d*x + c) +
e^(-d*x - c))^4 + 15*b^3*(e^(d*x + c) + e^(-d*x - c))^4 + 60*a*b^2*(e^(d*x
+ c) + e^(-d*x - c))^2 + 20*b^3*(e^(d*x + c) + e^(-d*x - c))^2 + 48*b^3)/(d
*(e^(d*x + c) + e^(-d*x - c))^5)
```

3.22 $\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=70

$$\frac{b^2(a+b)\tanh^3(c+dx)}{d}-\frac{3b(a+b)^2\tanh(c+dx)}{d}-\frac{(a+b)^3\coth(c+dx)}{d}-\frac{b^3\tanh^5(c+dx)}{5d}$$

```
[Out] -(((a + b)^3*Coth[c + d*x])/d) - (3*b*(a + b)^2*Tanh[c + d*x])/d + (b^2*(a
+ b)*Tanh[c + d*x]^3)/d - (b^3*Tanh[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0734432, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4132, 270}

$$\frac{b^2(a+b)\tanh^3(c+dx)}{d}-\frac{3b(a+b)^2\tanh(c+dx)}{d}-\frac{(a+b)^3\coth(c+dx)}{d}-\frac{b^3\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -(((a + b)^3*Coth[c + d*x])/d) - (3*b*(a + b)^2*Tanh[c + d*x])/d + (b^2*(a
+ b)*Tanh[c + d*x]^3)/d - (b^3*Tanh[c + d*x]^5)/(5*d)
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\left(a+b-bx^2\right)^3}{x^2}\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(-3b(a+b)^2+\frac{(a+b)^3}{x^2}+3b^2(a+b)x^2-b^3x^4\right)dx,x,\tanh(c+d\right.}{d}\\ &= -\frac{(a+b)^3\coth(c+dx)}{d}-\frac{3b(a+b)^2\tanh(c+dx)}{d}+\frac{b^2(a+b)\tanh^3(c+}{d}\end{aligned}$$

Mathematica [B] time = 2.73583, size = 380, normalized size = 5.43

$$-\frac{\operatorname{csch}(c)\operatorname{sech}(c)\coth(c+dx)\left(10a\left(5a^2+12ab+8b^2\right)\sinh(2c)-10\left(18a^2b+5a^3+20ab^2+8b^3\right)\sinh(2dx)-120a^2b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -(Coth[c + d*x]*Csch[c]*Sech[c]*(a + b*Sech[c + d*x]^2)^3*(10*a*(5*a^2 + 12
*a*b + 8*b^2)*Sinh[2*c] - 10*(5*a^3 + 18*a^2*b + 20*a*b^2 + 8*b^3)*Sinh[2*d
*x] - 25*a^3*Sinh[2*(c + d*x)] + 50*a*b^2*Sinh[2*(c + d*x)] + 30*b^3*Sinh[2
*(c + d*x)] - 20*a^3*Sinh[4*(c + d*x)] + 40*a*b^2*Sinh[4*(c + d*x)] + 24*b^
3*Sinh[4*(c + d*x)] - 5*a^3*Sinh[6*(c + d*x)] + 10*a*b^2*Sinh[6*(c + d*x)]
+ 6*b^3*Sinh[6*(c + d*x)] - 25*a^3*Sinh[2*(c + 2*d*x)] - 120*a^2*b*Sinh[2*(
c + 2*d*x)] - 160*a*b^2*Sinh[2*(c + 2*d*x)] - 64*b^3*Sinh[2*(c + 2*d*x)] +
25*a^3*Sinh[4*c + 2*d*x] + 30*a^2*b*Sinh[4*c + 2*d*x] + 5*a^3*Sinh[6*c + 4*
d*x] - 5*a^3*Sinh[4*c + 6*d*x] - 30*a^2*b*Sinh[4*c + 6*d*x] - 40*a*b^2*Sinh
[4*c + 6*d*x] - 16*b^3*Sinh[4*c + 6*d*x]))/(40*d*(a + 2*b + a*Cosh[2*(c + d
*x)])^3)
```

Maple [B] time = 0.043, size = 148, normalized size = 2.1

$$\frac{1}{d}\left(-a^3\coth(dx+c)+3a^2b\left(-\frac{1}{\cosh(dx+c)\sinh(dx+c)}-2\tanh(dx+c)\right)+3ab^2\left(-\frac{1}{\sinh(dx+c)(\cosh(dx+c))^3}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(-a^3*coth(d*x+c)+3*a^2*b*(-1/sinh(d*x+c)/cosh(d*x+c)-2*tanh(d*x+c))+3*
a*b^2*(-1/sinh(d*x+c)/cosh(d*x+c)^3-4*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c))+
b^3*(-1/sinh(d*x+c)/cosh(d*x+c)^5-6*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c
)^2)*tanh(d*x+c)))
```

Maxima [B] time = 1.05741, size = 483, normalized size = 6.9

$$-\frac{32}{5}b^3\left(\frac{4e^{(-2dx-2c)}}{d\left(4e^{(-2dx-2c)}+5e^{(-4dx-4c)}-5e^{(-8dx-8c)}-4e^{(-10dx-10c)}-e^{(-12dx-12c)}+1\right)}+\frac{5}{d\left(4e^{(-2dx-2c)}+5e^{(-4dx-4c)}-5e^{(-}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -32/5*b^3*(4*e^(-2*d*x - 2*c)/(d*(4*e^(-2*d*x - 2*c) + 5*e^(-4*d*x - 4*c) -
 5*e^(-8*d*x - 8*c) - 4*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) + 1)) + 5*e
^(-4*d*x - 4*c)/(d*(4*e^(-2*d*x - 2*c) + 5*e^(-4*d*x - 4*c) - 5*e^(-8*d*x -
 8*c) - 4*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) + 1)) + 1/(d*(4*e^(-2*d*x
 - 2*c) + 5*e^(-4*d*x - 4*c) - 5*e^(-8*d*x - 8*c) - 4*e^(-10*d*x - 10*c) -
e^(-12*d*x - 12*c) + 1))) - 16*a*b^2*(2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x -
2*c) - 2*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) + 1)) + 1/(d*(2*e^(-2*d*x - 2*
c) - 2*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) + 1))) + 2*a^3/(d*(e^(-2*d*x - 2
*c) - 1)) + 12*a^2*b/(d*(e^(-4*d*x - 4*c) - 1))
```

Fricas [B] time = 2.54886, size = 1580, normalized size = 22.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -4/5*((5*a^3 + 15*a^2*b + 20*a*b^2 + 8*b^3)*cosh(d*x + c)^5 + 5*(5*a^3 + 15
*a^2*b + 20*a*b^2 + 8*b^3)*cosh(d*x + c)*sinh(d*x + c)^4 - (15*a^2*b + 20*a
*b^2 + 8*b^3)*sinh(d*x + c)^5 + (25*a^3 + 75*a^2*b + 80*a*b^2 + 32*b^3)*cos
h(d*x + c)^3 - (45*a^2*b + 80*a*b^2 + 32*b^3 + 10*(15*a^2*b + 20*a*b^2 + 8*
b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (10*(5*a^3 + 15*a^2*b + 20*a*b^2 +
8*b^3)*cosh(d*x + c)^3 + 3*(25*a^3 + 75*a^2*b + 80*a*b^2 + 32*b^3)*cosh(d*x
 + c))*sinh(d*x + c)^2 + 10*(5*a^3 + 15*a^2*b + 14*a*b^2 + 4*b^3)*cosh(d*x
+ c) - (5*(15*a^2*b + 20*a*b^2 + 8*b^3)*cosh(d*x + c)^4 + 30*a^2*b + 60*a*b
^2 + 40*b^3 + 3*(45*a^2*b + 80*a*b^2 + 32*b^3)*cosh(d*x + c)^2)*sinh(d*x +
c))/(d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c
)^7 + 3*d*cosh(d*x + c)^5 + (21*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^5 +
5*(7*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^4 + d*cosh(d*x +
c)^3 + (35*d*cosh(d*x + c)^4 + 50*d*cosh(d*x + c)^2 + 9*d)*sinh(d*x + c)^3
+ 3*(7*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x
 + c)^2 - 5*d*cosh(d*x + c) + (7*d*cosh(d*x + c)^6 + 25*d*cosh(d*x + c)^4 +
 27*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\operatorname{sech}^2\left(c + dx\right)\right)^3 \operatorname{csch}^2\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*csch(c + d*x)**2, x)
```

Giac [B] time = 1.19468, size = 338, normalized size = 4.83

$$-\frac{2\left(a^3 + 3a^2b + 3ab^2 + b^3\right)}{d\left(e^{(2dx+2c)} - 1\right)} + \frac{2\left(15a^2be^{(8dx+8c)} + 15ab^2e^{(8dx+8c)} + 5b^3e^{(8dx+8c)} + 60a^2be^{(6dx+6c)} + 90ab^2e^{(6dx+6c)} + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)/(d*(e^(2*d*x + 2*c) - 1)) + 2/5*(15*a^2*
b*e^(8*d*x + 8*c) + 15*a*b^2*e^(8*d*x + 8*c) + 5*b^3*e^(8*d*x + 8*c) + 60*a
^2*b*e^(6*d*x + 6*c) + 90*a*b^2*e^(6*d*x + 6*c) + 30*b^3*e^(6*d*x + 6*c) +
90*a^2*b*e^(4*d*x + 4*c) + 160*a*b^2*e^(4*d*x + 4*c) + 80*b^3*e^(4*d*x + 4*
c) + 60*a^2*b*e^(2*d*x + 2*c) + 110*a*b^2*e^(2*d*x + 2*c) + 50*b^3*e^(2*d*x
 + 2*c) + 15*a^2*b + 25*a*b^2 + 11*b^3)/(d*(e^(2*d*x + 2*c) + 1)^5)
```

3.23 $\int \mathbf{csch}^3(c+dx)\left(a+b\mathbf{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=144

$$-\frac{b\left(6a^2+15ab+7b^2\right)\operatorname{sech}^3(c+dx)}{6d}-\frac{b^2(5a+7b)\operatorname{sech}^5(c+dx)}{10d}-\frac{(a+b)^2(a+7b)\operatorname{sech}(c+dx)}{2d}+\frac{(a+b)^2(a+7b)\tanh}{2d}$$

```
[Out] ((a + b)^2*(a + 7*b)*ArcTanh[Cosh[c + d*x]])/(2*d) - ((a + b)^2*(a + 7*b)*S
ech[c + d*x])/(2*d) - (b*(6*a^2 + 15*a*b + 7*b^2)*Sech[c + d*x]^3)/(6*d) -
(b^2*(5*a + 7*b)*Sech[c + d*x]^5)/(10*d) - ((a + b)*(b + a*Cosh[c + d*x]^2)
^2*Csch[c + d*x]^2*Sech[c + d*x]^5)/(2*d)
```

Rubi [A] time = 0.178248, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4133, 468, 570, 207}

$$-\frac{b\left(6a^2+15ab+7b^2\right)\operatorname{sech}^3(c+dx)}{6d}-\frac{b^2(5a+7b)\operatorname{sech}^5(c+dx)}{10d}-\frac{(a+b)^2(a+7b)\operatorname{sech}(c+dx)}{2d}+\frac{(a+b)^2(a+7b)\tanh}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + b)^2*(a + 7*b)*ArcTanh[Cosh[c + d*x]])/(2*d) - ((a + b)^2*(a + 7*b)*S
ech[c + d*x])/(2*d) - (b*(6*a^2 + 15*a*b + 7*b^2)*Sech[c + d*x]^3)/(6*d) -
(b^2*(5*a + 7*b)*Sech[c + d*x]^5)/(10*d) - ((a + b)*(b + a*Cosh[c + d*x]^2)
^2*Csch[c + d*x]^2*Sech[c + d*x]^5)/(2*d)
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \text{csch}^3(c+dx)\left(a+b\,\text{sech}^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(b+ax^2\right)^3}{x^6\left(1-x^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{(a+b)\left(b+a\cosh^2(c+dx)\right)^2\text{csch}^2(c+dx)\text{sech}^5(c+dx)}{2d} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{(a+b)\left(b+a\cosh^2(c+dx)\right)^2\text{csch}^2(c+dx)\text{sech}^5(c+dx)}{2d} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{(a+b)^2(a+7b)\text{sech}(c+dx)}{2d} - \frac{b\left(6a^2+15ab+7b^2\right)\text{sech}^3(c+dx)}{6d} -$$

$$= \frac{(a+b)^2(a+7b)\tanh^{-1}(\cosh(c+dx))}{2d} - \frac{(a+b)^2(a+7b)\text{sech}(c+dx)}{2d} -$$

Mathematica [A] time = 3.73439, size = 224, normalized size = 1.56

$$-\frac{\text{sech}^6(c+dx)\left(a\cosh^2(c+dx)+b\right)^3\left(\frac{3}{4}\left(195a^2b+75a^3+165ab^2+77b^3\right)\sinh(4(c+dx))\text{csch}^3(c+dx)+\coth(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -((b + a*Cosh[c + d*x]^2)^3*Sech[c + d*x]^6*((150*a^3 + 270*a^2*b - 30*a*b^
2 - 206*b^3 + 10*(9*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3)*Cosh[4*(c + d*x)] +
 15*(a + b)^2*(a + 7*b)*Cosh[6*(c + d*x)])*Coth[c + d*x]*Csch[c + d*x] - 48
0*(a + b)^2*(a + 7*b)*Cosh[c + d*x]^6*(Log[Cosh[(c + d*x)/2]] - Log[Sinh[(c
 + d*x)/2]]) + (3*(75*a^3 + 195*a^2*b + 165*a*b^2 + 77*b^3)*Csch[c + d*x]^3
*Sinh[4*(c + d*x)])/4))/(120*d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)
```

Maple [A] time = 0.05, size = 192, normalized size = 1.3

$$\frac{1}{d}\left(a^3\left(-\frac{\text{csch}\,(dx+c)\coth\,(dx+c)}{2}+\text{Artanh}\left(e^{dx+c}\right)\right)+3\,a^2b\left(-1/2\,\frac{1}{\left(\sinh\,(dx+c)\right)^2\cosh\,(dx+c)}-3/2\,(\cosh\,(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+3*a^2*b*(-1/2/s
inh(d*x+c)^2/cosh(d*x+c)-3/2/cosh(d*x+c)+3*arctanh(exp(d*x+c)))+3*a*b^2*(-1
/2/sinh(d*x+c)^2/cosh(d*x+c)^3-5/6/cosh(d*x+c)^3-5/2/cosh(d*x+c)+5*arctanh(
exp(d*x+c)))+b^3*(-1/2/sinh(d*x+c)^2/cosh(d*x+c)^5-7/10/cosh(d*x+c)^5-7/6/c
osh(d*x+c)^3-7/2/cosh(d*x+c)+7*arctanh(exp(d*x+c))))
```

Maxima [B] time = 1.06963, size = 751, normalized size = 5.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/30*b^3*(105*log(e^(-d*x - c) + 1)/d - 105*log(e^(-d*x - c) - 1)/d - 2*(10
5*e^(-d*x - c) + 350*e^(-3*d*x - 3*c) + 231*e^(-5*d*x - 5*c) - 412*e^(-7*d*
x - 7*c) + 231*e^(-9*d*x - 9*c) + 350*e^(-11*d*x - 11*c) + 105*e^(-13*d*x -
 13*c))/(d*(3*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) - 5*e^(-6*d*x - 6*c) - 5*
e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 3*e^(-12*d*x - 12*c) + e^(-14*d*x -
 14*c) + 1))) + 1/2*a*b^2*(15*log(e^(-d*x - c) + 1)/d - 15*log(e^(-d*x - c)
 - 1)/d - 2*(15*e^(-d*x - c) + 20*e^(-3*d*x - 3*c) - 22*e^(-5*d*x - 5*c) +
20*e^(-7*d*x - 7*c) + 15*e^(-9*d*x - 9*c))/(d*(e^(-2*d*x - 2*c) - 2*e^(-4*d
*x - 4*c) - 2*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)
)) + 3/2*a^2*b*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d + 2*(
3*e^(-d*x - c) - 2*e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c))/(d*(e^(-2*d*x - 2
*c) + e^(-4*d*x - 4*c) - e^(-6*d*x - 6*c) - 1))) + 1/2*a^3*(log(e^(-d*x - c
) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d
*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

Fricas [B] time = 3.63043, size = 17022, normalized size = 118.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/30*(30*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^13 + 390*(a^3 +
9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)*sinh(d*x + c)^12 + 30*(a^3 + 9*a^
2*b + 15*a*b^2 + 7*b^3)*sinh(d*x + c)^13 + 20*(9*a^3 + 45*a^2*b + 75*a*b^2
+ 35*b^3)*cosh(d*x + c)^11 + 20*(9*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3 + 117
*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^11 + 220
*(39*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^3 + (9*a^3 + 45*a^2*b
 + 75*a*b^2 + 35*b^3)*cosh(d*x + c))*sinh(d*x + c)^10 + 6*(75*a^3 + 195*a^2
*b + 165*a*b^2 + 77*b^3)*cosh(d*x + c)^9 + 2*(10725*(a^3 + 9*a^2*b + 15*a*b
^2 + 7*b^3)*cosh(d*x + c)^4 + 225*a^3 + 585*a^2*b + 495*a*b^2 + 231*b^3 + 5
50*(9*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^9
+ 6*(6435*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^5 + 550*(9*a^3 +
 45*a^2*b + 75*a*b^2 + 35*b^3)*cosh(d*x + c)^3 + 9*(75*a^3 + 195*a^2*b + 16
5*a*b^2 + 77*b^3)*cosh(d*x + c))*sinh(d*x + c)^8 + 8*(75*a^3 + 135*a^2*b -
15*a*b^2 - 103*b^3)*cosh(d*x + c)^7 + 8*(6435*(a^3 + 9*a^2*b + 15*a*b^2 + 7
*b^3)*cosh(d*x + c)^6 + 825*(9*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3)*cosh(d*x
 + c)^4 + 75*a^3 + 135*a^2*b - 15*a*b^2 - 103*b^3 + 27*(75*a^3 + 195*a^2*b
+ 165*a*b^2 + 77*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 8*(6435*(a^3 + 9*a
^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^7 + 1155*(9*a^3 + 45*a^2*b + 75*a*b^
2 + 35*b^3)*cosh(d*x + c)^5 + 63*(75*a^3 + 195*a^2*b + 165*a*b^2 + 77*b^3)*
cosh(d*x + c)^3 + 7*(75*a^3 + 135*a^2*b - 15*a*b^2 - 103*b^3)*cosh(d*x + c)
)*sinh(d*x + c)^6 + 6*(75*a^3 + 195*a^2*b + 165*a*b^2 + 77*b^3)*cosh(d*x +
c)^5 + 6*(6435*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^8 + 1540*(9
*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3)*cosh(d*x + c)^6 + 126*(75*a^3 + 195*a^
2*b + 165*a*b^2 + 77*b^3)*cosh(d*x + c)^4 + 75*a^3 + 195*a^2*b + 165*a*b^2
```

```
+ 77*b^3 + 28*(75*a^3 + 135*a^2*b - 15*a*b^2 - 103*b^3)*cosh(d*x + c)^2)*si
nh(d*x + c)^5 + 2*(10725*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^9
 + 3300*(9*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3)*cosh(d*x + c)^7 + 378*(75*a^
3 + 195*a^2*b + 165*a*b^2 + 77*b^3)*cosh(d*x + c)^5 + 140*(75*a^3 + 135*a^2
*b - 15*a*b^2 - 103*b^3)*cosh(d*x + c)^3 + 15*(75*a^3 + 195*a^2*b + 165*a*b
^2 + 77*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 20*(9*a^3 + 45*a^2*b + 75*a*b
^2 + 35*b^3)*cosh(d*x + c)^3 + 4*(2145*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*c
osh(d*x + c)^10 + 825*(9*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3)*cosh(d*x + c)^
8 + 126*(75*a^3 + 195*a^2*b + 165*a*b^2 + 77*b^3)*cosh(d*x + c)^6 + 70*(75*
a^3 + 135*a^2*b - 15*a*b^2 - 103*b^3)*cosh(d*x + c)^4 + 45*a^3 + 225*a^2*b
+ 375*a*b^2 + 175*b^3 + 15*(75*a^3 + 195*a^2*b + 165*a*b^2 + 77*b^3)*cosh(d
*x + c)^2)*sinh(d*x + c)^3 + 4*(585*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh
(d*x + c)^11 + 275*(9*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3)*cosh(d*x + c)^9 +
 54*(75*a^3 + 195*a^2*b + 165*a*b^2 + 77*b^3)*cosh(d*x + c)^7 + 42*(75*a^3
+ 135*a^2*b - 15*a*b^2 - 103*b^3)*cosh(d*x + c)^5 + 15*(75*a^3 + 195*a^2*b
+ 165*a*b^2 + 77*b^3)*cosh(d*x + c)^3 + 15*(9*a^3 + 45*a^2*b + 75*a*b^2 + 3
5*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 30*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^
3)*cosh(d*x + c) - 15*((a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^14
+ 14*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)*sinh(d*x + c)^13 + (a
^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*sinh(d*x + c)^14 + 3*(a^3 + 9*a^2*b + 15*a
*b^2 + 7*b^3)*cosh(d*x + c)^12 + (3*a^3 + 27*a^2*b + 45*a*b^2 + 21*b^3 + 91
*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^12 + 4*(
91*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^3 + 9*(a^3 + 9*a^2*b +
15*a*b^2 + 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^11 + (a^3 + 9*a^2*b + 15*a*b
^2 + 7*b^3)*cosh(d*x + c)^10 + (1001*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cos
h(d*x + c)^4 + a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3 + 198*(a^3 + 9*a^2*b + 15*a
*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 2*(1001*(a^3 + 9*a^2*b +
15*a*b^2 + 7*b^3)*cosh(d*x + c)^5 + 330*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*
cosh(d*x + c)^3 + 5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c))*sinh(
d*x + c)^9 - 5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^8 + (3003*(
a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^6 + 1485*(a^3 + 9*a^2*b + 1
5*a*b^2 + 7*b^3)*cosh(d*x + c)^4 - 5*a^3 - 45*a^2*b - 75*a*b^2 - 35*b^3 + 4
5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(
429*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^7 + 297*(a^3 + 9*a^2*b
 + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^5 + 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3
)*cosh(d*x + c)^3 - 5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c))*sin
h(d*x + c)^7 - 5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^6 + (3003
*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^8 + 2772*(a^3 + 9*a^2*b +
 15*a*b^2 + 7*b^3)*cosh(d*x + c)^6 + 210*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)
*cosh(d*x + c)^4 - 5*a^3 - 45*a^2*b - 75*a*b^2 - 35*b^3 - 140*(a^3 + 9*a^2*
b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 2*(1001*(a^3 + 9*a
^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^9 + 1188*(a^3 + 9*a^2*b + 15*a*b^2 +
 7*b^3)*cosh(d*x + c)^7 + 126*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x +
 c)^5 - 140*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^3 - 15*(a^3 +
9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + (a^3 + 9*a^2*b
 + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^4 + (1001*(a^3 + 9*a^2*b + 15*a*b^2 + 7*
b^3)*cosh(d*x + c)^10 + 1485*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x +
c)^8 + 210*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^6 - 350*(a^3 +
9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^4 + a^3 + 9*a^2*b + 15*a*b^2 + 7*
b^3 - 75*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^
4 + 4*(91*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^11 + 165*(a^3 +
9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^9 + 30*(a^3 + 9*a^2*b + 15*a*b^2
+ 7*b^3)*cosh(d*x + c)^7 - 70*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x +
 c)^5 - 25*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^3 + (a^3 + 9*a^
2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + a^3 + 9*a^2*b + 15
*a*b^2 + 7*b^3 + 3*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2 + (91
*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^12 + 198*(a^3 + 9*a^2*b +
 15*a*b^2 + 7*b^3)*cosh(d*x + c)^10 + 45*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)
*cosh(d*x + c)^8 - 140*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^6 -
```

75*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^4 + 3*a^3 + 27*a^2*b + 45*a*b^2 + 21*b^3 + 6*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(7*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^13 + 18*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^11 + 5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^9 - 20*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^7 - 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^5 + 2*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^3 + 3*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 15*((a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^14 + 14*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)*sinh(d*x + c)^13 + (a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*sinh(d*x + c)^14 + 3*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^12 + (3*a^3 + 27*a^2*b + 45*a*b^2 + 21*b^3 + 91*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^12 + 4*(91*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^3 + 9*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^11 + (a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^10 + (1001*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^4 + a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3 + 198*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 2*(1001*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^5 + 330*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^3 + 5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 - 5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^8 + (3003*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^6 + 1485*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^4 - 5*a^3 - 45*a^2*b - 75*a*b^2 - 35*b^3 + 45*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(429*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^7 + 297*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^5 + 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^3 - 5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 - 5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^6 + (3003*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^8 + 2772*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^6 + 210*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^4 - 5*a^3 - 45*a^2*b - 75*a*b^2 - 35*b^3 - 140*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 2*(1001*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^9 + 1188*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^7 + 126*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^5 - 140*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^3 - 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + (a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^4 + (1001*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^10 + 1485*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^8 + 210*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^6 - 350*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^4 + a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3 - 75*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(91*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^11 + 165*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^9 + 30*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^7 - 70*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^5 - 25*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^3 + (a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3 + 3*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2 + (91*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^12 + 198*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^10 + 45*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^8 - 140*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^6 - 75*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^4 + 3*a^3 + 27*a^2*b + 45*a*b^2 + 21*b^3 + 6*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(7*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^13 + 18*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^11 + 5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^9 - 20*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^7 - 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^5 + 2*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^3 + 3*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(195*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^12 + 110*(

```
9*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3)*cosh(d*x + c)^10 + 27*(75*a^3 + 195*a
^2*b + 165*a*b^2 + 77*b^3)*cosh(d*x + c)^8 + 28*(75*a^3 + 135*a^2*b - 15*a*
b^2 - 103*b^3)*cosh(d*x + c)^6 + 15*(75*a^3 + 195*a^2*b + 165*a*b^2 + 77*b^
3)*cosh(d*x + c)^4 + 15*a^3 + 135*a^2*b + 225*a*b^2 + 105*b^3 + 30*(9*a^3 +
 45*a^2*b + 75*a*b^2 + 35*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x
+ c)^14 + 14*d*cosh(d*x + c)*sinh(d*x + c)^13 + d*sinh(d*x + c)^14 + 3*d*co
sh(d*x + c)^12 + (91*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^12 + 4*(91*d*co
sh(d*x + c)^3 + 9*d*cosh(d*x + c))*sinh(d*x + c)^11 + d*cosh(d*x + c)^10 +
(1001*d*cosh(d*x + c)^4 + 198*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^10 + 2*(
1001*d*cosh(d*x + c)^5 + 330*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*
x + c)^9 - 5*d*cosh(d*x + c)^8 + (3003*d*cosh(d*x + c)^6 + 1485*d*cosh(d*x
+ c)^4 + 45*d*cosh(d*x + c)^2 - 5*d)*sinh(d*x + c)^8 + 8*(429*d*cosh(d*x +
c)^7 + 297*d*cosh(d*x + c)^5 + 15*d*cosh(d*x + c)^3 - 5*d*cosh(d*x + c))*si
nh(d*x + c)^7 - 5*d*cosh(d*x + c)^6 + (3003*d*cosh(d*x + c)^8 + 2772*d*cosh
(d*x + c)^6 + 210*d*cosh(d*x + c)^4 - 140*d*cosh(d*x + c)^2 - 5*d)*sinh(d*x
 + c)^6 + 2*(1001*d*cosh(d*x + c)^9 + 1188*d*cosh(d*x + c)^7 + 126*d*cosh(d
*x + c)^5 - 140*d*cosh(d*x + c)^3 - 15*d*cosh(d*x + c))*sinh(d*x + c)^5 + d
*cosh(d*x + c)^4 + (1001*d*cosh(d*x + c)^10 + 1485*d*cosh(d*x + c)^8 + 210*
d*cosh(d*x + c)^6 - 350*d*cosh(d*x + c)^4 - 75*d*cosh(d*x + c)^2 + d)*sinh(
d*x + c)^4 + 4*(91*d*cosh(d*x + c)^11 + 165*d*cosh(d*x + c)^9 + 30*d*cosh(d
*x + c)^7 - 70*d*cosh(d*x + c)^5 - 25*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*
sinh(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + (91*d*cosh(d*x + c)^12 + 198*d*cosh
(d*x + c)^10 + 45*d*cosh(d*x + c)^8 - 140*d*cosh(d*x + c)^6 - 75*d*cosh(d*x
 + c)^4 + 6*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^2 + 2*(7*d*cosh(d*x + c)
^13 + 18*d*cosh(d*x + c)^11 + 5*d*cosh(d*x + c)^9 - 20*d*cosh(d*x + c)^7 -
15*d*cosh(d*x + c)^5 + 2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x +
c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\,\mathrm{sech}^2\left(c + dx\right)\right)^3 \mathrm{csch}^3\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*csch(c + d*x)**3, x)
```

Giac [B] time = 1.16802, size = 470, normalized size = 3.26

$$\frac{\left(a^3 + 9\,a^2 b + 15\,ab^2 + 7\,b^3\right)\log\left(e^{(dx+c)} + e^{(-dx-c)} + 2\right)}{4\,d} - \frac{\left(a^3 + 9\,a^2 b + 15\,ab^2 + 7\,b^3\right)\log\left(e^{(dx+c)} + e^{(-dx-c)} - 2\right)}{4\,d} - \frac{a^3\left(e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/4*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*log(e^(d*x + c) + e^(-d*x - c) + 2)/
d - 1/4*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*log(e^(d*x + c) + e^(-d*x - c) -
 2)/d - (a^3*(e^(d*x + c) + e^(-d*x - c)) + 3*a^2*b*(e^(d*x + c) + e^(-d*x
- c)) + 3*a*b^2*(e^(d*x + c) + e^(-d*x - c)) + b^3*(e^(d*x + c) + e^(-d*x -
 c)))/(((e^(d*x + c) + e^(-d*x - c))^2 - 4)*d) - 2/15*(45*a^2*b*(e^(d*x + c
) + e^(-d*x - c))^4 + 90*a*b^2*(e^(d*x + c) + e^(-d*x - c))^4 + 45*b^3*(e^(
```

```
d*x + c) + e^(-d*x - c))^4 + 60*a*b^2*(e^(d*x + c) + e^(-d*x - c))^2 + 40*b
^3*(e^(d*x + c) + e^(-d*x - c))^2 + 48*b^3)/(d*(e^(d*x + c) + e^(-d*x - c))
^5)
```

3.24 $\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3 dx$

Optimal. Leaf size=104

$$-\frac{b^2(3a+4b)\tanh^3(c+dx)}{3d}+\frac{3b(a+b)(a+2b)\tanh(c+dx)}{d}-\frac{(a+b)^3\coth^3(c+dx)}{3d}+\frac{(a+b)^2(a+4b)\coth(c+d}{d}$$

```
[Out] ((a + b)^2*(a + 4*b)*Coth[c + d*x])/d - ((a + b)^3*Coth[c + d*x]^3)/(3*d) +
 (3*b*(a + b)*(a + 2*b)*Tanh[c + d*x])/d - (b^2*(3*a + 4*b)*Tanh[c + d*x]^3
)/(3*d) + (b^3*Tanh[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.105049, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4132, 448}

$$-\frac{b^2(3a+4b)\tanh^3(c+dx)}{3d}+\frac{3b(a+b)(a+2b)\tanh(c+dx)}{d}-\frac{(a+b)^3\coth^3(c+dx)}{3d}+\frac{(a+b)^2(a+4b)\coth(c+d}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + b)^2*(a + 4*b)*Coth[c + d*x])/d - ((a + b)^3*Coth[c + d*x]^3)/(3*d) +
 (3*b*(a + b)*(a + 2*b)*Tanh[c + d*x])/d - (b^2*(3*a + 4*b)*Tanh[c + d*x]^3
)/(3*d) + (b^3*Tanh[c + d*x]^5)/(5*d)
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3 dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)(a+b-bx^2)^3}{x^4}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(3b(a+b)(a+2b)+\frac{(a+b)^3}{x^4}-\frac{(a+b)^2(a+4b)}{x^2}-b^2(3a+4b)x^2+b^3x\right.\right.}{d}\\ &= \frac{(a+b)^2(a+4b)\coth(c+dx)}{d}-\frac{(a+b)^3\coth^3(c+dx)}{3d}+\frac{3b(a+b)(a+}{}\end{aligned}$$

Mathematica [B] time = 2.30334, size = 213, normalized size = 2.05

$$-\frac{8\tanh(c)\operatorname{sech}^5(c+dx)\left(a\cosh^2(c+dx)+b\right)^3\left(\operatorname{csch}(c)\sinh(dx)\cosh^4(c+dx)\left(5(a+b)^2(2a+11b)\coth(c)\coth(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (-8*(b + a*Cosh[c + d*x]^2)^3*Sech[c + d*x]^5*(-3*b^3*Cosh[c + d*x] + Cosh[
c + d*x]^3*(-(b^2*(15*a + 14*b)) + 5*(a + b)^3*Coth[c]^2*Coth[c + d*x]^2) -
 3*b^3*Csch[c]*Sinh[d*x] + Cosh[c + d*x]^4*(-(b*(45*a^2 + 120*a*b + 73*b^2)
) + 5*(a + b)^2*(2*a + 11*b)*Coth[c]*Coth[c + d*x])*Csch[c]*Sinh[d*x] - Cos
h[c + d*x]^2*(b^2*(15*a + 14*b) + 5*(a + b)^3*Coth[c]*Coth[c + d*x]^3)*Csch
[c]*Sinh[d*x])*Tanh[c])/(15*d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)
```

Maple [B] time = 0.051, size = 213, normalized size = 2.1

$$\frac{1}{d}\left(a^3\left(\frac{2}{3}-\frac{(\operatorname{csch}(dx+c))^2}{3}\right)\coth(dx+c)+3a^2b\left(-1/3\frac{1}{(\sinh(dx+c))^3\cosh(dx+c)}+4/3\frac{1}{\cosh(dx+c)\sinh(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+3*a^2*b*(-1/3/sinh(d*x+c)^3/co
sh(d*x+c)+4/3/sinh(d*x+c)/cosh(d*x+c)+8/3*tanh(d*x+c))+3*a*b^2*(-1/3/sinh(d
*x+c)^3/cosh(d*x+c)^3+2/sinh(d*x+c)/cosh(d*x+c)^3+8*(2/3+1/3*sech(d*x+c)^2)
*tanh(d*x+c))+b^3*(-1/3/sinh(d*x+c)^3/cosh(d*x+c)^5+8/3/sinh(d*x+c)/cosh(d*
x+c)^5+16*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c)))
```

Maxima [B] time = 1.09577, size = 896, normalized size = 8.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 4/3*a^3*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e
^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) - 1))) + 256/15*b^3*(2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c
) - 2*e^(-4*d*x - 4*c) - 6*e^(-6*d*x - 6*c) + 6*e^(-10*d*x - 10*c) + 2*e^(-
12*d*x - 12*c) - 2*e^(-14*d*x - 14*c) - e^(-16*d*x - 16*c) + 1)) - 2*e^(-4*
d*x - 4*c)/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-4*d*x - 4*c) - 6*e^(-6*d*x - 6*c)
 + 6*e^(-10*d*x - 10*c) + 2*e^(-12*d*x - 12*c) - 2*e^(-14*d*x - 14*c) - e^(
-16*d*x - 16*c) + 1)) - 6*e^(-6*d*x - 6*c)/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-4
*d*x - 4*c) - 6*e^(-6*d*x - 6*c) + 6*e^(-10*d*x - 10*c) + 2*e^(-12*d*x - 12
*c) - 2*e^(-14*d*x - 14*c) - e^(-16*d*x - 16*c) + 1)) + 1/(d*(2*e^(-2*d*x -
 2*c) - 2*e^(-4*d*x - 4*c) - 6*e^(-6*d*x - 6*c) + 6*e^(-10*d*x - 10*c) + 2*
e^(-12*d*x - 12*c) - 2*e^(-14*d*x - 14*c) - e^(-16*d*x - 16*c) + 1))) + 16*
a^2*b*(2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-6*d*x - 6*c) + e^(
-8*d*x - 8*c) - 1)) - 1/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-6*d*x - 6*c) + e^(-8
*d*x - 8*c) - 1))) + 32*a*b^2*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-4*d*x - 4*c) -
3*e^(-8*d*x - 8*c) + e^(-12*d*x - 12*c) - 1)) - 1/(d*(3*e^(-4*d*x - 4*c) -
3*e^(-8*d*x - 8*c) + e^(-12*d*x - 12*c) - 1)))
```

Fricas [B] time = 2.55822, size = 2480, normalized size = 23.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -8/15*((5*a^3 - 30*a^2*b - 60*a*b^2 - 32*b^3)*cosh(d*x + c)^6 + 12*(5*a^3 +
 15*a^2*b + 30*a*b^2 + 16*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (5*a^3 - 30*
a^2*b - 60*a*b^2 - 32*b^3)*sinh(d*x + c)^6 + 2*(15*a^3 - 60*a*b^2 - 32*b^3)
*cosh(d*x + c)^4 + (30*a^3 - 120*a*b^2 - 64*b^3 + 15*(5*a^3 - 30*a^2*b - 60
*a*b^2 - 32*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(5*(5*a^3 + 15*a^2*b
+ 30*a*b^2 + 16*b^3)*cosh(d*x + c)^3 + 4*(5*a^3 + 15*a^2*b + 15*a*b^2 + 8*b
^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 50*a^3 + 240*a^2*b + 360*a*b^2 + 192*b
^3 + (75*a^3 + 270*a^2*b + 300*a*b^2 + 64*b^3)*cosh(d*x + c)^2 + (15*(5*a^3
 - 30*a^2*b - 60*a*b^2 - 32*b^3)*cosh(d*x + c)^4 + 75*a^3 + 270*a^2*b + 300
*a*b^2 + 64*b^3 + 12*(15*a^3 - 60*a*b^2 - 32*b^3)*cosh(d*x + c)^2)*sinh(d*x
 + c)^2 + 4*(3*(5*a^3 + 15*a^2*b + 30*a*b^2 + 16*b^3)*cosh(d*x + c)^5 + 8*(
5*a^3 + 15*a^2*b + 15*a*b^2 + 8*b^3)*cosh(d*x + c)^3 + (25*a^3 + 75*a^2*b +
 30*a*b^2 - 32*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^10 + 10*
d*cosh(d*x + c)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 + 2*d*cosh(d*x + c)^8
+ (45*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^8 + 8*(15*d*cosh(d*x + c)^3 +
2*d*cosh(d*x + c))*sinh(d*x + c)^7 - 3*d*cosh(d*x + c)^6 + (210*d*cosh(d*x
+ c)^4 + 56*d*cosh(d*x + c)^2 - 3*d)*sinh(d*x + c)^6 + 2*(126*d*cosh(d*x +
c)^5 + 56*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^5 - 8*d*cosh
(d*x + c)^4 + (210*d*cosh(d*x + c)^6 + 140*d*cosh(d*x + c)^4 - 45*d*cosh(d*
x + c)^2 - 8*d)*sinh(d*x + c)^4 + 4*(30*d*cosh(d*x + c)^7 + 28*d*cosh(d*x +
 c)^5 - 5*d*cosh(d*x + c)^3 - 4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 2*d*cosh
(d*x + c)^2 + (45*d*cosh(d*x + c)^8 + 56*d*cosh(d*x + c)^6 - 45*d*cosh(d*x
+ c)^4 - 48*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^2 + 2*(5*d*cosh(d*x + c)
^9 + 8*d*cosh(d*x + c)^7 - 3*d*cosh(d*x + c)^5 - 8*d*cosh(d*x + c)^3 - 2*d*
cosh(d*x + c))*sinh(d*x + c) + 6*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\,\mathrm{sech}^2\,(c + dx)\right)^3 \mathrm{csch}^4\,(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*csch(c + d*x)**4, x)
```

Giac [B] time = 1.23254, size = 481, normalized size = 4.62

$$\frac{2\left(9\,a^2be^{(4\,dx+4\,c)} + 18\,ab^2e^{(4\,dx+4\,c)} + 9\,b^3e^{(4\,dx+4\,c)} - 6\,a^3e^{(2\,dx+2\,c)} - 36\,a^2be^{(2\,dx+2\,c)} - 54\,ab^2e^{(2\,dx+2\,c)} - 24\,b^3e^{(2\,dx+2\,c)} + \ldots\right.}{3\,d\left(e^{(2\,dx+2\,c)} - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 2/3*(9*a^2*b*e^(4*d*x + 4*c) + 18*a*b^2*e^(4*d*x + 4*c) + 9*b^3*e^(4*d*x +
4*c) - 6*a^3*e^(2*d*x + 2*c) - 36*a^2*b*e^(2*d*x + 2*c) - 54*a*b^2*e^(2*d*x
 + 2*c) - 24*b^3*e^(2*d*x + 2*c) + 2*a^3 + 15*a^2*b + 24*a*b^2 + 11*b^3)/(d
*(e^(2*d*x + 2*c) - 1)^3) - 2/15*(45*a^2*b*e^(8*d*x + 8*c) + 90*a*b^2*e^(8*
d*x + 8*c) + 45*b^3*e^(8*d*x + 8*c) + 180*a^2*b*e^(6*d*x + 6*c) + 450*a*b^2
*e^(6*d*x + 6*c) + 240*b^3*e^(6*d*x + 6*c) + 270*a^2*b*e^(4*d*x + 4*c) + 75
0*a*b^2*e^(4*d*x + 4*c) + 490*b^3*e^(4*d*x + 4*c) + 180*a^2*b*e^(2*d*x + 2*
c) + 510*a*b^2*e^(2*d*x + 2*c) + 320*b^3*e^(2*d*x + 2*c) + 45*a^2*b + 120*a
*b^2 + 73*b^3)/(d*(e^(2*d*x + 2*c) + 1)^5)
```

3.25 $\int \frac{\sinh^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=117

$$\frac{x(3a^2+12ab+8b^2)}{8a^3}-\frac{\sqrt{b}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^3d}-\frac{(5a+4b)\sinh(c+dx)\cosh(c+dx)}{8a^2d}+\frac{\sinh(c+dx)\cos\dots}{4ad}$$

```
[Out] ((3*a^2 + 12*a*b + 8*b^2)*x)/(8*a^3) - (Sqrt[b]*(a + b)^(3/2)*ArcTanh[(Sqrt
[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a^3*d) - ((5*a + 4*b)*Cosh[c + d*x]*Sinh[
c + d*x])/(8*a^2*d) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*a*d)
```

Rubi [A] time = 0.195511, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4132, 470, 527, 522, 206, 208}

$$\frac{x(3a^2+12ab+8b^2)}{8a^3}-\frac{\sqrt{b}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^3d}-\frac{(5a+4b)\sinh(c+dx)\cosh(c+dx)}{8a^2d}+\frac{\sinh(c+dx)\cos\dots}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```


```
[Out] ((3*a^2 + 12*a*b + 8*b^2)*x)/(8*a^3) - (Sqrt[b]*(a + b)^(3/2)*ArcTanh[(Sqrt
[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a^3*d) - ((5*a + 4*b)*Cosh[c + d*x]*Sinh[
c + d*x])/(8*a^2*d) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*a*d)
```


Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)^3(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad} - \frac{\operatorname{Subst}\left(\int \frac{a+b+(4a+3b)x^2}{(1-x^2)^2(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{4ad}$$

$$= -\frac{(5a+4b)\cosh(c+dx)\sinh(c+dx)}{8a^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad} - \frac{\operatorname{Subst}\left(\int \frac{-(a+b)(3a+4b)}{(1-x^2)(a+\cdots}\right.}{\cdots}$$

$$= -\frac{(5a+4b)\cosh(c+dx)\sinh(c+dx)}{8a^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad} - \frac{(b(a+b)^2)\operatorname{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{(3a^2+12ab+8b^2)x}{8a^3} - \frac{\sqrt{b}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^3d} - \frac{(5a+4b)\cosh(c+dx)\sinh(c+\cdots}{8a^2d}$$

Mathematica [B] time = 2.40738, size = 294, normalized size = 2.51

$$-\frac{\operatorname{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\sqrt{b}\left(34a^2b+3a^3+64ab^2+32b^3\right)(\cosh(2c)-\sinh(2c))\tanh^{-1}\left(\frac{(\cosh(2c)-\sin\cdots}{\cdots}\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(Sqrt[b]*(3*a^3 + 34*a^2*
b + 64*a*b^2 + 32*b^3)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b
)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])
^4])]*(Cosh[2*c] - Sinh[2*c]) - Sqrt[b*(Cosh[c] - Sinh[c])^4]*(a^2*(3*a + 2
*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]] + Sqrt[b]*Sqrt[a + b]*(-2*
a^2*c + 12*a^2*d*x + 48*a*b*d*x + 32*b^2*d*x - 8*a*(a + b)*Sinh[2*(c + d*x)
] + a^2*Sinh[4*(c + d*x)]))))/(64*a^3*Sqrt[b]*Sqrt[a + b]*d*(a + b*Sech[c +
 d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])
```

Maple [B] time = 0.089, size = 708, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/4/d/a/(tanh(1/2*d*x+1/2*c)+1)^4+1/2/d/a/(tanh(1/2*d*x+1/2*c)+1)^3+1/8/d/
a/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/a^2/(tanh(1/2*d*x+1/2*c)+1)^2*b-3/8/d/a/(
tanh(1/2*d*x+1/2*c)+1)-1/2/d/a^2/(tanh(1/2*d*x+1/2*c)+1)*b+3/8/d/a*ln(tanh(
1/2*d*x+1/2*c)+1)+3/2/d/a^2*ln(tanh(1/2*d*x+1/2*c)+1)*b+1/d/a^3*ln(tanh(1/2
*d*x+1/2*c)+1)*b^2-1/2/d*b^(1/2)/a/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+
1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-1/d*b^(3/2)/a^2/(a+b)^(
1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+
b)^(1/2))-1/2/d*b^(5/2)/a^3/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^
2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/2/d*b^(1/2)/a/(a+b)^(1/2)*ln
((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2
))+1/d*b^(3/2)/a^2/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(
1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/2/d*b^(5/2)/a^3/(a+b)^(1/2)*ln((a+b)^
(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/4/
d/a/(tanh(1/2*d*x+1/2*c)-1)^4+1/2/d/a/(tanh(1/2*d*x+1/2*c)-1)^3-1/8/d/a/(ta
nh(1/2*d*x+1/2*c)-1)^2-1/2/d/a^2/(tanh(1/2*d*x+1/2*c)-1)^2*b-3/8/d/a/(tanh(
1/2*d*x+1/2*c)-1)-1/2/d/a^2/(tanh(1/2*d*x+1/2*c)-1)*b-3/8/d/a*ln(tanh(1/2*d
*x+1/2*c)-1)-3/2/d/a^2*ln(tanh(1/2*d*x+1/2*c)-1)*b-1/d/a^3*ln(tanh(1/2*d*x+
1/2*c)-1)*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.96917, size = 4338, normalized size = 37.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/64*(a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh
(d*x + c)^8 + 8*(3*a^2 + 12*a*b + 8*b^2)*d*x*cosh(d*x + c)^4 - 8*(a^2 + a*b
)*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 - 2*a^2 - 2*a*b)*sinh(d*x + c)
^6 + 8*(7*a^2*cosh(d*x + c)^3 - 6*(a^2 + a*b)*cosh(d*x + c))*sinh(d*x + c)^
5 + 2*(35*a^2*cosh(d*x + c)^4 + 4*(3*a^2 + 12*a*b + 8*b^2)*d*x - 60*(a^2 +
a*b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^5 + 4*(3*a^2
 + 12*a*b + 8*b^2)*d*x*cosh(d*x + c) - 20*(a^2 + a*b)*cosh(d*x + c)^3)*sinh
(d*x + c)^3 + 8*(a^2 + a*b)*cosh(d*x + c)^2 + 4*(7*a^2*cosh(d*x + c)^6 + 12
*(3*a^2 + 12*a*b + 8*b^2)*d*x*cosh(d*x + c)^2 - 30*(a^2 + a*b)*cosh(d*x + c
```

```
)^4 + 2*a^2 + 2*a*b)*sinh(d*x + c)^2 + 32*((a + b)*cosh(d*x + c)^4 + 4*(a +
 b)*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a + b)*cosh(d*x + c)^2*sinh(d*x + c)
^2 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4)*sqr
t(a*b + b^2)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3
 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*
x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh
(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x +
c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(
a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh
(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*
b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d
*x + c) + a)) - a^2 + 8*(a^2*cosh(d*x + c)^7 + 4*(3*a^2 + 12*a*b + 8*b^2)*d
*x*cosh(d*x + c)^3 - 6*(a^2 + a*b)*cosh(d*x + c)^5 + 2*(a^2 + a*b)*cosh(d*x
 + c))*sinh(d*x + c))/(a^3*d*cosh(d*x + c)^4 + 4*a^3*d*cosh(d*x + c)^3*sinh
(d*x + c) + 6*a^3*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a^3*d*cosh(d*x + c)
*sinh(d*x + c)^3 + a^3*d*sinh(d*x + c)^4), 1/64*(a^2*cosh(d*x + c)^8 + 8*a^
2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 + 8*(3*a^2 + 12*a*b +
 8*b^2)*d*x*cosh(d*x + c)^4 - 8*(a^2 + a*b)*cosh(d*x + c)^6 + 4*(7*a^2*cosh
(d*x + c)^2 - 2*a^2 - 2*a*b)*sinh(d*x + c)^6 + 8*(7*a^2*cosh(d*x + c)^3 - 6
*(a^2 + a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^2*cosh(d*x + c)^4 + 4
*(3*a^2 + 12*a*b + 8*b^2)*d*x - 60*(a^2 + a*b)*cosh(d*x + c)^2)*sinh(d*x +
c)^4 + 8*(7*a^2*cosh(d*x + c)^5 + 4*(3*a^2 + 12*a*b + 8*b^2)*d*x*cosh(d*x +
 c) - 20*(a^2 + a*b)*cosh(d*x + c)^3)*sinh(d*x + c)^3 + 8*(a^2 + a*b)*cosh(
d*x + c)^2 + 4*(7*a^2*cosh(d*x + c)^6 + 12*(3*a^2 + 12*a*b + 8*b^2)*d*x*cos
h(d*x + c)^2 - 30*(a^2 + a*b)*cosh(d*x + c)^4 + 2*a^2 + 2*a*b)*sinh(d*x + c
)^2 - 64*((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)^3*sinh(d*x + c)
 + 6*(a + b)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a + b)*cosh(d*x + c)*sinh
(d*x + c)^3 + (a + b)*sinh(d*x + c)^4)*sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(
d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)
*sqrt(-a*b - b^2)/(a*b + b^2)) - a^2 + 8*(a^2*cosh(d*x + c)^7 + 4*(3*a^2 +
12*a*b + 8*b^2)*d*x*cosh(d*x + c)^3 - 6*(a^2 + a*b)*cosh(d*x + c)^5 + 2*(a^
2 + a*b)*cosh(d*x + c))*sinh(d*x + c))/(a^3*d*cosh(d*x + c)^4 + 4*a^3*d*cos
h(d*x + c)^3*sinh(d*x + c) + 6*a^3*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a^
3*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*d*sinh(d*x + c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17853, size = 312, normalized size = 2.67

$$\frac{(3a^2+12ab+8b^2)(dx+c)}{8a^3d} - \frac{(18a^2e^{(4dx+4c)}+72abe^{(4dx+4c)}+48b^2e^{(4dx+4c)}-8a^2e^{(2dx+2c)}-8abe^{(2dx+2c)}+a^2)e^{(-4dx}}{64a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(3*a^2 + 12*a*b + 8*b^2)*(d*x + c)/(a^3*d) - 1/64*(18*a^2*e^(4*d*x + 4*
c) + 72*a*b*e^(4*d*x + 4*c) + 48*b^2*e^(4*d*x + 4*c) - 8*a^2*e^(2*d*x + 2*c
) - 8*a*b*e^(2*d*x + 2*c) + a^2)*e^(-4*d*x - 4*c)/(a^3*d) - (a^2*b + 2*a*b^
2 + b^3)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-
a*b - b^2)*a^3*d) + 1/64*(a*d*e^(4*d*x + 4*c) - 8*a*d*e^(2*d*x + 2*c) - 8*b
*d*e^(2*d*x + 2*c))/(a^2*d^2)
```

3.26 $\int \frac{\sinh^3(c+dx)}{a+b\,\mathbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=71

$$-\frac{(a+b)\cosh(c+dx)}{a^2d}+\frac{\sqrt{b}(a+b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{a^{5/2}d}+\frac{\cosh^3(c+dx)}{3ad}$$

```
[Out] (Sqrt[b]*(a + b)*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(a^(5/2)*d) - ((a
 + b)*Cosh[c + d*x])/(a^2*d) + Cosh[c + d*x]^3/(3*a*d)
```

Rubi [A] time = 0.10468, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4133, 459, 321, 205}

$$-\frac{(a+b)\cosh(c+dx)}{a^2d}+\frac{\sqrt{b}(a+b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{a^{5/2}d}+\frac{\cosh^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (Sqrt[b]*(a + b)*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(a^(5/2)*d) - ((a
 + b)*Cosh[c + d*x])/(a^2*d) + Cosh[c + d*x]^3/(3*a*d)
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{x^2(1-x^2)}{b+ax^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\cosh^3(c+dx)}{3ad} - \frac{(a+b)\,\text{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx, x, \cosh(c+dx)\right)}{ad}$$

$$= -\frac{(a+b)\cosh(c+dx)}{a^2d} + \frac{\cosh^3(c+dx)}{3ad} + \frac{(b(a+b))\,\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cosh(c+dx)\right)}{a^2d}$$

$$= \frac{\sqrt{b}(a+b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{a^{5/2}d} - \frac{(a+b)\cosh(c+dx)}{a^2d} + \frac{\cosh^3(c+dx)}{3ad}$$

Mathematica [C] time = 2.14591, size = 372, normalized size = 5.24

$$(a\cosh(2(c+dx))+a+2b)\left(3\left(a^2+8ab+8b^2\right)\tan^{-1}\left(\frac{\sinh(c)\tanh\left(\frac{dx}{2}\right)\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\right)+\cosh(c)\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\cosh(\right.}{\sqrt{b}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*(3*(a^2 + 8*a*b + 8*b^2)*ArcTan[((Sqrt[a]
- I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c
]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt
[b]] + 3*(a^2 + 8*a*b + 8*b^2)*ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[
c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*
Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] - 3*a^2*(ArcTan[(Sqrt[
a] - I*Sqrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]] + ArcTan[(Sqrt[a] + I*Sqrt[a
 + b]*Tanh[(c + d*x)/2])/Sqrt[b]]) - 6*Sqrt[a]*Sqrt[b]*(3*a + 4*b)*Cosh[c +
 d*x] + 2*a^(3/2)*Sqrt[b]*Cosh[3*(c + d*x)]))/(48*a^(5/2)*Sqrt[b]*d*(b + a*
Cosh[c + d*x]^2))
```

Maple [B] time = 0.061, size = 261, normalized size = 3.7

$$\frac{1}{3\,da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3} - \frac{1}{2\,da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2} - \frac{1}{2\,da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1} - \frac{b}{da^2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/3/d/a/(tanh(1/2*d*x+1/2*c)+1)^3-1/2/d/a/(tanh(1/2*d*x+1/2*c)+1)^2-1/2/d/a
/(tanh(1/2*d*x+1/2*c)+1)-1/d/a^2/(tanh(1/2*d*x+1/2*c)+1)*b+1/d*b/a/(a*b)^(1
/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))+1/d*b^2
/a^2/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(
1/2))-1/3/d/a/(tanh(1/2*d*x+1/2*c)-1)^3-1/2/d/a/(tanh(1/2*d*x+1/2*c)-1)^2+1
/2/d/a/(tanh(1/2*d*x+1/2*c)-1)+1/d/a^2/(tanh(1/2*d*x+1/2*c)-1)*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(3\left(3\,ae^{(4\,c)}+4\,be^{(4\,c)}\right)e^{(4\,dx)}+3\left(3\,ae^{(2\,c)}+4\,be^{(2\,c)}\right)e^{(2\,dx)}-ae^{(6\,dx+6\,c)}-a\right)e^{(-3\,dx-3\,c)}}{24\,a^2d}+\frac{1}{8}\int\frac{16\left(\left(abe^{(3\,c)}+b^2e^{(3\,c)}\right)e^{(3\,dx)}-\right.}{a^3e^{(4\,dx+4\,c)}+a^3+2\left(a^3e^{(2\,c)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/24*(3*(3*a*e^(4*c) + 4*b*e^(4*c))*e^(4*d*x) + 3*(3*a*e^(2*c) + 4*b*e^(2*
c))*e^(2*d*x) - a*e^(6*d*x + 6*c) - a)*e^(-3*d*x - 3*c)/(a^2*d) + 1/8*integ
rate(16*((a*b*e^(3*c) + b^2*e^(3*c))*e^(3*d*x) - (a*b*e^c + b^2*e^c)*e^(d*x
))/(a^3*e^(4*d*x + 4*c) + a^3 + 2*(a^3*e^(2*c) + 2*a^2*b*e^(2*c))*e^(2*d*x)
), x)
```

Fricas [B] time = 2.95373, size = 3343, normalized size = 47.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/24*(a*cosh(d*x + c)^6 + 6*a*cosh(d*x + c)*sinh(d*x + c)^5 + a*sinh(d*x +
 c)^6 - 3*(3*a + 4*b)*cosh(d*x + c)^4 + 3*(5*a*cosh(d*x + c)^2 - 3*a - 4*b)
*sinh(d*x + c)^4 + 4*(5*a*cosh(d*x + c)^3 - 3*(3*a + 4*b)*cosh(d*x + c))*si
nh(d*x + c)^3 - 3*(3*a + 4*b)*cosh(d*x + c)^2 + 3*(5*a*cosh(d*x + c)^4 - 6*
(3*a + 4*b)*cosh(d*x + c)^2 - 3*a - 4*b)*sinh(d*x + c)^2 + 12*((a + b)*cosh
(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a + b)*cosh(d*x
+ c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3)*sqrt(-b/a)*log((a*cosh(d*x
+ c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a - 2*b
)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a - 2*b)*sinh(d*x + c)^2 + 4*(
a*cosh(d*x + c)^3 + (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x
+ c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + a*cosh(d*x
 + c) + (3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))*sqrt(-b/a) + a)/(a*cosh(d*
x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2
*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4
*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 6*(a*c
osh(d*x + c)^5 - 2*(3*a + 4*b)*cosh(d*x + c)^3 - (3*a + 4*b)*cosh(d*x + c))
*sinh(d*x + c) + a)/(a^2*d*cosh(d*x + c)^3 + 3*a^2*d*cosh(d*x + c)^2*sinh(d
*x + c) + 3*a^2*d*cosh(d*x + c)*sinh(d*x + c)^2 + a^2*d*sinh(d*x + c)^3), 1
/24*(a*cosh(d*x + c)^6 + 6*a*cosh(d*x + c)*sinh(d*x + c)^5 + a*sinh(d*x + c
)^6 - 3*(3*a + 4*b)*cosh(d*x + c)^4 + 3*(5*a*cosh(d*x + c)^2 - 3*a - 4*b)*s
inh(d*x + c)^4 + 4*(5*a*cosh(d*x + c)^3 - 3*(3*a + 4*b)*cosh(d*x + c))*sinh
(d*x + c)^3 - 3*(3*a + 4*b)*cosh(d*x + c)^2 + 3*(5*a*cosh(d*x + c)^4 - 6*(3
*a + 4*b)*cosh(d*x + c)^2 - 3*a - 4*b)*sinh(d*x + c)^2 - 24*((a + b)*cosh(d
*x + c)^3 + 3*(a + b)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a + b)*cosh(d*x +
c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3)*sqrt(b/a)*arctan(1/2*(a*cosh(
d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (a + 4
*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a + 4*b)*sinh(d*x + c))*sqrt(b/a
)/b) + 24*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)^2*sinh(d*x + c
) + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3)*sqrt
(b/a)*arctan(1/2*(a*cosh(d*x + c) + a*sinh(d*x + c))*sqrt(b/a)/b) + 6*(a*co
sh(d*x + c)^5 - 2*(3*a + 4*b)*cosh(d*x + c)^3 - (3*a + 4*b)*cosh(d*x + c))*
sinh(d*x + c) + a)/(a^2*d*cosh(d*x + c)^3 + 3*a^2*d*cosh(d*x + c)^2*sinh(d*
x + c) + 3*a^2*d*cosh(d*x + c)*sinh(d*x + c)^2 + a^2*d*sinh(d*x + c)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^3(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(sinh(c + d*x)**3/(a + b*sech(c + d*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.27 $\int \frac{\sinh^2(c+dx)}{a+b\,\mathbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=75

$$\frac{\sqrt{b}\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^2d}-\frac{x(a+2b)}{2a^2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2ad}$$

```
[Out] -((a + 2*b)*x)/(2*a^2) + (Sqrt[b]*Sqrt[a + b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x
])/Sqrt[a + b]])/(a^2*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.110428, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4132, 471, 522, 206, 208}

$$\frac{\sqrt{b}\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^2d}-\frac{x(a+2b)}{2a^2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((a + 2*b)*x)/(2*a^2) + (Sqrt[b]*Sqrt[a + b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x
])/Sqrt[a + b]])/(a^2*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*a*d)
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)^2(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad} - \frac{\text{Subst}\left(\int \frac{a+b+bx^2}{(1-x^2)(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{2ad}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad} + \frac{(b(a+b))\,\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \tanh(c+dx)\right)}{a^2d} - \frac{(a+2b)\,\text{S}}{}$$

$$= -\frac{(a+2b)x}{2a^2} + \frac{\sqrt{b}\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^2d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2ad}$$

Mathematica [B] time = 0.903549, size = 236, normalized size = 3.15

$$\frac{\operatorname{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\frac{\frac{(a^2+8ab+8b^2)(\cosh(2c)-\sinh(2c))\tanh^{-1}\left(\frac{(\cosh(2c)-\sinh(2c))\operatorname{sech}(dx)((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)}{d\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}} - 4x(}{a^2}\right.}{16\left(a+b\operatorname{sech}^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(-(ArcTanh[(Sqrt[b]*Tanh[c
 + d*x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*d)) + (-4*(a + 2*b)*x + ((a^2 +
8*a*b + 8*b^2)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d
*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(C
osh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (2*a
*Cosh[2*d*x]*Sinh[2*c])/d + (2*a*Cosh[2*c]*Sinh[2*d*x])/d)/a^2))/(16*(a + b
*Sech[c + d*x]^2))
```

Maple [B] time = 0.068, size = 383, normalized size = 5.1

$$-\frac{1}{2da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2} + \frac{1}{2da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1} - \frac{1}{2da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right) - \frac{b}{da^2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/2/d/a/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/a/(tanh(1/2*d*x+1/2*c)+1)-1/2/d/a*
ln(tanh(1/2*d*x+1/2*c)+1)-1/d/a^2*ln(tanh(1/2*d*x+1/2*c)+1)*b+1/2/d*b^(1/2)
```

```
/a/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b
^(1/2)+(a+b)^(1/2))-1/2/d*b^(1/2)/a/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x
+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/2/d*b^(3/2)/a^2/(a+b
)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+
(a+b)^(1/2))-1/2/d*b^(3/2)/a^2/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*
c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/2/d/a/(tanh(1/2*d*x+1/2*c
)-1)^2+1/2/d/a/(tanh(1/2*d*x+1/2*c)-1)+1/2/d/a*ln(tanh(1/2*d*x+1/2*c)-1)+1/
d/a^2*ln(tanh(1/2*d*x+1/2*c)-1)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.85626, size = 2132, normalized size = 28.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(a + 2*b)*d*x*cosh(d*x + c)^2 - a*cosh(d*x + c)^4 - 4*a*cosh(d*x +
 c)*sinh(d*x + c)^3 - a*sinh(d*x + c)^4 + 2*(2*(a + 2*b)*d*x - 3*a*cosh(d*x
 + c)^2)*sinh(d*x + c)^2 - 4*sqrt(a*b + b^2)*(cosh(d*x + c)^2 + 2*cosh(d*x
+ c)*sinh(d*x + c) + sinh(d*x + c)^2)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh
(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x
+ c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*
a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*
x + c) - 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*
x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)
*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a
*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2
*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 4*(2*(a + 2*b)*d*x*cosh(d*x + c) -
 a*cosh(d*x + c)^3)*sinh(d*x + c) + a)/(a^2*d*cosh(d*x + c)^2 + 2*a^2*d*cos
h(d*x + c)*sinh(d*x + c) + a^2*d*sinh(d*x + c)^2), -1/8*(4*(a + 2*b)*d*x*co
sh(d*x + c)^2 - a*cosh(d*x + c)^4 - 4*a*cosh(d*x + c)*sinh(d*x + c)^3 - a*s
inh(d*x + c)^4 + 2*(2*(a + 2*b)*d*x - 3*a*cosh(d*x + c)^2)*sinh(d*x + c)^2
- 8*sqrt(-a*b - b^2)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sin
h(d*x + c)^2)*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x +
c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-a*b - b^2)/(a*b + b^2)) + 4*(2*(a +
 2*b)*d*x*cosh(d*x + c) - a*cosh(d*x + c)^3)*sinh(d*x + c) + a)/(a^2*d*cosh
(d*x + c)^2 + 2*a^2*d*cosh(d*x + c)*sinh(d*x + c) + a^2*d*sinh(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(sinh(c + d*x)**2/(a + b*sech(c + d*x)**2), x)
```

Giac [B] time = 1.21563, size = 186, normalized size = 2.48

$$-\frac{(dx+c)(a+2b)}{2a^2d}+\frac{e^{(2dx+2c)}}{8ad}+\frac{\left(2ae^{(2dx+2c)}+4be^{(2dx+2c)}-a\right)e^{(-2dx-2c)}}{8a^2d}+\frac{\left(ab+b^2\right)\arctan\left(\frac{ae^{(2dx+2c)}+a+2b}{2\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*(d*x + c)*(a + 2*b)/(a^2*d) + 1/8*e^(2*d*x + 2*c)/(a*d) + 1/8*(2*a*e^(
2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) - a)*e^(-2*d*x - 2*c)/(a^2*d) + (a*b + b
^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b -
 b^2)*a^2*d)
```

3.28 $\int \frac{\sinh(c+dx)}{a+b\,\mathbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=47

$$\frac{\cosh(c+dx)}{ad}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{a^{3/2}d}$$

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(a^(3/2)*d)) + Cosh[c +
 d*x]/(a*d)
```

Rubi [A] time = 0.0484901, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4133, 321, 205}

$$\frac{\cosh(c+dx)}{ad}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(a^(3/2)*d)) + Cosh[c +
 d*x]/(a*d)
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx = \frac{\mathrm{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\cosh(c+dx)}{ad} - \frac{b\,\mathrm{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cosh(c+dx)\right)}{ad}$$
$$= -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{a^{3/2}d} + \frac{\cosh(c+dx)}{ad}$$

Mathematica [C] time = 1.04528, size = 328, normalized size = 6.98

$$\frac{\mathrm{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\frac{a\left(\tan^{-1}\left(\frac{\sqrt{a}-i\sqrt{a+b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)+\tan^{-1}\left(\frac{\sqrt{a}+i\sqrt{a+b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)\right)}{\sqrt{b}} - \frac{(a+4b)\left(\tan^{-1}\left(\frac{\sinh(c)\tan}{\cdots}\right.\right.}{\cdots}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((-(((a + 4*b)*(ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2
])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] -
 Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] + ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sq
rt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqr
t[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]))/Sqrt[b]) +
(a*(ArcTan[(Sqrt[a] - I*Sqrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]] + ArcTan[(S
qrt[a] + I*Sqrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]]))/Sqrt[b] + 4*Sqrt[a]*Co
sh[c + d*x])*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2)/(8*a^(3/2)*d*
(a + b*Sech[c + d*x]^2))
```

Maple [A] time = 0.023, size = 44, normalized size = 0.9

$$\frac{1}{da\,\mathrm{sech}\,(dx+c)} + \frac{b}{da}\arctan\left(b\,\mathrm{sech}\,(dx+c)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/d/a/sech(d*x+c)+1/d/a*b/(a*b)^(1/2)*arctan(sech(d*x+c)*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(e^{(2\,dx+2\,c)}+1\right)e^{(-dx-c)}}{2\,ad} - \frac{1}{2}\int \frac{4\left(be^{(3\,dx+3\,c)} - be^{(dx+c)}\right)}{a^2e^{(4\,dx+4\,c)} + a^2 + 2\left(a^2e^{(2\,c)} + 2\,abe^{(2\,c)}\right)e^{(2\,dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*d*x + 2*c) + 1)*e^(-d*x - c)/(a*d) - 1/2*integrate(4*(b*e^(3*d*x
+ 3*c) - b*e^(d*x + c))/(a^2*e^(4*d*x + 4*c) + a^2 + 2*(a^2*e^(2*c) + 2*a*b
*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.78275, size = 1643, normalized size = 34.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-b/a)*(cosh(d*x + c) + sinh(d*x + c))*log((a*cosh(d*x + c)^4 + 4
*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a - 2*b)*cosh(d*x
 + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x
 + c)^3 + (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(d*x + c)^3 + 3
*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + a*cosh(d*x + c) + (3
*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))*sqrt(-b/a) + a)/(a*cosh(d*x + c)^4 +
 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d
*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d
*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + cosh(d*x + c)^2
+ 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)/(a*d*cosh(d*x + c) +
 a*d*sinh(d*x + c)), 1/2*(2*sqrt(b/a)*(cosh(d*x + c) + sinh(d*x + c))*arcta
n(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x +
 c)^3 + (a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a + 4*b)*sinh(d*x
+ c))*sqrt(b/a)/b) - 2*sqrt(b/a)*(cosh(d*x + c) + sinh(d*x + c))*arctan(1/2
*(a*cosh(d*x + c) + a*sinh(d*x + c))*sqrt(b/a)/b) + cosh(d*x + c)^2 + 2*cos
h(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)/(a*d*cosh(d*x + c) + a*d*si
nh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(c + dx)}{a + b\operatorname{sech}^2(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(sinh(c + d*x)/(a + b*sech(c + d*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.29 $\int \frac{\operatorname{csch}(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=55

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}d(a+b)}-\frac{\tanh^{-1}(\cosh(c+dx))}{d(a+b)}$$

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(Sqrt[a]*(a + b)*d) - Arc
Tanh[Cosh[c + d*x]]/((a + b)*d)
```

Rubi [A] time = 0.081834, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4133, 481, 206, 205}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}d(a+b)}-\frac{\tanh^{-1}(\cosh(c+dx))}{d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(Sqrt[a]*(a + b)*d) - Arc
Tanh[Cosh[c + d*x]]/((a + b)*d)
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)(b+ax^2)}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(c+dx)\right)}{(a+b)d} + \frac{b\,\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cosh(c+dx)\right)}{(a+b)d}$$

$$= \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)d} - \frac{\tanh^{-1}(\cosh(c+dx))}{(a+b)d}$$

Mathematica [C] time = 0.848711, size = 232, normalized size = 4.22

$$\frac{\frac{\sqrt{b}\tan^{-1}\left(\frac{\sinh(c)\tanh\left(\frac{dx}{2}\right)\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\right)+\cosh(c)\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\tanh\left(\frac{dx}{2}\right)\right)}{\sqrt{b}}\right)}{\sqrt{a}} + \frac{\sqrt{b}\tan^{-1}\left(\frac{\sinh(c)\tanh\left(\frac{dx}{2}\right)\left(\sqrt{a}+i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh\ldots}\right.}{}\right.}{}}{d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((Sqrt[b]*ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sin
h[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[
c])^2]*Tanh[(d*x)/2]))/Sqrt[b]])/Sqrt[a] + (Sqrt[b]*ArcTan[((Sqrt[a] + I*Sq
rt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqr
t[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]])/
Sqrt[a] - Log[Cosh[(c + d*x)/2]] + Log[Sinh[(c + d*x)/2]])/((a + b)*d)
```

Maple [A] time = 0.051, size = 67, normalized size = 1.2

$$\frac{b}{d(a+b)}\arctan\left(\frac{1}{4}\left(2\,(a+b)(\tanh(1/2\,dx+c/2))^2+2\,a-2\,b\right)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + \frac{1}{d(a+b)}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/d*b/(a+b)/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/
(a*b)^(1/2))+1/d/(a+b)*ln(tanh(1/2*d*x+1/2*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(\left(e^{(dx+c)}+1\right)e^{(-c)}\right)}{ad+bd} + \frac{\log\left(\left(e^{(dx+c)}-1\right)e^{(-c)}\right)}{ad+bd} + 2\int \frac{be^{(3\,dx+3\,c)} - be^{(dx+c)}}{a^2+ab+\left(a^2e^{(4\,c)}+abe^{(4\,c)}\right)e^{(4\,dx)}+2\left(a^2e^{(2\,c)}+3\,abe^{(2\,c)}+2\,b^2e^{\ldots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -log((e^(d*x + c) + 1)*e^(-c))/(a*d + b*d) + log((e^(d*x + c) - 1)*e^(-c))/
(a*d + b*d) + 2*integrate((b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^2 + a*b +
```

```
(a^2*e^(4*c) + a*b*e^(4*c))*e^(4*d*x) + 2*(a^2*e^(2*c) + 3*a*b*e^(2*c) + 2*
b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.88902, size = 1461, normalized size = 26.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-b/a)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3
 + a*sinh(d*x + c)^4 + 2*(a - 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2
 + a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a - 2*b)*cosh(d*x + c
))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2
 + a*sinh(d*x + c)^3 + a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a)*sinh(d*x
 + c))*sqrt(-b/a) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)
^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)
^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x +
 c))*sinh(d*x + c) + a)) - 2*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 2*log
(cosh(d*x + c) + sinh(d*x + c) - 1))/((a + b)*d), -(sqrt(b/a)*arctan(1/2*(a
*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 +
(a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a + 4*b)*sinh(d*x + c))*sq
rt(b/a)/b) - sqrt(b/a)*arctan(1/2*(a*cosh(d*x + c) + a*sinh(d*x + c))*sqrt(
b/a)/b) + log(cosh(d*x + c) + sinh(d*x + c) + 1) - log(cosh(d*x + c) + sinh
(d*x + c) - 1))/((a + b)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(csch(c + d*x)/(a + b*sech(c + d*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.30 $\int \frac{\operatorname{csch}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{d(a+b)^{3/2}}-\frac{\coth(c+dx)}{d(a+b)}$$

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/((a + b)^(3/2)*d) -
Coth[c + d*x]/((a + b)*d)
```

Rubi [A] time = 0.0709458, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4132, 325, 208}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{d(a+b)^{3/2}}-\frac{\coth(c+dx)}{d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/((a + b)^(3/2)*d) -
Coth[c + d*x]/((a + b)*d)
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{\coth(c+dx)}{(a+b)d} + \frac{b\operatorname{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)d}$$
$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}d} - \frac{\coth(c+dx)}{(a+b)d}$$

Mathematica [B] time = 0.70113, size = 179, normalized size = 3.38

$$\frac{\operatorname{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\sqrt{a+b}\operatorname{csch}(c)\sinh(dx)\sqrt{b(\cosh(c)-\sinh(c))^4}\operatorname{csch}(c+dx)+b(\cosh(2c)\right.}{2d(a+b)^{3/2}\sqrt{b(\cosh(c)-\sinh(c))^4}\left(a+b\operatorname{sech}^2(c+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(b*ArcTanh[(Sech[d*x]*(Cos
h[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a +
b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]) + Sqrt[a + b]*Cs
ch[c]*Csch[c + d*x]*Sqrt[b*(Cosh[c] - Sinh[c])^4]*Sinh[d*x]))/(2*(a + b)^(3
/2)*d*(a + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])
```

Maple [B] time = 0.061, size = 147, normalized size = 2.8

$$-\frac{1}{2d(a+b)}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{2d}\sqrt{b}\ln\left(\sqrt{a+b}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+2\tanh(1/2\,dx+c/2)\sqrt{b}+\sqrt{a+b}\right)(a+b)^{-\frac{3}{2}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/2/d/(a+b)*tanh(1/2*d*x+1/2*c)+1/2/d*b^(1/2)/(a+b)^(3/2)*ln((a+b)^(1/2)*t
anh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-1/2/d*b^(1/
2)/(a+b)^(3/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*
b^(1/2)-(a+b)^(1/2))-1/2/d/(a+b)/tanh(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.80905, size = 1589, normalized size = 29.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 -
1)*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x
+ c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*c
osh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^
2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a^2 +
a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a
*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c
)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*c
osh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*c
osh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - 4)/((a + b)
*d*cosh(d*x + c)^2 + 2*(a + b)*d*cosh(d*x + c)*sinh(d*x + c) + (a + b)*d*si
nh(d*x + c)^2 - (a + b)*d), ((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x +
c) + sinh(d*x + c)^2 - 1)*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 +
2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a +
 b))/b) - 2)/((a + b)*d*cosh(d*x + c)^2 + 2*(a + b)*d*cosh(d*x + c)*sinh(d*
x + c) + (a + b)*d*sinh(d*x + c)^2 - (a + b)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^{2}(c+dx)}{a+b\operatorname{sech}^{2}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(csch(c + d*x)**2/(a + b*sech(c + d*x)**2), x)
```

Giac [A] time = 1.2643, size = 107, normalized size = 2.02

$$\frac{b\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}(ad+bd)}-\frac{2}{(ad+bd)\left(e^{(2\,dx+2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] b*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b - b
^2)*(a*d + b*d)) - 2/((a*d + b*d)*(e^(2*d*x + 2*c) - 1))
```

3.31 $\int \frac{\operatorname{csch}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=87

$$-\frac{\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{d(a+b)^2}+\frac{(a-b)\tanh^{-1}(\cosh(c+dx))}{2d(a+b)^2}-\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2d(a+b)}$$

```
[Out] -((Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/((a + b)^2*d))
+ ((a - b)*ArcTanh[Cosh[c + d*x]])/(2*(a + b)^2*d) - (Coth[c + d*x]*Csch[c
+ d*x])/(2*(a + b)*d)
```

Rubi [A] time = 0.11841, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4133, 471, 522, 206, 205}

$$-\frac{\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{d(a+b)^2}+\frac{(a-b)\tanh^{-1}(\cosh(c+dx))}{2d(a+b)^2}-\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/((a + b)^2*d))
+ ((a - b)*ArcTanh[Cosh[c + d*x]])/(2*(a + b)^2*d) - (Coth[c + d*x]*Csch[c
+ d*x])/(2*(a + b)*d)
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)^2(b+ax^2)}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2(a+b)d} - \frac{\operatorname{Subst}\left(\int \frac{b-ax^2}{(1-x^2)(b+ax^2)}\,dx, x, \cosh(c+dx)\right)}{2(a+b)d}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2(a+b)d} + \frac{(a-b)\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(c+dx)\right)}{2(a+b)^2d} - \frac{(ab)\operatorname{Subst}\left(\int \frac{1}{b+\cdots}\right.}{(\cdots}$$

$$= -\frac{\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{(a+b)^2d} + \frac{(a-b)\tanh^{-1}(\cosh(c+dx))}{2(a+b)^2d} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2(a+b)d}$$

Mathematica [C] time = 1.95443, size = 338, normalized size = 3.89

$$-\frac{\operatorname{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b)\left((a+b)\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)+(a+b)\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)+8\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sinh(c\cdots}{\cdots}\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((a + 2*b + a*Cosh[2*(c + d*x)])*(8*Sqrt[a]*Sqrt[b]*ArcTan[((Sqrt[a] - I*S
qrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sq
rt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]
+ 8*Sqrt[a]*Sqrt[b]*ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c
])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[
c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] + (a + b)*Csch[(c + d*x)/2]^2 - 4
*a*Log[Cosh[(c + d*x)/2]] + 4*b*Log[Cosh[(c + d*x)/2]] + 4*a*Log[Sinh[(c +
d*x)/2]] - 4*b*Log[Sinh[(c + d*x)/2]] + (a + b)*Sech[(c + d*x)/2]^2)*Sech[c
 + d*x]^2)/(16*(a + b)^2*d*(a + b*Sech[c + d*x]^2))
```

Maple [A] time = 0.058, size = 134, normalized size = 1.5

$$\frac{1}{8d(a+b)}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 - \frac{ab}{d(a+b)^2}\arctan\left(\frac{1}{4}\left(2\,(a+b)\,(\tanh(1/2\,dx+c/2))^2+2\,a-2\,b\right)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} - \frac{1}{8d(a+\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/8/d*tanh(1/2*d*x+1/2*c)^2/(a+b)-1/d*a*b/(a+b)^2/(a*b)^(1/2)*arctan(1/4*(2
*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))-1/8/d/(a+b)/tanh(1/2*d*x
```

```
+1/2*c)^2-1/2/d/(a+b)^2*ln(tanh(1/2*d*x+1/2*c))*a+1/2/d/(a+b)^2*ln(tanh(1/2
*d*x+1/2*c))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(a-b)\log\left(\left(e^{(dx+c)}+1\right)e^{(-c)}\right)}{2\left(a^2d+2\,abd+b^2d\right)}-\frac{(a-b)\log\left(\left(e^{(dx+c)}-1\right)e^{(-c)}\right)}{2\left(a^2d+2\,abd+b^2d\right)}-\frac{e^{(3\,dx+3\,c)}+e^{(dx+c)}}{ad+bd+\left(ade^{(4\,c)}+bde^{(4\,c)}\right)e^{(4\,dx)}-2\left(ade^{(2\,c)}+bde^{(2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(a - b)*log((e^(d*x + c) + 1)*e^(-c))/(a^2*d + 2*a*b*d + b^2*d) - 1/2*(
a - b)*log((e^(d*x + c) - 1)*e^(-c))/(a^2*d + 2*a*b*d + b^2*d) - (e^(3*d*x
+ 3*c) + e^(d*x + c))/(a*d + b*d + (a*d*e^(4*c) + b*d*e^(4*c))*e^(4*d*x) -
2*(a*d*e^(2*c) + b*d*e^(2*c))*e^(2*d*x)) - 8*integrate(1/4*(a*b*e^(3*d*x +
3*c) - a*b*e^(d*x + c))/(a^3 + 2*a^2*b + a*b^2 + (a^3*e^(4*c) + 2*a^2*b*e^(
4*c) + a*b^2*e^(4*c))*e^(4*d*x) + 2*(a^3*e^(2*c) + 4*a^2*b*e^(2*c) + 5*a*b^
2*e^(2*c) + 2*b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 3.18611, size = 5080, normalized size = 58.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a + b)*cosh(d*x + c)^3 + 6*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2
+ 2*(a + b)*sinh(d*x + c)^3 - (cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x +
 c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cos
h(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(
-a*b)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d
*x + c)^4 + 2*(a - 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a - 2*b)
*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a - 2*b)*cosh(d*x + c))*sinh(d*x
 + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c
)^3 + (3*cosh(d*x + c)^2 + 1)*sinh(d*x + c) + cosh(d*x + c))*sqrt(-a*b) + a
)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^
4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*
x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) +
a)) + 2*(a + b)*cosh(d*x + c) - ((a - b)*cosh(d*x + c)^4 + 4*(a - b)*cosh(d
*x + c)*sinh(d*x + c)^3 + (a - b)*sinh(d*x + c)^4 - 2*(a - b)*cosh(d*x + c)
^2 + 2*(3*(a - b)*cosh(d*x + c)^2 - a + b)*sinh(d*x + c)^2 + 4*((a - b)*cos
h(d*x + c)^3 - (a - b)*cosh(d*x + c))*sinh(d*x + c) + a - b)*log(cosh(d*x +
 c) + sinh(d*x + c) + 1) + ((a - b)*cosh(d*x + c)^4 + 4*(a - b)*cosh(d*x +
c)*sinh(d*x + c)^3 + (a - b)*sinh(d*x + c)^4 - 2*(a - b)*cosh(d*x + c)^2 +
2*(3*(a - b)*cosh(d*x + c)^2 - a + b)*sinh(d*x + c)^2 + 4*((a - b)*cosh(d*x
 + c)^3 - (a - b)*cosh(d*x + c))*sinh(d*x + c) + a - b)*log(cosh(d*x + c) +
 sinh(d*x + c) - 1) + 2*(3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))/
((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*d*cosh(d*x +
 c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*sinh(d*x + c)^4 - 2*(a^2 + 2*a*
b + b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 - (
a^2 + 2*a*b + b^2)*d)*sinh(d*x + c)^2 + (a^2 + 2*a*b + b^2)*d + 4*((a^2 + 2
*a*b + b^2)*d*cosh(d*x + c)^3 - (a^2 + 2*a*b + b^2)*d*cosh(d*x + c))*sinh(d
```

```
*x + c)), -1/2*(2*(a + b)*cosh(d*x + c)^3 + 6*(a + b)*cosh(d*x + c)*sinh(d*
x + c)^2 + 2*(a + b)*sinh(d*x + c)^3 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)
*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c
)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c)
 + 1)*sqrt(a*b)*arctan(1/2*sqrt(a*b)*(cosh(d*x + c) + sinh(d*x + c))/b) - 2
*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(
3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x +
c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(a*b)*arctan(1/2*(a*cosh(d*x +
 c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (a + 4*b)*c
osh(d*x + c) + (3*a*cosh(d*x + c)^2 + a + 4*b)*sinh(d*x + c))*sqrt(a*b)/(a*
b)) + 2*(a + b)*cosh(d*x + c) - ((a - b)*cosh(d*x + c)^4 + 4*(a - b)*cosh(d
*x + c)*sinh(d*x + c)^3 + (a - b)*sinh(d*x + c)^4 - 2*(a - b)*cosh(d*x + c)
^2 + 2*(3*(a - b)*cosh(d*x + c)^2 - a + b)*sinh(d*x + c)^2 + 4*((a - b)*cos
h(d*x + c)^3 - (a - b)*cosh(d*x + c))*sinh(d*x + c) + a - b)*log(cosh(d*x +
 c) + sinh(d*x + c) + 1) + ((a - b)*cosh(d*x + c)^4 + 4*(a - b)*cosh(d*x +
c)*sinh(d*x + c)^3 + (a - b)*sinh(d*x + c)^4 - 2*(a - b)*cosh(d*x + c)^2 +
2*(3*(a - b)*cosh(d*x + c)^2 - a + b)*sinh(d*x + c)^2 + 4*((a - b)*cosh(d*x
 + c)^3 - (a - b)*cosh(d*x + c))*sinh(d*x + c) + a - b)*log(cosh(d*x + c) +
 sinh(d*x + c) - 1) + 2*(3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))/
((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*d*cosh(d*x +
 c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*sinh(d*x + c)^4 - 2*(a^2 + 2*a*
b + b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 - (
a^2 + 2*a*b + b^2)*d)*sinh(d*x + c)^2 + (a^2 + 2*a*b + b^2)*d + 4*((a^2 + 2
*a*b + b^2)*d*cosh(d*x + c)^3 - (a^2 + 2*a*b + b^2)*d*cosh(d*x + c))*sinh(d
*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(csch(c + d*x)**3/(a + b*sech(c + d*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.32 $\int \frac{\mathrm{csch}^4(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=75

$$-\frac{a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{d(a+b)^{5/2}}-\frac{\coth^3(c+dx)}{3d(a+b)}+\frac{a\coth(c+dx)}{d(a+b)^2}$$

```
[Out] -((a*Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/((a + b)^(5/2)*d
)) + (a*Coth[c + d*x])/((a + b)^2*d) - Coth[c + d*x]^3/(3*(a + b)*d)
```

Rubi [A] time = 0.0989715, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4132, 453, 325, 208}

$$-\frac{a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{d(a+b)^{5/2}}-\frac{\coth^3(c+dx)}{3d(a+b)}+\frac{a\coth(c+dx)}{d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```


```
[Out] -((a*Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/((a + b)^(5/2)*d
)) + (a*Coth[c + d*x])/((a + b)^2*d) - Coth[c + d*x]^3/(3*(a + b)*d)
```


Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{x^4(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{\coth^3(c+dx)}{3(a+b)d} - \frac{a\operatorname{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{(a+b)d}$$

$$= \frac{a\coth(c+dx)}{(a+b)^2d} - \frac{\coth^3(c+dx)}{3(a+b)d} - \frac{(ab)\operatorname{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)^2d}$$

$$= -\frac{a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}d} + \frac{a\coth(c+dx)}{(a+b)^2d} - \frac{\coth^3(c+dx)}{3(a+b)d}$$

Mathematica [B] time = 2.0267, size = 216, normalized size = 2.88

$$\frac{\operatorname{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\frac{1}{4}\sqrt{a+b}\operatorname{csch}(c)\sqrt{b(\cosh(c)-\sinh(c))^4}\operatorname{csch}^3(c+dx)((b-2a)\sinh(2c+3dx)\right.}{6d(a+b)^{5/2}\sqrt{b(\cosh(c)-\mathrm{si}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(3*a*b*ArcTanh[(Sech[d*x]*
(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[
a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(-Cosh[2*c] + Sinh[2*c]) + (Sqrt[a +
 b]*Csch[c]*Csch[c + d*x]^3*Sqrt[b*(Cosh[c] - Sinh[c])^4]*(6*a*Sinh[d*x] -
3*b*Sinh[2*c + d*x] + (-2*a + b)*Sinh[2*c + 3*d*x]))/4))/(6*(a + b)^(5/2)*d
*(a + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])
```

Maple [B] time = 0.076, size = 258, normalized size = 3.4

$$-\frac{a}{24\,d\,(a+b)^2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{b}{24\,d\,(a+b)^2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 + \frac{3\,a}{8\,d\,(a+b)^2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right) - \frac{b}{8\,d\,(a+b)^2}\tanh\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/24/d/(a+b)^2*a*tanh(1/2*d*x+1/2*c)^3-1/24/d/(a+b)^2*b*tanh(1/2*d*x+1/2*c
)^3+3/8/d/(a+b)^2*a*tanh(1/2*d*x+1/2*c)-1/8/d/(a+b)^2*tanh(1/2*d*x+1/2*c)*b
-1/2/d*a*b^(1/2)/(a+b)^(5/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/
2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/2/d*a*b^(1/2)/(a+b)^(5/2)*ln(-(a+b)^(1/
2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2))-1/24/d/
(a+b)/tanh(1/2*d*x+1/2*c)^3+3/8/d/(a+b)^2/tanh(1/2*d*x+1/2*c)*a-1/8/d/(a+b)
^2/tanh(1/2*d*x+1/2*c)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.00318, size = 4602, normalized size = 61.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/6*(12*b*cosh(d*x + c)^4 + 48*b*cosh(d*x + c)*sinh(d*x + c)^3 + 12*b*sin
h(d*x + c)^4 + 24*a*cosh(d*x + c)^2 + 24*(3*b*cosh(d*x + c)^2 + a)*sinh(d*x
 + c)^2 - 3*(a*cosh(d*x + c)^6 + 6*a*cosh(d*x + c)*sinh(d*x + c)^5 + a*sinh
(d*x + c)^6 - 3*a*cosh(d*x + c)^4 + 3*(5*a*cosh(d*x + c)^2 - a)*sinh(d*x +
c)^4 + 4*(5*a*cosh(d*x + c)^3 - 3*a*cosh(d*x + c))*sinh(d*x + c)^3 + 3*a*co
sh(d*x + c)^2 + 3*(5*a*cosh(d*x + c)^4 - 6*a*cosh(d*x + c)^2 + a)*sinh(d*x
+ c)^2 + 6*(a*cosh(d*x + c)^5 - 2*a*cosh(d*x + c)^3 + a*cosh(d*x + c))*sinh
(d*x + c) - a)*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x +
c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2
+ 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8
*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)
+ 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c
) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*
cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2
*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c
)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) +
 48*(b*cosh(d*x + c)^3 + a*cosh(d*x + c))*sinh(d*x + c) - 8*a + 4*b)/((a^2
+ 2*a*b + b^2)*d*cosh(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)*si
nh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*d*sinh(d*x + c)^6 - 3*(a^2 + 2*a*b + b^
2)*d*cosh(d*x + c)^4 + 3*(5*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 - (a^2 +
2*a*b + b^2)*d)*sinh(d*x + c)^4 + 3*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 +
 4*(5*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^3 - 3*(a^2 + 2*a*b + b^2)*d*cosh(
d*x + c))*sinh(d*x + c)^3 + 3*(5*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^4 - 6*
(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 + (a^2 + 2*a*b + b^2)*d)*sinh(d*x + c
)^2 - (a^2 + 2*a*b + b^2)*d + 6*((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^5 - 2*
(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*cosh(d*x + c)
)*sinh(d*x + c)), -1/3*(6*b*cosh(d*x + c)^4 + 24*b*cosh(d*x + c)*sinh(d*x +
 c)^3 + 6*b*sinh(d*x + c)^4 + 12*a*cosh(d*x + c)^2 + 12*(3*b*cosh(d*x + c)^
2 + a)*sinh(d*x + c)^2 + 3*(a*cosh(d*x + c)^6 + 6*a*cosh(d*x + c)*sinh(d*x
+ c)^5 + a*sinh(d*x + c)^6 - 3*a*cosh(d*x + c)^4 + 3*(5*a*cosh(d*x + c)^2 -
 a)*sinh(d*x + c)^4 + 4*(5*a*cosh(d*x + c)^3 - 3*a*cosh(d*x + c))*sinh(d*x
+ c)^3 + 3*a*cosh(d*x + c)^2 + 3*(5*a*cosh(d*x + c)^4 - 6*a*cosh(d*x + c)^2
 + a)*sinh(d*x + c)^2 + 6*(a*cosh(d*x + c)^5 - 2*a*cosh(d*x + c)^3 + a*cosh
(d*x + c))*sinh(d*x + c) - a)*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^
2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/
(a + b))/b) + 24*(b*cosh(d*x + c)^3 + a*cosh(d*x + c))*sinh(d*x + c) - 4*a
+ 2*b)/((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*d*cos
h(d*x + c)*sinh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*d*sinh(d*x + c)^6 - 3*(a^2
 + 2*a*b + b^2)*d*cosh(d*x + c)^4 + 3*(5*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c
)^2 - (a^2 + 2*a*b + b^2)*d)*sinh(d*x + c)^4 + 3*(a^2 + 2*a*b + b^2)*d*cosh
(d*x + c)^2 + 4*(5*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^3 - 3*(a^2 + 2*a*b +
 b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*(a^2 + 2*a*b + b^2)*d*cosh(d*
x + c)^4 - 6*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 + (a^2 + 2*a*b + b^2)*d)
```

```
*sinh(d*x + c)^2 - (a^2 + 2*a*b + b^2)*d + 6*((a^2 + 2*a*b + b^2)*d*cosh(d*
x + c)^5 - 2*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*
cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(csch(c + d*x)**4/(a + b*sech(c + d*x)**2), x)
```

Giac [A] time = 1.28063, size = 173, normalized size = 2.31

$$-\frac{ab\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\left(a^2d+2\,abd+b^2d\right)\sqrt{-ab-b^2}}-\frac{2\left(3\,be^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}-2\,a+b\right)}{3\left(a^2d+2\,abd+b^2d\right)\left(e^{(2\,dx+2\,c)}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -a*b*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/((a^2*d + 2
*a*b*d + b^2*d)*sqrt(-a*b - b^2)) - 2/3*(3*b*e^(4*d*x + 4*c) + 6*a*e^(2*d*x
 + 2*c) - 2*a + b)/((a^2*d + 2*a*b*d + b^2*d)*(e^(2*d*x + 2*c) - 1)^3)
```

3.33 $\int \frac{\sinh^4(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=194

$$\frac{3x\left(a^2+8ab+8b^2\right)}{8a^4}-\frac{3\sqrt{b}\sqrt{a+b}(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^4d}-\frac{3b(3a+4b)\tanh(c+dx)}{8a^3d\left(a-b\tanh^2(c+dx)+b\right)}-\frac{(5a+6b)\sinh(c}{8a^2d\left(a-b\,\mathrm{ta}\right.}$$

```
[Out] (3*(a^2 + 8*a*b + 8*b^2)*x)/(8*a^4) - (3*Sqrt[b]*Sqrt[a + b]*(a + 2*b)*ArcT
anh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(2*a^4*d) - ((5*a + 6*b)*Cosh[c +
 d*x]*Sinh[c + d*x])/(8*a^2*d*(a + b - b*Tanh[c + d*x]^2)) + (Cosh[c + d*x]
^3*Sinh[c + d*x])/(4*a*d*(a + b - b*Tanh[c + d*x]^2)) - (3*b*(3*a + 4*b)*Ta
nh[c + d*x])/(8*a^3*d*(a + b - b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.273275, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4132, 470, 527, 522, 206, 208}

$$\frac{3x\left(a^2+8ab+8b^2\right)}{8a^4}-\frac{3\sqrt{b}\sqrt{a+b}(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^4d}-\frac{3b(3a+4b)\tanh(c+dx)}{8a^3d\left(a-b\tanh^2(c+dx)+b\right)}-\frac{(5a+6b)\sinh(c}{8a^2d\left(a-b\,\mathrm{ta}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (3*(a^2 + 8*a*b + 8*b^2)*x)/(8*a^4) - (3*Sqrt[b]*Sqrt[a + b]*(a + 2*b)*ArcT
anh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(2*a^4*d) - ((5*a + 6*b)*Cosh[c +
 d*x]*Sinh[c + d*x])/(8*a^2*d*(a + b - b*Tanh[c + d*x]^2)) + (Cosh[c + d*x]
^3*Sinh[c + d*x])/(4*a*d*(a + b - b*Tanh[c + d*x]^2)) - (3*b*(3*a + 4*b)*Ta
nh[c + d*x])/(8*a^3*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^3\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{a+b+(4a+5b)x^2}{(1-x^2)^2\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{4ad}$$

$$= -\frac{(5a+6b)\cosh(c+dx)\sinh(c+dx)}{8a^2d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-3(a+b}{(1}\right.}{}$$

$$= -\frac{(5a+6b)\cosh(c+dx)\sinh(c+dx)}{8a^2d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)} - \frac{3b(3a+4b)\,\mathrm{t}}{8a^3d\left(a+b-b\,\mathrm{t}\right.}$$

$$= -\frac{(5a+6b)\cosh(c+dx)\sinh(c+dx)}{8a^2d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)} - \frac{3b(3a+4b)\,\mathrm{t}}{8a^3d\left(a+b-b\,\mathrm{t}\right.}$$

$$= \frac{3\left(a^2+8ab+8b^2\right)x}{8a^4} - \frac{3\sqrt{b}\sqrt{a+b}(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^4d} - \frac{(5a+6b)\cosh(c+d}{8a^2d\left(a+b-b\tan\right.}$$

Mathematica [C] time = 14.197, size = 1330, normalized size = 6.86

$$-\frac{(\cosh(2c+2dx)a+a+2b)^2\left(16x+\frac{\left(a^3-6ba^2-24b^2a-16b^3\right)\tanh^{-1}\left(\frac{\operatorname{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)(\cosh(2c)-\sinh(2c))}{b(a+b)^{3/2}d\sqrt{b(\cosh(c)-\sinh(c))^4}} - \right.}{256a^2\left(b\operatorname{sech}^2(c+dx)+a\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -((a + 2*b + a*Cosh[2*c + 2*d*x])^2*Sech[c + d*x]^4*(16*x + ((a^3 - 6*a^2*b
 - 24*a*b^2 - 16*b^3)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)
*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^
4])]*(Cosh[2*c] - Sinh[2*c]))/(b*(a + b)^(3/2)*d*Sqrt[b*(Cosh[c] - Sinh[c])
^4]) + ((a^2 + 8*a*b + 8*b^2)*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x
]))/(b*(a + b)*d*(a + 2*b + a*Cosh[2*(c + d*x)]))))/(256*a^2*(a + b*Sech[c
+ d*x]^2)^2) + (3*(a + 2*b + a*Cosh[2*c + 2*d*x])^2*Sech[c + d*x]^4*(((a +
2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*b^(3/2)*(a + b)^(3/2)
*d) - (a*Sinh[2*(c + d*x)])/(8*b*(a + b)*d*(a + 2*b + a*Cosh[2*(c + d*x)]))
))/(128*(a + b*Sech[c + d*x]^2)^2) + ((a + 2*b + a*Cosh[2*c + 2*d*x])^2*Sec
h[c + d*x]^4*(((a^5 - 30*a^4*b - 480*a^3*b^2 - 1600*a^2*b^3 - 1920*a*b^4 -
768*b^5)*(((-I/8)*ArcTan[Sech[d*x]*(((-I/2)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b*
Cosh[4*c] - b*Sinh[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c]
 - b*Sinh[4*c]]))*(-(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x])]*Cos
h[2*c])/(a^4*b*Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) + ((I/8)*ArcT
an[Sech[d*x]*(((-I/2)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c
]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]))*(-(a
*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x])]*Sinh[2*c])/(a^4*b*Sqrt[a
+ b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]])))/(a + b) + (Sech[2*c]*(160*a^4*b*d
*x*Cosh[2*c] + 1248*a^3*b^2*d*x*Cosh[2*c] + 3392*a^2*b^3*d*x*Cosh[2*c] + 38
40*a*b^4*d*x*Cosh[2*c] + 1536*b^5*d*x*Cosh[2*c] + 80*a^4*b*d*x*Cosh[2*d*x]
+ 464*a^3*b^2*d*x*Cosh[2*d*x] + 768*a^2*b^3*d*x*Cosh[2*d*x] + 384*a*b^4*d*x
*Cosh[2*d*x] + 80*a^4*b*d*x*Cosh[4*c + 2*d*x] + 464*a^3*b^2*d*x*Cosh[4*c +
2*d*x] + 768*a^2*b^3*d*x*Cosh[4*c + 2*d*x] + 384*a*b^4*d*x*Cosh[4*c + 2*d*x
] + a^5*Sinh[2*c] + 34*a^4*b*Sinh[2*c] + 224*a^3*b^2*Sinh[2*c] + 576*a^2*b^
3*Sinh[2*c] + 640*a*b^4*Sinh[2*c] + 256*b^5*Sinh[2*c] - a^5*Sinh[2*d*x] - 6
2*a^4*b*Sinh[2*d*x] - 318*a^3*b^2*Sinh[2*d*x] - 512*a^2*b^3*Sinh[2*d*x] - 2
56*a*b^4*Sinh[2*d*x] - 30*a^4*b*Sinh[4*c + 2*d*x] - 158*a^3*b^2*Sinh[4*c +
2*d*x] - 256*a^2*b^3*Sinh[4*c + 2*d*x] - 128*a*b^4*Sinh[4*c + 2*d*x] - 12*a
^4*b*Sinh[2*c + 4*d*x] - 36*a^3*b^2*Sinh[2*c + 4*d*x] - 24*a^2*b^3*Sinh[2*c
 + 4*d*x] - 12*a^4*b*Sinh[6*c + 4*d*x] - 36*a^3*b^2*Sinh[6*c + 4*d*x] - 24*
a^2*b^3*Sinh[6*c + 4*d*x] + 2*a^4*b*Sinh[4*c + 6*d*x] + 2*a^3*b^2*Sinh[4*c
+ 6*d*x] + 2*a^4*b*Sinh[8*c + 6*d*x] + 2*a^3*b^2*Sinh[8*c + 6*d*x]))/(8*a^4
*b*(a + b)*d*(a + 2*b + a*Cosh[2*c + 2*d*x]))))/(128*(a + b*Sech[c + d*x]^2
)^2)
```

Maple [B] time = 0.109, size = 1025, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/d*b/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+
1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*tanh(1/2*d*x+1/2*c)-3/4/d*b^(1/2)
/a^2/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)
*b^(1/2)+(a+b)^(1/2))+3/4/d*b^(1/2)/a^2/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2
*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+3/8/d/a^2*ln(tanh(
1/2*d*x+1/2*c)+1)-3/8/d/a^2*ln(tanh(1/2*d*x+1/2*c)-1)+1/4/d/a^2/(tanh(1/2*d
*x+1/2*c)-1)^4+1/2/d/a^2/(tanh(1/2*d*x+1/2*c)-1)^3-1/8/d/a^2/(tanh(1/2*d*x+
1/2*c)-1)^2-3/8/d/a^2/(tanh(1/2*d*x+1/2*c)-1)-1/4/d/a^2/(tanh(1/2*d*x+1/2*c
)+1)^4+1/2/d/a^2/(tanh(1/2*d*x+1/2*c)+1)^3+1/8/d/a^2/(tanh(1/2*d*x+1/2*c)+1
```

```
)^2-3/8/d/a^2/(tanh(1/2*d*x+1/2*c)+1)-1/d*b/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*
tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a
+b)*tanh(1/2*d*x+1/2*c)^3-1/d*b^2/a^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d
*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*tanh(1
/2*d*x+1/2*c)-1/d/a^3/(tanh(1/2*d*x+1/2*c)-1)*b-3/d/a^3*ln(tanh(1/2*d*x+1/2
*c)-1)*b-3/d/a^4*ln(tanh(1/2*d*x+1/2*c)-1)*b^2-1/d/a^3/(tanh(1/2*d*x+1/2*c)
+1)*b+3/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)*b+3/d/a^4*ln(tanh(1/2*d*x+1/2*c)+1)
*b^2-1/d/a^3/(tanh(1/2*d*x+1/2*c)-1)^2*b+1/d/a^3/(tanh(1/2*d*x+1/2*c)+1)^2*
b+9/4/d*b^(3/2)/a^3/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh
(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-3/2/d*b^(5/2)/a^4/(a+b)^(1/2)*ln((a+b)
^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+3/2
/d*b^(5/2)/a^4/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*
d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-1/d*b^2/a^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh
(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*
tanh(1/2*d*x+1/2*c)^3-9/4/d*b^(3/2)/a^3/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2
*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.92577, size = 12945, normalized size = 66.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/64*(a^3*cosh(d*x + c)^12 + 12*a^3*cosh(d*x + c)*sinh(d*x + c)^11 + a^3*s
inh(d*x + c)^12 - 6*(a^3 + 2*a^2*b)*cosh(d*x + c)^10 + 6*(11*a^3*cosh(d*x +
 c)^2 - a^3 - 2*a^2*b)*sinh(d*x + c)^10 + 20*(11*a^3*cosh(d*x + c)^3 - 3*(a
^3 + 2*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^9 - (15*a^3 + 64*a^2*b + 64*a*b^
2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^8 + (495*a^3*cosh(d*x +
 c)^4 - 15*a^3 - 64*a^2*b - 64*a*b^2 + 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x - 2
70*(a^3 + 2*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(99*a^3*cosh(d*x +
c)^5 - 90*(a^3 + 2*a^2*b)*cosh(d*x + c)^3 - (15*a^3 + 64*a^2*b + 64*a*b^2 -
 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 + 16*(4*a
^2*b + 12*a*b^2 + 8*b^3 + 3*(a^3 + 10*a^2*b + 24*a*b^2 + 16*b^3)*d*x)*cosh(
d*x + c)^6 + 4*(231*a^3*cosh(d*x + c)^6 - 315*(a^3 + 2*a^2*b)*cosh(d*x + c)
^4 + 16*a^2*b + 48*a*b^2 + 32*b^3 + 12*(a^3 + 10*a^2*b + 24*a*b^2 + 16*b^3)
*d*x - 7*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*
cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(99*a^3*cosh(d*x + c)^7 - 189*(a^3 + 2
*a^2*b)*cosh(d*x + c)^5 - 7*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24*(a^3 + 8*a^2
*b + 8*a*b^2)*d*x)*cosh(d*x + c)^3 + 12*(4*a^2*b + 12*a*b^2 + 8*b^3 + 3*(a^
3 + 10*a^2*b + 24*a*b^2 + 16*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + (15
*a^3 + 128*a^2*b + 128*a*b^2 + 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x +
 c)^4 + (495*a^3*cosh(d*x + c)^8 - 1260*(a^3 + 2*a^2*b)*cosh(d*x + c)^6 - 7
0*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*
```

```
x + c)^4 + 15*a^3 + 128*a^2*b + 128*a*b^2 + 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*
x + 240*(4*a^2*b + 12*a*b^2 + 8*b^3 + 3*(a^3 + 10*a^2*b + 24*a*b^2 + 16*b^3
)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(55*a^3*cosh(d*x + c)^9 - 180*(
a^3 + 2*a^2*b)*cosh(d*x + c)^7 - 14*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24*(a^3
 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^5 + 80*(4*a^2*b + 12*a*b^2 + 8*b^3
 + 3*(a^3 + 10*a^2*b + 24*a*b^2 + 16*b^3)*d*x)*cosh(d*x + c)^3 + (15*a^3 +
128*a^2*b + 128*a*b^2 + 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c))*si
nh(d*x + c)^3 - a^3 + 6*(a^3 + 2*a^2*b)*cosh(d*x + c)^2 + 2*(33*a^3*cosh(d*
x + c)^10 - 135*(a^3 + 2*a^2*b)*cosh(d*x + c)^8 - 14*(15*a^3 + 64*a^2*b + 6
4*a*b^2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^6 + 120*(4*a^2*b
+ 12*a*b^2 + 8*b^3 + 3*(a^3 + 10*a^2*b + 24*a*b^2 + 16*b^3)*d*x)*cosh(d*x +
 c)^4 + 3*a^3 + 6*a^2*b + 3*(15*a^3 + 128*a^2*b + 128*a*b^2 + 24*(a^3 + 8*a
^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 48*((a^2 + 2*a*b)*c
osh(d*x + c)^8 + 8*(a^2 + 2*a*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a
*b)*sinh(d*x + c)^8 + 2*(a^2 + 4*a*b + 4*b^2)*cosh(d*x + c)^6 + 2*(14*(a^2
+ 2*a*b)*cosh(d*x + c)^2 + a^2 + 4*a*b + 4*b^2)*sinh(d*x + c)^6 + 4*(14*(a^
2 + 2*a*b)*cosh(d*x + c)^3 + 3*(a^2 + 4*a*b + 4*b^2)*cosh(d*x + c))*sinh(d*
x + c)^5 + (a^2 + 2*a*b)*cosh(d*x + c)^4 + (70*(a^2 + 2*a*b)*cosh(d*x + c)^
4 + 30*(a^2 + 4*a*b + 4*b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^4
 + 4*(14*(a^2 + 2*a*b)*cosh(d*x + c)^5 + 10*(a^2 + 4*a*b + 4*b^2)*cosh(d*x
+ c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(14*(a^2 + 2*a*b)
*cosh(d*x + c)^6 + 15*(a^2 + 4*a*b + 4*b^2)*cosh(d*x + c)^4 + 3*(a^2 + 2*a*
b)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(2*(a^2 + 2*a*b)*cosh(d*x + c)^7 +
3*(a^2 + 4*a*b + 4*b^2)*cosh(d*x + c)^5 + (a^2 + 2*a*b)*cosh(d*x + c)^3)*si
nh(d*x + c))*sqrt(a*b + b^2)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)
*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 +
2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b
^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) +
4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2
+ a + 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x
 + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x
 + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(
d*x + c))*sinh(d*x + c) + a)) + 4*(3*a^3*cosh(d*x + c)^11 - 15*(a^3 + 2*a^2
*b)*cosh(d*x + c)^9 - 2*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24*(a^3 + 8*a^2*b +
 8*a*b^2)*d*x)*cosh(d*x + c)^7 + 24*(4*a^2*b + 12*a*b^2 + 8*b^3 + 3*(a^3 +
10*a^2*b + 24*a*b^2 + 16*b^3)*d*x)*cosh(d*x + c)^5 + (15*a^3 + 128*a^2*b +
128*a*b^2 + 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^3 + 3*(a^3 + 2*
a^2*b)*cosh(d*x + c))*sinh(d*x + c))/(a^5*d*cosh(d*x + c)^8 + 8*a^5*d*cosh(
d*x + c)*sinh(d*x + c)^7 + a^5*d*sinh(d*x + c)^8 + a^5*d*cosh(d*x + c)^4 +
2*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^6 + 2*(14*a^5*d*cosh(d*x + c)^2 + (a^5 +
2*a^4*b)*d)*sinh(d*x + c)^6 + 4*(14*a^5*d*cosh(d*x + c)^3 + 3*(a^5 + 2*a^4*
b)*d*cosh(d*x + c))*sinh(d*x + c)^5 + (70*a^5*d*cosh(d*x + c)^4 + a^5*d + 3
0*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(14*a^5*d*cosh(d*x
 + c)^5 + a^5*d*cosh(d*x + c) + 10*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^3)*sinh(
d*x + c)^3 + 2*(14*a^5*d*cosh(d*x + c)^6 + 3*a^5*d*cosh(d*x + c)^2 + 15*(a^
5 + 2*a^4*b)*d*cosh(d*x + c)^4)*sinh(d*x + c)^2 + 4*(2*a^5*d*cosh(d*x + c)^
7 + a^5*d*cosh(d*x + c)^3 + 3*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^5)*sinh(d*x +
 c)), 1/64*(a^3*cosh(d*x + c)^12 + 12*a^3*cosh(d*x + c)*sinh(d*x + c)^11 +
a^3*sinh(d*x + c)^12 - 6*(a^3 + 2*a^2*b)*cosh(d*x + c)^10 + 6*(11*a^3*cosh(
d*x + c)^2 - a^3 - 2*a^2*b)*sinh(d*x + c)^10 + 20*(11*a^3*cosh(d*x + c)^3 -
 3*(a^3 + 2*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^9 - (15*a^3 + 64*a^2*b + 64
*a*b^2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^8 + (495*a^3*cosh(
d*x + c)^4 - 15*a^3 - 64*a^2*b - 64*a*b^2 + 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*
x - 270*(a^3 + 2*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(99*a^3*cosh(d
*x + c)^5 - 90*(a^3 + 2*a^2*b)*cosh(d*x + c)^3 - (15*a^3 + 64*a^2*b + 64*a*
b^2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 + 16
*(4*a^2*b + 12*a*b^2 + 8*b^3 + 3*(a^3 + 10*a^2*b + 24*a*b^2 + 16*b^3)*d*x)*
cosh(d*x + c)^6 + 4*(231*a^3*cosh(d*x + c)^6 - 315*(a^3 + 2*a^2*b)*cosh(d*x
 + c)^4 + 16*a^2*b + 48*a*b^2 + 32*b^3 + 12*(a^3 + 10*a^2*b + 24*a*b^2 + 16
```

```
*b^3)*d*x - 7*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*
d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(99*a^3*cosh(d*x + c)^7 - 189*(a^
3 + 2*a^2*b)*cosh(d*x + c)^5 - 7*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24*(a^3 +
8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^3 + 12*(4*a^2*b + 12*a*b^2 + 8*b^3 +
3*(a^3 + 10*a^2*b + 24*a*b^2 + 16*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5
+ (15*a^3 + 128*a^2*b + 128*a*b^2 + 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(
d*x + c)^4 + (495*a^3*cosh(d*x + c)^8 - 1260*(a^3 + 2*a^2*b)*cosh(d*x + c)^
6 - 70*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*co
sh(d*x + c)^4 + 15*a^3 + 128*a^2*b + 128*a*b^2 + 24*(a^3 + 8*a^2*b + 8*a*b^
2)*d*x + 240*(4*a^2*b + 12*a*b^2 + 8*b^3 + 3*(a^3 + 10*a^2*b + 24*a*b^2 + 1
6*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(55*a^3*cosh(d*x + c)^9 -
180*(a^3 + 2*a^2*b)*cosh(d*x + c)^7 - 14*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24
*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^5 + 80*(4*a^2*b + 12*a*b^2 +
8*b^3 + 3*(a^3 + 10*a^2*b + 24*a*b^2 + 16*b^3)*d*x)*cosh(d*x + c)^3 + (15*a
^3 + 128*a^2*b + 128*a*b^2 + 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c
))*sinh(d*x + c)^3 - a^3 + 6*(a^3 + 2*a^2*b)*cosh(d*x + c)^2 + 2*(33*a^3*co
sh(d*x + c)^10 - 135*(a^3 + 2*a^2*b)*cosh(d*x + c)^8 - 14*(15*a^3 + 64*a^2*
b + 64*a*b^2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^6 + 120*(4*a
^2*b + 12*a*b^2 + 8*b^3 + 3*(a^3 + 10*a^2*b + 24*a*b^2 + 16*b^3)*d*x)*cosh(
d*x + c)^4 + 3*a^3 + 6*a^2*b + 3*(15*a^3 + 128*a^2*b + 128*a*b^2 + 24*(a^3
+ 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 96*((a^2 + 2*a
*b)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2
+ 2*a*b)*sinh(d*x + c)^8 + 2*(a^2 + 4*a*b + 4*b^2)*cosh(d*x + c)^6 + 2*(14*
(a^2 + 2*a*b)*cosh(d*x + c)^2 + a^2 + 4*a*b + 4*b^2)*sinh(d*x + c)^6 + 4*(1
4*(a^2 + 2*a*b)*cosh(d*x + c)^3 + 3*(a^2 + 4*a*b + 4*b^2)*cosh(d*x + c))*si
nh(d*x + c)^5 + (a^2 + 2*a*b)*cosh(d*x + c)^4 + (70*(a^2 + 2*a*b)*cosh(d*x
+ c)^4 + 30*(a^2 + 4*a*b + 4*b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x +
 c)^4 + 4*(14*(a^2 + 2*a*b)*cosh(d*x + c)^5 + 10*(a^2 + 4*a*b + 4*b^2)*cosh
(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(14*(a^2 + 2
*a*b)*cosh(d*x + c)^6 + 15*(a^2 + 4*a*b + 4*b^2)*cosh(d*x + c)^4 + 3*(a^2 +
 2*a*b)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(2*(a^2 + 2*a*b)*cosh(d*x + c)
^7 + 3*(a^2 + 4*a*b + 4*b^2)*cosh(d*x + c)^5 + (a^2 + 2*a*b)*cosh(d*x + c)^
3)*sinh(d*x + c))*sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh
(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-a*b - b^2)/(a*
b + b^2)) + 4*(3*a^3*cosh(d*x + c)^11 - 15*(a^3 + 2*a^2*b)*cosh(d*x + c)^9
- 2*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(
d*x + c)^7 + 24*(4*a^2*b + 12*a*b^2 + 8*b^3 + 3*(a^3 + 10*a^2*b + 24*a*b^2
+ 16*b^3)*d*x)*cosh(d*x + c)^5 + (15*a^3 + 128*a^2*b + 128*a*b^2 + 24*(a^3
+ 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^3 + 3*(a^3 + 2*a^2*b)*cosh(d*x + c)
)*sinh(d*x + c))/(a^5*d*cosh(d*x + c)^8 + 8*a^5*d*cosh(d*x + c)*sinh(d*x +
c)^7 + a^5*d*sinh(d*x + c)^8 + a^5*d*cosh(d*x + c)^4 + 2*(a^5 + 2*a^4*b)*d*
cosh(d*x + c)^6 + 2*(14*a^5*d*cosh(d*x + c)^2 + (a^5 + 2*a^4*b)*d)*sinh(d*x
 + c)^6 + 4*(14*a^5*d*cosh(d*x + c)^3 + 3*(a^5 + 2*a^4*b)*d*cosh(d*x + c))*
sinh(d*x + c)^5 + (70*a^5*d*cosh(d*x + c)^4 + a^5*d + 30*(a^5 + 2*a^4*b)*d*
cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(14*a^5*d*cosh(d*x + c)^5 + a^5*d*cosh
(d*x + c) + 10*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^3)*sinh(d*x + c)^3 + 2*(14*a
^5*d*cosh(d*x + c)^6 + 3*a^5*d*cosh(d*x + c)^2 + 15*(a^5 + 2*a^4*b)*d*cosh(
d*x + c)^4)*sinh(d*x + c)^2 + 4*(2*a^5*d*cosh(d*x + c)^7 + a^5*d*cosh(d*x +
 c)^3 + 3*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^5)*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a+b*sech(d*x+c)**2)**2,x)
```

[Out] Timed out

Giac [A] time = 1.18934, size = 454, normalized size = 2.34

$$\frac{3\left(a^{2}+8\,ab+8\,b^{2}\right)(dx+c)}{8\,a^{4}d}-\frac{\left(18\,a^{2}e^{(4\,dx+4\,c)}+144\,abe^{(4\,dx+4\,c)}+144\,b^{2}e^{(4\,dx+4\,c)}-8\,a^{2}e^{(2\,dx+2\,c)}-16\,abe^{(2\,dx+2\,c)}+a^{2}\right)e}{64\,a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 3/8*(a^2 + 8*a*b + 8*b^2)*(d*x + c)/(a^4*d) - 1/64*(18*a^2*e^(4*d*x + 4*c)
+ 144*a*b*e^(4*d*x + 4*c) + 144*b^2*e^(4*d*x + 4*c) - 8*a^2*e^(2*d*x + 2*c)
 - 16*a*b*e^(2*d*x + 2*c) + a^2)*e^(-4*d*x - 4*c)/(a^4*d) - 3/2*(a^2*b + 3*
a*b^2 + 2*b^3)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(
sqrt(-a*b - b^2)*a^4*d) + 1/64*(a^2*d*e^(4*d*x + 4*c) - 8*a^2*d*e^(2*d*x +
2*c) - 16*a*b*d*e^(2*d*x + 2*c))/(a^4*d^2) + (a^2*b*e^(2*d*x + 2*c) + 3*a*b
^2*e^(2*d*x + 2*c) + 2*b^3*e^(2*d*x + 2*c) + a^2*b + a*b^2)/((a*e^(4*d*x +
4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)*a^4*d)
```

3.34 $\int \frac{\sinh^3(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=114

$$-\frac{b(a+b)\cosh(c+dx)}{2a^3d\left(a\cosh^2(c+dx)+b\right)}-\frac{(a+2b)\cosh(c+dx)}{a^3d}+\frac{\sqrt{b}(3a+5b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{7/2}d}+\frac{\cosh^3(c+dx)}{3a^2d}$$

```
[Out] (Sqrt[b]*(3*a + 5*b)*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(2*a^(7/2)*d)
 - ((a + 2*b)*Cosh[c + d*x])/(a^3*d) + Cosh[c + d*x]^3/(3*a^2*d) - (b*(a +
b)*Cosh[c + d*x])/(2*a^3*d*(b + a*Cosh[c + d*x]^2))
```

Rubi [A] time = 0.144245, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4133, 455, 1153, 205}

$$-\frac{b(a+b)\cosh(c+dx)}{2a^3d\left(a\cosh^2(c+dx)+b\right)}-\frac{(a+2b)\cosh(c+dx)}{a^3d}+\frac{\sqrt{b}(3a+5b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{7/2}d}+\frac{\cosh^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```


```
[Out] (Sqrt[b]*(3*a + 5*b)*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(2*a^(7/2)*d)
 - ((a + 2*b)*Cosh[c + d*x])/(a^3*d) + Cosh[c + d*x]^3/(3*a^2*d) - (b*(a +
b)*Cosh[c + d*x])/(2*a^3*d*(b + a*Cosh[c + d*x]^2))
```


Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{x^4(1-x^2)}{(b+ax^2)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{b(a+b)\cosh(c+dx)}{2a^3d\left(b+a\cosh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{b(a+b)-2a(a+b)x^2+2a^2x^4}{b+ax^2}\,dx, x, \cosh(c+dx)\right)}{2a^3d}$$

$$= -\frac{b(a+b)\cosh(c+dx)}{2a^3d\left(b+a\cosh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \left(-2(a+2b)+2ax^2+\frac{3ab+5b^2}{b+ax^2}\right)dx, x, \cosh(c\right.}{2a^3d}$$

$$= -\frac{(a+2b)\cosh(c+dx)}{a^3d} + \frac{\cosh^3(c+dx)}{3a^2d} - \frac{b(a+b)\cosh(c+dx)}{2a^3d\left(b+a\cosh^2(c+dx)\right)} + \frac{(b(3a+5b))}{}$$

$$= \frac{\sqrt{b}(3a+5b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{7/2}d} - \frac{(a+2b)\cosh(c+dx)}{a^3d} + \frac{\cosh^3(c+dx)}{3a^2d} - \frac{b(a+}{2a^3d\left(b\right.}$$

Mathematica [C] time = 4.67833, size = 861, normalized size = 7.55

$$\frac{(\cosh(2(c+dx))a+a+2b)^2\operatorname{sech}^4(c+dx)\left(\frac{9\tan^{-1}\left(\frac{\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\right)\sinh(c)\tanh\left(\frac{dx}{2}\right)+\cosh(c)\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\tanh\right.}{\sqrt{b}}\right)}{b^{3/2}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]^4*((9*a^3*ArcTan[((Sqrt[a]
 - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[
c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqr
t[b]])/b^(3/2) + 576*a*Sqrt[b]*ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[
c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*
Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] + 960*b^(3/2)*ArcTan[(
(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2]
 + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/
2]))/Sqrt[b]] + (9*a^3*ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sin
h[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Co
sh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]])/b^(3/2) + 576*a*Sqrt[b]*ArcTa
n[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)
/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*
x)/2]))/Sqrt[b]] + 960*b^(3/2)*ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[
c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*
Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] - (9*a^3*ArcTan[(Sqrt[
a] - I*Sqrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]])/b^(3/2) - (9*a^3*ArcTan[(Sq
rt[a] + I*Sqrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]])/b^(3/2) - 96*Sqrt[a]*(3*
a + 8*b)*Cosh[c]*Cosh[d*x] + 32*a^(3/2)*Cosh[3*c]*Cosh[3*d*x] - (384*a^(3/2
)*b*Cosh[c + d*x])/(a + 2*b + a*Cosh[2*(c + d*x)]) - (384*Sqrt[a]*b^2*Cosh[
```

```
c + d*x])/(a + 2*b + a*Cosh[2*(c + d*x)]) - 288*a^(3/2)*Sinh[c]*Sinh[d*x] -
 768*Sqrt[a]*b*Sinh[c]*Sinh[d*x] + 32*a^(3/2)*Sinh[3*c]*Sinh[3*d*x]))/(1536
*a^(7/2)*d*(a + b*Sech[c + d*x]^2)^2)
```

Maple [B] time = 0.082, size = 561, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/3/d/a^2/(tanh(1/2*d*x+1/2*c)+1)^3-1/2/d/a^2/(tanh(1/2*d*x+1/2*c)+1)^2-1/2
/d/a^2/(tanh(1/2*d*x+1/2*c)+1)-2/d/a^3/(tanh(1/2*d*x+1/2*c)+1)*b-1/d*b/a^2/
(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-
2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*tanh(1/2*d*x+1/2*c)^2+1/d*b^2/a^3/(tanh(1/2*
d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2
*d*x+1/2*c)^2*b+a+b)*tanh(1/2*d*x+1/2*c)^2-1/d*b/a^2/(tanh(1/2*d*x+1/2*c)^4
*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^
2*b+a+b)-1/d*b^2/a^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tan
h(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+3/2/d*b/a^2/(a*b)^(1/2)
*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))+5/2/d*b^2/
a^3/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1
/2))-1/3/d/a^2/(tanh(1/2*d*x+1/2*c)-1)^3-1/2/d/a^2/(tanh(1/2*d*x+1/2*c)-1)^
2+1/2/d/a^2/(tanh(1/2*d*x+1/2*c)-1)+2/d/a^3/(tanh(1/2*d*x+1/2*c)-1)*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2 e^{(10\,dx+10\,c)} + a^2 - \left(7\,a^2 e^{(8\,c)} + 20\,abe^{(8\,c)}\right)e^{(8\,dx)} - 2\left(13\,a^2 e^{(6\,c)} + 66\,abe^{(6\,c)} + 60\,b^2 e^{(6\,c)}\right)e^{(6\,dx)} - 2\left(13\,a^2 e^{(4\,c)} + 66\,abe^{(4\,c)} \cdots\right.}{24\left(a^4 de^{(7\,dx+7\,c)} + a^4 de^{(3\,dx+3\,c)} + 2\left(a^4 de^{(5\,c)} + 2\,a^3 bde^{(5\,c)}\right)e^{(5\,dx)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/24*(a^2*e^(10*d*x + 10*c) + a^2 - (7*a^2*e^(8*c) + 20*a*b*e^(8*c))*e^(8*d
*x) - 2*(13*a^2*e^(6*c) + 66*a*b*e^(6*c) + 60*b^2*e^(6*c))*e^(6*d*x) - 2*(1
3*a^2*e^(4*c) + 66*a*b*e^(4*c) + 60*b^2*e^(4*c))*e^(4*d*x) - (7*a^2*e^(2*c)
 + 20*a*b*e^(2*c))*e^(2*d*x))/(a^4*d*e^(7*d*x + 7*c) + a^4*d*e^(3*d*x + 3*c
) + 2*(a^4*d*e^(5*c) + 2*a^3*b*d*e^(5*c))*e^(5*d*x)) + 1/8*integrate(8*((3*
a*b*e^(3*c) + 5*b^2*e^(3*c))*e^(3*d*x) - (3*a*b*e^c + 5*b^2*e^c)*e^(d*x))/(
a^4*e^(4*d*x + 4*c) + a^4 + 2*(a^4*e^(2*c) + 2*a^3*b*e^(2*c))*e^(2*d*x)), x
)
```

Fricas [B] time = 3.98992, size = 9646, normalized size = 84.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/24*(a^2*cosh(d*x + c)^10 + 10*a^2*cosh(d*x + c)*sinh(d*x + c)^9 + a^2*si
nh(d*x + c)^10 - (7*a^2 + 20*a*b)*cosh(d*x + c)^8 + (45*a^2*cosh(d*x + c)^2
 - 7*a^2 - 20*a*b)*sinh(d*x + c)^8 + 8*(15*a^2*cosh(d*x + c)^3 - (7*a^2 + 2
0*a*b)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(13*a^2 + 66*a*b + 60*b^2)*cosh(d
*x + c)^6 + 2*(105*a^2*cosh(d*x + c)^4 - 14*(7*a^2 + 20*a*b)*cosh(d*x + c)^
2 - 13*a^2 - 66*a*b - 60*b^2)*sinh(d*x + c)^6 + 4*(63*a^2*cosh(d*x + c)^5 -
 14*(7*a^2 + 20*a*b)*cosh(d*x + c)^3 - 3*(13*a^2 + 66*a*b + 60*b^2)*cosh(d*
x + c))*sinh(d*x + c)^5 - 2*(13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)^4 + 2*
(105*a^2*cosh(d*x + c)^6 - 35*(7*a^2 + 20*a*b)*cosh(d*x + c)^4 - 15*(13*a^2
 + 66*a*b + 60*b^2)*cosh(d*x + c)^2 - 13*a^2 - 66*a*b - 60*b^2)*sinh(d*x +
c)^4 + 8*(15*a^2*cosh(d*x + c)^7 - 7*(7*a^2 + 20*a*b)*cosh(d*x + c)^5 - 5*(
13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)^3 - (13*a^2 + 66*a*b + 60*b^2)*cosh
(d*x + c))*sinh(d*x + c)^3 - (7*a^2 + 20*a*b)*cosh(d*x + c)^2 + (45*a^2*cos
h(d*x + c)^8 - 28*(7*a^2 + 20*a*b)*cosh(d*x + c)^6 - 30*(13*a^2 + 66*a*b +
60*b^2)*cosh(d*x + c)^4 - 12*(13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)^2 - 7
*a^2 - 20*a*b)*sinh(d*x + c)^2 + 6*((3*a^2 + 5*a*b)*cosh(d*x + c)^7 + 7*(3*
a^2 + 5*a*b)*cosh(d*x + c)*sinh(d*x + c)^6 + (3*a^2 + 5*a*b)*sinh(d*x + c)^
7 + 2*(3*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^5 + (21*(3*a^2 + 5*a*b)*cosh(
d*x + c)^2 + 6*a^2 + 22*a*b + 20*b^2)*sinh(d*x + c)^5 + 5*(7*(3*a^2 + 5*a*b
)*cosh(d*x + c)^3 + 2*(3*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c))*sinh(d*x + c
)^4 + (3*a^2 + 5*a*b)*cosh(d*x + c)^3 + (35*(3*a^2 + 5*a*b)*cosh(d*x + c)^4
 + 20*(3*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^2 + 3*a^2 + 5*a*b)*sinh(d*x +
 c)^3 + (21*(3*a^2 + 5*a*b)*cosh(d*x + c)^5 + 20*(3*a^2 + 11*a*b + 10*b^2)*
cosh(d*x + c)^3 + 3*(3*a^2 + 5*a*b)*cosh(d*x + c))*sinh(d*x + c)^2 + (7*(3*
a^2 + 5*a*b)*cosh(d*x + c)^6 + 10*(3*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^4
 + 3*(3*a^2 + 5*a*b)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(-b/a)*log((a*cosh
(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a
- 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a - 2*b)*sinh(d*x + c)^2
+ 4*(a*cosh(d*x + c)^3 + (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh
(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + a*cos
h(d*x + c) + (3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))*sqrt(-b/a) + a)/(a*co
sh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(
a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^
2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + a
^2 + 2*(5*a^2*cosh(d*x + c)^9 - 4*(7*a^2 + 20*a*b)*cosh(d*x + c)^7 - 6*(13*
a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)^5 - 4*(13*a^2 + 66*a*b + 60*b^2)*cosh(
d*x + c)^3 - (7*a^2 + 20*a*b)*cosh(d*x + c))*sinh(d*x + c))/(a^4*d*cosh(d*x
 + c)^7 + 7*a^4*d*cosh(d*x + c)*sinh(d*x + c)^6 + a^4*d*sinh(d*x + c)^7 + a
^4*d*cosh(d*x + c)^3 + 2*(a^4 + 2*a^3*b)*d*cosh(d*x + c)^5 + (21*a^4*d*cosh
(d*x + c)^2 + 2*(a^4 + 2*a^3*b)*d)*sinh(d*x + c)^5 + 5*(7*a^4*d*cosh(d*x +
c)^3 + 2*(a^4 + 2*a^3*b)*d*cosh(d*x + c))*sinh(d*x + c)^4 + (35*a^4*d*cosh(
d*x + c)^4 + a^4*d + 20*(a^4 + 2*a^3*b)*d*cosh(d*x + c)^2)*sinh(d*x + c)^3
+ (21*a^4*d*cosh(d*x + c)^5 + 3*a^4*d*cosh(d*x + c) + 20*(a^4 + 2*a^3*b)*d*
cosh(d*x + c)^3)*sinh(d*x + c)^2 + (7*a^4*d*cosh(d*x + c)^6 + 3*a^4*d*cosh(
d*x + c)^2 + 10*(a^4 + 2*a^3*b)*d*cosh(d*x + c)^4)*sinh(d*x + c)), 1/24*(a^
2*cosh(d*x + c)^10 + 10*a^2*cosh(d*x + c)*sinh(d*x + c)^9 + a^2*sinh(d*x +
c)^10 - (7*a^2 + 20*a*b)*cosh(d*x + c)^8 + (45*a^2*cosh(d*x + c)^2 - 7*a^2
- 20*a*b)*sinh(d*x + c)^8 + 8*(15*a^2*cosh(d*x + c)^3 - (7*a^2 + 20*a*b)*co
sh(d*x + c))*sinh(d*x + c)^7 - 2*(13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)^6
 + 2*(105*a^2*cosh(d*x + c)^4 - 14*(7*a^2 + 20*a*b)*cosh(d*x + c)^2 - 13*a^
2 - 66*a*b - 60*b^2)*sinh(d*x + c)^6 + 4*(63*a^2*cosh(d*x + c)^5 - 14*(7*a^
2 + 20*a*b)*cosh(d*x + c)^3 - 3*(13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c))*s
inh(d*x + c)^5 - 2*(13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)^4 + 2*(105*a^2*
cosh(d*x + c)^6 - 35*(7*a^2 + 20*a*b)*cosh(d*x + c)^4 - 15*(13*a^2 + 66*a*b
 + 60*b^2)*cosh(d*x + c)^2 - 13*a^2 - 66*a*b - 60*b^2)*sinh(d*x + c)^4 + 8*
(15*a^2*cosh(d*x + c)^7 - 7*(7*a^2 + 20*a*b)*cosh(d*x + c)^5 - 5*(13*a^2 +
66*a*b + 60*b^2)*cosh(d*x + c)^3 - (13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)
)*sinh(d*x + c)^3 - (7*a^2 + 20*a*b)*cosh(d*x + c)^2 + (45*a^2*cosh(d*x + c
)^8 - 28*(7*a^2 + 20*a*b)*cosh(d*x + c)^6 - 30*(13*a^2 + 66*a*b + 60*b^2)*c
```

```
osh(d*x + c)^4 - 12*(13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)^2 - 7*a^2 - 20
*a*b)*sinh(d*x + c)^2 - 12*((3*a^2 + 5*a*b)*cosh(d*x + c)^7 + 7*(3*a^2 + 5*
a*b)*cosh(d*x + c)*sinh(d*x + c)^6 + (3*a^2 + 5*a*b)*sinh(d*x + c)^7 + 2*(3
*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^5 + (21*(3*a^2 + 5*a*b)*cosh(d*x + c)
^2 + 6*a^2 + 22*a*b + 20*b^2)*sinh(d*x + c)^5 + 5*(7*(3*a^2 + 5*a*b)*cosh(d
*x + c)^3 + 2*(3*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + (3
*a^2 + 5*a*b)*cosh(d*x + c)^3 + (35*(3*a^2 + 5*a*b)*cosh(d*x + c)^4 + 20*(3
*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^2 + 3*a^2 + 5*a*b)*sinh(d*x + c)^3 +
(21*(3*a^2 + 5*a*b)*cosh(d*x + c)^5 + 20*(3*a^2 + 11*a*b + 10*b^2)*cosh(d*x
 + c)^3 + 3*(3*a^2 + 5*a*b)*cosh(d*x + c))*sinh(d*x + c)^2 + (7*(3*a^2 + 5*
a*b)*cosh(d*x + c)^6 + 10*(3*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^4 + 3*(3*
a^2 + 5*a*b)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*(a*cosh(d
*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (a + 4*
b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a + 4*b)*sinh(d*x + c))*sqrt(b/a)
/b) + 12*((3*a^2 + 5*a*b)*cosh(d*x + c)^7 + 7*(3*a^2 + 5*a*b)*cosh(d*x + c)
*sinh(d*x + c)^6 + (3*a^2 + 5*a*b)*sinh(d*x + c)^7 + 2*(3*a^2 + 11*a*b + 10
*b^2)*cosh(d*x + c)^5 + (21*(3*a^2 + 5*a*b)*cosh(d*x + c)^2 + 6*a^2 + 22*a*
b + 20*b^2)*sinh(d*x + c)^5 + 5*(7*(3*a^2 + 5*a*b)*cosh(d*x + c)^3 + 2*(3*a
^2 + 11*a*b + 10*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + (3*a^2 + 5*a*b)*cosh
(d*x + c)^3 + (35*(3*a^2 + 5*a*b)*cosh(d*x + c)^4 + 20*(3*a^2 + 11*a*b + 10
*b^2)*cosh(d*x + c)^2 + 3*a^2 + 5*a*b)*sinh(d*x + c)^3 + (21*(3*a^2 + 5*a*b
)*cosh(d*x + c)^5 + 20*(3*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^3 + 3*(3*a^2
 + 5*a*b)*cosh(d*x + c))*sinh(d*x + c)^2 + (7*(3*a^2 + 5*a*b)*cosh(d*x + c)
^6 + 10*(3*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^4 + 3*(3*a^2 + 5*a*b)*cosh(
d*x + c)^2)*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*(a*cosh(d*x + c) + a*sinh(d
*x + c))*sqrt(b/a)/b) + a^2 + 2*(5*a^2*cosh(d*x + c)^9 - 4*(7*a^2 + 20*a*b)
*cosh(d*x + c)^7 - 6*(13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)^5 - 4*(13*a^2
 + 66*a*b + 60*b^2)*cosh(d*x + c)^3 - (7*a^2 + 20*a*b)*cosh(d*x + c))*sinh(
d*x + c))/(a^4*d*cosh(d*x + c)^7 + 7*a^4*d*cosh(d*x + c)*sinh(d*x + c)^6 +
a^4*d*sinh(d*x + c)^7 + a^4*d*cosh(d*x + c)^3 + 2*(a^4 + 2*a^3*b)*d*cosh(d*
x + c)^5 + (21*a^4*d*cosh(d*x + c)^2 + 2*(a^4 + 2*a^3*b)*d)*sinh(d*x + c)^5
 + 5*(7*a^4*d*cosh(d*x + c)^3 + 2*(a^4 + 2*a^3*b)*d*cosh(d*x + c))*sinh(d*x
 + c)^4 + (35*a^4*d*cosh(d*x + c)^4 + a^4*d + 20*(a^4 + 2*a^3*b)*d*cosh(d*x
 + c)^2)*sinh(d*x + c)^3 + (21*a^4*d*cosh(d*x + c)^5 + 3*a^4*d*cosh(d*x + c
) + 20*(a^4 + 2*a^3*b)*d*cosh(d*x + c)^3)*sinh(d*x + c)^2 + (7*a^4*d*cosh(d
*x + c)^6 + 3*a^4*d*cosh(d*x + c)^2 + 10*(a^4 + 2*a^3*b)*d*cosh(d*x + c)^4)
*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*sech(d*x+c)**2)**2,x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.35 $\int \frac{\sinh^2(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=131

$$\frac{\sqrt{b}(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^3d\sqrt{a+b}}+\frac{b\tanh(c+dx)}{a^2d\left(a-b\tanh^2(c+dx)+b\right)}-\frac{x(a+4b)}{2a^3}+\frac{\sinh(c+dx)\cosh(c+dx)}{2ad\left(a-b\tanh^2(c+dx)+b\right)}$$

```
[Out] -((a + 4*b)*x)/(2*a^3) + (Sqrt[b]*(3*a + 4*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x
])/Sqrt[a + b]])/(2*a^3*Sqrt[a + b]*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*a
*d*(a + b - b*Tanh[c + d*x]^2)) + (b*Tanh[c + d*x])/(a^2*d*(a + b - b*Tanh[
c + d*x]^2))
```

Rubi [A] time = 0.19647, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4132, 471, 527, 522, 206, 208}

$$\frac{\sqrt{b}(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^3d\sqrt{a+b}}+\frac{b\tanh(c+dx)}{a^2d\left(a-b\tanh^2(c+dx)+b\right)}-\frac{x(a+4b)}{2a^3}+\frac{\sinh(c+dx)\cosh(c+dx)}{2ad\left(a-b\tanh^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -((a + 4*b)*x)/(2*a^3) + (Sqrt[b]*(3*a + 4*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x
])/Sqrt[a + b]])/(2*a^3*Sqrt[a + b]*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*a
*d*(a + b - b*Tanh[c + d*x]^2)) + (b*Tanh[c + d*x])/(a^2*d*(a + b - b*Tanh[
c + d*x]^2))
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)^2\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{a+b+3bx^2}{\left(1-x^2\right)\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{2ad}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)} + \frac{b\tanh(c+dx)}{a^2d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{-2(a+b)(a+2}{\left(1-x^2\right)(a}\right.}{4}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)} + \frac{b\tanh(c+dx)}{a^2d\left(a+b-b\tanh^2(c+dx)\right)} - \frac{(a+4b)\,\text{Subst}\left(\int \frac{1}{1-}\right.}{2}$$

$$= -\frac{(a+4b)x}{2a^3} + \frac{\sqrt{b}(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^3\sqrt{a+b}\,d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)} + \frac{}{a}$$

Mathematica [B] time = 11.1801, size = 791, normalized size = 6.04

$$\frac{\text{sech}^4(c+dx)(a\cosh(2c+2dx)+a+2b)^2\left(\frac{\left(a^2+8ab+8b^2\right)\text{sech}(2c)((a+2b)\sinh(2c)-a\sinh(2dx))}{bd(a+b)(a\cosh(2(c+dx))+a+2b)} + \frac{\left(-6a^2b+a^3-24ab^2-16b^3\right)(\cosh(2c)-\sin}{b}\right.}{128a^2\left(a+b\text{sech}^2(c+dx)\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*c + 2*d*x])^2*Sech[c + d*x]^4*(16*x + ((a^3 - 6*a^2*b
- 24*a*b^2 - 16*b^3)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*
Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4
```

```
])]*(Cosh[2*c] - Sinh[2*c]))/(b*(a + b)^(3/2)*d*Sqrt[b*(Cosh[c] - Sinh[c])^
4]) + ((a^2 + 8*a*b + 8*b^2)*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]
))/(b*(a + b)*d*(a + 2*b + a*Cosh[2*(c + d*x)]))))/(128*a^2*(a + b*Sech[c +
 d*x]^2)^2) + ((a + 2*b + a*Cosh[2*c + 2*d*x])^2*Sech[c + d*x]^4*(-64*(a +
2*b)*x + ((-a^4 + 16*a^3*b + 144*a^2*b^2 + 256*a*b^3 + 128*b^4)*ArcTanh[(Se
ch[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/
(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]))/(b*
(a + b)^(3/2)*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (16*a*Cosh[2*d*x]*Sinh[2*c
])/d + (16*a*Cosh[2*c]*Sinh[2*d*x])/d - ((a^3 + 18*a^2*b + 48*a*b^2 + 32*b^
3)*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/(b*(a + b)*d*(a + 2*b +
 a*Cosh[2*(c + d*x)]))))/(256*a^3*(a + b*Sech[c + d*x]^2)^2) - ((a + 2*b +
a*Cosh[2*c + 2*d*x])^2*Sech[c + d*x]^4*(-((a*ArcTanh[(Sqrt[b]*Tanh[c + d*x]
)/Sqrt[a + b]])/(a + b)^(3/2)) + (Sqrt[b]*(a + 2*b)*Sinh[2*(c + d*x)])/((a
+ b)*(a + 2*b + a*Cosh[2*(c + d*x)]))))/(256*b^(3/2)*d*(a + b*Sech[c + d*x]
^2)^2) + ((a + 2*b + a*Cosh[2*c + 2*d*x])^2*Sech[c + d*x]^4*(-((a + 2*b)*Ar
cTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*b^(3/2)*(a + b)^(3/2)*d) + (
a*Sinh[2*(c + d*x)])/(8*b*(a + b)*d*(a + 2*b + a*Cosh[2*(c + d*x)]))))/(16*
(a + b*Sech[c + d*x]^2)^2)
```

Maple [B] time = 0.094, size = 537, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/2/d/a^2/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/a^2/(tanh(1/2*d*x+1/2*c)+1)-1/2/
d/a^2*ln(tanh(1/2*d*x+1/2*c)+1)-2/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)*b+1/d*b/a
^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2
*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*tanh(1/2*d*x+1/2*c)^3+1/d*b/a^2/(tanh(1/2
*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/
2*d*x+1/2*c)^2*b+a+b)*tanh(1/2*d*x+1/2*c)+3/4/d*b^(1/2)/a^2/(a+b)^(1/2)*ln(
(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2)
)-3/4/d*b^(1/2)/a^2/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh
(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/d*b^(3/2)/a^3/(a+b)^(1/2)*ln((a+b)^(
1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-1/d*b
^(3/2)/a^3/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+
1/2*c)*b^(1/2)+(a+b)^(1/2))+1/2/d/a^2/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/a^2/(
tanh(1/2*d*x+1/2*c)-1)+1/2/d/a^2*ln(tanh(1/2*d*x+1/2*c)-1)+2/d/a^3*ln(tanh(
1/2*d*x+1/2*c)-1)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.88621, size = 7262, normalized size = 55.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(
d*x + c)^8 - 2*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^6 + 2*(14*
a^2*cosh(d*x + c)^2 - 2*(a^2 + 4*a*b)*d*x + a^2 + 2*a*b)*sinh(d*x + c)^6 +
4*(14*a^2*cosh(d*x + c)^3 - 3*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x
+ c))*sinh(d*x + c)^5 - 8*((a^2 + 6*a*b + 8*b^2)*d*x + a*b + 2*b^2)*cosh(d*
x + c)^4 + 2*(35*a^2*cosh(d*x + c)^4 - 4*(a^2 + 6*a*b + 8*b^2)*d*x - 15*(2*
(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^2 - 4*a*b - 8*b^2)*sinh(d*x
+ c)^4 + 8*(7*a^2*cosh(d*x + c)^5 - 5*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*c
osh(d*x + c)^3 - 4*((a^2 + 6*a*b + 8*b^2)*d*x + a*b + 2*b^2)*cosh(d*x + c))
*sinh(d*x + c)^3 - 2*(2*(a^2 + 4*a*b)*d*x + a^2 + 6*a*b)*cosh(d*x + c)^2 +
2*(14*a^2*cosh(d*x + c)^6 - 15*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x
 + c)^4 - 2*(a^2 + 4*a*b)*d*x - 24*((a^2 + 6*a*b + 8*b^2)*d*x + a*b + 2*b^2
)*cosh(d*x + c)^2 - a^2 - 6*a*b)*sinh(d*x + c)^2 + 2*((3*a^2 + 4*a*b)*cosh(
d*x + c)^6 + 6*(3*a^2 + 4*a*b)*cosh(d*x + c)*sinh(d*x + c)^5 + (3*a^2 + 4*a
*b)*sinh(d*x + c)^6 + 2*(3*a^2 + 10*a*b + 8*b^2)*cosh(d*x + c)^4 + (15*(3*a
^2 + 4*a*b)*cosh(d*x + c)^2 + 6*a^2 + 20*a*b + 16*b^2)*sinh(d*x + c)^4 + 4*
(5*(3*a^2 + 4*a*b)*cosh(d*x + c)^3 + 2*(3*a^2 + 10*a*b + 8*b^2)*cosh(d*x +
c))*sinh(d*x + c)^3 + (3*a^2 + 4*a*b)*cosh(d*x + c)^2 + (15*(3*a^2 + 4*a*b)
*cosh(d*x + c)^4 + 12*(3*a^2 + 10*a*b + 8*b^2)*cosh(d*x + c)^2 + 3*a^2 + 4*
a*b)*sinh(d*x + c)^2 + 2*(3*(3*a^2 + 4*a*b)*cosh(d*x + c)^5 + 4*(3*a^2 + 10
*a*b + 8*b^2)*cosh(d*x + c)^3 + (3*a^2 + 4*a*b)*cosh(d*x + c))*sinh(d*x + c
))*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x
+ c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*c
osh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^
2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a^2 +
a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a
*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c
)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*c
osh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*c
osh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - a^2 + 4*(2*
a^2*cosh(d*x + c)^7 - 3*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^5
 - 8*((a^2 + 6*a*b + 8*b^2)*d*x + a*b + 2*b^2)*cosh(d*x + c)^3 - (2*(a^2 +
4*a*b)*d*x + a^2 + 6*a*b)*cosh(d*x + c))*sinh(d*x + c))/(a^4*d*cosh(d*x + c
)^6 + 6*a^4*d*cosh(d*x + c)*sinh(d*x + c)^5 + a^4*d*sinh(d*x + c)^6 + a^4*d
*cosh(d*x + c)^2 + 2*(a^4 + 2*a^3*b)*d*cosh(d*x + c)^4 + (15*a^4*d*cosh(d*x
 + c)^2 + 2*(a^4 + 2*a^3*b)*d)*sinh(d*x + c)^4 + 4*(5*a^4*d*cosh(d*x + c)^3
 + 2*(a^4 + 2*a^3*b)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (15*a^4*d*cosh(d*x
+ c)^4 + a^4*d + 12*(a^4 + 2*a^3*b)*d*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*
(3*a^4*d*cosh(d*x + c)^5 + a^4*d*cosh(d*x + c) + 4*(a^4 + 2*a^3*b)*d*cosh(d
*x + c)^3)*sinh(d*x + c)), 1/8*(a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*s
inh(d*x + c)^7 + a^2*sinh(d*x + c)^8 - 2*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b
)*cosh(d*x + c)^6 + 2*(14*a^2*cosh(d*x + c)^2 - 2*(a^2 + 4*a*b)*d*x + a^2 +
 2*a*b)*sinh(d*x + c)^6 + 4*(14*a^2*cosh(d*x + c)^3 - 3*(2*(a^2 + 4*a*b)*d*
x - a^2 - 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^5 - 8*((a^2 + 6*a*b + 8*b^2)*
d*x + a*b + 2*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*cosh(d*x + c)^4 - 4*(a^2 + 6
*a*b + 8*b^2)*d*x - 15*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^2
- 4*a*b - 8*b^2)*sinh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^5 - 5*(2*(a^2 + 4
*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^3 - 4*((a^2 + 6*a*b + 8*b^2)*d*x + a
*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(2*(a^2 + 4*a*b)*d*x + a^2 +
 6*a*b)*cosh(d*x + c)^2 + 2*(14*a^2*cosh(d*x + c)^6 - 15*(2*(a^2 + 4*a*b)*d
*x - a^2 - 2*a*b)*cosh(d*x + c)^4 - 2*(a^2 + 4*a*b)*d*x - 24*((a^2 + 6*a*b
```

```
+ 8*b^2)*d*x + a*b + 2*b^2)*cosh(d*x + c)^2 - a^2 - 6*a*b)*sinh(d*x + c)^2
+ 4*((3*a^2 + 4*a*b)*cosh(d*x + c)^6 + 6*(3*a^2 + 4*a*b)*cosh(d*x + c)*sinh
(d*x + c)^5 + (3*a^2 + 4*a*b)*sinh(d*x + c)^6 + 2*(3*a^2 + 10*a*b + 8*b^2)*
cosh(d*x + c)^4 + (15*(3*a^2 + 4*a*b)*cosh(d*x + c)^2 + 6*a^2 + 20*a*b + 16
*b^2)*sinh(d*x + c)^4 + 4*(5*(3*a^2 + 4*a*b)*cosh(d*x + c)^3 + 2*(3*a^2 + 1
0*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + (3*a^2 + 4*a*b)*cosh(d*x +
c)^2 + (15*(3*a^2 + 4*a*b)*cosh(d*x + c)^4 + 12*(3*a^2 + 10*a*b + 8*b^2)*co
sh(d*x + c)^2 + 3*a^2 + 4*a*b)*sinh(d*x + c)^2 + 2*(3*(3*a^2 + 4*a*b)*cosh(
d*x + c)^5 + 4*(3*a^2 + 10*a*b + 8*b^2)*cosh(d*x + c)^3 + (3*a^2 + 4*a*b)*c
osh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2
 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(
a + b))/b) - a^2 + 4*(2*a^2*cosh(d*x + c)^7 - 3*(2*(a^2 + 4*a*b)*d*x - a^2
- 2*a*b)*cosh(d*x + c)^5 - 8*((a^2 + 6*a*b + 8*b^2)*d*x + a*b + 2*b^2)*cosh
(d*x + c)^3 - (2*(a^2 + 4*a*b)*d*x + a^2 + 6*a*b)*cosh(d*x + c))*sinh(d*x +
 c))/(a^4*d*cosh(d*x + c)^6 + 6*a^4*d*cosh(d*x + c)*sinh(d*x + c)^5 + a^4*d
*sinh(d*x + c)^6 + a^4*d*cosh(d*x + c)^2 + 2*(a^4 + 2*a^3*b)*d*cosh(d*x + c
)^4 + (15*a^4*d*cosh(d*x + c)^2 + 2*(a^4 + 2*a^3*b)*d)*sinh(d*x + c)^4 + 4*
(5*a^4*d*cosh(d*x + c)^3 + 2*(a^4 + 2*a^3*b)*d*cosh(d*x + c))*sinh(d*x + c)
^3 + (15*a^4*d*cosh(d*x + c)^4 + a^4*d + 12*(a^4 + 2*a^3*b)*d*cosh(d*x + c)
^2)*sinh(d*x + c)^2 + 2*(3*a^4*d*cosh(d*x + c)^5 + a^4*d*cosh(d*x + c) + 4*
(a^4 + 2*a^3*b)*d*cosh(d*x + c)^3)*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*sech(d*x+c)**2)**2,x)

[Out] Timed out
```

Giac [A] time = 1.18342, size = 325, normalized size = 2.48

$$-\frac{(dx+c)(a+4b)}{2a^3d}+\frac{e^{(2dx+2c)}}{8a^2d}+\frac{\left(3ab+4b^2\right)\arctan\left(\frac{ae^{(2dx+2c)}+a+2b}{2\sqrt{-ab-b^2}}\right)}{2\sqrt{-ab-b^2}a^3d}+\frac{2a^2e^{(6dx+6c)}+8abe^{(6dx+6c)}+a^2e^{(4dx+4c)}-16b}{24\left(ae^{(6dx+6c)}+2ae^{(4dx+4c)}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] -1/2*(d*x + c)*(a + 4*b)/(a^3*d) + 1/8*e^(2*d*x + 2*c)/(a^2*d) + 1/2*(3*a*b
 + 4*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(
-a*b - b^2)*a^3*d) + 1/24*(2*a^2*e^(6*d*x + 6*c) + 8*a*b*e^(6*d*x + 6*c) +
a^2*e^(4*d*x + 4*c) - 16*b^2*e^(4*d*x + 4*c) - 4*a^2*e^(2*d*x + 2*c) - 28*a
*b*e^(2*d*x + 2*c) - 3*a^2)/((a*e^(6*d*x + 6*c) + 2*a*e^(4*d*x + 4*c) + 4*b
*e^(4*d*x + 4*c) + a*e^(2*d*x + 2*c))*a^3*d)
```

3.36 $\int \frac{\sinh(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=84

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{5/2}d}+\frac{3\cosh(c+dx)}{2a^2d}-\frac{\cosh^3(c+dx)}{2ad\left(a\cosh^2(c+dx)+b\right)}$$

```
[Out] (-3*Sqrt[b]*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(2*a^(5/2)*d) + (3*Cos
h[c + d*x])/(2*a^2*d) - Cosh[c + d*x]^3/(2*a*d*(b + a*Cosh[c + d*x]^2))
```

Rubi [A] time = 0.0647587, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4133, 288, 321, 205}

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{5/2}d}+\frac{3\cosh(c+dx)}{2a^2d}-\frac{\cosh^3(c+dx)}{2ad\left(a\cosh^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (-3*Sqrt[b]*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(2*a^(5/2)*d) + (3*Cos
h[c + d*x])/(2*a^2*d) - Cosh[c + d*x]^3/(2*a*d*(b + a*Cosh[c + d*x]^2))
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x^4}{\left(b+ax^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}\\
&= -\frac{\cosh^3(c+dx)}{2ad\left(b+a\cosh^2(c+dx)\right)} + \frac{3\operatorname{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx, x, \cosh(c+dx)\right)}{2ad}\\
&= \frac{3\cosh(c+dx)}{2a^2d} - \frac{\cosh^3(c+dx)}{2ad\left(b+a\cosh^2(c+dx)\right)} - \frac{(3b)\operatorname{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cosh(c+dx)\right)}{2a^2d}\\
&= -\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{5/2}d} + \frac{3\cosh(c+dx)}{2a^2d} - \frac{\cosh^3(c+dx)}{2ad\left(b+a\cosh^2(c+dx)\right)}
\end{aligned}
$$

Mathematica [C] time = 2.70154, size = 479, normalized size = 5.7

$$
\frac{\operatorname{sech}^4(c+dx)(a\cosh(2(c+dx))+a+2b)^2\left(\frac{2\left(-\left(a^2+24b^2\right)\tan^{-1}\left(\frac{\sinh(c)\tanh\left(\frac{dx}{2}\right)\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\right)+\cosh(c)\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(\ldots}{\sqrt{b}}\right)\ldots}{\ldots}\right.}{\ldots}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]^4*((32*Cosh[c]*Cosh[d*x])/
a^2 + (32*b*Cosh[c + d*x])/(a^2*(a + 2*b + a*Cosh[2*(c + d*x)])) + (2*(-((a
^2 + 24*b^2)*ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*
Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Si
nh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]) - a^2*ArcTan[((Sqrt[a] + I*Sqrt[a + b]*
Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*S
qrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] - 24*b^2*Ar
cTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d
*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[
(d*x)/2]))/Sqrt[b]] + a^2*ArcTan[(Sqrt[a] - I*Sqrt[a + b]*Tanh[(c + d*x)/2]
)/Sqrt[b]] + a^2*ArcTan[(Sqrt[a] + I*Sqrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]
] + 16*Sqrt[a]*b^(3/2)*Sinh[c]*Sinh[d*x]))/(a^(5/2)*b^(3/2))))/(128*d*(a +
b*Sech[c + d*x]^2)^2)
```

Maple [A] time = 0.031, size = 74, normalized size = 0.9

$$
\frac{b\operatorname{sech}(dx+c)}{2\,da^2\left(a+b\left(\operatorname{sech}(dx+c)\right)^2\right)} + \frac{3\,b}{2\,da^2}\arctan\left(b\operatorname{sech}(dx+c)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + \frac{1}{da^2\operatorname{sech}(dx+c)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x)
```

[Out] 1/2/d/a^2*b*sech(d*x+c)/(a+b*sech(d*x+c)^2)+3/2/d/a^2*b/(a*b)^(1/2)*arctan(
sech(d*x+c)*b/(a*b)^(1/2))+1/d/a^2/sech(d*x+c)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\left(ae^{(4c)}+2\,be^{(4c)}\right)e^{(4\,dx)}+3\left(ae^{(2c)}+2\,be^{(2c)}\right)e^{(2\,dx)}+ae^{(6\,dx+6\,c)}+a}{2\left(a^3de^{(5\,dx+5\,c)}+a^3de^{(dx+c)}+2\left(a^3de^{(3\,c)}+2\,a^2bde^{(3\,c)}\right)e^{(3\,dx)}\right)}-\frac{1}{2}\int\frac{6\left(be^{(3\,dx+3\,c)}-be^{(dx+c)}\right)}{a^3e^{(4\,dx+4\,c)}+a^3+2\left(a^3e^{(2\,c)}+2\,a^2be^{(2\,c)}\right)e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out] 1/2*(3*(a*e^(4*c) + 2*b*e^(4*c))*e^(4*d*x) + 3*(a*e^(2*c) + 2*b*e^(2*c))*e^
(2*d*x) + a*e^(6*d*x + 6*c) + a)/(a^3*d*e^(5*d*x + 5*c) + a^3*d*e^(d*x + c)
 + 2*(a^3*d*e^(3*c) + 2*a^2*b*d*e^(3*c))*e^(3*d*x)) - 1/2*integrate(6*(b*e^
(3*d*x + 3*c) - b*e^(d*x + c))/(a^3*e^(4*d*x + 4*c) + a^3 + 2*(a^3*e^(2*c)
+ 2*a^2*b*e^(2*c))*e^(2*d*x)), x)

Fricas [B] time = 2.87968, size = 4732, normalized size = 56.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [1/4*(2*a*cosh(d*x + c)^6 + 12*a*cosh(d*x + c)*sinh(d*x + c)^5 + 2*a*sinh(d
x + c)^6 + 6(a + 2*b)*cosh(d*x + c)^4 + 6*(5*a*cosh(d*x + c)^2 + a + 2*b)
*sinh(d*x + c)^4 + 8*(5*a*cosh(d*x + c)^3 + 3*(a + 2*b)*cosh(d*x + c))*sinh
(d*x + c)^3 + 6*(a + 2*b)*cosh(d*x + c)^2 + 6*(5*a*cosh(d*x + c)^4 + 6*(a +
 2*b)*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 3*(a*cosh(d*x + c)^5 + 5
*a*cosh(d*x + c)*sinh(d*x + c)^4 + a*sinh(d*x + c)^5 + 2*(a + 2*b)*cosh(d*x
 + c)^3 + 2*(5*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^3 + 2*(5*a*cosh(d
x + c)^3 + 3(a + 2*b)*cosh(d*x + c))*sinh(d*x + c)^2 + a*cosh(d*x + c) +
(5*a*cosh(d*x + c)^4 + 6*(a + 2*b)*cosh(d*x + c)^2 + a)*sinh(d*x + c))*sqrt
(-b/a)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(
d*x + c)^4 + 2*(a - 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a - 2*b
)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a - 2*b)*cosh(d*x + c))*sinh(d*
x + c) - 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(
d*x + c)^3 + a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))*sqr
t(-b/a) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sin
h(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2
*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(
d*x + c) + a)) + 12*(a*cosh(d*x + c)^5 + 2*(a + 2*b)*cosh(d*x + c)^3 + (a +
 2*b)*cosh(d*x + c))*sinh(d*x + c) + 2*a)/(a^3*d*cosh(d*x + c)^5 + 5*a^3*d*
cosh(d*x + c)*sinh(d*x + c)^4 + a^3*d*sinh(d*x + c)^5 + a^3*d*cosh(d*x + c)
 + 2*(a^3 + 2*a^2*b)*d*cosh(d*x + c)^3 + 2*(5*a^3*d*cosh(d*x + c)^2 + (a^3
+ 2*a^2*b)*d)*sinh(d*x + c)^3 + 2*(5*a^3*d*cosh(d*x + c)^3 + 3*(a^3 + 2*a^2
*b)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (5*a^3*d*cosh(d*x + c)^4 + a^3*d + 6
*(a^3 + 2*a^2*b)*d*cosh(d*x + c)^2)*sinh(d*x + c)), 1/2*(a*cosh(d*x + c)^6
+ 6*a*cosh(d*x + c)*sinh(d*x + c)^5 + a*sinh(d*x + c)^6 + 3*(a + 2*b)*cosh(
d*x + c)^4 + 3*(5*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^4 + 4*(5*a*cos
h(d*x + c)^3 + 3*(a + 2*b)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a + 2*b)*cos
h(d*x + c)^2 + 3*(5*a*cosh(d*x + c)^4 + 6*(a + 2*b)*cosh(d*x + c)^2 + a + 2

```
*b)*sinh(d*x + c)^2 + 3*(a*cosh(d*x + c)^5 + 5*a*cosh(d*x + c)*sinh(d*x + c
)^4 + a*sinh(d*x + c)^5 + 2*(a + 2*b)*cosh(d*x + c)^3 + 2*(5*a*cosh(d*x + c
)^2 + a + 2*b)*sinh(d*x + c)^3 + 2*(5*a*cosh(d*x + c)^3 + 3*(a + 2*b)*cosh(
d*x + c))*sinh(d*x + c)^2 + a*cosh(d*x + c) + (5*a*cosh(d*x + c)^4 + 6*(a +
 2*b)*cosh(d*x + c)^2 + a)*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*(a*cosh(d*x
+ c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (a + 4*b)*
cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a + 4*b)*sinh(d*x + c))*sqrt(b/a)/b)
 - 3*(a*cosh(d*x + c)^5 + 5*a*cosh(d*x + c)*sinh(d*x + c)^4 + a*sinh(d*x +
c)^5 + 2*(a + 2*b)*cosh(d*x + c)^3 + 2*(5*a*cosh(d*x + c)^2 + a + 2*b)*sinh
(d*x + c)^3 + 2*(5*a*cosh(d*x + c)^3 + 3*(a + 2*b)*cosh(d*x + c))*sinh(d*x
+ c)^2 + a*cosh(d*x + c) + (5*a*cosh(d*x + c)^4 + 6*(a + 2*b)*cosh(d*x + c)
^2 + a)*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*(a*cosh(d*x + c) + a*sinh(d*x +
 c))*sqrt(b/a)/b) + 6*(a*cosh(d*x + c)^5 + 2*(a + 2*b)*cosh(d*x + c)^3 + (a
 + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)/(a^3*d*cosh(d*x + c)^5 + 5*a^3*d*
cosh(d*x + c)*sinh(d*x + c)^4 + a^3*d*sinh(d*x + c)^5 + a^3*d*cosh(d*x + c)
 + 2*(a^3 + 2*a^2*b)*d*cosh(d*x + c)^3 + 2*(5*a^3*d*cosh(d*x + c)^2 + (a^3
+ 2*a^2*b)*d)*sinh(d*x + c)^3 + 2*(5*a^3*d*cosh(d*x + c)^3 + 3*(a^3 + 2*a^2
*b)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (5*a^3*d*cosh(d*x + c)^4 + a^3*d + 6
*(a^3 + 2*a^2*b)*d*cosh(d*x + c)^2)*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.37 $\int \frac{\mathbf{csch}(c+dx)}{\left(a+b\mathbf{sech}^2(c+dx)\right)^2} dx$

Optimal. Leaf size=99

$$\frac{\sqrt{b}(3a+b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{3/2}d(a+b)^2} - \frac{b\cosh(c+dx)}{2ad(a+b)\left(a\cosh^2(c+dx)+b\right)} - \frac{\tanh^{-1}(\cosh(c+dx))}{d(a+b)^2}$$

```
[Out] (Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(2*a^(3/2)*(a +
 b)^2*d) - ArcTanh[Cosh[c + d*x]]/((a + b)^2*d) - (b*Cosh[c + d*x])/(2*a*(a
 + b)*d*(b + a*Cosh[c + d*x]^2))
```

Rubi [A] time = 0.129906, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4133, 470, 522, 206, 205}

$$\frac{\sqrt{b}(3a+b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{3/2}d(a+b)^2} - \frac{b\cosh(c+dx)}{2ad(a+b)\left(a\cosh^2(c+dx)+b\right)} - \frac{\tanh^{-1}(\cosh(c+dx))}{d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(2*a^(3/2)*(a +
 b)^2*d) - ArcTanh[Cosh[c + d*x]]/((a + b)^2*d) - (b*Cosh[c + d*x])/(2*a*(a
 + b)*d*(b + a*Cosh[c + d*x]^2))
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)\left(b+ax^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{b\cosh(c+dx)}{2a(a+b)d\left(b+a\cosh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{b+(-2a-b)x^2}{(1-x^2)\left(b+ax^2\right)}\,dx, x, \cosh(c+dx)\right)}{2a(a+b)d}$$

$$= -\frac{b\cosh(c+dx)}{2a(a+b)d\left(b+a\cosh^2(c+dx)\right)} - \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(c+dx)\right)}{(a+b)^2d} + \frac{(b(3a+b))\operatorname{Su}}{}$$

$$= \frac{\sqrt{b}(3a+b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{3/2}(a+b)^2d} - \frac{\tanh^{-1}(\cosh(c+dx))}{(a+b)^2d} - \frac{b\cosh(c+dx)}{2a(a+b)d\left(b+a\cosh^2(c+d\right.}$$

Mathematica [C] time = 1.12061, size = 377, normalized size = 3.81

$$\frac{\operatorname{sech}^3(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\frac{\sqrt{b}(3a+b)\operatorname{sech}(c+dx)(a\cosh(2(c+dx))+a+2b)\tan^{-1}\left(\frac{\sinh(c)\tanh\left(\frac{dx}{2}\right)\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\right)+\text{cc}}{\sqrt{b}}\right)}{a^{3/2}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^3*((-2*b*(a + b))/a + (Sqrt[
b]*(3*a + b)*ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*
Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Si
nh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c +
 d*x])/a^(3/2) + (Sqrt[b]*(3*a + b)*ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(
Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a
+ b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cosh
[2*(c + d*x)])*Sech[c + d*x])/a^(3/2) - 2*(a + 2*b + a*Cosh[2*(c + d*x)])*L
og[Cosh[(c + d*x)/2]]*Sech[c + d*x] + 2*(a + 2*b + a*Cosh[2*(c + d*x)])*Log
[Sinh[(c + d*x)/2]]*Sech[c + d*x]))/(8*(a + b)^2*d*(a + b*Sech[c + d*x]^2)^
2)
```

Maple [B] time = 0.07, size = 431, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/d*b/(a+b)^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*
d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*tanh(1/2*d*x+1/2*c)^2+1/d*b^2
/(a+b)^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/
2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)/a*tanh(1/2*d*x+1/2*c)^2-1/d*b/(a+b)
^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2
*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)-1/d*b^2/(a+b)^2/(tanh(1/2*d*x+1/2*c)^4*a+
b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b
+a+b)/a+3/2/d*b/(a+b)^2/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)
^2+2*a-2*b)/(a*b)^(1/2))+1/2/d*b^2/(a+b)^2/a/(a*b)^(1/2)*arctan(1/4*(2*(a+b
)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))+1/d/(a+b)^2*ln(tanh(1/2*d*x+1
/2*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{be^{(3\,dx+3\,c)}+be^{(dx+c)}}{a^3d+a^2bd+\left(a^3de^{(4\,c)}+a^2bde^{(4\,c)}\right)e^{(4\,dx)}+2\left(a^3de^{(2\,c)}+3\,a^2bde^{(2\,c)}+2\,ab^2de^{(2\,c)}\right)e^{(2\,dx)}}-\frac{\log\left(\left(e^{(dx+c)}+1\right)e^{(-c)}\right)}{a^2d+2\,abd+b^2d}+\frac{\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -(b*e^(3*d*x + 3*c) + b*e^(d*x + c))/(a^3*d + a^2*b*d + (a^3*d*e^(4*c) + a^
2*b*d*e^(4*c))*e^(4*d*x) + 2*(a^3*d*e^(2*c) + 3*a^2*b*d*e^(2*c) + 2*a*b^2*d
*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c) + 1)*e^(-c))/(a^2*d + 2*a*b*d + b^2
*d) + log((e^(d*x + c) - 1)*e^(-c))/(a^2*d + 2*a*b*d + b^2*d) + 2*integrate
(1/2*((3*a*b*e^(3*c) + b^2*e^(3*c))*e^(3*d*x) - (3*a*b*e^c + b^2*e^c)*e^(d*
x))/(a^4 + 2*a^3*b + a^2*b^2 + (a^4*e^(4*c) + 2*a^3*b*e^(4*c) + a^2*b^2*e^(
4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) + 4*a^3*b*e^(2*c) + 5*a^2*b^2*e^(2*c) + 2*
a*b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 3.3364, size = 5986, normalized size = 60.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a*b + b^2)*cosh(d*x + c)^3 + 12*(a*b + b^2)*cosh(d*x + c)*sinh(d*
x + c)^2 + 4*(a*b + b^2)*sinh(d*x + c)^3 - ((3*a^2 + a*b)*cosh(d*x + c)^4 +
 4*(3*a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 + a*b)*sinh(d*x + c
)^4 + 2*(3*a^2 + 7*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a^2 + a*b)*cosh(d
*x + c)^2 + 3*a^2 + 7*a*b + 2*b^2)*sinh(d*x + c)^2 + 3*a^2 + a*b + 4*((3*a^
2 + a*b)*cosh(d*x + c)^3 + (3*a^2 + 7*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x
+ c))*sqrt(-b/a)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3
 + a*sinh(d*x + c)^4 + 2*(a - 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2
 + a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a - 2*b)*cosh(d*x + c
))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2
 + a*sinh(d*x + c)^3 + a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a)*sinh(d*x
 + c))*sqrt(-b/a) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)
```

```
^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)
^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x +
 c))*sinh(d*x + c) + a)) + 4*(a*b + b^2)*cosh(d*x + c) + 4*(a^2*cosh(d*x +
c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 +
 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x
+ c)^2 + a^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d
*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 4*(a^2*cosh(d*x + c)^4 +
4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)
*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2
+ a^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)
)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 4*(3*(a*b + b^2)*cosh(d*x + c)^2
 + a*b + b^2)*sinh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 +
 4*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^3
*b + a^2*b^2)*d*sinh(d*x + c)^4 + 2*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d
*cosh(d*x + c)^2 + 2*(3*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (a^4
+ 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d)*sinh(d*x + c)^2 + (a^4 + 2*a^3*b + a^2*
b^2)*d + 4*((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^3 + (a^4 + 4*a^3*b +
5*a^2*b^2 + 2*a*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*(a*b + b^2)*c
osh(d*x + c)^3 + 6*(a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(a*b + b^2
)*sinh(d*x + c)^3 + ((3*a^2 + a*b)*cosh(d*x + c)^4 + 4*(3*a^2 + a*b)*cosh(d
*x + c)*sinh(d*x + c)^3 + (3*a^2 + a*b)*sinh(d*x + c)^4 + 2*(3*a^2 + 7*a*b
+ 2*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a^2 + a*b)*cosh(d*x + c)^2 + 3*a^2 + 7*a
*b + 2*b^2)*sinh(d*x + c)^2 + 3*a^2 + a*b + 4*((3*a^2 + a*b)*cosh(d*x + c)^
3 + (3*a^2 + 7*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*arctan(
1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c
)^3 + (a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a + 4*b)*sinh(d*x +
c))*sqrt(b/a)/b) - ((3*a^2 + a*b)*cosh(d*x + c)^4 + 4*(3*a^2 + a*b)*cosh(d*
x + c)*sinh(d*x + c)^3 + (3*a^2 + a*b)*sinh(d*x + c)^4 + 2*(3*a^2 + 7*a*b +
 2*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a^2 + a*b)*cosh(d*x + c)^2 + 3*a^2 + 7*a*
b + 2*b^2)*sinh(d*x + c)^2 + 3*a^2 + a*b + 4*((3*a^2 + a*b)*cosh(d*x + c)^3
 + (3*a^2 + 7*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*arctan(1
/2*(a*cosh(d*x + c) + a*sinh(d*x + c))*sqrt(b/a)/b) + 2*(a*b + b^2)*cosh(d*
x + c) + 2*(a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2
*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)
^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2
*a*b)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1)
- 2*(a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d
*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^
2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*c
osh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(3*
(a*b + b^2)*cosh(d*x + c)^2 + a*b + b^2)*sinh(d*x + c))/((a^4 + 2*a^3*b + a
^2*b^2)*d*cosh(d*x + c)^4 + 4*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sin
h(d*x + c)^3 + (a^4 + 2*a^3*b + a^2*b^2)*d*sinh(d*x + c)^4 + 2*(a^4 + 4*a^3
*b + 5*a^2*b^2 + 2*a*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^4 + 2*a^3*b + a^2*b^2
)*d*cosh(d*x + c)^2 + (a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d)*sinh(d*x + c
)^2 + (a^4 + 2*a^3*b + a^2*b^2)*d + 4*((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x
 + c)^3 + (a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*cosh(d*x + c))*sinh(d*x +
 c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(csch(c + d*x)/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.38 $\int \frac{\text{csch}^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=92

$$\frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2d(a+b)^{5/2}}-\frac{3\coth(c+dx)}{2d(a+b)^2}+\frac{\coth(c+dx)}{2d(a+b)\left(a-b\tanh^2(c+dx)+b\right)}$$

```
[Out] (3*Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(2*(a + b)^(5/2)*d
) - (3*Coth[c + d*x])/(2*(a + b)^2*d) + Coth[c + d*x]/(2*(a + b)*d*(a + b -
 b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.0844725, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4132, 290, 325, 208}

$$\frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2d(a+b)^{5/2}}-\frac{3\coth(c+dx)}{2d(a+b)^2}+\frac{\coth(c+dx)}{2d(a+b)\left(a-b\tanh^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (3*Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(2*(a + b)^(5/2)*d
) - (3*Coth[c + d*x])/(2*(a + b)^2*d) + Coth[c + d*x]/(2*(a + b)*d*(a + b -
 b*Tanh[c + d*x]^2))
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{x^2\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\coth(c+dx)}{2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{3\operatorname{Subst}\left(\int \frac{1}{x^2\left(a+b-bx^2\right)}\,dx, x, \tanh(c+dx)\right)}{2(a+b)d}$$

$$= -\frac{3\coth(c+dx)}{2(a+b)^2d} + \frac{\coth(c+dx)}{2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{(3b)\operatorname{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x,\right.}{2(a+b)^2d}$$

$$= \frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}d} - \frac{3\coth(c+dx)}{2(a+b)^2d} + \frac{\coth(c+dx)}{2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)}$$

Mathematica [B] time = 2.46124, size = 220, normalized size = 2.39

$$\frac{\operatorname{sech}^4(c+dx)(a\cosh(2(c+dx))+a+2b)\left(2\operatorname{csch}(c)\sinh(dx)\operatorname{csch}(c+dx)(a\cosh(2(c+dx))+a+2b)+\frac{3b(\cosh(2c)-\text{si}}{}\right.}{8d(a+b)^2\left(a+b\text{se}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*((3*b*ArcTanh[(Sech[d*x]*(
Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a
 + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])*(Cosh
[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + 2*(a + 2*
b + a*Cosh[2*(c + d*x)])*Csch[c]*Csch[c + d*x]*Sinh[d*x] + b*Sech[2*c]*Sinh
[2*d*x] - (b*(a + 2*b)*Tanh[2*c])/a))/(8*(a + b)^2*d*(a + b*Sech[c + d*x]^2
)^2)
```

Maple [B] time = 0.085, size = 310, normalized size = 3.4

$$-\frac{1}{2d\left(a^2+2ab+b^2\right)}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{b}{d(a+b)^2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4a+b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/2/d/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)+1/d*b/(a+b)^2/(tanh(1/2*d*x+1/2*
c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2
*c)^2*b+a+b)*tanh(1/2*d*x+1/2*c)^3+1/d*b/(a+b)^2/(tanh(1/2*d*x+1/2*c)^4*a+b
```

```
*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+
a+b)*tanh(1/2*d*x+1/2*c)+3/4/d*b^(1/2)/(a+b)^(5/2)*ln((a+b)^(1/2)*tanh(1/2*
d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-3/4/d*b^(1/2)/(a+b)
^(5/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(
a+b)^(1/2))-1/2/d/(a+b)^2/tanh(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.46985, size = 5806, normalized size = 63.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(2*a^2 + a*b + 2*b^2)*cosh(d*x + c)^4 + 16*(2*a^2 + a*b + 2*b^2)*c
osh(d*x + c)*sinh(d*x + c)^3 + 4*(2*a^2 + a*b + 2*b^2)*sinh(d*x + c)^4 + 8*
(2*a^2 + 4*a*b - b^2)*cosh(d*x + c)^2 + 8*(3*(2*a^2 + a*b + 2*b^2)*cosh(d*x
 + c)^2 + 2*a^2 + 4*a*b - b^2)*sinh(d*x + c)^2 - 3*(a^2*cosh(d*x + c)^6 + 6
*a^2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d*x + c)^6 + (a^2 + 4*a*b)*co
sh(d*x + c)^4 + (15*a^2*cosh(d*x + c)^2 + a^2 + 4*a*b)*sinh(d*x + c)^4 + 4*
(5*a^2*cosh(d*x + c)^3 + (a^2 + 4*a*b)*cosh(d*x + c))*sinh(d*x + c)^3 - (a^
2 + 4*a*b)*cosh(d*x + c)^2 + (15*a^2*cosh(d*x + c)^4 + 6*(a^2 + 4*a*b)*cosh
(d*x + c)^2 - a^2 - 4*a*b)*sinh(d*x + c)^2 - a^2 + 2*(3*a^2*cosh(d*x + c)^5
 + 2*(a^2 + 4*a*b)*cosh(d*x + c)^3 - (a^2 + 4*a*b)*cosh(d*x + c))*sinh(d*x
+ c))*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d
*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^
2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*
(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a^2
 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2
+ a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x
+ c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b
)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(
a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 8*a^2 -
4*a*b + 16*((2*a^2 + a*b + 2*b^2)*cosh(d*x + c)^3 + (2*a^2 + 4*a*b - b^2)*c
osh(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^6 +
 6*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^4 + 2*a^3
*b + a^2*b^2)*d*sinh(d*x + c)^6 + (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d*c
osh(d*x + c)^4 + (15*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (a^4 + 6
*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d)*sinh(d*x + c)^4 - (a^4 + 6*a^3*b + 9*a^2*b
^2 + 4*a*b^3)*d*cosh(d*x + c)^2 + 4*(5*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x
 + c)^3 + (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d*cosh(d*x + c))*sinh(d*x +
 c)^3 + (15*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 6*(a^4 + 6*a^3*b
+ 9*a^2*b^2 + 4*a*b^3)*d*cosh(d*x + c)^2 - (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a
*b^3)*d)*sinh(d*x + c)^2 - (a^4 + 2*a^3*b + a^2*b^2)*d + 2*(3*(a^4 + 2*a^3*
b + a^2*b^2)*d*cosh(d*x + c)^5 + 2*(a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d*
```

```
cosh(d*x + c)^3 - (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d*cosh(d*x + c))*si
nh(d*x + c)), -1/2*(2*(2*a^2 + a*b + 2*b^2)*cosh(d*x + c)^4 + 8*(2*a^2 + a*
b + 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(2*a^2 + a*b + 2*b^2)*sinh(d*x
 + c)^4 + 4*(2*a^2 + 4*a*b - b^2)*cosh(d*x + c)^2 + 4*(3*(2*a^2 + a*b + 2*b
^2)*cosh(d*x + c)^2 + 2*a^2 + 4*a*b - b^2)*sinh(d*x + c)^2 - 3*(a^2*cosh(d*
x + c)^6 + 6*a^2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d*x + c)^6 + (a^2
 + 4*a*b)*cosh(d*x + c)^4 + (15*a^2*cosh(d*x + c)^2 + a^2 + 4*a*b)*sinh(d*x
 + c)^4 + 4*(5*a^2*cosh(d*x + c)^3 + (a^2 + 4*a*b)*cosh(d*x + c))*sinh(d*x
+ c)^3 - (a^2 + 4*a*b)*cosh(d*x + c)^2 + (15*a^2*cosh(d*x + c)^4 + 6*(a^2 +
 4*a*b)*cosh(d*x + c)^2 - a^2 - 4*a*b)*sinh(d*x + c)^2 - a^2 + 2*(3*a^2*cos
h(d*x + c)^5 + 2*(a^2 + 4*a*b)*cosh(d*x + c)^3 - (a^2 + 4*a*b)*cosh(d*x + c
))*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh
(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b)
+ 4*a^2 - 2*a*b + 8*((2*a^2 + a*b + 2*b^2)*cosh(d*x + c)^3 + (2*a^2 + 4*a*b
 - b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x
 + c)^6 + 6*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^
4 + 2*a^3*b + a^2*b^2)*d*sinh(d*x + c)^6 + (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a
*b^3)*d*cosh(d*x + c)^4 + (15*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 +
 (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d)*sinh(d*x + c)^4 - (a^4 + 6*a^3*b
+ 9*a^2*b^2 + 4*a*b^3)*d*cosh(d*x + c)^2 + 4*(5*(a^4 + 2*a^3*b + a^2*b^2)*d
*cosh(d*x + c)^3 + (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d*cosh(d*x + c))*s
inh(d*x + c)^3 + (15*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 6*(a^4 +
 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d*cosh(d*x + c)^2 - (a^4 + 6*a^3*b + 9*a^2*
b^2 + 4*a*b^3)*d)*sinh(d*x + c)^2 - (a^4 + 2*a^3*b + a^2*b^2)*d + 2*(3*(a^4
 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^5 + 2*(a^4 + 6*a^3*b + 9*a^2*b^2 + 4*
a*b^3)*d*cosh(d*x + c)^3 - (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d*cosh(d*x
 + c))*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(csch(c + d*x)**2/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [B] time = 1.36499, size = 328, normalized size = 3.57

$$\frac{3\,b\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{2\,\left(a^2d+2\,abd+b^2d\right)\sqrt{-ab-b^2}} - \frac{2\,a^2e^{(4\,dx+4\,c)}+abe^{(4\,dx+4\,c)}+2\,b^2e^{(4\,dx+4\,c)}+4\,a^2e^{(2\,dx+2\,c)}+8\,abe^{(2\,dx+2\,c)}-2\,b^2e^{(2\,dx\cdots}}{\left(a^3d+2\,a^2bd+ab^2d\right)\left(ae^{(6\,dx+6\,c)}+ae^{(4\,dx+4\,c)}+4\,be^{(4\,dx+4\,c)}-ae^{(2\,dx+2\,c)}-4\,be^{\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 3/2*b*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/((a^2*d +
2*a*b*d + b^2*d)*sqrt(-a*b - b^2)) - (2*a^2*e^(4*d*x + 4*c) + a*b*e^(4*d*x
+ 4*c) + 2*b^2*e^(4*d*x + 4*c) + 4*a^2*e^(2*d*x + 2*c) + 8*a*b*e^(2*d*x + 2
*c) - 2*b^2*e^(2*d*x + 2*c) + 2*a^2 - a*b)/((a^3*d + 2*a^2*b*d + a*b^2*d)*(
a*e^(6*d*x + 6*c) + a*e^(4*d*x + 4*c) + 4*b*e^(4*d*x + 4*c) - a*e^(2*d*x +
```

```
2*c) - 4*b*e^(2*d*x + 2*c) - a))
```

3.39 $\int \frac{\text{csch}^3(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=147

$$-\frac{(a-b)\cosh(c+dx)}{2d(a+b)^2\left(a\cosh^2(c+dx)+b\right)}-\frac{\sqrt{b}(3a-b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2\sqrt{a}d(a+b)^3}+\frac{(a-3b)\tanh^{-1}(\cosh(c+dx))}{2d(a+b)^3}-\frac{\coth(c+}{2d(a+b)\left(a\right.}$$

```
[Out] -((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(2*Sqrt[a]*(a
+ b)^3*d) + ((a - 3*b)*ArcTanh[Cosh[c + d*x]])/(2*(a + b)^3*d) - ((a - b)*C
osh[c + d*x])/(2*(a + b)^2*d*(b + a*Cosh[c + d*x]^2)) - (Coth[c + d*x]*Csch
[c + d*x])/(2*(a + b)*d*(b + a*Cosh[c + d*x]^2))
```

Rubi [A] time = 0.205658, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4133, 470, 527, 522, 206, 205}

$$-\frac{(a-b)\cosh(c+dx)}{2d(a+b)^2\left(a\cosh^2(c+dx)+b\right)}-\frac{\sqrt{b}(3a-b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2\sqrt{a}d(a+b)^3}+\frac{(a-3b)\tanh^{-1}(\cosh(c+dx))}{2d(a+b)^3}-\frac{\coth(c+}{2d(a+b)\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```


```
[Out] -((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(2*Sqrt[a]*(a
+ b)^3*d) + ((a - 3*b)*ArcTanh[Cosh[c + d*x]])/(2*(a + b)^3*d) - ((a - b)*C
osh[c + d*x])/(2*(a + b)^2*d*(b + a*Cosh[c + d*x]^2)) - (Coth[c + d*x]*Csch
[c + d*x])/(2*(a + b)*d*(b + a*Cosh[c + d*x]^2))
```


Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
```

```
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)^2(b+ax^2)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2(a+b)d\left(b+a\cosh^2(c+dx)\right)} - \frac{\operatorname{Subst}\left(\int \frac{b+(-a+2b)x^2}{(1-x^2)(b+ax^2)^2}\,dx, x, \cosh(c+dx)\right)}{2(a+b)d}$$

$$= -\frac{(a-b)\cosh(c+dx)}{2(a+b)^2d\left(b+a\cosh^2(c+dx)\right)} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2(a+b)d\left(b+a\cosh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{-4b^2+2(a\cdots}{(1-x^2)(b\cdots}\right.}{4b\cdots}$$

$$= -\frac{(a-b)\cosh(c+dx)}{2(a+b)^2d\left(b+a\cosh^2(c+dx)\right)} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2(a+b)d\left(b+a\cosh^2(c+dx)\right)} + \frac{(a-3b)\operatorname{Subst}\left(\int\right.}{2\cdots}$$

$$= -\frac{(3a-b)\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2\sqrt{a}(a+b)^3d} + \frac{(a-3b)\tanh^{-1}(\cosh(c+dx))}{2(a+b)^3d} - \frac{(a-b)\cosh(\cdots}{2(a+b)^2d\left(b+a\operatorname{co}\cdots\right.}$$

Mathematica [C] time = 2.15362, size = 462, normalized size = 3.14

$$\frac{\operatorname{sech}^3(c+dx)(a\cosh(2(c+dx))+a+2b)\left(-(a+b)\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)\operatorname{sech}(c+dx)(a\cosh(2(c+dx))+a+2b)+4(a-3\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^3*(8*b*(a + b) + (4*Sqrt[b]*
(-3*a + b)*ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Si
nh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh
[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d
*x])/Sqrt[a] + (4*Sqrt[b]*(-3*a + b)*ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[
(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a
 + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos
h[2*(c + d*x)])*Sech[c + d*x])/Sqrt[a] - (a + b)*(a + 2*b + a*Cosh[2*(c + d
*x)])*Csch[(c + d*x)/2]^2*Sech[c + d*x] + 4*(a - 3*b)*(a + 2*b + a*Cosh[2*(
c + d*x)])*Log[Cosh[(c + d*x)/2]]*Sech[c + d*x] - 4*(a - 3*b)*(a + 2*b + a*
Cosh[2*(c + d*x)])*Log[Sinh[(c + d*x)/2]]*Sech[c + d*x] - (a + b)*(a + 2*b
+ a*Cosh[2*(c + d*x)])*Sech[(c + d*x)/2]^2*Sech[c + d*x]))/(32*(a + b)^3*d*
(a + b*Sech[c + d*x]^2)^2)
```

Maple [B] time = 0.083, size = 496, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/8/d*tanh(1/2*d*x+1/2*c)^2/(a^2+2*a*b+b^2)+1/d*b/(a+b)^3/(tanh(1/2*d*x+1/2
*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/
2*c)^2*b+a+b)*a*tanh(1/2*d*x+1/2*c)^2-1/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^
4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)
^2*b+a+b)*tanh(1/2*d*x+1/2*c)^2+1/d*b/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*ta
nh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b
)*a+1/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh
(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)-3/2/d*b/(a+b)^3/(a*b)^(1
/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))*a+1/2/d
*b^2/(a+b)^3/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)
/(a*b)^(1/2))-1/8/d/(a+b)^2/tanh(1/2*d*x+1/2*c)^2-1/2/d/(a+b)^3*ln(tanh(1/2
*d*x+1/2*c))*a+3/2/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a - 3*b)*log((e^(d*x + c) + 1)*e^(-c))/(a^3*d + 3*a^2*b*d + 3*a*b^2*d
+ b^3*d) - 1/2*(a - 3*b)*log((e^(d*x + c) - 1)*e^(-c))/(a^3*d + 3*a^2*b*d +
 3*a*b^2*d + b^3*d) - ((a*e^(7*c) - b*e^(7*c))*e^(7*d*x) + (3*a*e^(5*c) + 5
*b*e^(5*c))*e^(5*d*x) + (3*a*e^(3*c) + 5*b*e^(3*c))*e^(3*d*x) + (a*e^c - b*
e^c)*e^(d*x))/(a^3*d + 2*a^2*b*d + a*b^2*d + (a^3*d*e^(8*c) + 2*a^2*b*d*e^(
8*c) + a*b^2*d*e^(8*c))*e^(8*d*x) + 4*(a^2*b*d*e^(6*c) + 2*a*b^2*d*e^(6*c)
+ b^3*d*e^(6*c))*e^(6*d*x) - 2*(a^3*d*e^(4*c) + 6*a^2*b*d*e^(4*c) + 9*a*b^2
*d*e^(4*c) + 4*b^3*d*e^(4*c))*e^(4*d*x) + 4*(a^2*b*d*e^(2*c) + 2*a*b^2*d*e^
(2*c) + b^3*d*e^(2*c))*e^(2*d*x)) - 8*integrate(1/8*((3*a*b*e^(3*c) - b^2*e
^(3*c))*e^(3*d*x) - (3*a*b*e^c - b^2*e^c)*e^(d*x))/(a^4 + 3*a^3*b + 3*a^2*b
^2 + a*b^3 + (a^4*e^(4*c) + 3*a^3*b*e^(4*c) + 3*a^2*b^2*e^(4*c) + a*b^3*e^(
```

```
4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) + 5*a^3*b*e^(2*c) + 9*a^2*b^2*e^(2*c) + 7*
a*b^3*e^(2*c) + 2*b^4*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 3.48575, size = 16629, normalized size = 113.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a^2 - b^2)*cosh(d*x + c)^7 + 28*(a^2 - b^2)*cosh(d*x + c)*sinh(d*
x + c)^6 + 4*(a^2 - b^2)*sinh(d*x + c)^7 + 4*(3*a^2 + 8*a*b + 5*b^2)*cosh(d
*x + c)^5 + 4*(21*(a^2 - b^2)*cosh(d*x + c)^2 + 3*a^2 + 8*a*b + 5*b^2)*sinh
(d*x + c)^5 + 20*(7*(a^2 - b^2)*cosh(d*x + c)^3 + (3*a^2 + 8*a*b + 5*b^2)*c
osh(d*x + c))*sinh(d*x + c)^4 + 4*(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x + c)^3 +
 4*(35*(a^2 - b^2)*cosh(d*x + c)^4 + 10*(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x +
c)^2 + 3*a^2 + 8*a*b + 5*b^2)*sinh(d*x + c)^3 + 4*(21*(a^2 - b^2)*cosh(d*x
+ c)^5 + 10*(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 + 8*a*b + 5*
b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + ((3*a^2 - a*b)*cosh(d*x + c)^8 + 8*(3
*a^2 - a*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^2 - a*b)*sinh(d*x + c)^8 +
 4*(3*a*b - b^2)*cosh(d*x + c)^6 + 4*(7*(3*a^2 - a*b)*cosh(d*x + c)^2 + 3*a
*b - b^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2 - a*b)*cosh(d*x + c)^3 + 3*(3*a*b -
 b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(3*a^2 + 11*a*b - 4*b^2)*cosh(d*x
+ c)^4 + 2*(35*(3*a^2 - a*b)*cosh(d*x + c)^4 + 30*(3*a*b - b^2)*cosh(d*x +
c)^2 - 3*a^2 - 11*a*b + 4*b^2)*sinh(d*x + c)^4 + 8*(7*(3*a^2 - a*b)*cosh(d*
x + c)^5 + 10*(3*a*b - b^2)*cosh(d*x + c)^3 - (3*a^2 + 11*a*b - 4*b^2)*cosh
(d*x + c))*sinh(d*x + c)^3 + 4*(3*a*b - b^2)*cosh(d*x + c)^2 + 4*(7*(3*a^2
- a*b)*cosh(d*x + c)^6 + 15*(3*a*b - b^2)*cosh(d*x + c)^4 - 3*(3*a^2 + 11*a
*b - 4*b^2)*cosh(d*x + c)^2 + 3*a*b - b^2)*sinh(d*x + c)^2 + 3*a^2 - a*b +
8*((3*a^2 - a*b)*cosh(d*x + c)^7 + 3*(3*a*b - b^2)*cosh(d*x + c)^5 - (3*a^2
 + 11*a*b - 4*b^2)*cosh(d*x + c)^3 + (3*a*b - b^2)*cosh(d*x + c))*sinh(d*x
+ c))*sqrt(-b/a)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3
 + a*sinh(d*x + c)^4 + 2*(a - 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2
 + a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a - 2*b)*cosh(d*x + c
))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2
 + a*sinh(d*x + c)^3 + a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a)*sinh(d*x
 + c))*sqrt(-b/a) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)
^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)
^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x +
 c))*sinh(d*x + c) + a)) + 4*(a^2 - b^2)*cosh(d*x + c) - 2*((a^2 - 3*a*b)*c
osh(d*x + c)^8 + 8*(a^2 - 3*a*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 - 3*a
*b)*sinh(d*x + c)^8 + 4*(a*b - 3*b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 - 3*a*b)*
cosh(d*x + c)^2 + a*b - 3*b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 - 3*a*b)*cosh(d*
x + c)^3 + 3*(a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(a^2 + a*b -
12*b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 - 3*a*b)*cosh(d*x + c)^4 + 30*(a*b - 3
*b^2)*cosh(d*x + c)^2 - a^2 - a*b + 12*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2 - 3
*a*b)*cosh(d*x + c)^5 + 10*(a*b - 3*b^2)*cosh(d*x + c)^3 - (a^2 + a*b - 12*
b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a*b - 3*b^2)*cosh(d*x + c)^2 + 4*(
7*(a^2 - 3*a*b)*cosh(d*x + c)^6 + 15*(a*b - 3*b^2)*cosh(d*x + c)^4 - 3*(a^2
 + a*b - 12*b^2)*cosh(d*x + c)^2 + a*b - 3*b^2)*sinh(d*x + c)^2 + a^2 - 3*a
*b + 8*((a^2 - 3*a*b)*cosh(d*x + c)^7 + 3*(a*b - 3*b^2)*cosh(d*x + c)^5 - (
a^2 + a*b - 12*b^2)*cosh(d*x + c)^3 + (a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x
 + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 2*((a^2 - 3*a*b)*cosh(d*x +
 c)^8 + 8*(a^2 - 3*a*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 - 3*a*b)*sinh(
d*x + c)^8 + 4*(a*b - 3*b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 - 3*a*b)*cosh(d*x
+ c)^2 + a*b - 3*b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 - 3*a*b)*cosh(d*x + c)^3
```

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+ 3*(a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(a^2 + a*b - 12*b^2)*c
osh(d*x + c)^4 + 2*(35*(a^2 - 3*a*b)*cosh(d*x + c)^4 + 30*(a*b - 3*b^2)*cos
h(d*x + c)^2 - a^2 - a*b + 12*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2 - 3*a*b)*cos
h(d*x + c)^5 + 10*(a*b - 3*b^2)*cosh(d*x + c)^3 - (a^2 + a*b - 12*b^2)*cosh
(d*x + c))*sinh(d*x + c)^3 + 4*(a*b - 3*b^2)*cosh(d*x + c)^2 + 4*(7*(a^2 -
3*a*b)*cosh(d*x + c)^6 + 15*(a*b - 3*b^2)*cosh(d*x + c)^4 - 3*(a^2 + a*b -
12*b^2)*cosh(d*x + c)^2 + a*b - 3*b^2)*sinh(d*x + c)^2 + a^2 - 3*a*b + 8*((
a^2 - 3*a*b)*cosh(d*x + c)^7 + 3*(a*b - 3*b^2)*cosh(d*x + c)^5 - (a^2 + a*b
 - 12*b^2)*cosh(d*x + c)^3 + (a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c))*lo
g(cosh(d*x + c) + sinh(d*x + c) - 1) + 4*(7*(a^2 - b^2)*cosh(d*x + c)^6 + 5
*(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x + c)^4 + 3*(3*a^2 + 8*a*b + 5*b^2)*cosh(d
*x + c)^2 + a^2 - b^2)*sinh(d*x + c))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*
d*cosh(d*x + c)^8 + 8*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)*s
inh(d*x + c)^7 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*sinh(d*x + c)^8 + 4*
(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^4 + 3*a^3*b
 + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^2 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^
4)*d)*sinh(d*x + c)^6 - 2*(a^4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d
*cosh(d*x + c)^4 + 8*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)
^3 + 3*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5
 + 2*(35*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^4 + 30*(a^3*b
+ 3*a^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c)^2 - (a^4 + 7*a^3*b + 15*a^2*b^
2 + 13*a*b^3 + 4*b^4)*d)*sinh(d*x + c)^4 + 4*(a^3*b + 3*a^2*b^2 + 3*a*b^3 +
 b^4)*d*cosh(d*x + c)^2 + 8*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d
*x + c)^5 + 10*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c)^3 - (a^4
 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^
3 + 4*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^6 + 15*(a^3*b
+ 3*a^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c)^4 - 3*(a^4 + 7*a^3*b + 15*a^2*
b^2 + 13*a*b^3 + 4*b^4)*d*cosh(d*x + c)^2 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 +
b^4)*d)*sinh(d*x + c)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d + 8*((a^4 +
 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^7 + 3*(a^3*b + 3*a^2*b^2 + 3*
a*b^3 + b^4)*d*cosh(d*x + c)^5 - (a^4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4
*b^4)*d*cosh(d*x + c)^3 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x +
c))*sinh(d*x + c)), -1/2*(2*(a^2 - b^2)*cosh(d*x + c)^7 + 14*(a^2 - b^2)*co
sh(d*x + c)*sinh(d*x + c)^6 + 2*(a^2 - b^2)*sinh(d*x + c)^7 + 2*(3*a^2 + 8*
a*b + 5*b^2)*cosh(d*x + c)^5 + 2*(21*(a^2 - b^2)*cosh(d*x + c)^2 + 3*a^2 +
8*a*b + 5*b^2)*sinh(d*x + c)^5 + 10*(7*(a^2 - b^2)*cosh(d*x + c)^3 + (3*a^2
 + 8*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 2*(3*a^2 + 8*a*b + 5*b^2
)*cosh(d*x + c)^3 + 2*(35*(a^2 - b^2)*cosh(d*x + c)^4 + 10*(3*a^2 + 8*a*b +
 5*b^2)*cosh(d*x + c)^2 + 3*a^2 + 8*a*b + 5*b^2)*sinh(d*x + c)^3 + 2*(21*(a
^2 - b^2)*cosh(d*x + c)^5 + 10*(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x + c)^3 + 3*
(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - ((3*a^2 - a*b)*cos
h(d*x + c)^8 + 8*(3*a^2 - a*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^2 - a*b
)*sinh(d*x + c)^8 + 4*(3*a*b - b^2)*cosh(d*x + c)^6 + 4*(7*(3*a^2 - a*b)*co
sh(d*x + c)^2 + 3*a*b - b^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2 - a*b)*cosh(d*x
+ c)^3 + 3*(3*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(3*a^2 + 11*a*b
 - 4*b^2)*cosh(d*x + c)^4 + 2*(35*(3*a^2 - a*b)*cosh(d*x + c)^4 + 30*(3*a*b
 - b^2)*cosh(d*x + c)^2 - 3*a^2 - 11*a*b + 4*b^2)*sinh(d*x + c)^4 + 8*(7*(3
*a^2 - a*b)*cosh(d*x + c)^5 + 10*(3*a*b - b^2)*cosh(d*x + c)^3 - (3*a^2 + 1
1*a*b - 4*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a*b - b^2)*cosh(d*x +
c)^2 + 4*(7*(3*a^2 - a*b)*cosh(d*x + c)^6 + 15*(3*a*b - b^2)*cosh(d*x + c)^
4 - 3*(3*a^2 + 11*a*b - 4*b^2)*cosh(d*x + c)^2 + 3*a*b - b^2)*sinh(d*x + c)
^2 + 3*a^2 - a*b + 8*((3*a^2 - a*b)*cosh(d*x + c)^7 + 3*(3*a*b - b^2)*cosh(
d*x + c)^5 - (3*a^2 + 11*a*b - 4*b^2)*cosh(d*x + c)^3 + (3*a*b - b^2)*cosh(
d*x + c))*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh
(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (a + 4*b)*cosh(d*x + c) + (
3*a*cosh(d*x + c)^2 + a + 4*b)*sinh(d*x + c))*sqrt(b/a)/b) + ((3*a^2 - a*b)
*cosh(d*x + c)^8 + 8*(3*a^2 - a*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^2 -
 a*b)*sinh(d*x + c)^8 + 4*(3*a*b - b^2)*cosh(d*x + c)^6 + 4*(7*(3*a^2 - a*b
)*cosh(d*x + c)^2 + 3*a*b - b^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2 - a*b)*cosh(
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d*x + c)^3 + 3*(3*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(3*a^2 + 11
*a*b - 4*b^2)*cosh(d*x + c)^4 + 2*(35*(3*a^2 - a*b)*cosh(d*x + c)^4 + 30*(3
*a*b - b^2)*cosh(d*x + c)^2 - 3*a^2 - 11*a*b + 4*b^2)*sinh(d*x + c)^4 + 8*(
7*(3*a^2 - a*b)*cosh(d*x + c)^5 + 10*(3*a*b - b^2)*cosh(d*x + c)^3 - (3*a^2
 + 11*a*b - 4*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a*b - b^2)*cosh(d*
x + c)^2 + 4*(7*(3*a^2 - a*b)*cosh(d*x + c)^6 + 15*(3*a*b - b^2)*cosh(d*x +
 c)^4 - 3*(3*a^2 + 11*a*b - 4*b^2)*cosh(d*x + c)^2 + 3*a*b - b^2)*sinh(d*x
+ c)^2 + 3*a^2 - a*b + 8*((3*a^2 - a*b)*cosh(d*x + c)^7 + 3*(3*a*b - b^2)*c
osh(d*x + c)^5 - (3*a^2 + 11*a*b - 4*b^2)*cosh(d*x + c)^3 + (3*a*b - b^2)*c
osh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*(a*cosh(d*x + c) + a*sinh
(d*x + c))*sqrt(b/a)/b) + 2*(a^2 - b^2)*cosh(d*x + c) - ((a^2 - 3*a*b)*cosh
(d*x + c)^8 + 8*(a^2 - 3*a*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 - 3*a*b)
*sinh(d*x + c)^8 + 4*(a*b - 3*b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 - 3*a*b)*cos
h(d*x + c)^2 + a*b - 3*b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 - 3*a*b)*cosh(d*x +
 c)^3 + 3*(a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(a^2 + a*b - 12*
b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 - 3*a*b)*cosh(d*x + c)^4 + 30*(a*b - 3*b^
2)*cosh(d*x + c)^2 - a^2 - a*b + 12*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2 - 3*a*
b)*cosh(d*x + c)^5 + 10*(a*b - 3*b^2)*cosh(d*x + c)^3 - (a^2 + a*b - 12*b^2
)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a*b - 3*b^2)*cosh(d*x + c)^2 + 4*(7*(
a^2 - 3*a*b)*cosh(d*x + c)^6 + 15*(a*b - 3*b^2)*cosh(d*x + c)^4 - 3*(a^2 +
a*b - 12*b^2)*cosh(d*x + c)^2 + a*b - 3*b^2)*sinh(d*x + c)^2 + a^2 - 3*a*b
+ 8*((a^2 - 3*a*b)*cosh(d*x + c)^7 + 3*(a*b - 3*b^2)*cosh(d*x + c)^5 - (a^2
 + a*b - 12*b^2)*cosh(d*x + c)^3 + (a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x +
c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + ((a^2 - 3*a*b)*cosh(d*x + c)^8
 + 8*(a^2 - 3*a*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 - 3*a*b)*sinh(d*x +
 c)^8 + 4*(a*b - 3*b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 - 3*a*b)*cosh(d*x + c)^
2 + a*b - 3*b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 - 3*a*b)*cosh(d*x + c)^3 + 3*(
a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(a^2 + a*b - 12*b^2)*cosh(d
*x + c)^4 + 2*(35*(a^2 - 3*a*b)*cosh(d*x + c)^4 + 30*(a*b - 3*b^2)*cosh(d*x
 + c)^2 - a^2 - a*b + 12*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2 - 3*a*b)*cosh(d*x
 + c)^5 + 10*(a*b - 3*b^2)*cosh(d*x + c)^3 - (a^2 + a*b - 12*b^2)*cosh(d*x
+ c))*sinh(d*x + c)^3 + 4*(a*b - 3*b^2)*cosh(d*x + c)^2 + 4*(7*(a^2 - 3*a*b
)*cosh(d*x + c)^6 + 15*(a*b - 3*b^2)*cosh(d*x + c)^4 - 3*(a^2 + a*b - 12*b^
2)*cosh(d*x + c)^2 + a*b - 3*b^2)*sinh(d*x + c)^2 + a^2 - 3*a*b + 8*((a^2 -
 3*a*b)*cosh(d*x + c)^7 + 3*(a*b - 3*b^2)*cosh(d*x + c)^5 - (a^2 + a*b - 12
*b^2)*cosh(d*x + c)^3 + (a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cos
h(d*x + c) + sinh(d*x + c) - 1) + 2*(7*(a^2 - b^2)*cosh(d*x + c)^6 + 5*(3*a
^2 + 8*a*b + 5*b^2)*cosh(d*x + c)^4 + 3*(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x +
c)^2 + a^2 - b^2)*sinh(d*x + c))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cos
h(d*x + c)^8 + 8*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)*sinh(d
*x + c)^7 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*sinh(d*x + c)^8 + 4*(a^3*
b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^4 + 3*a^3*b + 3*
a^2*b^2 + a*b^3)*d*cosh(d*x + c)^2 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d)
*sinh(d*x + c)^6 - 2*(a^4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d*cosh
(d*x + c)^4 + 8*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^3 +
3*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*
(35*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^4 + 30*(a^3*b + 3*a
^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c)^2 - (a^4 + 7*a^3*b + 15*a^2*b^2 + 1
3*a*b^3 + 4*b^4)*d)*sinh(d*x + c)^4 + 4*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)
*d*cosh(d*x + c)^2 + 8*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x +
c)^5 + 10*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c)^3 - (a^4 + 7*
a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4
*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^6 + 15*(a^3*b + 3*a
^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c)^4 - 3*(a^4 + 7*a^3*b + 15*a^2*b^2 +
 13*a*b^3 + 4*b^4)*d*cosh(d*x + c)^2 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*
d)*sinh(d*x + c)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d + 8*((a^4 + 3*a^
3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^7 + 3*(a^3*b + 3*a^2*b^2 + 3*a*b^3
 + b^4)*d*cosh(d*x + c)^5 - (a^4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)
*d*cosh(d*x + c)^3 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c))*s
```

```
inh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^{3}(c+dx)}{\left(a+b\operatorname{sech}^{2}(c+dx)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(csch(c + d*x)**3/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.40 $\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2} dx$

Optimal. Leaf size=123

$$-\frac{\sqrt{b}(3a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2d(a+b)^{7/2}}-\frac{ab\tanh(c+dx)}{2d(a+b)^3\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\coth^3(c+dx)}{3d(a+b)^2}+\frac{(a-b)\coth(c+dx)}{d(a+b)^3}$$

```
[Out] -((3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(2*(a +
 b)^(7/2)*d) + ((a - b)*Coth[c + d*x])/((a + b)^3*d) - Coth[c + d*x]^3/(3*(
a + b)^2*d) - (a*b*Tanh[c + d*x])/(2*(a + b)^3*d*(a + b - b*Tanh[c + d*x]^2
))
```

Rubi [A] time = 0.195059, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4132, 456, 1261, 208}

$$-\frac{\sqrt{b}(3a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2d(a+b)^{7/2}}-\frac{ab\tanh(c+dx)}{2d(a+b)^3\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\coth^3(c+dx)}{3d(a+b)^2}+\frac{(a-b)\coth(c+dx)}{d(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```


```
[Out] -((3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(2*(a +
 b)^(7/2)*d) + ((a - b)*Coth[c + d*x])/((a + b)^3*d) - Coth[c + d*x]^3/(3*(
a + b)^2*d) - (a*b*Tanh[c + d*x])/(2*(a + b)^3*d*(a + b - b*Tanh[c + d*x]^2
))
```


Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{x^4\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{ab\tanh(c+dx)}{2(a+b)^3d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{b\operatorname{Subst}\left(\int \frac{\frac{2}{b(a+b)}-\frac{2ax^2}{b(a+b)^2}-\frac{ax^4}{(a+b)^3}}{x^4\left(a+b-bx^2\right)}\,dx, x, \tanh(c+\right.}{2d}$$

$$= -\frac{ab\tanh(c+dx)}{2(a+b)^3d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{b\operatorname{Subst}\left(\int\left(\frac{2}{b(a+b)^2x^4}-\frac{2(a-b)}{b(a+b)^3x^2}+\frac{-3a+2b}{(a+b)^3(a+b-b}\right.\right.}{2d}$$

$$= \frac{(a-b)\coth(c+dx)}{(a+b)^3d} - \frac{\coth^3(c+dx)}{3(a+b)^2d} - \frac{ab\tanh(c+dx)}{2(a+b)^3d\left(a+b-b\tanh^2(c+dx)\right)} - \frac{((3a-}{}$$

$$= -\frac{(3a-2b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2(a+b)^{7/2}d} + \frac{(a-b)\coth(c+dx)}{(a+b)^3d} - \frac{\coth^3(c+dx)}{3(a+b)^2d} - \frac{}{2(a+b}$$

Mathematica [B] time = 5.93029, size = 295, normalized size = 2.4

$$\frac{\operatorname{sech}^4(c+dx)(a\cosh(2(c+dx))+a+2b)\left(-3ab\operatorname{sech}(2c)\sinh(2dx)-2(a+b)\coth(c)\operatorname{csch}^2(c+dx)(a\cosh(2(c+dx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*(-2*(a + b)*(a + 2*b + a*C
osh[2*(c + d*x)])*Coth[c]*Csch[c + d*x]^2 - (3*(3*a - 2*b)*b*ArcTanh[(Sech[
d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*
Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])
*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) - 4*(
a - 2*b)*(a + 2*b + a*Cosh[2*(c + d*x)])*Csch[c]*Csch[c + d*x]*Sinh[d*x] +
2*(a + b)*(a + 2*b + a*Cosh[2*(c + d*x)])*Csch[c]*Csch[c + d*x]^3*Sinh[d*x]
 - 3*a*b*Sech[2*c]*Sinh[2*d*x] + 3*b*(a + 2*b)*Tanh[2*c]))/(24*(a + b)^3*d*
(a + b*Sech[c + d*x]^2)^2)
```

Maple [B] time = 0.102, size = 577, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/24/d/(a+b)/(a^2+2*a*b+b^2)*a*tanh(1/2*d*x+1/2*c)^3-1/24/d/(a+b)/(a^2+2*a
*b+b^2)*b*tanh(1/2*d*x+1/2*c)^3+3/8/d/(a+b)/(a^2+2*a*b+b^2)*a*tanh(1/2*d*x+
1/2*c)-5/8/d/(a+b)/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)*b-1/d*b/(a+b)^3/(tan
h(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*ta
nh(1/2*d*x+1/2*c)^2*b+a+b)*a*tanh(1/2*d*x+1/2*c)^3-1/d*b/(a+b)^3/(tanh(1/2*
d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2
*d*x+1/2*c)^2*b+a+b)*a*tanh(1/2*d*x+1/2*c)-3/4/d*b^(1/2)/(a+b)^(7/2)*a*ln((
a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))
+3/4/d*b^(1/2)/(a+b)^(7/2)*a*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1
/2*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2))+1/2/d*b^(3/2)/(a+b)^(7/2)*ln((a+b)^(1/2)
*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-1/2/d*b^(
3/2)/(a+b)^(7/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c
)*b^(1/2)-(a+b)^(1/2))-1/24/d/(a+b)^2/tanh(1/2*d*x+1/2*c)^3+3/8/d/(a+b)^3/t
anh(1/2*d*x+1/2*c)*a-5/8/d/(a+b)^3/tanh(1/2*d*x+1/2*c)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.99228, size = 14893, normalized size = 121.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(12*(3*a*b - 2*b^2)*cosh(d*x + c)^8 + 96*(3*a*b - 2*b^2)*cosh(d*x +
c)*sinh(d*x + c)^7 + 12*(3*a*b - 2*b^2)*sinh(d*x + c)^8 + 24*(2*a^2 + 3*a*b
 + 11*b^2)*cosh(d*x + c)^6 + 24*(14*(3*a*b - 2*b^2)*cosh(d*x + c)^2 + 2*a^2
 + 3*a*b + 11*b^2)*sinh(d*x + c)^6 + 48*(14*(3*a*b - 2*b^2)*cosh(d*x + c)^3
 + 3*(2*a^2 + 3*a*b + 11*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 8*(10*a^2 +
22*a*b - 33*b^2)*cosh(d*x + c)^4 + 8*(105*(3*a*b - 2*b^2)*cosh(d*x + c)^4 +
 45*(2*a^2 + 3*a*b + 11*b^2)*cosh(d*x + c)^2 + 10*a^2 + 22*a*b - 33*b^2)*si
nh(d*x + c)^4 + 32*(21*(3*a*b - 2*b^2)*cosh(d*x + c)^5 + 15*(2*a^2 + 3*a*b
+ 11*b^2)*cosh(d*x + c)^3 + (10*a^2 + 22*a*b - 33*b^2)*cosh(d*x + c))*sinh(
d*x + c)^3 + 8*(2*a^2 - 9*a*b + 19*b^2)*cosh(d*x + c)^2 + 8*(42*(3*a*b - 2*
b^2)*cosh(d*x + c)^6 + 45*(2*a^2 + 3*a*b + 11*b^2)*cosh(d*x + c)^4 + 6*(10*
a^2 + 22*a*b - 33*b^2)*cosh(d*x + c)^2 + 2*a^2 - 9*a*b + 19*b^2)*sinh(d*x +
 c)^2 + 3*((3*a^2 - 2*a*b)*cosh(d*x + c)^10 + 10*(3*a^2 - 2*a*b)*cosh(d*x +
 c)*sinh(d*x + c)^9 + (3*a^2 - 2*a*b)*sinh(d*x + c)^10 - (3*a^2 - 14*a*b +
8*b^2)*cosh(d*x + c)^8 + (45*(3*a^2 - 2*a*b)*cosh(d*x + c)^2 - 3*a^2 + 14*a
*b - 8*b^2)*sinh(d*x + c)^8 + 8*(15*(3*a^2 - 2*a*b)*cosh(d*x + c)^3 - (3*a^
2 - 14*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(3*a^2 + 16*a*b - 12
*b^2)*cosh(d*x + c)^6 + 2*(105*(3*a^2 - 2*a*b)*cosh(d*x + c)^4 - 14*(3*a^2
- 14*a*b + 8*b^2)*cosh(d*x + c)^2 - 3*a^2 - 16*a*b + 12*b^2)*sinh(d*x + c)^
```

```
6 + 4*(63*(3*a^2 - 2*a*b)*cosh(d*x + c)^5 - 14*(3*a^2 - 14*a*b + 8*b^2)*cos
h(d*x + c)^3 - 3*(3*a^2 + 16*a*b - 12*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 +
 2*(3*a^2 + 16*a*b - 12*b^2)*cosh(d*x + c)^4 + 2*(105*(3*a^2 - 2*a*b)*cosh(
d*x + c)^6 - 35*(3*a^2 - 14*a*b + 8*b^2)*cosh(d*x + c)^4 - 15*(3*a^2 + 16*a
*b - 12*b^2)*cosh(d*x + c)^2 + 3*a^2 + 16*a*b - 12*b^2)*sinh(d*x + c)^4 + 8
*(15*(3*a^2 - 2*a*b)*cosh(d*x + c)^7 - 7*(3*a^2 - 14*a*b + 8*b^2)*cosh(d*x
+ c)^5 - 5*(3*a^2 + 16*a*b - 12*b^2)*cosh(d*x + c)^3 + (3*a^2 + 16*a*b - 12
*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + (3*a^2 - 14*a*b + 8*b^2)*cosh(d*x +
c)^2 + (45*(3*a^2 - 2*a*b)*cosh(d*x + c)^8 - 28*(3*a^2 - 14*a*b + 8*b^2)*co
sh(d*x + c)^6 - 30*(3*a^2 + 16*a*b - 12*b^2)*cosh(d*x + c)^4 + 12*(3*a^2 +
16*a*b - 12*b^2)*cosh(d*x + c)^2 + 3*a^2 - 14*a*b + 8*b^2)*sinh(d*x + c)^2
- 3*a^2 + 2*a*b + 2*(5*(3*a^2 - 2*a*b)*cosh(d*x + c)^9 - 4*(3*a^2 - 14*a*b
+ 8*b^2)*cosh(d*x + c)^7 - 6*(3*a^2 + 16*a*b - 12*b^2)*cosh(d*x + c)^5 + 4*
(3*a^2 + 16*a*b - 12*b^2)*cosh(d*x + c)^3 + (3*a^2 - 14*a*b + 8*b^2)*cosh(d
*x + c))*sinh(d*x + c))*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*co
sh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*
x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 +
8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(
d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sin
h(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a +
 b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x +
 c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sin
h(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c
) + a)) - 16*a^2 + 44*a*b + 16*(6*(3*a*b - 2*b^2)*cosh(d*x + c)^7 + 9*(2*a^
2 + 3*a*b + 11*b^2)*cosh(d*x + c)^5 + 2*(10*a^2 + 22*a*b - 33*b^2)*cosh(d*x
 + c)^3 + (2*a^2 - 9*a*b + 19*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 3*
a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^10 + 10*(a^4 + 3*a^3*b + 3*a^2*b
^2 + a*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^4 + 3*a^3*b + 3*a^2*b^2 +
a*b^3)*d*sinh(d*x + c)^10 - (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*
cosh(d*x + c)^8 + (45*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^2
 - (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d)*sinh(d*x + c)^8 - 2*(a^4
 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*cosh(d*x + c)^6 + 8*(15*(a^4
+ 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^3 - (a^4 - a^3*b - 9*a^2*b^2
 - 11*a*b^3 - 4*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^4 + 3*a^3
*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^4 - 14*(a^4 - a^3*b - 9*a^2*b^2 - 1
1*a*b^3 - 4*b^4)*d*cosh(d*x + c)^2 - (a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3
 + 6*b^4)*d)*sinh(d*x + c)^6 + 2*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6
*b^4)*d*cosh(d*x + c)^4 + 4*(63*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(
d*x + c)^5 - 14*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*cosh(d*x + c
)^3 - 3*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*cosh(d*x + c))*si
nh(d*x + c)^5 + 2*(105*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^
6 - 35*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*cosh(d*x + c)^4 - 15*
(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*cosh(d*x + c)^2 + (a^4 +
9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d)*sinh(d*x + c)^4 + (a^4 - a^3*b
- 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*cosh(d*x + c)^2 + 8*(15*(a^4 + 3*a^3*b +
3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^7 - 7*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^
3 - 4*b^4)*d*cosh(d*x + c)^5 - 5*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6
*b^4)*d*cosh(d*x + c)^3 + (a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d
*cosh(d*x + c))*sinh(d*x + c)^3 + (45*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d
*cosh(d*x + c)^8 - 28*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*cosh(d
*x + c)^6 - 30*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*cosh(d*x +
 c)^4 + 12*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*cosh(d*x + c)^
2 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d)*sinh(d*x + c)^2 - (a^4
+ 3*a^3*b + 3*a^2*b^2 + a*b^3)*d + 2*(5*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)
*d*cosh(d*x + c)^9 - 4*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*cosh(
d*x + c)^7 - 6*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*cosh(d*x +
 c)^5 + 4*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*cosh(d*x + c)^3
 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*cosh(d*x + c))*sinh(d*x +
 c)), -1/6*(6*(3*a*b - 2*b^2)*cosh(d*x + c)^8 + 48*(3*a*b - 2*b^2)*cosh(d*x
```

+ c)*sinh(d*x + c)^7 + 6*(3*a*b - 2*b^2)*sinh(d*x + c)^8 + 12*(2*a^2 + 3*a
*b + 11*b^2)*cosh(d*x + c)^6 + 12*(14*(3*a*b - 2*b^2)*cosh(d*x + c)^2 + 2*a
^2 + 3*a*b + 11*b^2)*sinh(d*x + c)^6 + 24*(14*(3*a*b - 2*b^2)*cosh(d*x + c)
^3 + 3*(2*a^2 + 3*a*b + 11*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*(10*a^2
+ 22*a*b - 33*b^2)*cosh(d*x + c)^4 + 4*(105*(3*a*b - 2*b^2)*cosh(d*x + c)^4
+ 45*(2*a^2 + 3*a*b + 11*b^2)*cosh(d*x + c)^2 + 10*a^2 + 22*a*b - 33*b^2)*
sinh(d*x + c)^4 + 16*(21*(3*a*b - 2*b^2)*cosh(d*x + c)^5 + 15*(2*a^2 + 3*a*
b + 11*b^2)*cosh(d*x + c)^3 + (10*a^2 + 22*a*b - 33*b^2)*cosh(d*x + c))*sin
h(d*x + c)^3 + 4*(2*a^2 - 9*a*b + 19*b^2)*cosh(d*x + c)^2 + 4*(42*(3*a*b -
2*b^2)*cosh(d*x + c)^6 + 45*(2*a^2 + 3*a*b + 11*b^2)*cosh(d*x + c)^4 + 6*(1
0*a^2 + 22*a*b - 33*b^2)*cosh(d*x + c)^2 + 2*a^2 - 9*a*b + 19*b^2)*sinh(d*x
+ c)^2 + 3*((3*a^2 - 2*a*b)*cosh(d*x + c)^10 + 10*(3*a^2 - 2*a*b)*cosh(d*x
+ c)*sinh(d*x + c)^9 + (3*a^2 - 2*a*b)*sinh(d*x + c)^10 - (3*a^2 - 14*a*b
+ 8*b^2)*cosh(d*x + c)^8 + (45*(3*a^2 - 2*a*b)*cosh(d*x + c)^2 - 3*a^2 + 14
*a*b - 8*b^2)*sinh(d*x + c)^8 + 8*(15*(3*a^2 - 2*a*b)*cosh(d*x + c)^3 - (3*
a^2 - 14*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(3*a^2 + 16*a*b -
12*b^2)*cosh(d*x + c)^6 + 2*(105*(3*a^2 - 2*a*b)*cosh(d*x + c)^4 - 14*(3*a^
2 - 14*a*b + 8*b^2)*cosh(d*x + c)^2 - 3*a^2 - 16*a*b + 12*b^2)*sinh(d*x + c
)^6 + 4*(63*(3*a^2 - 2*a*b)*cosh(d*x + c)^5 - 14*(3*a^2 - 14*a*b + 8*b^2)*c
osh(d*x + c)^3 - 3*(3*a^2 + 16*a*b - 12*b^2)*cosh(d*x + c))*sinh(d*x + c)^5
+ 2*(3*a^2 + 16*a*b - 12*b^2)*cosh(d*x + c)^4 + 2*(105*(3*a^2 - 2*a*b)*cos
h(d*x + c)^6 - 35*(3*a^2 - 14*a*b + 8*b^2)*cosh(d*x + c)^4 - 15*(3*a^2 + 16
*a*b - 12*b^2)*cosh(d*x + c)^2 + 3*a^2 + 16*a*b - 12*b^2)*sinh(d*x + c)^4 +
8*(15*(3*a^2 - 2*a*b)*cosh(d*x + c)^7 - 7*(3*a^2 - 14*a*b + 8*b^2)*cosh(d*
x + c)^5 - 5*(3*a^2 + 16*a*b - 12*b^2)*cosh(d*x + c)^3 + (3*a^2 + 16*a*b -
12*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + (3*a^2 - 14*a*b + 8*b^2)*cosh(d*x
+ c)^2 + (45*(3*a^2 - 2*a*b)*cosh(d*x + c)^8 - 28*(3*a^2 - 14*a*b + 8*b^2)*
cosh(d*x + c)^6 - 30*(3*a^2 + 16*a*b - 12*b^2)*cosh(d*x + c)^4 + 12*(3*a^2
+ 16*a*b - 12*b^2)*cosh(d*x + c)^2 + 3*a^2 - 14*a*b + 8*b^2)*sinh(d*x + c)^
2 - 3*a^2 + 2*a*b + 2*(5*(3*a^2 - 2*a*b)*cosh(d*x + c)^9 - 4*(3*a^2 - 14*a*
b + 8*b^2)*cosh(d*x + c)^7 - 6*(3*a^2 + 16*a*b - 12*b^2)*cosh(d*x + c)^5 +
4*(3*a^2 + 16*a*b - 12*b^2)*cosh(d*x + c)^3 + (3*a^2 - 14*a*b + 8*b^2)*cosh
(d*x + c))*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 +
2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a +
b))/b) - 8*a^2 + 22*a*b + 8*(6*(3*a*b - 2*b^2)*cosh(d*x + c)^7 + 9*(2*a^2
+ 3*a*b + 11*b^2)*cosh(d*x + c)^5 + 2*(10*a^2 + 22*a*b - 33*b^2)*cosh(d*x +
c)^3 + (2*a^2 - 9*a*b + 19*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 3*a^
3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^10 + 10*(a^4 + 3*a^3*b + 3*a^2*b^2
+ a*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*
b^3)*d*sinh(d*x + c)^10 - (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*co
sh(d*x + c)^8 + (45*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^2 -
(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d)*sinh(d*x + c)^8 - 2*(a^4 +
9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*cosh(d*x + c)^6 + 8*(15*(a^4 +
3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^3 - (a^4 - a^3*b - 9*a^2*b^2 -
11*a*b^3 - 4*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^4 + 3*a^3*b
+ 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^4 - 14*(a^4 - a^3*b - 9*a^2*b^2 - 11*
a*b^3 - 4*b^4)*d*cosh(d*x + c)^2 - (a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 +
6*b^4)*d)*sinh(d*x + c)^6 + 2*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b
^4)*d*cosh(d*x + c)^4 + 4*(63*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*
x + c)^5 - 14*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*cosh(d*x + c)^
3 - 3*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*cosh(d*x + c))*sinh
(d*x + c)^5 + 2*(105*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^6
- 35*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*cosh(d*x + c)^4 - 15*(a
^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*cosh(d*x + c)^2 + (a^4 + 9*
a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d)*sinh(d*x + c)^4 + (a^4 - a^3*b -
9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*cosh(d*x + c)^2 + 8*(15*(a^4 + 3*a^3*b + 3*
a^2*b^2 + a*b^3)*d*cosh(d*x + c)^7 - 7*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3
- 4*b^4)*d*cosh(d*x + c)^5 - 5*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b
^4)*d*cosh(d*x + c)^3 + (a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*c

```
osh(d*x + c))*sinh(d*x + c)^3 + (45*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*c
osh(d*x + c)^8 - 28*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*cosh(d*x
 + c)^6 - 30*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*cosh(d*x + c
)^4 + 12*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*cosh(d*x + c)^2
+ (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d)*sinh(d*x + c)^2 - (a^4 +
3*a^3*b + 3*a^2*b^2 + a*b^3)*d + 2*(5*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d
*cosh(d*x + c)^9 - 4*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*cosh(d*
x + c)^7 - 6*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*cosh(d*x + c
)^5 + 4*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*cosh(d*x + c)^3 +
 (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*cosh(d*x + c))*sinh(d*x + c
))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(csch(c + d*x)**4/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [B] time = 1.37426, size = 358, normalized size = 2.91

$$-\frac{\left(3\,ab-2\,b^2\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{2\,\left(a^3d+3\,a^2bd+3\,ab^2d+b^3d\right)\sqrt{-ab-b^2}}+\frac{abe^{(2\,dx+2\,c)}+2\,b^2e^{(2\,dx+2\,c)}+ab}{\left(a^3d+3\,a^2bd+3\,ab^2d+b^3d\right)\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*a*b - 2*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b
^2))/((a^3*d + 3*a^2*b*d + 3*a*b^2*d + b^3*d)*sqrt(-a*b - b^2)) + (a*b*e^(2
*d*x + 2*c) + 2*b^2*e^(2*d*x + 2*c) + a*b)/((a^3*d + 3*a^2*b*d + 3*a*b^2*d
+ b^3*d)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a
)) - 4/3*(3*b*e^(4*d*x + 4*c) + 3*a*e^(2*d*x + 2*c) - 3*b*e^(2*d*x + 2*c) -
 a + 2*b)/((a^3*d + 3*a^2*b*d + 3*a*b^2*d + b^3*d)*(e^(2*d*x + 2*c) - 1)^3)
```

3.41 $\int \frac{\sinh^4(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=242

$$-\frac{3\sqrt{b}\left(5a^2+20ab+16b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^5d\sqrt{a+b}}+\frac{3x\left(a^2+12ab+16b^2\right)}{8a^5}-\frac{3b(a+2b)\tanh(c+dx)}{2a^4d\left(a-b\tanh^2(c+dx)+b\right)}-\frac{b(7a+}{8a^3d\left(a-\right.}$$

```
[Out] (3*(a^2 + 12*a*b + 16*b^2)*x)/(8*a^5) - (3*Sqrt[b]*(5*a^2 + 20*a*b + 16*b^2
)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*a^5*Sqrt[a + b]*d) - ((5
*a + 8*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*a^2*d*(a + b - b*Tanh[c + d*x]^2)
^2) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*a*d*(a + b - b*Tanh[c + d*x]^2)^2)
 - (b*(7*a + 12*b)*Tanh[c + d*x])/(8*a^3*d*(a + b - b*Tanh[c + d*x]^2)^2) -
 (3*b*(a + 2*b)*Tanh[c + d*x])/(2*a^4*d*(a + b - b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.39915, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4132, 470, 527, 522, 206, 208}

$$-\frac{3\sqrt{b}\left(5a^2+20ab+16b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^5d\sqrt{a+b}}+\frac{3x\left(a^2+12ab+16b^2\right)}{8a^5}-\frac{3b(a+2b)\tanh(c+dx)}{2a^4d\left(a-b\tanh^2(c+dx)+b\right)}-\frac{b(7a+}{8a^3d\left(a-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```


```
[Out] (3*(a^2 + 12*a*b + 16*b^2)*x)/(8*a^5) - (3*Sqrt[b]*(5*a^2 + 20*a*b + 16*b^2
)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*a^5*Sqrt[a + b]*d) - ((5
*a + 8*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*a^2*d*(a + b - b*Tanh[c + d*x]^2)
^2) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*a*d*(a + b - b*Tanh[c + d*x]^2)^2)
 - (b*(7*a + 12*b)*Tanh[c + d*x])/(8*a^3*d*(a + b - b*Tanh[c + d*x]^2)^2) -
 (3*b*(a + 2*b)*Tanh[c + d*x])/(2*a^4*d*(a + b - b*Tanh[c + d*x]^2))
```


Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)^3\left(a+b-bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{a+b+(4a+7b)x^2}{\left(1-x^2\right)^2\left(a+b-bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{4ad}$$

$$= -\frac{(5a+8b)\cosh(c+dx)\sinh(c+dx)}{8a^2d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-(}{}\right.}{}$$

$$= -\frac{(5a+8b)\cosh(c+dx)\sinh(c+dx)}{8a^2d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{b(7a+1}{8a^3d\left(a+b\right.}$$

$$= -\frac{(5a+8b)\cosh(c+dx)\sinh(c+dx)}{8a^2d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{b(7a+1}{8a^3d\left(a+b\right.}$$

$$= -\frac{(5a+8b)\cosh(c+dx)\sinh(c+dx)}{8a^2d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{b(7a+1}{8a^3d\left(a+b\right.}$$

$$= \frac{3\left(a^2+12ab+16b^2\right)x}{8a^5} - \frac{3\sqrt{b}\left(5a^2+20ab+16b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^5\sqrt{a+b}d} - \frac{(5a+8b)}{8a^2d\left(a\right.}$$

Mathematica [B] time = 26.7117, size = 3080, normalized size = 12.73

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (3*(a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6*(((3*a^2 + 8*a*b + 8*b
^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b
]*(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d
*x)])/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2)))/(16384*b^(5/2)*d*(a +
 b*Sech[c + d*x]^2)^3) + ((a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6
*((-3*a*(a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/
2) + (Sqrt[b]*(3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*
b^2)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/((a + b)^2*(a + 2*b + a*Cosh[2*(
c + d*x)])^2)))/(16384*b^(5/2)*d*(a + b*Sech[c + d*x]^2)^3) - (3*(a + 2*b +
 a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6*((-2*(3*a^5 - 10*a^4*b + 80*a^3*b^2
 + 480*a^2*b^3 + 640*a*b^4 + 256*b^5)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[
2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos
h[c] - Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c]
- Sinh[c])^4]) + (Sech[2*c]*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*d*x*
Cosh[2*c] + 512*a*b^2*(a + b)^2*(a + 2*b)*d*x*Cosh[2*d*x] + 128*a^4*b^2*d*x
*Cosh[2*(c + 2*d*x)] + 256*a^3*b^3*d*x*Cosh[2*(c + 2*d*x)] + 128*a^2*b^4*d*
x*Cosh[2*(c + 2*d*x)] + 512*a^4*b^2*d*x*Cosh[4*c + 2*d*x] + 2048*a^3*b^3*d*
x*Cosh[4*c + 2*d*x] + 2560*a^2*b^4*d*x*Cosh[4*c + 2*d*x] + 1024*a*b^5*d*x*C
osh[4*c + 2*d*x] + 128*a^4*b^2*d*x*Cosh[6*c + 4*d*x] + 256*a^3*b^3*d*x*Cosh
[6*c + 4*d*x] + 128*a^2*b^4*d*x*Cosh[6*c + 4*d*x] - 9*a^6*Sinh[2*c] + 12*a^
5*b*Sinh[2*c] + 684*a^4*b^2*Sinh[2*c] + 2880*a^3*b^3*Sinh[2*c] + 5280*a^2*b
^4*Sinh[2*c] + 4608*a*b^5*Sinh[2*c] + 1536*b^6*Sinh[2*c] + 9*a^6*Sinh[2*d*x
] - 14*a^5*b*Sinh[2*d*x] - 608*a^4*b^2*Sinh[2*d*x] - 2112*a^3*b^3*Sinh[2*d*
x] - 2560*a^2*b^4*Sinh[2*d*x] - 1024*a*b^5*Sinh[2*d*x] + 3*a^6*Sinh[2*(c +
2*d*x)] - 12*a^5*b*Sinh[2*(c + 2*d*x)] - 204*a^4*b^2*Sinh[2*(c + 2*d*x)] -
384*a^3*b^3*Sinh[2*(c + 2*d*x)] - 192*a^2*b^4*Sinh[2*(c + 2*d*x)] - 3*a^6*S
inh[4*c + 2*d*x] + 10*a^5*b*Sinh[4*c + 2*d*x] + 304*a^4*b^2*Sinh[4*c + 2*d*
x] + 1056*a^3*b^3*Sinh[4*c + 2*d*x] + 1280*a^2*b^4*Sinh[4*c + 2*d*x] + 512*
a*b^5*Sinh[4*c + 2*d*x]))/(a + 2*b + a*Cosh[2*(c + d*x)])^2))/(65536*a^3*b^
2*(a + b)^2*d*(a + b*Sech[c + d*x]^2)^3) + ((a + 2*b + a*Cosh[2*c + 2*d*x])
^3*Sech[c + d*x]^6*((6*(a^6 - 8*a^5*b + 120*a^4*b^2 + 1280*a^3*b^3 + 3200*a
^2*b^4 + 3072*a*b^5 + 1024*b^6)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*
((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] -
 Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh
[c])^4]) + (Sech[2*c]*(-1536*b^2*(a + b)^2*(3*a^3 + 14*a^2*b + 24*a*b^2 + 1
6*b^3)*d*x*Cosh[2*c] - 3072*a*b^2*(a^2 + 3*a*b + 2*b^2)^2*d*x*Cosh[2*d*x] -
 768*a^5*b^2*d*x*Cosh[2*(c + 2*d*x)] - 3072*a^4*b^3*d*x*Cosh[2*(c + 2*d*x)]
 - 3840*a^3*b^4*d*x*Cosh[2*(c + 2*d*x)] - 1536*a^2*b^5*d*x*Cosh[2*(c + 2*d*
x)] - 3072*a^5*b^2*d*x*Cosh[4*c + 2*d*x] - 18432*a^4*b^3*d*x*Cosh[4*c + 2*d
*x] - 39936*a^3*b^4*d*x*Cosh[4*c + 2*d*x] - 36864*a^2*b^5*d*x*Cosh[4*c + 2*
d*x] - 12288*a*b^6*d*x*Cosh[4*c + 2*d*x] - 768*a^5*b^2*d*x*Cosh[6*c + 4*d*x
] - 3072*a^4*b^3*d*x*Cosh[6*c + 4*d*x] - 3840*a^3*b^4*d*x*Cosh[6*c + 4*d*x]
 - 1536*a^2*b^5*d*x*Cosh[6*c + 4*d*x] + 9*a^7*Sinh[2*c] - 54*a^6*b*Sinh[2*c
] - 2392*a^5*b^2*Sinh[2*c] - 13968*a^4*b^3*Sinh[2*c] - 36480*a^3*b^4*Sinh[2
*c] - 50432*a^2*b^5*Sinh[2*c] - 35840*a*b^6*Sinh[2*c] - 10240*b^7*Sinh[2*c]
 - 9*a^7*Sinh[2*d*x] + 56*a^6*b*Sinh[2*d*x] + 2552*a^5*b^2*Sinh[2*d*x] + 13
184*a^4*b^3*Sinh[2*d*x] + 27072*a^3*b^4*Sinh[2*d*x] + 24576*a^2*b^5*Sinh[2*
d*x] + 8192*a*b^6*Sinh[2*d*x] - 3*a^7*Sinh[2*(c + 2*d*x)] + 26*a^6*b*Sinh[2
*(c + 2*d*x)] + 992*a^5*b^2*Sinh[2*(c + 2*d*x)] + 3648*a^4*b^3*Sinh[2*(c +
2*d*x)] + 4480*a^3*b^4*Sinh[2*(c + 2*d*x)] + 1792*a^2*b^5*Sinh[2*(c + 2*d*x
)] + 3*a^7*Sinh[4*c + 2*d*x] - 24*a^6*b*Sinh[4*c + 2*d*x] - 600*a^5*b^2*Sin
h[4*c + 2*d*x] - 3200*a^4*b^3*Sinh[4*c + 2*d*x] - 6720*a^3*b^4*Sinh[4*c + 2
*d*x] - 6144*a^2*b^5*Sinh[4*c + 2*d*x] - 2048*a*b^6*Sinh[4*c + 2*d*x] + 256
*a^5*b^2*Sinh[6*c + 4*d*x] + 1024*a^4*b^3*Sinh[6*c + 4*d*x] + 1280*a^3*b^4*
Sinh[6*c + 4*d*x] + 512*a^2*b^5*Sinh[6*c + 4*d*x] + 64*a^5*b^2*Sinh[4*c + 6
```

```
*d*x] + 128*a^4*b^3*Sinh[4*c + 6*d*x] + 64*a^3*b^4*Sinh[4*c + 6*d*x] + 64*a
^5*b^2*Sinh[8*c + 6*d*x] + 128*a^4*b^3*Sinh[8*c + 6*d*x] + 64*a^3*b^4*Sinh[
8*c + 6*d*x]))/(a + 2*b + a*Cosh[2*(c + d*x)])^2))/(32768*a^4*b^2*(a + b)^2
*d*(a + b*Sech[c + d*x]^2)^3) - ((a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c +
 d*x]^6*((6*a^2*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[
d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(
Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (a*Se
ch[2*c]*((-9*a^4 - 16*a^3*b + 48*a^2*b^2 + 128*a*b^3 + 64*b^4)*Sinh[2*d*x]
+ a*(-3*a^3 + 2*a^2*b + 24*a*b^2 + 16*b^3)*Sinh[2*(c + 2*d*x)] + (3*a^4 - 6
4*a^2*b^2 - 128*a*b^3 - 64*b^4)*Sinh[4*c + 2*d*x]) + (9*a^5 + 18*a^4*b - 64
*a^3*b^2 - 256*a^2*b^3 - 320*a*b^4 - 128*b^5)*Tanh[2*c])/(a^2*(a + 2*b + a*
Cosh[2*(c + d*x)])^2)))/(8192*b^2*(a + b)^2*d*(a + b*Sech[c + d*x]^2)^3) +
((a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6*(768*(7*a^2 + 32*a*b + 3
2*b^2)*x - (3*(a^7 - 14*a^6*b + 336*a^5*b^2 + 5600*a^4*b^3 + 22400*a^3*b^4
+ 37632*a^2*b^5 + 28672*a*b^6 + 8192*b^7)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - S
inh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*
(Cosh[c] - Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]))/(b^2*(a + b)^(5/2)*d*Sqrt
[b*(Cosh[c] - Sinh[c])^4]) - (4*(a^5 + 50*a^4*b + 400*a^3*b^2 + 1120*a^2*b^
3 + 1280*a*b^4 + 512*b^5)*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/
(b*(a + b)*d*(a + 2*b + a*Cosh[2*(c + d*x)])^2) + (768*a*(a + 2*b)*(Cosh[2*
(c + d*x)] - Sinh[2*(c + d*x)]))/d - (768*a*(a + 2*b)*(Cosh[2*(c + d*x)] +
Sinh[2*(c + d*x)]))/d + (128*a^2*Sinh[4*(c + d*x)])/d + (a*(3*a^6 - 44*a^5*
b - 1900*a^4*b^2 - 10880*a^3*b^3 - 23360*a^2*b^4 - 21504*a*b^5 - 7168*b^6)*
Sech[2*c]*Sinh[2*d*x] + (-3*a^7 + 42*a^6*b + 2192*a^5*b^2 + 16480*a^4*b^3 +
 51200*a^3*b^4 + 77824*a^2*b^5 + 57344*a*b^6 + 16384*b^7)*Tanh[2*c])/(b^2*(
a + b)^2*d*(a + 2*b + a*Cosh[2*(c + d*x)]))))/(32768*a^5*(a + b*Sech[c + d*
x]^2)^3)
```

Maple [B] time = 0.125, size = 1677, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -27/4/d*b/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d
*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^3-35/4/d
*b^2/a^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/
2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^3+3/d*b^3/a^4
/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a
-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^3-9/4/d*b/a^2/(tanh(1
/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(
1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)-21/4/d*b^2/a^3/(tanh(1/2*d*x+
1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x
+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)-3/d*b^3/a^4/(tanh(1/2*d*x+1/2*c)^4*a
+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*
b+a+b)^2*tanh(1/2*d*x+1/2*c)-15/16/d*b^(1/2)/a^3/(a+b)^(1/2)*ln((a+b)^(1/2)
*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+15/16/d*b
^(1/2)/a^3/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x
+1/2*c)*b^(1/2)-(a+b)^(1/2))-15/4/d*b^(3/2)/a^4/(a+b)^(1/2)*ln((a+b)^(1/2)*
tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+15/4/d*b^(
3/2)/a^4/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1
/2*c)*b^(1/2)-(a+b)^(1/2))-3/d*b^(5/2)/a^5/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(
1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+3/d*b^(5/2)/a^5
/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^
(1/2)-(a+b)^(1/2))-3/8/d/a^3*ln(tanh(1/2*d*x+1/2*c)-1)-21/4/d*b^2/a^3/(tanh
```

```
(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tan
h(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7-3/d*b^3/a^4/(tanh(1/2*d*x
+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*
x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7-27/4/d*b/a^2/(tanh(1/2*d*x+1/2*c)
^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c
)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^5-35/4/d*b^2/a^3/(tanh(1/2*d*x+1/2*c)^4*a+
b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b
+a+b)^2*tanh(1/2*d*x+1/2*c)^5+3/d*b^3/a^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1
/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*
tanh(1/2*d*x+1/2*c)^5-9/2/d/a^4*ln(tanh(1/2*d*x+1/2*c)-1)*b-6/d/a^5*ln(tanh
(1/2*d*x+1/2*c)-1)*b^2+3/2/d/a^4/(tanh(1/2*d*x+1/2*c)+1)^2*b-3/2/d/a^4/(tan
h(1/2*d*x+1/2*c)+1)*b+9/2/d/a^4*ln(tanh(1/2*d*x+1/2*c)+1)*b+6/d/a^5*ln(tanh
(1/2*d*x+1/2*c)+1)*b^2-3/2/d/a^4/(tanh(1/2*d*x+1/2*c)-1)^2*b-3/2/d/a^4/(tan
h(1/2*d*x+1/2*c)-1)*b+3/8/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)-1/4/d/a^3/(tanh(1
/2*d*x+1/2*c)+1)^4+1/2/d/a^3/(tanh(1/2*d*x+1/2*c)+1)^3+1/8/d/a^3/(tanh(1/2*
d*x+1/2*c)+1)^2-3/8/d/a^3/(tanh(1/2*d*x+1/2*c)+1)+1/4/d/a^3/(tanh(1/2*d*x+1
/2*c)-1)^4+1/2/d/a^3/(tanh(1/2*d*x+1/2*c)-1)^3-1/8/d/a^3/(tanh(1/2*d*x+1/2*
c)-1)^2-3/8/d/a^3/(tanh(1/2*d*x+1/2*c)-1)-9/4/d*b/a^2/(tanh(1/2*d*x+1/2*c)^
4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)
^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.98083, size = 30538, normalized size = 126.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/64*(a^4*cosh(d*x + c)^16 + 16*a^4*cosh(d*x + c)*sinh(d*x + c)^15 + a^4*s
inh(d*x + c)^16 - 4*(a^4 + 4*a^3*b)*cosh(d*x + c)^14 + 4*(30*a^4*cosh(d*x +
 c)^2 - a^4 - 4*a^3*b)*sinh(d*x + c)^14 + 56*(10*a^4*cosh(d*x + c)^3 - (a^4
 + 4*a^3*b)*cosh(d*x + c))*sinh(d*x + c)^13 - 2*(13*a^4 + 72*a^3*b + 88*a^2
*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^12 + 2*(910*a^4*
cosh(d*x + c)^4 - 13*a^4 - 72*a^3*b - 88*a^2*b^2 + 12*(a^4 + 12*a^3*b + 16*
a^2*b^2)*d*x - 182*(a^4 + 4*a^3*b)*cosh(d*x + c)^2)*sinh(d*x + c)^12 + 8*(5
46*a^4*cosh(d*x + c)^5 - 182*(a^4 + 4*a^3*b)*cosh(d*x + c)^3 - 3*(13*a^4 +
72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)
)*sinh(d*x + c)^11 - 4*(9*a^4 + 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4
+ 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^10 + 4*(2002*a^4*cos
h(d*x + c)^6 - 1001*(a^4 + 4*a^3*b)*cosh(d*x + c)^4 - 9*a^4 - 24*a^3*b + 16
*a^2*b^2 + 32*a*b^3 + 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x - 33*
(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cos
h(d*x + c)^2)*sinh(d*x + c)^10 + 8*(1430*a^4*cosh(d*x + c)^7 - 1001*(a^4 +
4*a^3*b)*cosh(d*x + c)^5 - 55*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 1
```

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2*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^3 - 5*(9*a^4 + 24*a^3*b - 16*a^2*b
^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x +
 c))*sinh(d*x + c)^9 + 16*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b^4 + 3
*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*cosh(d*x + c)^
8 + 2*(6435*a^4*cosh(d*x + c)^8 - 6006*(a^4 + 4*a^3*b)*cosh(d*x + c)^6 - 49
5*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*c
osh(d*x + c)^4 + 216*a^3*b + 912*a^2*b^2 + 1472*a*b^3 + 896*b^4 + 24*(3*a^4
 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x - 90*(9*a^4 + 24*a^3*b
 - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)
*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 16*(715*a^4*cosh(d*x + c)^9 - 858*(a^4
+ 4*a^3*b)*cosh(d*x + c)^7 - 99*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 +
 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^5 - 30*(9*a^4 + 24*a^3*b - 16*a^
2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*
x + c)^3 + 8*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*
a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c
)^7 + 4*(9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3 + 24*(a^4 + 14*a^3*b +
 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^6 + 4*(2002*a^4*cosh(d*x + c)^10
 - 3003*(a^4 + 4*a^3*b)*cosh(d*x + c)^8 - 462*(13*a^4 + 72*a^3*b + 88*a^2*b
^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^6 - 210*(9*a^4 + 2
4*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^
3)*d*x)*cosh(d*x + c)^4 + 9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3 + 24*
(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x + 112*(27*a^3*b + 114*a^2*b^2
+ 184*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128
*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(546*a^4*cosh(d*x + c)^11 -
 1001*(a^4 + 4*a^3*b)*cosh(d*x + c)^9 - 198*(13*a^4 + 72*a^3*b + 88*a^2*b^2
 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^7 - 126*(9*a^4 + 24*
a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)
*d*x)*cosh(d*x + c)^5 + 112*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b^4 +
 3*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*cosh(d*x + c
)^3 + 3*(9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3 + 24*(a^4 + 14*a^3*b +
 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(13*a^4 + 1
44*a^3*b + 200*a^2*b^2 + 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c
)^4 + 2*(910*a^4*cosh(d*x + c)^12 - 2002*(a^4 + 4*a^3*b)*cosh(d*x + c)^10 -
 495*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x
)*cosh(d*x + c)^8 - 420*(9*a^4 + 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4
 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^6 + 560*(27*a^3*b +
 114*a^2*b^2 + 184*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 22
4*a*b^3 + 128*b^4)*d*x)*cosh(d*x + c)^4 + 13*a^4 + 144*a^3*b + 200*a^2*b^2
+ 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x + 30*(9*a^4 + 168*a^3*b + 496*a^2*b^
2 + 416*a*b^3 + 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x +
 c)^2)*sinh(d*x + c)^4 - a^4 + 8*(70*a^4*cosh(d*x + c)^13 - 182*(a^4 + 4*a^
3*b)*cosh(d*x + c)^11 - 55*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a
^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^9 - 60*(9*a^4 + 24*a^3*b - 16*a^2*b^2
 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c
)^7 + 112*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*a^3
*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*cosh(d*x + c)^5 + 10*(9*a^4 +
168*a^3*b + 496*a^2*b^2 + 416*a*b^3 + 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*
a*b^3)*d*x)*cosh(d*x + c)^3 + (13*a^4 + 144*a^3*b + 200*a^2*b^2 + 12*(a^4 +
 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 + 4*a^
3*b)*cosh(d*x + c)^2 + 4*(30*a^4*cosh(d*x + c)^14 - 91*(a^4 + 4*a^3*b)*cosh
(d*x + c)^12 - 33*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16
*a^2*b^2)*d*x)*cosh(d*x + c)^10 - 45*(9*a^4 + 24*a^3*b - 16*a^2*b^2 - 32*a*
b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^8 + 11
2*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*a^3*b + 152
*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*cosh(d*x + c)^6 + 15*(9*a^4 + 168*a^3*
b + 496*a^2*b^2 + 416*a*b^3 + 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d
*x)*cosh(d*x + c)^4 + a^4 + 4*a^3*b + 3*(13*a^4 + 144*a^3*b + 200*a^2*b^2 +
 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 1
2*((5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^12 + 12*(5*a^4 + 20*a^3*b
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+ 16*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^11 + (5*a^4 + 20*a^3*b + 16*a^2*b
^2)*sinh(d*x + c)^12 + 4*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*
x + c)^10 + 2*(10*a^4 + 60*a^3*b + 112*a^2*b^2 + 64*a*b^3 + 33*(5*a^4 + 20*
a^3*b + 16*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 20*(11*(5*a^4 + 20*
a^3*b + 16*a^2*b^2)*cosh(d*x + c)^3 + 2*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32
*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 + 2*(15*a^4 + 100*a^3*b + 248*a^2*b^
2 + 288*a*b^3 + 128*b^4)*cosh(d*x + c)^8 + (495*(5*a^4 + 20*a^3*b + 16*a^2*
b^2)*cosh(d*x + c)^4 + 30*a^4 + 200*a^3*b + 496*a^2*b^2 + 576*a*b^3 + 256*b
^4 + 180*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^2)*sinh(d
x + c)^8 + 8(99*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^5 + 60*(5*a
^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^3 + 2*(15*a^4 + 100*a^
3*b + 248*a^2*b^2 + 288*a*b^3 + 128*b^4)*cosh(d*x + c))*sinh(d*x + c)^7 + 4
*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^6 + 4*(231*(5*a^4
 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^6 + 210*(5*a^4 + 30*a^3*b + 56*a^2*
b^2 + 32*a*b^3)*cosh(d*x + c)^4 + 5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3
+ 14*(15*a^4 + 100*a^3*b + 248*a^2*b^2 + 288*a*b^3 + 128*b^4)*cosh(d*x + c)
^2)*sinh(d*x + c)^6 + 8*(99*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^7
 + 126*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^5 + 14*(15*
a^4 + 100*a^3*b + 248*a^2*b^2 + 288*a*b^3 + 128*b^4)*cosh(d*x + c)^3 + 3*(5
*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + (
5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^4 + (495*(5*a^4 + 20*a^3*b + 1
6*a^2*b^2)*cosh(d*x + c)^8 + 840*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)
*cosh(d*x + c)^6 + 140*(15*a^4 + 100*a^3*b + 248*a^2*b^2 + 288*a*b^3 + 128*
b^4)*cosh(d*x + c)^4 + 5*a^4 + 20*a^3*b + 16*a^2*b^2 + 60*(5*a^4 + 30*a^3*b
 + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(55*(5*a^4 +
 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^9 + 120*(5*a^4 + 30*a^3*b + 56*a^2*b^
2 + 32*a*b^3)*cosh(d*x + c)^7 + 28*(15*a^4 + 100*a^3*b + 248*a^2*b^2 + 288*
a*b^3 + 128*b^4)*cosh(d*x + c)^5 + 20*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a
*b^3)*cosh(d*x + c)^3 + (5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c))*sinh
(d*x + c)^3 + 2*(33*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^10 + 90*(
5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^8 + 28*(15*a^4 + 10
0*a^3*b + 248*a^2*b^2 + 288*a*b^3 + 128*b^4)*cosh(d*x + c)^6 + 30*(5*a^4 +
30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^4 + 3*(5*a^4 + 20*a^3*b + 1
6*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(3*(5*a^4 + 20*a^3*b + 16*a
^2*b^2)*cosh(d*x + c)^11 + 10*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*co
sh(d*x + c)^9 + 4*(15*a^4 + 100*a^3*b + 248*a^2*b^2 + 288*a*b^3 + 128*b^4)*
cosh(d*x + c)^7 + 6*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c
)^5 + (5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^3)*sinh(d*x + c))*sqrt(
b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 +
 a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x
+ c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d
*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cos
h(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh
(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*
a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x
+ c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x
+ c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 8*(2*a^4*cosh(d*x +
 c)^15 - 7*(a^4 + 4*a^3*b)*cosh(d*x + c)^13 - 3*(13*a^4 + 72*a^3*b + 88*a^2
b^2 - 12(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^11 - 5*(9*a^4 +
24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b
^3)*d*x)*cosh(d*x + c)^9 + 16*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b^4
 + 3*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*cosh(d*x +
 c)^7 + 3*(9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3 + 24*(a^4 + 14*a^3*b
 + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^5 + (13*a^4 + 144*a^3*b + 200*
a^2*b^2 + 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^3 + (a^4 + 4*
a^3*b)*cosh(d*x + c))*sinh(d*x + c))/(a^7*d*cosh(d*x + c)^12 + 12*a^7*d*cos
h(d*x + c)*sinh(d*x + c)^11 + a^7*d*sinh(d*x + c)^12 + a^7*d*cosh(d*x + c)^
4 + 4*(a^7 + 2*a^6*b)*d*cosh(d*x + c)^10 + 2*(33*a^7*d*cosh(d*x + c)^2 + 2*
(a^7 + 2*a^6*b)*d)*sinh(d*x + c)^10 + 2*(3*a^7 + 8*a^6*b + 8*a^5*b^2)*d*cos

h(d*x + c)^8 + 20*(11*a^7*d*cosh(d*x + c)^3 + 2*(a^7 + 2*a^6*b)*d*cosh(d*x
+ c))*sinh(d*x + c)^9 + (495*a^7*d*cosh(d*x + c)^4 + 180*(a^7 + 2*a^6*b)*d*
cosh(d*x + c)^2 + 2*(3*a^7 + 8*a^6*b + 8*a^5*b^2)*d)*sinh(d*x + c)^8 + 4*(a
^7 + 2*a^6*b)*d*cosh(d*x + c)^6 + 8*(99*a^7*d*cosh(d*x + c)^5 + 60*(a^7 + 2
*a^6*b)*d*cosh(d*x + c)^3 + 2*(3*a^7 + 8*a^6*b + 8*a^5*b^2)*d*cosh(d*x + c)
)*sinh(d*x + c)^7 + 4*(231*a^7*d*cosh(d*x + c)^6 + 210*(a^7 + 2*a^6*b)*d*co
sh(d*x + c)^4 + 14*(3*a^7 + 8*a^6*b + 8*a^5*b^2)*d*cosh(d*x + c)^2 + (a^7 +
 2*a^6*b)*d)*sinh(d*x + c)^6 + 8*(99*a^7*d*cosh(d*x + c)^7 + 126*(a^7 + 2*a
^6*b)*d*cosh(d*x + c)^5 + 14*(3*a^7 + 8*a^6*b + 8*a^5*b^2)*d*cosh(d*x + c)^
3 + 3*(a^7 + 2*a^6*b)*d*cosh(d*x + c))*sinh(d*x + c)^5 + (495*a^7*d*cosh(d*
x + c)^8 + a^7*d + 840*(a^7 + 2*a^6*b)*d*cosh(d*x + c)^6 + 140*(3*a^7 + 8*a
^6*b + 8*a^5*b^2)*d*cosh(d*x + c)^4 + 60*(a^7 + 2*a^6*b)*d*cosh(d*x + c)^2)
*sinh(d*x + c)^4 + 4*(55*a^7*d*cosh(d*x + c)^9 + a^7*d*cosh(d*x + c) + 120*
(a^7 + 2*a^6*b)*d*cosh(d*x + c)^7 + 28*(3*a^7 + 8*a^6*b + 8*a^5*b^2)*d*cosh
(d*x + c)^5 + 20*(a^7 + 2*a^6*b)*d*cosh(d*x + c)^3)*sinh(d*x + c)^3 + 2*(33
*a^7*d*cosh(d*x + c)^10 + 3*a^7*d*cosh(d*x + c)^2 + 90*(a^7 + 2*a^6*b)*d*co
sh(d*x + c)^8 + 28*(3*a^7 + 8*a^6*b + 8*a^5*b^2)*d*cosh(d*x + c)^6 + 30*(a^
7 + 2*a^6*b)*d*cosh(d*x + c)^4)*sinh(d*x + c)^2 + 4*(3*a^7*d*cosh(d*x + c)^
11 + a^7*d*cosh(d*x + c)^3 + 10*(a^7 + 2*a^6*b)*d*cosh(d*x + c)^9 + 4*(3*a^
7 + 8*a^6*b + 8*a^5*b^2)*d*cosh(d*x + c)^7 + 6*(a^7 + 2*a^6*b)*d*cosh(d*x +
 c)^5)*sinh(d*x + c)), 1/64*(a^4*cosh(d*x + c)^16 + 16*a^4*cosh(d*x + c)*si
nh(d*x + c)^15 + a^4*sinh(d*x + c)^16 - 4*(a^4 + 4*a^3*b)*cosh(d*x + c)^14
+ 4*(30*a^4*cosh(d*x + c)^2 - a^4 - 4*a^3*b)*sinh(d*x + c)^14 + 56*(10*a^4*
cosh(d*x + c)^3 - (a^4 + 4*a^3*b)*cosh(d*x + c))*sinh(d*x + c)^13 - 2*(13*a
^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x
 + c)^12 + 2*(910*a^4*cosh(d*x + c)^4 - 13*a^4 - 72*a^3*b - 88*a^2*b^2 + 12
*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x - 182*(a^4 + 4*a^3*b)*cosh(d*x + c)^2)*s
inh(d*x + c)^12 + 8*(546*a^4*cosh(d*x + c)^5 - 182*(a^4 + 4*a^3*b)*cosh(d*x
 + c)^3 - 3*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b
^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^11 - 4*(9*a^4 + 24*a^3*b - 16*a^2*b^2
 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c
)^10 + 4*(2002*a^4*cosh(d*x + c)^6 - 1001*(a^4 + 4*a^3*b)*cosh(d*x + c)^4 -
 9*a^4 - 24*a^3*b + 16*a^2*b^2 + 32*a*b^3 + 24*(a^4 + 14*a^3*b + 40*a^2*b^2
 + 32*a*b^3)*d*x - 33*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b
+ 16*a^2*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 8*(1430*a^4*cosh(d*x
 + c)^7 - 1001*(a^4 + 4*a^3*b)*cosh(d*x + c)^5 - 55*(13*a^4 + 72*a^3*b + 88
*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^3 - 5*(9*a^4
 + 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*
a*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^9 + 16*(27*a^3*b + 114*a^2*b^2 + 1
84*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^
4)*d*x)*cosh(d*x + c)^8 + 2*(6435*a^4*cosh(d*x + c)^8 - 6006*(a^4 + 4*a^3*b
)*cosh(d*x + c)^6 - 495*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*
b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^4 + 216*a^3*b + 912*a^2*b^2 + 1472*a*b^3
 + 896*b^4 + 24*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x
- 90*(9*a^4 + 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^
2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 16*(715*a^4*cosh(
d*x + c)^9 - 858*(a^4 + 4*a^3*b)*cosh(d*x + c)^7 - 99*(13*a^4 + 72*a^3*b +
88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^5 - 30*(9*
a^4 + 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 +
32*a*b^3)*d*x)*cosh(d*x + c)^3 + 8*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 11
2*b^4 + 3*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*cosh(
d*x + c))*sinh(d*x + c)^7 + 4*(9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3
+ 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^6 + 4*(200
2*a^4*cosh(d*x + c)^10 - 3003*(a^4 + 4*a^3*b)*cosh(d*x + c)^8 - 462*(13*a^4
 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x +
 c)^6 - 210*(9*a^4 + 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b
+ 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^4 + 9*a^4 + 168*a^3*b + 496*a^2
*b^2 + 416*a*b^3 + 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x + 112*(2
7*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*a^3*b + 152*a^2

```
*b^2 + 224*a*b^3 + 128*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(546*
a^4*cosh(d*x + c)^11 - 1001*(a^4 + 4*a^3*b)*cosh(d*x + c)^9 - 198*(13*a^4 +
 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c
)^7 - 126*(9*a^4 + 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b +
40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^5 + 112*(27*a^3*b + 114*a^2*b^2 +
 184*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*
b^4)*d*x)*cosh(d*x + c)^3 + 3*(9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3
+ 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c))*sinh(d*x
+ c)^5 + 2*(13*a^4 + 144*a^3*b + 200*a^2*b^2 + 12*(a^4 + 12*a^3*b + 16*a^2*
b^2)*d*x)*cosh(d*x + c)^4 + 2*(910*a^4*cosh(d*x + c)^12 - 2002*(a^4 + 4*a^3
*b)*cosh(d*x + c)^10 - 495*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a
^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^8 - 420*(9*a^4 + 24*a^3*b - 16*a^2*b^
2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x +
c)^6 + 560*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*a^
3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*cosh(d*x + c)^4 + 13*a^4 + 14
4*a^3*b + 200*a^2*b^2 + 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x + 30*(9*a^4 +
168*a^3*b + 496*a^2*b^2 + 416*a*b^3 + 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*
a*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - a^4 + 8*(70*a^4*cosh(d*x + c
)^13 - 182*(a^4 + 4*a^3*b)*cosh(d*x + c)^11 - 55*(13*a^4 + 72*a^3*b + 88*a^
2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^9 - 60*(9*a^4 +
 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*
b^3)*d*x)*cosh(d*x + c)^7 + 112*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b
^4 + 3*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*cosh(d*x
 + c)^5 + 10*(9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3 + 24*(a^4 + 14*a^
3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^3 + (13*a^4 + 144*a^3*b + 2
00*a^2*b^2 + 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c))*sinh(d*x
+ c)^3 + 4*(a^4 + 4*a^3*b)*cosh(d*x + c)^2 + 4*(30*a^4*cosh(d*x + c)^14 - 9
1*(a^4 + 4*a^3*b)*cosh(d*x + c)^12 - 33*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 1
2*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^10 - 45*(9*a^4 + 24*a^3*
b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x
)*cosh(d*x + c)^8 + 112*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b^4 + 3*(
3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*cosh(d*x + c)^6
+ 15*(9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3 + 24*(a^4 + 14*a^3*b + 40
*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^4 + a^4 + 4*a^3*b + 3*(13*a^4 + 144
*a^3*b + 200*a^2*b^2 + 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^
2)*sinh(d*x + c)^2 - 24*((5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^12 +
 12*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^11 + (5*a^4
 + 20*a^3*b + 16*a^2*b^2)*sinh(d*x + c)^12 + 4*(5*a^4 + 30*a^3*b + 56*a^2*b
^2 + 32*a*b^3)*cosh(d*x + c)^10 + 2*(10*a^4 + 60*a^3*b + 112*a^2*b^2 + 64*a
*b^3 + 33*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^10
 + 20*(11*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^3 + 2*(5*a^4 + 30*a
^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 + 2*(15*a^4 +
100*a^3*b + 248*a^2*b^2 + 288*a*b^3 + 128*b^4)*cosh(d*x + c)^8 + (495*(5*a^
4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^4 + 30*a^4 + 200*a^3*b + 496*a^2*b
^2 + 576*a*b^3 + 256*b^4 + 180*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*c
osh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(99*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*cos
h(d*x + c)^5 + 60*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^
3 + 2*(15*a^4 + 100*a^3*b + 248*a^2*b^2 + 288*a*b^3 + 128*b^4)*cosh(d*x + c
))*sinh(d*x + c)^7 + 4*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x
+ c)^6 + 4*(231*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^6 + 210*(5*a^
4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^4 + 5*a^4 + 30*a^3*b +
56*a^2*b^2 + 32*a*b^3 + 14*(15*a^4 + 100*a^3*b + 248*a^2*b^2 + 288*a*b^3 +
128*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(99*(5*a^4 + 20*a^3*b + 16*a^
2*b^2)*cosh(d*x + c)^7 + 126*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cos
h(d*x + c)^5 + 14*(15*a^4 + 100*a^3*b + 248*a^2*b^2 + 288*a*b^3 + 128*b^4)*
cosh(d*x + c)^3 + 3*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c
))*sinh(d*x + c)^5 + (5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^4 + (495
*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^8 + 840*(5*a^4 + 30*a^3*b +
56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^6 + 140*(15*a^4 + 100*a^3*b + 248*a^2*
```

```
b^2 + 288*a*b^3 + 128*b^4)*cosh(d*x + c)^4 + 5*a^4 + 20*a^3*b + 16*a^2*b^2
+ 60*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^2)*sinh(d*x +
 c)^4 + 4*(55*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^9 + 120*(5*a^4
+ 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^7 + 28*(15*a^4 + 100*a^3*
b + 248*a^2*b^2 + 288*a*b^3 + 128*b^4)*cosh(d*x + c)^5 + 20*(5*a^4 + 30*a^3
*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^3 + (5*a^4 + 20*a^3*b + 16*a^2*b^
2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(33*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*c
osh(d*x + c)^10 + 90*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x +
c)^8 + 28*(15*a^4 + 100*a^3*b + 248*a^2*b^2 + 288*a*b^3 + 128*b^4)*cosh(d*x
 + c)^6 + 30*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^4 + 3
*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(3*(5
*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^11 + 10*(5*a^4 + 30*a^3*b + 56*
a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^9 + 4*(15*a^4 + 100*a^3*b + 248*a^2*b^2 +
 288*a*b^3 + 128*b^4)*cosh(d*x + c)^7 + 6*(5*a^4 + 30*a^3*b + 56*a^2*b^2 +
32*a*b^3)*cosh(d*x + c)^5 + (5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^3
)*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(
d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b) +
 8*(2*a^4*cosh(d*x + c)^15 - 7*(a^4 + 4*a^3*b)*cosh(d*x + c)^13 - 3*(13*a^4
 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x +
 c)^11 - 5*(9*a^4 + 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b +
 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^9 + 16*(27*a^3*b + 114*a^2*b^2 +
 184*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*
b^4)*d*x)*cosh(d*x + c)^7 + 3*(9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3
+ 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^5 + (13*a^
4 + 144*a^3*b + 200*a^2*b^2 + 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*
x + c)^3 + (a^4 + 4*a^3*b)*cosh(d*x + c))*sinh(d*x + c))/(a^7*d*cosh(d*x +
c)^12 + 12*a^7*d*cosh(d*x + c)*sinh(d*x + c)^11 + a^7*d*sinh(d*x + c)^12 +
a^7*d*cosh(d*x + c)^4 + 4*(a^7 + 2*a^6*b)*d*cosh(d*x + c)^10 + 2*(33*a^7*d*
cosh(d*x + c)^2 + 2*(a^7 + 2*a^6*b)*d)*sinh(d*x + c)^10 + 2*(3*a^7 + 8*a^6*
b + 8*a^5*b^2)*d*cosh(d*x + c)^8 + 20*(11*a^7*d*cosh(d*x + c)^3 + 2*(a^7 +
2*a^6*b)*d*cosh(d*x + c))*sinh(d*x + c)^9 + (495*a^7*d*cosh(d*x + c)^4 + 18
0*(a^7 + 2*a^6*b)*d*cosh(d*x + c)^2 + 2*(3*a^7 + 8*a^6*b + 8*a^5*b^2)*d)*si
nh(d*x + c)^8 + 4*(a^7 + 2*a^6*b)*d*cosh(d*x + c)^6 + 8*(99*a^7*d*cosh(d*x
+ c)^5 + 60*(a^7 + 2*a^6*b)*d*cosh(d*x + c)^3 + 2*(3*a^7 + 8*a^6*b + 8*a^5*
b^2)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 4*(231*a^7*d*cosh(d*x + c)^6 + 210*
(a^7 + 2*a^6*b)*d*cosh(d*x + c)^4 + 14*(3*a^7 + 8*a^6*b + 8*a^5*b^2)*d*cosh
(d*x + c)^2 + (a^7 + 2*a^6*b)*d)*sinh(d*x + c)^6 + 8*(99*a^7*d*cosh(d*x + c
)^7 + 126*(a^7 + 2*a^6*b)*d*cosh(d*x + c)^5 + 14*(3*a^7 + 8*a^6*b + 8*a^5*b
^2)*d*cosh(d*x + c)^3 + 3*(a^7 + 2*a^6*b)*d*cosh(d*x + c))*sinh(d*x + c)^5
+ (495*a^7*d*cosh(d*x + c)^8 + a^7*d + 840*(a^7 + 2*a^6*b)*d*cosh(d*x + c)^
6 + 140*(3*a^7 + 8*a^6*b + 8*a^5*b^2)*d*cosh(d*x + c)^4 + 60*(a^7 + 2*a^6*b
)*d*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(55*a^7*d*cosh(d*x + c)^9 + a^7*d*
cosh(d*x + c) + 120*(a^7 + 2*a^6*b)*d*cosh(d*x + c)^7 + 28*(3*a^7 + 8*a^6*b
 + 8*a^5*b^2)*d*cosh(d*x + c)^5 + 20*(a^7 + 2*a^6*b)*d*cosh(d*x + c)^3)*sin
h(d*x + c)^3 + 2*(33*a^7*d*cosh(d*x + c)^10 + 3*a^7*d*cosh(d*x + c)^2 + 90*
(a^7 + 2*a^6*b)*d*cosh(d*x + c)^8 + 28*(3*a^7 + 8*a^6*b + 8*a^5*b^2)*d*cosh
(d*x + c)^6 + 30*(a^7 + 2*a^6*b)*d*cosh(d*x + c)^4)*sinh(d*x + c)^2 + 4*(3*
a^7*d*cosh(d*x + c)^11 + a^7*d*cosh(d*x + c)^3 + 10*(a^7 + 2*a^6*b)*d*cosh(
d*x + c)^9 + 4*(3*a^7 + 8*a^6*b + 8*a^5*b^2)*d*cosh(d*x + c)^7 + 6*(a^7 + 2
*a^6*b)*d*cosh(d*x + c)^5)*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a+b*sech(d*x+c)**2)**3,x)
```

[Out] Timed out

Giac [B] time = 1.22649, size = 714, normalized size = 2.95

$$\frac{3\left(a^2+12\,ab+16\,b^2\right)(dx+c)}{8\,a^5d}-\frac{3\left(5\,a^2b+20\,ab^2+16\,b^3\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{8\,\sqrt{-ab-b^2}a^5d}+\frac{a^3de^{(4\,dx+4\,c)}-8\,a^3de^{(2\,dx+2\,c)}-24\,a^2}{64\,a^6d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 3/8*(a^2 + 12*a*b + 16*b^2)*(d*x + c)/(a^5*d) - 3/8*(5*a^2*b + 20*a*b^2 + 1
6*b^3)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*
b - b^2)*a^5*d) + 1/64*(a^3*d*e^(4*d*x + 4*c) - 8*a^3*d*e^(2*d*x + 2*c) - 2
4*a^2*b*d*e^(2*d*x + 2*c))/(a^6*d^2) - 1/64*(6*a^4*e^(12*d*x + 12*c) + 72*a
^3*b*e^(12*d*x + 12*c) + 96*a^2*b^2*e^(12*d*x + 12*c) + 16*a^4*e^(10*d*x +
10*c) + 168*a^3*b*e^(10*d*x + 10*c) + 384*a^2*b^2*e^(10*d*x + 10*c) + 256*a
*b^3*e^(10*d*x + 10*c) + 5*a^4*e^(8*d*x + 8*c) - 64*a^3*b*e^(8*d*x + 8*c) -
 192*a^2*b^2*e^(8*d*x + 8*c) - 256*a*b^3*e^(8*d*x + 8*c) - 256*b^4*e^(8*d*x
 + 8*c) - 20*a^4*e^(6*d*x + 6*c) - 360*a^3*b*e^(6*d*x + 6*c) - 1024*a^2*b^2
*e^(6*d*x + 6*c) - 896*a*b^3*e^(6*d*x + 6*c) - 20*a^4*e^(4*d*x + 4*c) - 216
*a^3*b*e^(4*d*x + 4*c) - 304*a^2*b^2*e^(4*d*x + 4*c) - 4*a^4*e^(2*d*x + 2*c
) - 16*a^3*b*e^(2*d*x + 2*c) + a^4)/((a*e^(6*d*x + 6*c) + 2*a*e^(4*d*x + 4*
c) + 4*b*e^(4*d*x + 4*c) + a*e^(2*d*x + 2*c))^2*a^5*d)
```

3.42 $\int \frac{\sinh^3(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=154

$$\frac{b^2(a+b)\cosh(c+dx)}{4a^4d\left(a\cosh^2(c+dx)+b\right)^2}-\frac{b(9a+13b)\cosh(c+dx)}{8a^4d\left(a\cosh^2(c+dx)+b\right)}-\frac{(a+3b)\cosh(c+dx)}{a^4d}+\frac{5\sqrt{b}(3a+7b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx}{\sqrt{b}}\right.}{8a^{9/2}d}$$

```
[Out] (5*Sqrt[b]*(3*a + 7*b)*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(8*a^(9/2)*
d) - ((a + 3*b)*Cosh[c + d*x])/(a^4*d) + Cosh[c + d*x]^3/(3*a^3*d) + (b^2*(
a + b)*Cosh[c + d*x])/(4*a^4*d*(b + a*Cosh[c + d*x]^2)^2) - (b*(9*a + 13*b)
*Cosh[c + d*x])/(8*a^4*d*(b + a*Cosh[c + d*x]^2))
```

Rubi [A] time = 0.217989, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4133, 455, 1814, 1153, 205}

$$\frac{b^2(a+b)\cosh(c+dx)}{4a^4d\left(a\cosh^2(c+dx)+b\right)^2}-\frac{b(9a+13b)\cosh(c+dx)}{8a^4d\left(a\cosh^2(c+dx)+b\right)}-\frac{(a+3b)\cosh(c+dx)}{a^4d}+\frac{5\sqrt{b}(3a+7b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx}{\sqrt{b}}\right.}{8a^{9/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```


```
[Out] (5*Sqrt[b]*(3*a + 7*b)*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(8*a^(9/2)*
d) - ((a + 3*b)*Cosh[c + d*x])/(a^4*d) + Cosh[c + d*x]^3/(3*a^3*d) + (b^2*(
a + b)*Cosh[c + d*x])/(4*a^4*d*(b + a*Cosh[c + d*x]^2)^2) - (b*(9*a + 13*b)
*Cosh[c + d*x])/(8*a^4*d*(b + a*Cosh[c + d*x]^2))
```


Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
```

```
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^6(1-x^2)}{(b+ax^2)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{b^2(a+b)\cosh(c+dx)}{4a^4d\left(b+a\cosh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{-b^2(a+b)+4ab(a+b)x^2-4a^2(a+b)x^4+4a^3x^6}{(b+ax^2)^2}\,dx, x, \cosh(c+\right.}{4a^4d}$$

$$= \frac{b^2(a+b)\cosh(c+dx)}{4a^4d\left(b+a\cosh^2(c+dx)\right)^2} - \frac{b(9a+13b)\cosh(c+dx)}{8a^4d\left(b+a\cosh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-b^2(7a+11b)+8ab(a+2b)}{b+ax^2}\right.}{8a^4}$$

$$= \frac{b^2(a+b)\cosh(c+dx)}{4a^4d\left(b+a\cosh^2(c+dx)\right)^2} - \frac{b(9a+13b)\cosh(c+dx)}{8a^4d\left(b+a\cosh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \left(8b(a+3b)-8abx\right.\right.}{}$$

$$= -\frac{(a+3b)\cosh(c+dx)}{a^4d} + \frac{\cosh^3(c+dx)}{3a^3d} + \frac{b^2(a+b)\cosh(c+dx)}{4a^4d\left(b+a\cosh^2(c+dx)\right)^2} - \frac{b(9a+13b)\,\text{co}}{8a^4d\left(b+a\cos\right.}$$

$$= \frac{5\sqrt{b}(3a+7b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{9/2}d} - \frac{(a+3b)\cosh(c+dx)}{a^4d} + \frac{\cosh^3(c+dx)}{3a^3d} + \frac{b^2(a+}{4a^4d\left(b+\right.}$$

Mathematica [C] time = 10.7796, size = 1217, normalized size = 7.9

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^3*Sech[c + d*x]^6*((24*(3*a - 4*b)*(ArcTan
[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/
2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x
)/2]))/Sqrt[b]] + ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])
^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c]
 - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]))/(a^(3/2)*b^(5/2)) - (54*(ArcTan[(
Sqrt[a] - I*Sqrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]] + ArcTan[(Sqrt[a] + I*S
qrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]]))/(Sqrt[a]*b^(5/2)) - (36*Cosh[c + d
```

```
*x]*(3*a + 10*b + 3*a*Cosh[2*(c + d*x)]))/(b^2*(a + 2*b + a*Cosh[2*(c + d*x
)])^2) + (48*Cosh[c + d*x]*(3*a^2 + 6*a*b + 8*b^2 + a*(3*a - 4*b)*Cosh[2*(c
 + d*x)]))/(a*b^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2) + (3*(3*a^4 - 40*a^3*b
 + 720*a^2*b^2 + 6720*a*b^3 + 8960*b^4)*ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sq
rt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqr
t[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] + 3*(3*a^4 -
40*a^3*b + 720*a^2*b^2 + 6720*a*b^3 + 8960*b^4)*ArcTan[((Sqrt[a] + I*Sqrt[a
 + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a]
 + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] + (2*
Sqrt[a]*Sqrt[b]*Cosh[c + d*x]*(9*a^5 - 90*a^4*b - 10144*a^3*b^2 - 48672*a^2
*b^3 - 85120*a*b^4 - 53760*b^5 + a*(9*a^4 - 120*a^3*b - 12432*a^2*b^2 - 479
36*a*b^3 - 44800*b^4)*Cosh[2*(c + d*x)] - 128*a^2*b^2*(15*a + 28*b)*Cosh[4*
(c + d*x)] + 128*a^3*b^2*Cosh[6*(c + d*x)]))/(a + 2*b + a*Cosh[2*(c + d*x)]
)^2)/(a^(9/2)*b^(5/2)) + (9*((-3*(a^3 - 8*a^2*b + 80*a*b^2 + 320*b^3)*ArcTa
n[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)
/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*
x)/2]))/Sqrt[b]])/b^(5/2) - (3*(a^3 - 8*a^2*b + 80*a*b^2 + 320*b^3)*ArcTan[
((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2
] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)
/2]))/Sqrt[b]])/b^(5/2) + 512*Sqrt[a]*Cosh[c]*Cosh[d*x] - (8*Sqrt[a]*(a^3 +
 24*a^2*b + 80*a*b^2 + 64*b^3)*Cosh[c + d*x])/(b*(a + 2*b + a*Cosh[2*(c + d
*x)])^2) - (2*Sqrt[a]*(3*a^3 - 24*a^2*b - 400*a*b^2 - 576*b^3)*Cosh[c + d*x
])/(b^2*(a + 2*b + a*Cosh[2*(c + d*x)])) + 512*Sqrt[a]*Sinh[c]*Sinh[d*x]))/
a^(7/2)))/(49152*d*(a + b*Sech[c + d*x]^2)^3)
```

Maple [B] time = 0.098, size = 1296, normalized size = 8.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/3/d/a^3/(tanh(1/2*d*x+1/2*c)+1)^3-1/2/d/a^3/(tanh(1/2*d*x+1/2*c)+1)^2-1/2
/d/a^3/(tanh(1/2*d*x+1/2*c)+1)-3/d/a^4/(tanh(1/2*d*x+1/2*c)+1)*b-9/4/d*b/a^
2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*
a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^6+1/2/d*b^2/a^3/(tan
h(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*ta
nh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^6+11/4/d*b^3/a^4/(tanh(1/2
*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/
2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^6-27/4/d*b/a/(tanh(1/2*d*x+1/2*
c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2
*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^4-15/4/d*b^2/a^2/(tanh(1/2*d*x+1/2
*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/
2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^4-5/4/d*b^3/a^3/(tanh(1/2*d*x+1/2
*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/
2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^4-33/4/d*b^4/a^4/(tanh(1/2*d*x+1/
2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1
/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^4-27/4/d*b/a^2/(tanh(1/2*d*x+1/2
*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/
2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^2-5/2/d*b^2/a^3/(tanh(1/2*d*x+1/2*c)^4*
a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2
*b+a+b)^2*tanh(1/2*d*x+1/2*c)^2+33/4/d*b^3/a^4/(tanh(1/2*d*x+1/2*c)^4*a+b*t
anh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+
b)^2*tanh(1/2*d*x+1/2*c)^2-9/4/d*b/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*
d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2-5/d
*b^2/a^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/
```

```
2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2-11/4/d*b^3/a^4/(tanh(1/2*d*x+1/2*
c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2
*c)^2*b+a+b)^2+15/8/d*b/a^3/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/
2*c)^2+2*a-2*b)/(a*b)^(1/2))+35/8/d*b^2/a^4/(a*b)^(1/2)*arctan(1/4*(2*(a+b)
*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))-1/3/d/a^3/(tanh(1/2*d*x+1/2*c)
-1)^3-1/2/d/a^3/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/a^3/(tanh(1/2*d*x+1/2*c)-1)
+3/d/a^4/(tanh(1/2*d*x+1/2*c)-1)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out] Exception raised: RuntimeError
```

Fricas [B] time = 3.47859, size = 21811, normalized size = 141.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [1/48*(2*a^3*cosh(d*x + c)^14 + 28*a^3*cosh(d*x + c)*sinh(d*x + c)^13 + 2*a
^3*sinh(d*x + c)^14 - 2*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^12 + 2*(91*a^3*cos
h(d*x + c)^2 - 5*a^3 - 28*a^2*b)*sinh(d*x + c)^12 + 8*(91*a^3*cosh(d*x + c)
^3 - 3*(5*a^3 + 28*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^11 - 2*(39*a^3 + 290
*a^2*b + 350*a*b^2)*cosh(d*x + c)^10 + 2*(1001*a^3*cosh(d*x + c)^4 - 39*a^3
 - 290*a^2*b - 350*a*b^2 - 66*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^2)*sinh(d*x
+ c)^10 + 4*(1001*a^3*cosh(d*x + c)^5 - 110*(5*a^3 + 28*a^2*b)*cosh(d*x + c
)^3 - 5*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 - 1
0*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^8 + 2*(3003*a^3*
cosh(d*x + c)^6 - 495*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^4 - 85*a^3 - 730*a^2
*b - 1410*a*b^2 - 840*b^3 - 45*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x +
c)^2)*sinh(d*x + c)^8 + 16*(429*a^3*cosh(d*x + c)^7 - 99*(5*a^3 + 28*a^2*b)
*cosh(d*x + c)^5 - 15*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^3 - 5*
(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 -
 10*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^6 + 2*(3003*a^
3*cosh(d*x + c)^8 - 924*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^6 - 210*(39*a^3 +
290*a^2*b + 350*a*b^2)*cosh(d*x + c)^4 - 85*a^3 - 730*a^2*b - 1410*a*b^2 -
840*b^3 - 140*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^2)*s
inh(d*x + c)^6 + 4*(1001*a^3*cosh(d*x + c)^9 - 396*(5*a^3 + 28*a^2*b)*cosh(
d*x + c)^7 - 126*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^5 - 140*(17
*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^3 - 15*(17*a^3 + 146*
a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(39*a^3 + 2
90*a^2*b + 350*a*b^2)*cosh(d*x + c)^4 + 2*(1001*a^3*cosh(d*x + c)^10 - 495*
(5*a^3 + 28*a^2*b)*cosh(d*x + c)^8 - 210*(39*a^3 + 290*a^2*b + 350*a*b^2)*c
osh(d*x + c)^6 - 350*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x +
c)^4 - 39*a^3 - 290*a^2*b - 350*a*b^2 - 75*(17*a^3 + 146*a^2*b + 282*a*b^2
+ 168*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(91*a^3*cosh(d*x + c)^11 -
55*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^9 - 30*(39*a^3 + 290*a^2*b + 350*a*b^2)
*cosh(d*x + c)^7 - 70*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x +
```

```
c)^5 - 25*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^3 - (39
*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*a^3 - 2*(5
*a^3 + 28*a^2*b)*cosh(d*x + c)^2 + 2*(91*a^3*cosh(d*x + c)^12 - 66*(5*a^3 +
 28*a^2*b)*cosh(d*x + c)^10 - 45*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x
+ c)^8 - 140*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^6 - 7
5*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^4 - 5*a^3 - 28*a
^2*b - 6*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2
+ 15*((3*a^3 + 7*a^2*b)*cosh(d*x + c)^11 + 11*(3*a^3 + 7*a^2*b)*cosh(d*x +
c)*sinh(d*x + c)^10 + (3*a^3 + 7*a^2*b)*sinh(d*x + c)^11 + 4*(3*a^3 + 13*a^
2*b + 14*a*b^2)*cosh(d*x + c)^9 + (12*a^3 + 52*a^2*b + 56*a*b^2 + 55*(3*a^3
 + 7*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^9 + 3*(55*(3*a^3 + 7*a^2*b)*cosh
(d*x + c)^3 + 12*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c))*sinh(d*x + c)
^8 + 2*(9*a^3 + 45*a^2*b + 80*a*b^2 + 56*b^3)*cosh(d*x + c)^7 + 2*(165*(3*a
^3 + 7*a^2*b)*cosh(d*x + c)^4 + 9*a^3 + 45*a^2*b + 80*a*b^2 + 56*b^3 + 72*(
3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 14*(33*(3*a
^3 + 7*a^2*b)*cosh(d*x + c)^5 + 24*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x +
 c)^3 + (9*a^3 + 45*a^2*b + 80*a*b^2 + 56*b^3)*cosh(d*x + c))*sinh(d*x + c)
^6 + 4*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^5 + 2*(231*(3*a^3 + 7*a^
2*b)*cosh(d*x + c)^6 + 252*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^4 +
6*a^3 + 26*a^2*b + 28*a*b^2 + 21*(9*a^3 + 45*a^2*b + 80*a*b^2 + 56*b^3)*cos
h(d*x + c)^2)*sinh(d*x + c)^5 + 2*(165*(3*a^3 + 7*a^2*b)*cosh(d*x + c)^7 +
252*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^5 + 35*(9*a^3 + 45*a^2*b +
80*a*b^2 + 56*b^3)*cosh(d*x + c)^3 + 10*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(
d*x + c))*sinh(d*x + c)^4 + (3*a^3 + 7*a^2*b)*cosh(d*x + c)^3 + (165*(3*a^3
 + 7*a^2*b)*cosh(d*x + c)^8 + 336*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x +
c)^6 + 70*(9*a^3 + 45*a^2*b + 80*a*b^2 + 56*b^3)*cosh(d*x + c)^4 + 3*a^3 +
7*a^2*b + 40*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^3
 + (55*(3*a^3 + 7*a^2*b)*cosh(d*x + c)^9 + 144*(3*a^3 + 13*a^2*b + 14*a*b^2
)*cosh(d*x + c)^7 + 42*(9*a^3 + 45*a^2*b + 80*a*b^2 + 56*b^3)*cosh(d*x + c)
^5 + 40*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^3 + 3*(3*a^3 + 7*a^2*b)
*cosh(d*x + c))*sinh(d*x + c)^2 + (11*(3*a^3 + 7*a^2*b)*cosh(d*x + c)^10 +
36*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^8 + 14*(9*a^3 + 45*a^2*b + 8
0*a*b^2 + 56*b^3)*cosh(d*x + c)^6 + 20*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d
*x + c)^4 + 3*(3*a^3 + 7*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(-b/a)*
log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c
)^4 + 2*(a - 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a - 2*b)*sinh(
d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a - 2*b)*cosh(d*x + c))*sinh(d*x + c)
+ 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c
)^3 + a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))*sqrt(-b/a)
 + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x +
 c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sin
h(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c
) + a)) + 4*(7*a^3*cosh(d*x + c)^13 - 6*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^11
 - 5*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^9 - 20*(17*a^3 + 146*a^
2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^7 - 15*(17*a^3 + 146*a^2*b + 282*a
*b^2 + 168*b^3)*cosh(d*x + c)^5 - 2*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d
*x + c)^3 - (5*a^3 + 28*a^2*b)*cosh(d*x + c))*sinh(d*x + c))/(a^6*d*cosh(d*
x + c)^11 + 11*a^6*d*cosh(d*x + c)*sinh(d*x + c)^10 + a^6*d*sinh(d*x + c)^1
1 + 4*(a^6 + 2*a^5*b)*d*cosh(d*x + c)^9 + a^6*d*cosh(d*x + c)^3 + (55*a^6*d
*cosh(d*x + c)^2 + 4*(a^6 + 2*a^5*b)*d)*sinh(d*x + c)^9 + 2*(3*a^6 + 8*a^5*
b + 8*a^4*b^2)*d*cosh(d*x + c)^7 + 3*(55*a^6*d*cosh(d*x + c)^3 + 12*(a^6 +
2*a^5*b)*d*cosh(d*x + c))*sinh(d*x + c)^8 + 2*(165*a^6*d*cosh(d*x + c)^4 +
72*(a^6 + 2*a^5*b)*d*cosh(d*x + c)^2 + (3*a^6 + 8*a^5*b + 8*a^4*b^2)*d)*sin
h(d*x + c)^7 + 4*(a^6 + 2*a^5*b)*d*cosh(d*x + c)^5 + 14*(33*a^6*d*cosh(d*x
+ c)^5 + 24*(a^6 + 2*a^5*b)*d*cosh(d*x + c)^3 + (3*a^6 + 8*a^5*b + 8*a^4*b^
2)*d*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(231*a^6*d*cosh(d*x + c)^6 + 252*(a
^6 + 2*a^5*b)*d*cosh(d*x + c)^4 + 21*(3*a^6 + 8*a^5*b + 8*a^4*b^2)*d*cosh(d
*x + c)^2 + 2*(a^6 + 2*a^5*b)*d)*sinh(d*x + c)^5 + 2*(165*a^6*d*cosh(d*x +
c)^7 + 252*(a^6 + 2*a^5*b)*d*cosh(d*x + c)^5 + 35*(3*a^6 + 8*a^5*b + 8*a^4*
```

b^2)*d*cosh(d*x + c)^3 + 10*(a^6 + 2*a^5*b)*d*cosh(d*x + c))*sinh(d*x + c)^
4 + (165*a^6*d*cosh(d*x + c)^8 + 336*(a^6 + 2*a^5*b)*d*cosh(d*x + c)^6 + a^
6*d + 70*(3*a^6 + 8*a^5*b + 8*a^4*b^2)*d*cosh(d*x + c)^4 + 40*(a^6 + 2*a^5*
b)*d*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (55*a^6*d*cosh(d*x + c)^9 + 144*(a^
6 + 2*a^5*b)*d*cosh(d*x + c)^7 + 3*a^6*d*cosh(d*x + c) + 42*(3*a^6 + 8*a^5*
b + 8*a^4*b^2)*d*cosh(d*x + c)^5 + 40*(a^6 + 2*a^5*b)*d*cosh(d*x + c)^3)*si
nh(d*x + c)^2 + (11*a^6*d*cosh(d*x + c)^10 + 36*(a^6 + 2*a^5*b)*d*cosh(d*x
+ c)^8 + 3*a^6*d*cosh(d*x + c)^2 + 14*(3*a^6 + 8*a^5*b + 8*a^4*b^2)*d*cosh(
d*x + c)^6 + 20*(a^6 + 2*a^5*b)*d*cosh(d*x + c)^4)*sinh(d*x + c)), 1/24*(a^
3*cosh(d*x + c)^14 + 14*a^3*cosh(d*x + c)*sinh(d*x + c)^13 + a^3*sinh(d*x +
c)^14 - (5*a^3 + 28*a^2*b)*cosh(d*x + c)^12 + (91*a^3*cosh(d*x + c)^2 - 5*
a^3 - 28*a^2*b)*sinh(d*x + c)^12 + 4*(91*a^3*cosh(d*x + c)^3 - 3*(5*a^3 + 2
8*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^11 - (39*a^3 + 290*a^2*b + 350*a*b^2)
*cosh(d*x + c)^10 + (1001*a^3*cosh(d*x + c)^4 - 39*a^3 - 290*a^2*b - 350*a*
b^2 - 66*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 2*(1001*a^3
*cosh(d*x + c)^5 - 110*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^3 - 5*(39*a^3 + 290
*a^2*b + 350*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 - 5*(17*a^3 + 146*a^2*b
+ 282*a*b^2 + 168*b^3)*cosh(d*x + c)^8 + (3003*a^3*cosh(d*x + c)^6 - 495*(5
*a^3 + 28*a^2*b)*cosh(d*x + c)^4 - 85*a^3 - 730*a^2*b - 1410*a*b^2 - 840*b^
3 - 45*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^8 +
8*(429*a^3*cosh(d*x + c)^7 - 99*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^5 - 15*(39
*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^3 - 5*(17*a^3 + 146*a^2*b + 282
*a*b^2 + 168*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 - 5*(17*a^3 + 146*a^2*b +
282*a*b^2 + 168*b^3)*cosh(d*x + c)^6 + (3003*a^3*cosh(d*x + c)^8 - 924*(5*a
^3 + 28*a^2*b)*cosh(d*x + c)^6 - 210*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(
d*x + c)^4 - 85*a^3 - 730*a^2*b - 1410*a*b^2 - 840*b^3 - 140*(17*a^3 + 146*
a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 2*(1001*a^3
*cosh(d*x + c)^9 - 396*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^7 - 126*(39*a^3 + 2
90*a^2*b + 350*a*b^2)*cosh(d*x + c)^5 - 140*(17*a^3 + 146*a^2*b + 282*a*b^2
+ 168*b^3)*cosh(d*x + c)^3 - 15*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)
*cosh(d*x + c))*sinh(d*x + c)^5 - (39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x
+ c)^4 + (1001*a^3*cosh(d*x + c)^10 - 495*(5*a^3 + 28*a^2*b)*cosh(d*x + c)
^8 - 210*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^6 - 350*(17*a^3 + 1
46*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^4 - 39*a^3 - 290*a^2*b - 350*
a*b^2 - 75*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^2)*sinh
(d*x + c)^4 + 4*(91*a^3*cosh(d*x + c)^11 - 55*(5*a^3 + 28*a^2*b)*cosh(d*x +
c)^9 - 30*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^7 - 70*(17*a^3 +
146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^5 - 25*(17*a^3 + 146*a^2*b +
282*a*b^2 + 168*b^3)*cosh(d*x + c)^3 - (39*a^3 + 290*a^2*b + 350*a*b^2)*co
sh(d*x + c))*sinh(d*x + c)^3 + a^3 - (5*a^3 + 28*a^2*b)*cosh(d*x + c)^2 + (
91*a^3*cosh(d*x + c)^12 - 66*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^10 - 45*(39*a
^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^8 - 140*(17*a^3 + 146*a^2*b + 282
*a*b^2 + 168*b^3)*cosh(d*x + c)^6 - 75*(17*a^3 + 146*a^2*b + 282*a*b^2 + 16
8*b^3)*cosh(d*x + c)^4 - 5*a^3 - 28*a^2*b - 6*(39*a^3 + 290*a^2*b + 350*a*b
^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 15*((3*a^3 + 7*a^2*b)*cosh(d*x + c)^
11 + 11*(3*a^3 + 7*a^2*b)*cosh(d*x + c)*sinh(d*x + c)^10 + (3*a^3 + 7*a^2*b
)*sinh(d*x + c)^11 + 4*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^9 + (12*
a^3 + 52*a^2*b + 56*a*b^2 + 55*(3*a^3 + 7*a^2*b)*cosh(d*x + c)^2)*sinh(d*x
+ c)^9 + 3*(55*(3*a^3 + 7*a^2*b)*cosh(d*x + c)^3 + 12*(3*a^3 + 13*a^2*b + 1
4*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^8 + 2*(9*a^3 + 45*a^2*b + 80*a*b^2 +
56*b^3)*cosh(d*x + c)^7 + 2*(165*(3*a^3 + 7*a^2*b)*cosh(d*x + c)^4 + 9*a^3
+ 45*a^2*b + 80*a*b^2 + 56*b^3 + 72*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x
+ c)^2)*sinh(d*x + c)^7 + 14*(33*(3*a^3 + 7*a^2*b)*cosh(d*x + c)^5 + 24*(3*
a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^3 + (9*a^3 + 45*a^2*b + 80*a*b^2 +
56*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 4*(3*a^3 + 13*a^2*b + 14*a*b^2)*c
osh(d*x + c)^5 + 2*(231*(3*a^3 + 7*a^2*b)*cosh(d*x + c)^6 + 252*(3*a^3 + 13
*a^2*b + 14*a*b^2)*cosh(d*x + c)^4 + 6*a^3 + 26*a^2*b + 28*a*b^2 + 21*(9*a^
3 + 45*a^2*b + 80*a*b^2 + 56*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 2*(165
*(3*a^3 + 7*a^2*b)*cosh(d*x + c)^7 + 252*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh

(d*x + c)^5 + 35*(9*a^3 + 45*a^2*b + 80*a*b^2 + 56*b^3)*cosh(d*x + c)^3 + 1
0*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + (3*a^3 + 7
*a^2*b)*cosh(d*x + c)^3 + (165*(3*a^3 + 7*a^2*b)*cosh(d*x + c)^8 + 336*(3*a
^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^6 + 70*(9*a^3 + 45*a^2*b + 80*a*b^2
 + 56*b^3)*cosh(d*x + c)^4 + 3*a^3 + 7*a^2*b + 40*(3*a^3 + 13*a^2*b + 14*a*
b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (55*(3*a^3 + 7*a^2*b)*cosh(d*x + c)
^9 + 144*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^7 + 42*(9*a^3 + 45*a^2
*b + 80*a*b^2 + 56*b^3)*cosh(d*x + c)^5 + 40*(3*a^3 + 13*a^2*b + 14*a*b^2)*
cosh(d*x + c)^3 + 3*(3*a^3 + 7*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^2 + (11*
(3*a^3 + 7*a^2*b)*cosh(d*x + c)^10 + 36*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(
d*x + c)^8 + 14*(9*a^3 + 45*a^2*b + 80*a*b^2 + 56*b^3)*cosh(d*x + c)^6 + 20
*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^4 + 3*(3*a^3 + 7*a^2*b)*cosh(d
*x + c)^2)*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cos
h(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (a + 4*b)*cosh(d*x + c) +
(3*a*cosh(d*x + c)^2 + a + 4*b)*sinh(d*x + c))*sqrt(b/a)/b) + 15*((3*a^3 +
7*a^2*b)*cosh(d*x + c)^11 + 11*(3*a^3 + 7*a^2*b)*cosh(d*x + c)*sinh(d*x + c
)^10 + (3*a^3 + 7*a^2*b)*sinh(d*x + c)^11 + 4*(3*a^3 + 13*a^2*b + 14*a*b^2)
*cosh(d*x + c)^9 + (12*a^3 + 52*a^2*b + 56*a*b^2 + 55*(3*a^3 + 7*a^2*b)*cos
h(d*x + c)^2)*sinh(d*x + c)^9 + 3*(55*(3*a^3 + 7*a^2*b)*cosh(d*x + c)^3 + 1
2*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^8 + 2*(9*a^3 +
 45*a^2*b + 80*a*b^2 + 56*b^3)*cosh(d*x + c)^7 + 2*(165*(3*a^3 + 7*a^2*b)*c
osh(d*x + c)^4 + 9*a^3 + 45*a^2*b + 80*a*b^2 + 56*b^3 + 72*(3*a^3 + 13*a^2*
b + 14*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 14*(33*(3*a^3 + 7*a^2*b)*c
osh(d*x + c)^5 + 24*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^3 + (9*a^3
+ 45*a^2*b + 80*a*b^2 + 56*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 4*(3*a^3 +
 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^5 + 2*(231*(3*a^3 + 7*a^2*b)*cosh(d*x +
 c)^6 + 252*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^4 + 6*a^3 + 26*a^2*
b + 28*a*b^2 + 21*(9*a^3 + 45*a^2*b + 80*a*b^2 + 56*b^3)*cosh(d*x + c)^2)*s
inh(d*x + c)^5 + 2*(165*(3*a^3 + 7*a^2*b)*cosh(d*x + c)^7 + 252*(3*a^3 + 13
*a^2*b + 14*a*b^2)*cosh(d*x + c)^5 + 35*(9*a^3 + 45*a^2*b + 80*a*b^2 + 56*b
^3)*cosh(d*x + c)^3 + 10*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c))*sinh(
d*x + c)^4 + (3*a^3 + 7*a^2*b)*cosh(d*x + c)^3 + (165*(3*a^3 + 7*a^2*b)*cos
h(d*x + c)^8 + 336*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^6 + 70*(9*a^
3 + 45*a^2*b + 80*a*b^2 + 56*b^3)*cosh(d*x + c)^4 + 3*a^3 + 7*a^2*b + 40*(3
*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (55*(3*a^3 +
 7*a^2*b)*cosh(d*x + c)^9 + 144*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)
^7 + 42*(9*a^3 + 45*a^2*b + 80*a*b^2 + 56*b^3)*cosh(d*x + c)^5 + 40*(3*a^3
+ 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^3 + 3*(3*a^3 + 7*a^2*b)*cosh(d*x + c))
*sinh(d*x + c)^2 + (11*(3*a^3 + 7*a^2*b)*cosh(d*x + c)^10 + 36*(3*a^3 + 13*
a^2*b + 14*a*b^2)*cosh(d*x + c)^8 + 14*(9*a^3 + 45*a^2*b + 80*a*b^2 + 56*b^
3)*cosh(d*x + c)^6 + 20*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^4 + 3*(
3*a^3 + 7*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*(a*co
sh(d*x + c) + a*sinh(d*x + c))*sqrt(b/a)/b) + 2*(7*a^3*cosh(d*x + c)^13 - 6
*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^11 - 5*(39*a^3 + 290*a^2*b + 350*a*b^2)*c
osh(d*x + c)^9 - 20*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c
)^7 - 15*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^5 - 2*(39
*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^3 - (5*a^3 + 28*a^2*b)*cosh(d*x
 + c))*sinh(d*x + c))/(a^6*d*cosh(d*x + c)^11 + 11*a^6*d*cosh(d*x + c)*sinh
(d*x + c)^10 + a^6*d*sinh(d*x + c)^11 + 4*(a^6 + 2*a^5*b)*d*cosh(d*x + c)^9
 + a^6*d*cosh(d*x + c)^3 + (55*a^6*d*cosh(d*x + c)^2 + 4*(a^6 + 2*a^5*b)*d)
*sinh(d*x + c)^9 + 2*(3*a^6 + 8*a^5*b + 8*a^4*b^2)*d*cosh(d*x + c)^7 + 3*(5
5*a^6*d*cosh(d*x + c)^3 + 12*(a^6 + 2*a^5*b)*d*cosh(d*x + c))*sinh(d*x + c)
^8 + 2*(165*a^6*d*cosh(d*x + c)^4 + 72*(a^6 + 2*a^5*b)*d*cosh(d*x + c)^2 +
(3*a^6 + 8*a^5*b + 8*a^4*b^2)*d)*sinh(d*x + c)^7 + 4*(a^6 + 2*a^5*b)*d*cosh
(d*x + c)^5 + 14*(33*a^6*d*cosh(d*x + c)^5 + 24*(a^6 + 2*a^5*b)*d*cosh(d*x
+ c)^3 + (3*a^6 + 8*a^5*b + 8*a^4*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^6 + 2
*(231*a^6*d*cosh(d*x + c)^6 + 252*(a^6 + 2*a^5*b)*d*cosh(d*x + c)^4 + 21*(3
*a^6 + 8*a^5*b + 8*a^4*b^2)*d*cosh(d*x + c)^2 + 2*(a^6 + 2*a^5*b)*d)*sinh(d
x + c)^5 + 2(165*a^6*d*cosh(d*x + c)^7 + 252*(a^6 + 2*a^5*b)*d*cosh(d*x +

```
 c)^5 + 35*(3*a^6 + 8*a^5*b + 8*a^4*b^2)*d*cosh(d*x + c)^3 + 10*(a^6 + 2*a^
5*b)*d*cosh(d*x + c))*sinh(d*x + c)^4 + (165*a^6*d*cosh(d*x + c)^8 + 336*(a
^6 + 2*a^5*b)*d*cosh(d*x + c)^6 + a^6*d + 70*(3*a^6 + 8*a^5*b + 8*a^4*b^2)*
d*cosh(d*x + c)^4 + 40*(a^6 + 2*a^5*b)*d*cosh(d*x + c)^2)*sinh(d*x + c)^3 +
 (55*a^6*d*cosh(d*x + c)^9 + 144*(a^6 + 2*a^5*b)*d*cosh(d*x + c)^7 + 3*a^6*
d*cosh(d*x + c) + 42*(3*a^6 + 8*a^5*b + 8*a^4*b^2)*d*cosh(d*x + c)^5 + 40*(
a^6 + 2*a^5*b)*d*cosh(d*x + c)^3)*sinh(d*x + c)^2 + (11*a^6*d*cosh(d*x + c)
^10 + 36*(a^6 + 2*a^5*b)*d*cosh(d*x + c)^8 + 3*a^6*d*cosh(d*x + c)^2 + 14*(
3*a^6 + 8*a^5*b + 8*a^4*b^2)*d*cosh(d*x + c)^6 + 20*(a^6 + 2*a^5*b)*d*cosh(
d*x + c)^4)*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.43 $\int \frac{\sinh^2(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=187

$$\frac{\sqrt{b}\left(15a^2+40ab+24b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^4d(a+b)^{3/2}}+\frac{b(11a+12b)\tanh(c+dx)}{8a^3d(a+b)\left(a-b\tanh^2(c+dx)+b\right)}+\frac{3b\tanh(c+dx)}{4a^2d\left(a-b\tanh^2(c+dx)\right)}$$

```
[Out] -((a + 6*b)*x)/(2*a^4) + (Sqrt[b]*(15*a^2 + 40*a*b + 24*b^2)*ArcTanh[(Sqrt[
b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*a^4*(a + b)^(3/2)*d) + (Cosh[c + d*x]*Si
nh[c + d*x])/(2*a*d*(a + b - b*Tanh[c + d*x]^2)^2) + (3*b*Tanh[c + d*x])/(4
*a^2*d*(a + b - b*Tanh[c + d*x]^2)^2) + (b*(11*a + 12*b)*Tanh[c + d*x])/(8*
a^3*(a + b)*d*(a + b - b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.291762, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4132, 471, 527, 522, 206, 208}

$$\frac{\sqrt{b}\left(15a^2+40ab+24b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^4d(a+b)^{3/2}}+\frac{b(11a+12b)\tanh(c+dx)}{8a^3d(a+b)\left(a-b\tanh^2(c+dx)+b\right)}+\frac{3b\tanh(c+dx)}{4a^2d\left(a-b\tanh^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```


```
[Out] -((a + 6*b)*x)/(2*a^4) + (Sqrt[b]*(15*a^2 + 40*a*b + 24*b^2)*ArcTanh[(Sqrt[
b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*a^4*(a + b)^(3/2)*d) + (Cosh[c + d*x]*Si
nh[c + d*x])/(2*a*d*(a + b - b*Tanh[c + d*x]^2)^2) + (3*b*Tanh[c + d*x])/(4
*a^2*d*(a + b - b*Tanh[c + d*x]^2)^2) + (b*(11*a + 12*b)*Tanh[c + d*x])/(8*
a^3*(a + b)*d*(a + b - b*Tanh[c + d*x]^2))
```


Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)^2(a+b-bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{a+b+5bx^2}{(1-x^2)(a+b-bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{2ad}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{3b\tanh(c+dx)}{4a^2d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{-2(a+b)(2a+}{(1-x^2)(}\right.}{}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{3b\tanh(c+dx)}{4a^2d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{b(11a+12b)\,\mathrm{ta}}{8a^3(a+b)d\left(a+b-\right.}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{3b\tanh(c+dx)}{4a^2d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{b(11a+12b)\,\mathrm{ta}}{8a^3(a+b)d\left(a+b-\right.}$$

$$= -\frac{(a+6b)x}{2a^4} + \frac{\sqrt{b}\left(15a^2+40ab+24b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^4(a+b)^{3/2}d} + \frac{\cosh(c+dx)\sinh(c+}{2ad\left(a+b-b\tanh^2(c+\right.}$$

Mathematica [B] time = 18.7883, size = 2544, normalized size = 13.6

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (-5*(a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6*(((3*a^2 + 8*a*b + 8*
b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[
b]*(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cosh[2*(c + d*x)])*Sinh[2*(c +
d*x)])/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2)))/(8192*b^(5/2)*d*(a +
 b*Sech[c + d*x]^2)^3) - ((a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6
*((-3*a*(a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/
2) + (Sqrt[b]*(3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*
b^2)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/((a + b)^2*(a + 2*b + a*Cosh[2*(
c + d*x)])^2)))/(2048*b^(5/2)*d*(a + b*Sech[c + d*x]^2)^3) + ((a + 2*b + a*
Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6*((-2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 +
480*a^2*b^3 + 640*a*b^4 + 256*b^5)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c
])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c
] - Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - S
inh[c])^4]) + (Sech[2*c]*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*d*x*Cos
h[2*c] + 512*a*b^2*(a + b)^2*(a + 2*b)*d*x*Cosh[2*d*x] + 128*a^4*b^2*d*x*Co
sh[2*(c + 2*d*x)] + 256*a^3*b^3*d*x*Cosh[2*(c + 2*d*x)] + 128*a^2*b^4*d*x*C
osh[2*(c + 2*d*x)] + 512*a^4*b^2*d*x*Cosh[4*c + 2*d*x] + 2048*a^3*b^3*d*x*C
osh[4*c + 2*d*x] + 2560*a^2*b^4*d*x*Cosh[4*c + 2*d*x] + 1024*a*b^5*d*x*Cosh
[4*c + 2*d*x] + 128*a^4*b^2*d*x*Cosh[6*c + 4*d*x] + 256*a^3*b^3*d*x*Cosh[6*
c + 4*d*x] + 128*a^2*b^4*d*x*Cosh[6*c + 4*d*x] - 9*a^6*Sinh[2*c] + 12*a^5*b
*Sinh[2*c] + 684*a^4*b^2*Sinh[2*c] + 2880*a^3*b^3*Sinh[2*c] + 5280*a^2*b^4*
Sinh[2*c] + 4608*a*b^5*Sinh[2*c] + 1536*b^6*Sinh[2*c] + 9*a^6*Sinh[2*d*x] -
 14*a^5*b*Sinh[2*d*x] - 608*a^4*b^2*Sinh[2*d*x] - 2112*a^3*b^3*Sinh[2*d*x]
- 2560*a^2*b^4*Sinh[2*d*x] - 1024*a*b^5*Sinh[2*d*x] + 3*a^6*Sinh[2*(c + 2*d
*x)] - 12*a^5*b*Sinh[2*(c + 2*d*x)] - 204*a^4*b^2*Sinh[2*(c + 2*d*x)] - 384
*a^3*b^3*Sinh[2*(c + 2*d*x)] - 192*a^2*b^4*Sinh[2*(c + 2*d*x)] - 3*a^6*Sinh
[4*c + 2*d*x] + 10*a^5*b*Sinh[4*c + 2*d*x] + 304*a^4*b^2*Sinh[4*c + 2*d*x]
+ 1056*a^3*b^3*Sinh[4*c + 2*d*x] + 1280*a^2*b^4*Sinh[4*c + 2*d*x] + 512*a*b
^5*Sinh[4*c + 2*d*x]))/(a + 2*b + a*Cosh[2*(c + d*x)])^2))/(4096*a^3*b^2*(a
 + b)^2*d*(a + b*Sech[c + d*x]^2)^3) + ((a + 2*b + a*Cosh[2*c + 2*d*x])^3*S
ech[c + d*x]^6*((6*(a^6 - 8*a^5*b + 120*a^4*b^2 + 1280*a^3*b^3 + 3200*a^2*b
^4 + 3072*a*b^5 + 1024*b^6)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a
+ 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sin
h[c])^4])]*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])
^4]) + (Sech[2*c]*(-1536*b^2*(a + b)^2*(3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^
3)*d*x*Cosh[2*c] - 3072*a*b^2*(a^2 + 3*a*b + 2*b^2)^2*d*x*Cosh[2*d*x] - 768
*a^5*b^2*d*x*Cosh[2*(c + 2*d*x)] - 3072*a^4*b^3*d*x*Cosh[2*(c + 2*d*x)] - 3
840*a^3*b^4*d*x*Cosh[2*(c + 2*d*x)] - 1536*a^2*b^5*d*x*Cosh[2*(c + 2*d*x)]
- 3072*a^5*b^2*d*x*Cosh[4*c + 2*d*x] - 18432*a^4*b^3*d*x*Cosh[4*c + 2*d*x]
- 39936*a^3*b^4*d*x*Cosh[4*c + 2*d*x] - 36864*a^2*b^5*d*x*Cosh[4*c + 2*d*x]
 - 12288*a*b^6*d*x*Cosh[4*c + 2*d*x] - 768*a^5*b^2*d*x*Cosh[6*c + 4*d*x] -
3072*a^4*b^3*d*x*Cosh[6*c + 4*d*x] - 3840*a^3*b^4*d*x*Cosh[6*c + 4*d*x] - 1
536*a^2*b^5*d*x*Cosh[6*c + 4*d*x] + 9*a^7*Sinh[2*c] - 54*a^6*b*Sinh[2*c] -
2392*a^5*b^2*Sinh[2*c] - 13968*a^4*b^3*Sinh[2*c] - 36480*a^3*b^4*Sinh[2*c]
- 50432*a^2*b^5*Sinh[2*c] - 35840*a*b^6*Sinh[2*c] - 10240*b^7*Sinh[2*c] - 9
*a^7*Sinh[2*d*x] + 56*a^6*b*Sinh[2*d*x] + 2552*a^5*b^2*Sinh[2*d*x] + 13184*
a^4*b^3*Sinh[2*d*x] + 27072*a^3*b^4*Sinh[2*d*x] + 24576*a^2*b^5*Sinh[2*d*x]
 + 8192*a*b^6*Sinh[2*d*x] - 3*a^7*Sinh[2*(c + 2*d*x)] + 26*a^6*b*Sinh[2*(c
+ 2*d*x)] + 992*a^5*b^2*Sinh[2*(c + 2*d*x)] + 3648*a^4*b^3*Sinh[2*(c + 2*d*
x)] + 4480*a^3*b^4*Sinh[2*(c + 2*d*x)] + 1792*a^2*b^5*Sinh[2*(c + 2*d*x)] +
 3*a^7*Sinh[4*c + 2*d*x] - 24*a^6*b*Sinh[4*c + 2*d*x] - 600*a^5*b^2*Sinh[4*
c + 2*d*x] - 3200*a^4*b^3*Sinh[4*c + 2*d*x] - 6720*a^3*b^4*Sinh[4*c + 2*d*x
] - 6144*a^2*b^5*Sinh[4*c + 2*d*x] - 2048*a*b^6*Sinh[4*c + 2*d*x] + 256*a^5
*b^2*Sinh[6*c + 4*d*x] + 1024*a^4*b^3*Sinh[6*c + 4*d*x] + 1280*a^3*b^4*Sinh
[6*c + 4*d*x] + 512*a^2*b^5*Sinh[6*c + 4*d*x] + 64*a^5*b^2*Sinh[4*c + 6*d*x
] + 128*a^4*b^3*Sinh[4*c + 6*d*x] + 64*a^3*b^4*Sinh[4*c + 6*d*x] + 64*a^5*b
^2*Sinh[8*c + 6*d*x] + 128*a^4*b^3*Sinh[8*c + 6*d*x] + 64*a^3*b^4*Sinh[8*c
+ 6*d*x]))/(a + 2*b + a*Cosh[2*(c + d*x)])^2))/(16384*a^4*b^2*(a + b)^2*d*(
a + b*Sech[c + d*x]^2)^3) + ((a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x
```

```
]^6*((6*a^2*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x]
 - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(Cosh
[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (a*Sech[2
*c]*((-9*a^4 - 16*a^3*b + 48*a^2*b^2 + 128*a*b^3 + 64*b^4)*Sinh[2*d*x] + a*
(-3*a^3 + 2*a^2*b + 24*a*b^2 + 16*b^3)*Sinh[2*(c + 2*d*x)] + (3*a^4 - 64*a^
2*b^2 - 128*a*b^3 - 64*b^4)*Sinh[4*c + 2*d*x]) + (9*a^5 + 18*a^4*b - 64*a^3
*b^2 - 256*a^2*b^3 - 320*a*b^4 - 128*b^5)*Tanh[2*c])/(a^2*(a + 2*b + a*Cosh
[2*(c + d*x)])^2)))/(4096*b^2*(a + b)^2*d*(a + b*Sech[c + d*x]^2)^3)
```

Maple [B] time = 0.112, size = 1320, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -1/2/d/a^3/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/a^3/(tanh(1/2*d*x+1/2*c)+1)-1/2/
d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)-3/d/a^4*ln(tanh(1/2*d*x+1/2*c)+1)*b+9/4/d*b
/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)
^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7+2/d*b^2/a^3/(ta
nh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*t
anh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7+27/4/d*b/a/(tanh(1/2*d*
x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d
*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^5+23/4/d*b^2/a^2/(tanh(1/2*d
*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*
d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^5-2/d*b^3/a^3/(tanh(1/2*d*x
+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*
x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^5+27/4/d*b/a/(tanh(1/2*d*x+1/
2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1
/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^3+23/4/d*b^2/a^2/(tanh(1/2*d*x+1
/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+
1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^3-2/d*b^3/a^3/(tanh(1/2*d*x+1/2
*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/
2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^3+9/4/d*b/a^2/(tanh(1/2*d*x+1/2*c
)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*
c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)+2/d*b^2/a^3/(tanh(1/2*d*x+1/2*c)^4*a+b*ta
nh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b
)^2*tanh(1/2*d*x+1/2*c)+15/16/d*b^(1/2)/a^2/(a+b)^(3/2)*ln((a+b)^(1/2)*tanh
(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+5/2/d*b^(3/2)/
a^3/(a+b)^(3/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*
b^(1/2)+(a+b)^(1/2))+3/2/d*b^(5/2)/a^4/(a+b)^(3/2)*ln((a+b)^(1/2)*tanh(1/2*
d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-15/16/d*b^(1/2)/a^2
/(a+b)^(3/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(
1/2)+(a+b)^(1/2))-5/2/d*b^(3/2)/a^3/(a+b)^(3/2)*ln((a+b)^(1/2)*tanh(1/2*d*x
+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-3/2/d*b^(5/2)/a^4/(a+b
)^(3/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+
(a+b)^(1/2))+1/2/d/a^3/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/a^3/(tanh(1/2*d*x+1/
2*c)-1)+1/2/d/a^3*ln(tanh(1/2*d*x+1/2*c)-1)+3/d/a^4*ln(tanh(1/2*d*x+1/2*c)-
1)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 4.00341, size = 23448, normalized size = 125.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [1/16*(2*(a^4 + a^3*b)*cosh(d*x + c)^12 + 24*(a^4 + a^3*b)*cosh(d*x + c)*si
nh(d*x + c)^11 + 2*(a^4 + a^3*b)*sinh(d*x + c)^12 + 8*(a^4 + 3*a^3*b + 2*a^
2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c)^10 + 4*(2*a^4 + 6*a^
3*b + 4*a^2*b^2 - 2*(a^4 + 7*a^3*b + 6*a^2*b^2)*d*x + 33*(a^4 + a^3*b)*cosh
(d*x + c)^2)*sinh(d*x + c)^10 + 40*(11*(a^4 + a^3*b)*cosh(d*x + c)^3 + 2*(a
^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c))*
sinh(d*x + c)^9 + 2*(5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*
a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c)^8 + 2*(495*(a^4 + a^3*b)*
cosh(d*x + c)^4 + 5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3
*b + 20*a^2*b^2 + 12*a*b^3)*d*x + 180*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7
*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 16*(99*(a^4 + a
^3*b)*cosh(d*x + c)^5 + 60*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*
a^2*b^2)*d*x)*cosh(d*x + c)^3 + (5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 -
16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c
)^7 - 4*(27*a^3*b + 102*a^2*b^2 + 152*a*b^3 + 80*b^4 + 4*(3*a^4 + 29*a^3*b
+ 82*a^2*b^2 + 104*a*b^3 + 48*b^4)*d*x)*cosh(d*x + c)^6 + 4*(462*(a^4 + a^3
*b)*cosh(d*x + c)^6 + 420*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a
^2*b^2)*d*x)*cosh(d*x + c)^4 - 27*a^3*b - 102*a^2*b^2 - 152*a*b^3 - 80*b^4
- 4*(3*a^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*b^3 + 48*b^4)*d*x + 14*(5*a^4 +
3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3
)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(198*(a^4 + a^3*b)*cosh(d*x + c
)^7 + 252*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cos
h(d*x + c)^5 + 14*(5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^
3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c)^3 - 3*(27*a^3*b + 102*a^2*b
^2 + 152*a*b^3 + 80*b^4 + 4*(3*a^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*b^3 + 48
*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(5*a^4 + 75*a^3*b + 192*a^2*b
^2 + 128*a*b^3 + 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x +
 c)^4 + 2*(495*(a^4 + a^3*b)*cosh(d*x + c)^8 + 840*(a^4 + 3*a^3*b + 2*a^2*b
^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c)^6 + 70*(5*a^4 + 3*a^3*b
 - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*
cosh(d*x + c)^4 - 5*a^4 - 75*a^3*b - 192*a^2*b^2 - 128*a*b^3 - 16*(a^4 + 9*
a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x - 30*(27*a^3*b + 102*a^2*b^2 + 152*a*b^3
 + 80*b^4 + 4*(3*a^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*b^3 + 48*b^4)*d*x)*cos
h(d*x + c)^2)*sinh(d*x + c)^4 - 2*a^4 - 2*a^3*b + 8*(55*(a^4 + a^3*b)*cosh(
d*x + c)^9 + 120*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d
*x)*cosh(d*x + c)^7 + 14*(5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4
 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c)^5 - 10*(27*a^3*b + 1
02*a^2*b^2 + 152*a*b^3 + 80*b^4 + 4*(3*a^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*
b^3 + 48*b^4)*d*x)*cosh(d*x + c)^3 - (5*a^4 + 75*a^3*b + 192*a^2*b^2 + 128*
a*b^3 + 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c))*sinh
(d*x + c)^3 - 4*(2*a^4 + 15*a^3*b + 14*a^2*b^2 + 2*(a^4 + 7*a^3*b + 6*a^2*b
^2)*d*x)*cosh(d*x + c)^2 + 4*(33*(a^4 + a^3*b)*cosh(d*x + c)^10 + 90*(a^4 +
 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c)^8 + 1
4*(5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2
 + 12*a*b^3)*d*x)*cosh(d*x + c)^6 - 15*(27*a^3*b + 102*a^2*b^2 + 152*a*b^3
```

```
+ 80*b^4 + 4*(3*a^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*b^3 + 48*b^4)*d*x)*cosh
(d*x + c)^4 - 2*a^4 - 15*a^3*b - 14*a^2*b^2 - 2*(a^4 + 7*a^3*b + 6*a^2*b^2)
*d*x - 3*(5*a^4 + 75*a^3*b + 192*a^2*b^2 + 128*a*b^3 + 16*(a^4 + 9*a^3*b +
20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((15*a^4 + 4
0*a^3*b + 24*a^2*b^2)*cosh(d*x + c)^10 + 10*(15*a^4 + 40*a^3*b + 24*a^2*b^2
)*cosh(d*x + c)*sinh(d*x + c)^9 + (15*a^4 + 40*a^3*b + 24*a^2*b^2)*sinh(d*x
 + c)^10 + 4*(15*a^4 + 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^8 +
 (60*a^4 + 280*a^3*b + 416*a^2*b^2 + 192*a*b^3 + 45*(15*a^4 + 40*a^3*b + 24
*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(15*a^4 + 40*a^3*b + 24*
a^2*b^2)*cosh(d*x + c)^3 + 4*(15*a^4 + 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*c
osh(d*x + c))*sinh(d*x + c)^7 + 2*(45*a^4 + 240*a^3*b + 512*a^2*b^2 + 512*a
*b^3 + 192*b^4)*cosh(d*x + c)^6 + 2*(105*(15*a^4 + 40*a^3*b + 24*a^2*b^2)*c
osh(d*x + c)^4 + 45*a^4 + 240*a^3*b + 512*a^2*b^2 + 512*a*b^3 + 192*b^4 + 5
6*(15*a^4 + 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^2)*sinh(d*x +
c)^6 + 4*(63*(15*a^4 + 40*a^3*b + 24*a^2*b^2)*cosh(d*x + c)^5 + 56*(15*a^4
+ 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^3 + 3*(45*a^4 + 240*a^3*
b + 512*a^2*b^2 + 512*a*b^3 + 192*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*(
15*a^4 + 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^4 + 2*(105*(15*a^
4 + 40*a^3*b + 24*a^2*b^2)*cosh(d*x + c)^6 + 140*(15*a^4 + 70*a^3*b + 104*a
^2*b^2 + 48*a*b^3)*cosh(d*x + c)^4 + 30*a^4 + 140*a^3*b + 208*a^2*b^2 + 96*
a*b^3 + 15*(45*a^4 + 240*a^3*b + 512*a^2*b^2 + 512*a*b^3 + 192*b^4)*cosh(d*
x + c)^2)*sinh(d*x + c)^4 + 8*(15*(15*a^4 + 40*a^3*b + 24*a^2*b^2)*cosh(d*x
 + c)^7 + 28*(15*a^4 + 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^5 +
 5*(45*a^4 + 240*a^3*b + 512*a^2*b^2 + 512*a*b^3 + 192*b^4)*cosh(d*x + c)^3
 + 2*(15*a^4 + 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c))*sinh(d*x +
 c)^3 + (15*a^4 + 40*a^3*b + 24*a^2*b^2)*cosh(d*x + c)^2 + (45*(15*a^4 + 40
*a^3*b + 24*a^2*b^2)*cosh(d*x + c)^8 + 112*(15*a^4 + 70*a^3*b + 104*a^2*b^2
 + 48*a*b^3)*cosh(d*x + c)^6 + 30*(45*a^4 + 240*a^3*b + 512*a^2*b^2 + 512*a
*b^3 + 192*b^4)*cosh(d*x + c)^4 + 15*a^4 + 40*a^3*b + 24*a^2*b^2 + 24*(15*a
^4 + 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
2*(5*(15*a^4 + 40*a^3*b + 24*a^2*b^2)*cosh(d*x + c)^9 + 16*(15*a^4 + 70*a^3
*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^7 + 6*(45*a^4 + 240*a^3*b + 512*
a^2*b^2 + 512*a*b^3 + 192*b^4)*cosh(d*x + c)^5 + 8*(15*a^4 + 70*a^3*b + 104
*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^3 + (15*a^4 + 40*a^3*b + 24*a^2*b^2)*cos
h(d*x + c))*sinh(d*x + c))*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2
*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh
(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2
 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*si
nh(d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*
sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(
a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*
x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*
sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x
+ c) + a)) + 8*(3*(a^4 + a^3*b)*cosh(d*x + c)^11 + 10*(a^4 + 3*a^3*b + 2*a^
2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c)^9 + 2*(5*a^4 + 3*a^3
*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x
)*cosh(d*x + c)^7 - 3*(27*a^3*b + 102*a^2*b^2 + 152*a*b^3 + 80*b^4 + 4*(3*a
^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*b^3 + 48*b^4)*d*x)*cosh(d*x + c)^5 - (5*
a^4 + 75*a^3*b + 192*a^2*b^2 + 128*a*b^3 + 16*(a^4 + 9*a^3*b + 20*a^2*b^2 +
 12*a*b^3)*d*x)*cosh(d*x + c)^3 - (2*a^4 + 15*a^3*b + 14*a^2*b^2 + 2*(a^4 +
 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^7 + a^6*b)*d*c
osh(d*x + c)^10 + 10*(a^7 + a^6*b)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^7 +
 a^6*b)*d*sinh(d*x + c)^10 + 4*(a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh(d*x + c)^
8 + (45*(a^7 + a^6*b)*d*cosh(d*x + c)^2 + 4*(a^7 + 3*a^6*b + 2*a^5*b^2)*d)*
sinh(d*x + c)^8 + 2*(3*a^7 + 11*a^6*b + 16*a^5*b^2 + 8*a^4*b^3)*d*cosh(d*x
+ c)^6 + 8*(15*(a^7 + a^6*b)*d*cosh(d*x + c)^3 + 4*(a^7 + 3*a^6*b + 2*a^5*b
^2)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^7 + a^6*b)*d*cosh(d*x + c)
^4 + 56*(a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh(d*x + c)^2 + (3*a^7 + 11*a^6*b +
 16*a^5*b^2 + 8*a^4*b^3)*d)*sinh(d*x + c)^6 + 4*(a^7 + 3*a^6*b + 2*a^5*b^2)
```

```
*d*cosh(d*x + c)^4 + 4*(63*(a^7 + a^6*b)*d*cosh(d*x + c)^5 + 56*(a^7 + 3*a^
6*b + 2*a^5*b^2)*d*cosh(d*x + c)^3 + 3*(3*a^7 + 11*a^6*b + 16*a^5*b^2 + 8*a
^4*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^7 + a^6*b)*d*cosh(d*x
+ c)^6 + 140*(a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh(d*x + c)^4 + 15*(3*a^7 + 11
*a^6*b + 16*a^5*b^2 + 8*a^4*b^3)*d*cosh(d*x + c)^2 + 2*(a^7 + 3*a^6*b + 2*a
^5*b^2)*d)*sinh(d*x + c)^4 + (a^7 + a^6*b)*d*cosh(d*x + c)^2 + 8*(15*(a^7 +
 a^6*b)*d*cosh(d*x + c)^7 + 28*(a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh(d*x + c)^
5 + 5*(3*a^7 + 11*a^6*b + 16*a^5*b^2 + 8*a^4*b^3)*d*cosh(d*x + c)^3 + 2*(a^
7 + 3*a^6*b + 2*a^5*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (45*(a^7 + a^6*
b)*d*cosh(d*x + c)^8 + 112*(a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh(d*x + c)^6 +
30*(3*a^7 + 11*a^6*b + 16*a^5*b^2 + 8*a^4*b^3)*d*cosh(d*x + c)^4 + 24*(a^7
+ 3*a^6*b + 2*a^5*b^2)*d*cosh(d*x + c)^2 + (a^7 + a^6*b)*d)*sinh(d*x + c)^2
 + 2*(5*(a^7 + a^6*b)*d*cosh(d*x + c)^9 + 16*(a^7 + 3*a^6*b + 2*a^5*b^2)*d*
cosh(d*x + c)^7 + 6*(3*a^7 + 11*a^6*b + 16*a^5*b^2 + 8*a^4*b^3)*d*cosh(d*x
+ c)^5 + 8*(a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh(d*x + c)^3 + (a^7 + a^6*b)*d*
cosh(d*x + c))*sinh(d*x + c)), 1/8*((a^4 + a^3*b)*cosh(d*x + c)^12 + 12*(a^
4 + a^3*b)*cosh(d*x + c)*sinh(d*x + c)^11 + (a^4 + a^3*b)*sinh(d*x + c)^12
+ 4*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x
+ c)^10 + 2*(2*a^4 + 6*a^3*b + 4*a^2*b^2 - 2*(a^4 + 7*a^3*b + 6*a^2*b^2)*d*
x + 33*(a^4 + a^3*b)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 20*(11*(a^4 + a^3*
b)*cosh(d*x + c)^3 + 2*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*
b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^9 + (5*a^4 + 3*a^3*b - 32*a^2*b^2 -
32*a*b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c)^8
+ (495*(a^4 + a^3*b)*cosh(d*x + c)^4 + 5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*
b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x + 180*(a^4 + 3*a^3*b +
 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c
)^8 + 8*(99*(a^4 + a^3*b)*cosh(d*x + c)^5 + 60*(a^4 + 3*a^3*b + 2*a^2*b^2 -
 (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c)^3 + (5*a^4 + 3*a^3*b - 32*a
^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*
x + c))*sinh(d*x + c)^7 - 2*(27*a^3*b + 102*a^2*b^2 + 152*a*b^3 + 80*b^4 +
4*(3*a^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*b^3 + 48*b^4)*d*x)*cosh(d*x + c)^6
 + 2*(462*(a^4 + a^3*b)*cosh(d*x + c)^6 + 420*(a^4 + 3*a^3*b + 2*a^2*b^2 -
(a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c)^4 - 27*a^3*b - 102*a^2*b^2 -
 152*a*b^3 - 80*b^4 - 4*(3*a^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*b^3 + 48*b^4
)*d*x + 14*(5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3*b + 2
0*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(198*(a^4 +
 a^3*b)*cosh(d*x + c)^7 + 252*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b +
 6*a^2*b^2)*d*x)*cosh(d*x + c)^5 + 14*(5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*
b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c)^3 - 3*(
27*a^3*b + 102*a^2*b^2 + 152*a*b^3 + 80*b^4 + 4*(3*a^4 + 29*a^3*b + 82*a^2*
b^2 + 104*a*b^3 + 48*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - (5*a^4 + 75
*a^3*b + 192*a^2*b^2 + 128*a*b^3 + 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^
3)*d*x)*cosh(d*x + c)^4 + (495*(a^4 + a^3*b)*cosh(d*x + c)^8 + 840*(a^4 + 3
*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c)^6 + 70*
(5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2 +
 12*a*b^3)*d*x)*cosh(d*x + c)^4 - 5*a^4 - 75*a^3*b - 192*a^2*b^2 - 128*a*b^
3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x - 30*(27*a^3*b + 102*a^2
*b^2 + 152*a*b^3 + 80*b^4 + 4*(3*a^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*b^3 +
48*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - a^4 - a^3*b + 4*(55*(a^4 +
a^3*b)*cosh(d*x + c)^9 + 120*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b +
6*a^2*b^2)*d*x)*cosh(d*x + c)^7 + 14*(5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b
^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c)^5 - 10*(
27*a^3*b + 102*a^2*b^2 + 152*a*b^3 + 80*b^4 + 4*(3*a^4 + 29*a^3*b + 82*a^2*
b^2 + 104*a*b^3 + 48*b^4)*d*x)*cosh(d*x + c)^3 - (5*a^4 + 75*a^3*b + 192*a^
2*b^2 + 128*a*b^3 + 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*
x + c))*sinh(d*x + c)^3 - 2*(2*a^4 + 15*a^3*b + 14*a^2*b^2 + 2*(a^4 + 7*a^3
*b + 6*a^2*b^2)*d*x)*cosh(d*x + c)^2 + 2*(33*(a^4 + a^3*b)*cosh(d*x + c)^10
 + 90*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*
x + c)^8 + 14*(5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3*b
```

```
+ 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c)^6 - 15*(27*a^3*b + 102*a^2*b^2
+ 152*a*b^3 + 80*b^4 + 4*(3*a^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*b^3 + 48*b^
4)*d*x)*cosh(d*x + c)^4 - 2*a^4 - 15*a^3*b - 14*a^2*b^2 - 2*(a^4 + 7*a^3*b
+ 6*a^2*b^2)*d*x - 3*(5*a^4 + 75*a^3*b + 192*a^2*b^2 + 128*a*b^3 + 16*(a^4
+ 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
((15*a^4 + 40*a^3*b + 24*a^2*b^2)*cosh(d*x + c)^10 + 10*(15*a^4 + 40*a^3*b
+ 24*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^9 + (15*a^4 + 40*a^3*b + 24*a^2*b
^2)*sinh(d*x + c)^10 + 4*(15*a^4 + 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(
d*x + c)^8 + (60*a^4 + 280*a^3*b + 416*a^2*b^2 + 192*a*b^3 + 45*(15*a^4 + 4
0*a^3*b + 24*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(15*a^4 + 40
*a^3*b + 24*a^2*b^2)*cosh(d*x + c)^3 + 4*(15*a^4 + 70*a^3*b + 104*a^2*b^2 +
 48*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(45*a^4 + 240*a^3*b + 512*a^2
*b^2 + 512*a*b^3 + 192*b^4)*cosh(d*x + c)^6 + 2*(105*(15*a^4 + 40*a^3*b + 2
4*a^2*b^2)*cosh(d*x + c)^4 + 45*a^4 + 240*a^3*b + 512*a^2*b^2 + 512*a*b^3 +
 192*b^4 + 56*(15*a^4 + 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^6 + 4*(63*(15*a^4 + 40*a^3*b + 24*a^2*b^2)*cosh(d*x + c)^5 +
 56*(15*a^4 + 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^3 + 3*(45*a^
4 + 240*a^3*b + 512*a^2*b^2 + 512*a*b^3 + 192*b^4)*cosh(d*x + c))*sinh(d*x
+ c)^5 + 4*(15*a^4 + 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^4 + 2
*(105*(15*a^4 + 40*a^3*b + 24*a^2*b^2)*cosh(d*x + c)^6 + 140*(15*a^4 + 70*a
^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^4 + 30*a^4 + 140*a^3*b + 208*a
^2*b^2 + 96*a*b^3 + 15*(45*a^4 + 240*a^3*b + 512*a^2*b^2 + 512*a*b^3 + 192*
b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*(15*a^4 + 40*a^3*b + 24*a^2*b
^2)*cosh(d*x + c)^7 + 28*(15*a^4 + 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(
d*x + c)^5 + 5*(45*a^4 + 240*a^3*b + 512*a^2*b^2 + 512*a*b^3 + 192*b^4)*cos
h(d*x + c)^3 + 2*(15*a^4 + 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)
)*sinh(d*x + c)^3 + (15*a^4 + 40*a^3*b + 24*a^2*b^2)*cosh(d*x + c)^2 + (45*
(15*a^4 + 40*a^3*b + 24*a^2*b^2)*cosh(d*x + c)^8 + 112*(15*a^4 + 70*a^3*b +
 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^6 + 30*(45*a^4 + 240*a^3*b + 512*a^2
*b^2 + 512*a*b^3 + 192*b^4)*cosh(d*x + c)^4 + 15*a^4 + 40*a^3*b + 24*a^2*b^
2 + 24*(15*a^4 + 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^2)*sinh(d
*x + c)^2 + 2*(5*(15*a^4 + 40*a^3*b + 24*a^2*b^2)*cosh(d*x + c)^9 + 16*(15*
a^4 + 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^7 + 6*(45*a^4 + 240*
a^3*b + 512*a^2*b^2 + 512*a*b^3 + 192*b^4)*cosh(d*x + c)^5 + 8*(15*a^4 + 70
*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^3 + (15*a^4 + 40*a^3*b + 24*
a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(
d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)
*sqrt(-b/(a + b))/b) + 4*(3*(a^4 + a^3*b)*cosh(d*x + c)^11 + 10*(a^4 + 3*a^
3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c)^9 + 2*(5*a
^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*
a*b^3)*d*x)*cosh(d*x + c)^7 - 3*(27*a^3*b + 102*a^2*b^2 + 152*a*b^3 + 80*b^
4 + 4*(3*a^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*b^3 + 48*b^4)*d*x)*cosh(d*x +
c)^5 - (5*a^4 + 75*a^3*b + 192*a^2*b^2 + 128*a*b^3 + 16*(a^4 + 9*a^3*b + 20
*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c)^3 - (2*a^4 + 15*a^3*b + 14*a^2*b^2
+ 2*(a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^7 +
a^6*b)*d*cosh(d*x + c)^10 + 10*(a^7 + a^6*b)*d*cosh(d*x + c)*sinh(d*x + c)^
9 + (a^7 + a^6*b)*d*sinh(d*x + c)^10 + 4*(a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh
(d*x + c)^8 + (45*(a^7 + a^6*b)*d*cosh(d*x + c)^2 + 4*(a^7 + 3*a^6*b + 2*a^
5*b^2)*d)*sinh(d*x + c)^8 + 2*(3*a^7 + 11*a^6*b + 16*a^5*b^2 + 8*a^4*b^3)*d
*cosh(d*x + c)^6 + 8*(15*(a^7 + a^6*b)*d*cosh(d*x + c)^3 + 4*(a^7 + 3*a^6*b
 + 2*a^5*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^7 + a^6*b)*d*cos
h(d*x + c)^4 + 56*(a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh(d*x + c)^2 + (3*a^7 +
11*a^6*b + 16*a^5*b^2 + 8*a^4*b^3)*d)*sinh(d*x + c)^6 + 4*(a^7 + 3*a^6*b +
2*a^5*b^2)*d*cosh(d*x + c)^4 + 4*(63*(a^7 + a^6*b)*d*cosh(d*x + c)^5 + 56*(
a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh(d*x + c)^3 + 3*(3*a^7 + 11*a^6*b + 16*a^5
*b^2 + 8*a^4*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^7 + a^6*b)*d
*cosh(d*x + c)^6 + 140*(a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh(d*x + c)^4 + 15*(
3*a^7 + 11*a^6*b + 16*a^5*b^2 + 8*a^4*b^3)*d*cosh(d*x + c)^2 + 2*(a^7 + 3*a
^6*b + 2*a^5*b^2)*d)*sinh(d*x + c)^4 + (a^7 + a^6*b)*d*cosh(d*x + c)^2 + 8*
```

```
(15*(a^7 + a^6*b)*d*cosh(d*x + c)^7 + 28*(a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh
(d*x + c)^5 + 5*(3*a^7 + 11*a^6*b + 16*a^5*b^2 + 8*a^4*b^3)*d*cosh(d*x + c)
^3 + 2*(a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (45*(
a^7 + a^6*b)*d*cosh(d*x + c)^8 + 112*(a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh(d*x
 + c)^6 + 30*(3*a^7 + 11*a^6*b + 16*a^5*b^2 + 8*a^4*b^3)*d*cosh(d*x + c)^4
+ 24*(a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh(d*x + c)^2 + (a^7 + a^6*b)*d)*sinh(
d*x + c)^2 + 2*(5*(a^7 + a^6*b)*d*cosh(d*x + c)^9 + 16*(a^7 + 3*a^6*b + 2*a
^5*b^2)*d*cosh(d*x + c)^7 + 6*(3*a^7 + 11*a^6*b + 16*a^5*b^2 + 8*a^4*b^3)*d
*cosh(d*x + c)^5 + 8*(a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh(d*x + c)^3 + (a^7 +
 a^6*b)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18836, size = 517, normalized size = 2.76

$$\frac{\left(15\,a^2b+40\,ab^2+24\,b^3\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{8\,\left(a^5d+a^4bd\right)\sqrt{-ab-b^2}}-\frac{9\,a^3be^{(6\,dx+6\,c)}+32\,a^2b^2e^{(6\,dx+6\,c)}+24\,ab^3e^{(6\,dx+6\,c)}+27\,a^3be^{(4\,dx+4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(15*a^2*b + 40*a*b^2 + 24*b^3)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)
/sqrt(-a*b - b^2))/((a^5*d + a^4*b*d)*sqrt(-a*b - b^2)) - 1/4*(9*a^3*b*e^(6
*d*x + 6*c) + 32*a^2*b^2*e^(6*d*x + 6*c) + 24*a*b^3*e^(6*d*x + 6*c) + 27*a^
3*b*e^(4*d*x + 4*c) + 102*a^2*b^2*e^(4*d*x + 4*c) + 152*a*b^3*e^(4*d*x + 4*
c) + 80*b^4*e^(4*d*x + 4*c) + 27*a^3*b*e^(2*d*x + 2*c) + 80*a^2*b^2*e^(2*d*
x + 2*c) + 56*a*b^3*e^(2*d*x + 2*c) + 9*a^3*b + 10*a^2*b^2)/((a^5*d + a^4*b
*d)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)^2)
- 1/2*(d*x + c)*(a + 6*b)/(a^4*d) + 1/8*e^(2*d*x + 2*c)/(a^3*d) + 1/8*(2*a*
e^(2*d*x + 2*c) + 12*b*e^(2*d*x + 2*c) - a)*e^(-2*d*x - 2*c)/(a^4*d)
```

3.44 $\int \frac{\sinh(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=116

$$-\frac{5\cosh^3(c+dx)}{8a^2d\left(a\cosh^2(c+dx)+b\right)}-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{7/2}d}+\frac{15\cosh(c+dx)}{8a^3d}-\frac{\cosh^5(c+dx)}{4ad\left(a\cosh^2(c+dx)+b\right)^2}$$

```
[Out] (-15*Sqrt[b]*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(8*a^(7/2)*d) + (15*C
osh[c + d*x])/(8*a^3*d) - Cosh[c + d*x]^5/(4*a*d*(b + a*Cosh[c + d*x]^2)^2)
 - (5*Cosh[c + d*x]^3)/(8*a^2*d*(b + a*Cosh[c + d*x]^2))
```

Rubi [A] time = 0.0841521, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4133, 288, 321, 205}

$$-\frac{5\cosh^3(c+dx)}{8a^2d\left(a\cosh^2(c+dx)+b\right)}-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{7/2}d}+\frac{15\cosh(c+dx)}{8a^3d}-\frac{\cosh^5(c+dx)}{4ad\left(a\cosh^2(c+dx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (-15*Sqrt[b]*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(8*a^(7/2)*d) + (15*C
osh[c + d*x])/(8*a^3*d) - Cosh[c + d*x]^5/(4*a*d*(b + a*Cosh[c + d*x]^2)^2)
 - (5*Cosh[c + d*x]^3)/(8*a^2*d*(b + a*Cosh[c + d*x]^2))
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{\left(b+ax^2\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\cosh^5(c+dx)}{4ad\left(b+a\cosh^2(c+dx)\right)^2} + \frac{5\,\text{Subst}\left(\int \frac{x^4}{\left(b+ax^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{4ad}$$

$$= -\frac{\cosh^5(c+dx)}{4ad\left(b+a\cosh^2(c+dx)\right)^2} - \frac{5\cosh^3(c+dx)}{8a^2d\left(b+a\cosh^2(c+dx)\right)} + \frac{15\,\text{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx, x, c\right.}{8a^2d}$$

$$= \frac{15\cosh(c+dx)}{8a^3d} - \frac{\cosh^5(c+dx)}{4ad\left(b+a\cosh^2(c+dx)\right)^2} - \frac{5\cosh^3(c+dx)}{8a^2d\left(b+a\cosh^2(c+dx)\right)} - \frac{(15b)\,\text{Su}}{}$$

$$= -\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{7/2}d} + \frac{15\cosh(c+dx)}{8a^3d} - \frac{\cosh^5(c+dx)}{4ad\left(b+a\cosh^2(c+dx)\right)^2} - \frac{5}{8a^2d\,(}$$

Mathematica [C] time = 9.16809, size = 453, normalized size = 3.91

$$\text{sech}^6(c+dx)(a\cosh(2(c+dx))+a+2b)^3\left(\frac{8\cosh(c+dx)\left(3\left(a^4+48ab^3\right)\cosh(2(c+dx))+16b^3(9a+14b)\right)}{a^3b^2(a\cosh(2(c+dx))+a+2b)^2} - \frac{15\left(\left(a^3+64b^3\right)\tan^{-1}\left(\frac{\sinh(c)\tanh(}{}\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^3*Sech[c + d*x]^6*((-15*((a^3 + 64*b^3)*Ar
cTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d
*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[
(d*x)/2]))/Sqrt[b]] + (a^3 + 64*b^3)*ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[
(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a
 + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] - a^3*(ArcTan[(S
qrt[a] - I*Sqrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]] + ArcTan[(Sqrt[a] + I*Sq
rt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]])))/(a^(7/2)*b^(5/2)) + (512*Cosh[c]*C
osh[d*x])/a^3 + (8*Cosh[c + d*x]*(16*b^3*(9*a + 14*b) + 3*(a^4 + 48*a*b^3)*
Cosh[2*(c + d*x)]))/(a^3*b^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2) + (512*Sinh
[c]*Sinh[d*x])/a^3 - (6*a*Csch[c + d*x]*Sinh[4*(c + d*x)])/(b^2*(a + 2*b +
a*Cosh[2*(c + d*x)])^2)))/(4096*d*(a + b*Sech[c + d*x]^2)^3)
```

Maple [A] time = 0.035, size = 107, normalized size = 0.9

$$\frac{7b^2(\text{sech}(dx+c))^3}{8da^3\left(a+b(\text{sech}(dx+c))^2\right)^2} + \frac{9b\text{sech}(dx+c)}{8da^2\left(a+b(\text{sech}(dx+c))^2\right)^2} + \frac{15b}{8da^3}\arctan\left(b\text{sech}(dx+c)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + \frac{}{da^3\text{sech}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 7/8/d/a^3*b^2/(a+b*sech(d*x+c)^2)^2*sech(d*x+c)^3+9/8/d/a^2*b/(a+b*sech(d*x
+c)^2)^2*sech(d*x+c)+15/8/d/a^3*b/(a*b)^(1/2)*arctan(sech(d*x+c)*b/(a*b)^(1
/2))+1/d/a^3/sech(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,a^2e^{(10\,dx+10\,c)}+2\,a^2+5\left(2\,a^2e^{(8\,c)}+5\,abe^{(8\,c)}\right)e^{(8\,dx)}+5\left(4\,a^2e^{(6\,c)}+15\,abe^{(6\,c)}+12\,b^2e^{(6\,c)}\right)e^{(6\,dx)}+5\left(4\,a^2e^{(4\,c)}+15\,abe^{(4\,c}\right.}{4\left(a^5de^{(9\,dx+9\,c)}+a^5de^{(dx+c)}+4\left(a^5de^{(7\,c)}+2\,a^4bde^{(7\,c)}\right)e^{(7\,dx)}+2\left(3\,a^5de^{(5\,c)}+8\,a^4bde^{(5\,c)}+8\,a^3b^2de^{(5\,c)}\right)e^{(5\,d}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*a^2*e^(10*d*x + 10*c) + 2*a^2 + 5*(2*a^2*e^(8*c) + 5*a*b*e^(8*c))*e^
(8*d*x) + 5*(4*a^2*e^(6*c) + 15*a*b*e^(6*c) + 12*b^2*e^(6*c))*e^(6*d*x) + 5
*(4*a^2*e^(4*c) + 15*a*b*e^(4*c) + 12*b^2*e^(4*c))*e^(4*d*x) + 5*(2*a^2*e^(
2*c) + 5*a*b*e^(2*c))*e^(2*d*x))/(a^5*d*e^(9*d*x + 9*c) + a^5*d*e^(d*x + c)
 + 4*(a^5*d*e^(7*c) + 2*a^4*b*d*e^(7*c))*e^(7*d*x) + 2*(3*a^5*d*e^(5*c) + 8
*a^4*b*d*e^(5*c) + 8*a^3*b^2*d*e^(5*c))*e^(5*d*x) + 4*(a^5*d*e^(3*c) + 2*a^
4*b*d*e^(3*c))*e^(3*d*x)) - 1/2*integrate(15/2*(b*e^(3*d*x + 3*c) - b*e^(d*
x + c))/(a^4*e^(4*d*x + 4*c) + a^4 + 2*(a^4*e^(2*c) + 2*a^3*b*e^(2*c))*e^(2
*d*x)), x)
```

Fricas [B] time = 2.96016, size = 12176, normalized size = 104.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(8*a^2*cosh(d*x + c)^10 + 80*a^2*cosh(d*x + c)*sinh(d*x + c)^9 + 8*a^
2*sinh(d*x + c)^10 + 20*(2*a^2 + 5*a*b)*cosh(d*x + c)^8 + 20*(18*a^2*cosh(d
*x + c)^2 + 2*a^2 + 5*a*b)*sinh(d*x + c)^8 + 160*(6*a^2*cosh(d*x + c)^3 + (
2*a^2 + 5*a*b)*cosh(d*x + c))*sinh(d*x + c)^7 + 20*(4*a^2 + 15*a*b + 12*b^2
)*cosh(d*x + c)^6 + 20*(84*a^2*cosh(d*x + c)^4 + 28*(2*a^2 + 5*a*b)*cosh(d*
x + c)^2 + 4*a^2 + 15*a*b + 12*b^2)*sinh(d*x + c)^6 + 8*(252*a^2*cosh(d*x +
 c)^5 + 140*(2*a^2 + 5*a*b)*cosh(d*x + c)^3 + 15*(4*a^2 + 15*a*b + 12*b^2)*
cosh(d*x + c))*sinh(d*x + c)^5 + 20*(4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c)
^4 + 20*(84*a^2*cosh(d*x + c)^6 + 70*(2*a^2 + 5*a*b)*cosh(d*x + c)^4 + 15*(
4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c)^2 + 4*a^2 + 15*a*b + 12*b^2)*sinh(d*
x + c)^4 + 80*(12*a^2*cosh(d*x + c)^7 + 14*(2*a^2 + 5*a*b)*cosh(d*x + c)^5
+ 5*(4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c)^3 + (4*a^2 + 15*a*b + 12*b^2)*c
osh(d*x + c))*sinh(d*x + c)^3 + 20*(2*a^2 + 5*a*b)*cosh(d*x + c)^2 + 20*(18
*a^2*cosh(d*x + c)^8 + 28*(2*a^2 + 5*a*b)*cosh(d*x + c)^6 + 15*(4*a^2 + 15*
a*b + 12*b^2)*cosh(d*x + c)^4 + 6*(4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c)^2
 + 2*a^2 + 5*a*b)*sinh(d*x + c)^2 + 15*(a^2*cosh(d*x + c)^9 + 9*a^2*cosh(d*
x + c)*sinh(d*x + c)^8 + a^2*sinh(d*x + c)^9 + 4*(a^2 + 2*a*b)*cosh(d*x + c
)^7 + 4*(9*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^7 + 28*(3*a^2*c
osh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(3*a^2 +
```

```
8*a*b + 8*b^2)*cosh(d*x + c)^5 + 2*(63*a^2*cosh(d*x + c)^4 + 42*(a^2 + 2*a*
b)*cosh(d*x + c)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(d*x + c)^5 + 2*(63*a^2*cos
h(d*x + c)^5 + 70*(a^2 + 2*a*b)*cosh(d*x + c)^3 + 5*(3*a^2 + 8*a*b + 8*b^2)
*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^3 + 4*(21*a
^2*cosh(d*x + c)^6 + 35*(a^2 + 2*a*b)*cosh(d*x + c)^4 + 5*(3*a^2 + 8*a*b +
8*b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^3 + a^2*cosh(d*x + c) +
 4*(9*a^2*cosh(d*x + c)^7 + 21*(a^2 + 2*a*b)*cosh(d*x + c)^5 + 5*(3*a^2 + 8
*a*b + 8*b^2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c
)^2 + (9*a^2*cosh(d*x + c)^8 + 28*(a^2 + 2*a*b)*cosh(d*x + c)^6 + 10*(3*a^2
 + 8*a*b + 8*b^2)*cosh(d*x + c)^4 + 12*(a^2 + 2*a*b)*cosh(d*x + c)^2 + a^2)
*sinh(d*x + c))*sqrt(-b/a)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(
d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a - 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(
d*x + c)^2 + a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a - 2*b)*co
sh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(
d*x + c)^2 + a*sinh(d*x + c)^3 + a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a
)*sinh(d*x + c))*sqrt(-b/a) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sin
h(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cos
h(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*
cosh(d*x + c))*sinh(d*x + c) + a)) + 8*a^2 + 40*(2*a^2*cosh(d*x + c)^9 + 4*
(2*a^2 + 5*a*b)*cosh(d*x + c)^7 + 3*(4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c)
^5 + 2*(4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c)^3 + (2*a^2 + 5*a*b)*cosh(d*x
 + c))*sinh(d*x + c))/(a^5*d*cosh(d*x + c)^9 + 9*a^5*d*cosh(d*x + c)*sinh(d
*x + c)^8 + a^5*d*sinh(d*x + c)^9 + 4*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^7 + 4
*(9*a^5*d*cosh(d*x + c)^2 + (a^5 + 2*a^4*b)*d)*sinh(d*x + c)^7 + a^5*d*cosh
(d*x + c) + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c)^5 + 28*(3*a^5*d
*cosh(d*x + c)^3 + (a^5 + 2*a^4*b)*d*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(63
*a^5*d*cosh(d*x + c)^4 + 42*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^2 + (3*a^5 + 8*
a^4*b + 8*a^3*b^2)*d)*sinh(d*x + c)^5 + 4*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^3
 + 2*(63*a^5*d*cosh(d*x + c)^5 + 70*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^3 + 5*(
3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(21*a^5*d
*cosh(d*x + c)^6 + 35*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^4 + 5*(3*a^5 + 8*a^4*
b + 8*a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 + 2*a^4*b)*d)*sinh(d*x + c)^3 + 4*(
9*a^5*d*cosh(d*x + c)^7 + 21*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^5 + 5*(3*a^5 +
 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c)^3 + 3*(a^5 + 2*a^4*b)*d*cosh(d*x + c)
)*sinh(d*x + c)^2 + (9*a^5*d*cosh(d*x + c)^8 + 28*(a^5 + 2*a^4*b)*d*cosh(d*
x + c)^6 + a^5*d + 10*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c)^4 + 12*
(a^5 + 2*a^4*b)*d*cosh(d*x + c)^2)*sinh(d*x + c)), 1/8*(4*a^2*cosh(d*x + c)
^10 + 40*a^2*cosh(d*x + c)*sinh(d*x + c)^9 + 4*a^2*sinh(d*x + c)^10 + 10*(2
*a^2 + 5*a*b)*cosh(d*x + c)^8 + 10*(18*a^2*cosh(d*x + c)^2 + 2*a^2 + 5*a*b)
*sinh(d*x + c)^8 + 80*(6*a^2*cosh(d*x + c)^3 + (2*a^2 + 5*a*b)*cosh(d*x + c
))*sinh(d*x + c)^7 + 10*(4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c)^6 + 10*(84*
a^2*cosh(d*x + c)^4 + 28*(2*a^2 + 5*a*b)*cosh(d*x + c)^2 + 4*a^2 + 15*a*b +
 12*b^2)*sinh(d*x + c)^6 + 4*(252*a^2*cosh(d*x + c)^5 + 140*(2*a^2 + 5*a*b)
*cosh(d*x + c)^3 + 15*(4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c))*sinh(d*x + c
)^5 + 10*(4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c)^4 + 10*(84*a^2*cosh(d*x +
c)^6 + 70*(2*a^2 + 5*a*b)*cosh(d*x + c)^4 + 15*(4*a^2 + 15*a*b + 12*b^2)*co
sh(d*x + c)^2 + 4*a^2 + 15*a*b + 12*b^2)*sinh(d*x + c)^4 + 40*(12*a^2*cosh(
d*x + c)^7 + 14*(2*a^2 + 5*a*b)*cosh(d*x + c)^5 + 5*(4*a^2 + 15*a*b + 12*b^
2)*cosh(d*x + c)^3 + (4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c))*sinh(d*x + c)
^3 + 10*(2*a^2 + 5*a*b)*cosh(d*x + c)^2 + 10*(18*a^2*cosh(d*x + c)^8 + 28*(
2*a^2 + 5*a*b)*cosh(d*x + c)^6 + 15*(4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c)
^4 + 6*(4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c)^2 + 2*a^2 + 5*a*b)*sinh(d*x
+ c)^2 + 15*(a^2*cosh(d*x + c)^9 + 9*a^2*cosh(d*x + c)*sinh(d*x + c)^8 + a^
2*sinh(d*x + c)^9 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^7 + 4*(9*a^2*cosh(d*x + c
)^2 + a^2 + 2*a*b)*sinh(d*x + c)^7 + 28*(3*a^2*cosh(d*x + c)^3 + (a^2 + 2*a
*b)*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c
)^5 + 2*(63*a^2*cosh(d*x + c)^4 + 42*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 3*a^2
+ 8*a*b + 8*b^2)*sinh(d*x + c)^5 + 2*(63*a^2*cosh(d*x + c)^5 + 70*(a^2 + 2*
a*b)*cosh(d*x + c)^3 + 5*(3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*x +
```

```
c)^4 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^3 + 4*(21*a^2*cosh(d*x + c)^6 + 35*(a^
2 + 2*a*b)*cosh(d*x + c)^4 + 5*(3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)^2 + a^
2 + 2*a*b)*sinh(d*x + c)^3 + a^2*cosh(d*x + c) + 4*(9*a^2*cosh(d*x + c)^7 +
 21*(a^2 + 2*a*b)*cosh(d*x + c)^5 + 5*(3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)
^3 + 3*(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^2 + (9*a^2*cosh(d*x + c)^
8 + 28*(a^2 + 2*a*b)*cosh(d*x + c)^6 + 10*(3*a^2 + 8*a*b + 8*b^2)*cosh(d*x
+ c)^4 + 12*(a^2 + 2*a*b)*cosh(d*x + c)^2 + a^2)*sinh(d*x + c))*sqrt(b/a)*a
rctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d
*x + c)^3 + (a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a + 4*b)*sinh(
d*x + c))*sqrt(b/a)/b) - 15*(a^2*cosh(d*x + c)^9 + 9*a^2*cosh(d*x + c)*sinh
(d*x + c)^8 + a^2*sinh(d*x + c)^9 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^7 + 4*(9*
a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^7 + 28*(3*a^2*cosh(d*x + c
)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(3*a^2 + 8*a*b + 8*b
^2)*cosh(d*x + c)^5 + 2*(63*a^2*cosh(d*x + c)^4 + 42*(a^2 + 2*a*b)*cosh(d*x
 + c)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(d*x + c)^5 + 2*(63*a^2*cosh(d*x + c)^
5 + 70*(a^2 + 2*a*b)*cosh(d*x + c)^3 + 5*(3*a^2 + 8*a*b + 8*b^2)*cosh(d*x +
 c))*sinh(d*x + c)^4 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^3 + 4*(21*a^2*cosh(d*x
 + c)^6 + 35*(a^2 + 2*a*b)*cosh(d*x + c)^4 + 5*(3*a^2 + 8*a*b + 8*b^2)*cosh
(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^3 + a^2*cosh(d*x + c) + 4*(9*a^2*c
osh(d*x + c)^7 + 21*(a^2 + 2*a*b)*cosh(d*x + c)^5 + 5*(3*a^2 + 8*a*b + 8*b^
2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^2 + (9*a^
2*cosh(d*x + c)^8 + 28*(a^2 + 2*a*b)*cosh(d*x + c)^6 + 10*(3*a^2 + 8*a*b +
8*b^2)*cosh(d*x + c)^4 + 12*(a^2 + 2*a*b)*cosh(d*x + c)^2 + a^2)*sinh(d*x +
 c))*sqrt(b/a)*arctan(1/2*(a*cosh(d*x + c) + a*sinh(d*x + c))*sqrt(b/a)/b)
+ 4*a^2 + 20*(2*a^2*cosh(d*x + c)^9 + 4*(2*a^2 + 5*a*b)*cosh(d*x + c)^7 + 3
*(4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c)^5 + 2*(4*a^2 + 15*a*b + 12*b^2)*co
sh(d*x + c)^3 + (2*a^2 + 5*a*b)*cosh(d*x + c))*sinh(d*x + c))/(a^5*d*cosh(d
*x + c)^9 + 9*a^5*d*cosh(d*x + c)*sinh(d*x + c)^8 + a^5*d*sinh(d*x + c)^9 +
 4*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^7 + 4*(9*a^5*d*cosh(d*x + c)^2 + (a^5 +
2*a^4*b)*d)*sinh(d*x + c)^7 + a^5*d*cosh(d*x + c) + 2*(3*a^5 + 8*a^4*b + 8*
a^3*b^2)*d*cosh(d*x + c)^5 + 28*(3*a^5*d*cosh(d*x + c)^3 + (a^5 + 2*a^4*b)*
d*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(63*a^5*d*cosh(d*x + c)^4 + 42*(a^5 +
2*a^4*b)*d*cosh(d*x + c)^2 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*d)*sinh(d*x + c)
^5 + 4*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^3 + 2*(63*a^5*d*cosh(d*x + c)^5 + 70
*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^3 + 5*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh
(d*x + c))*sinh(d*x + c)^4 + 4*(21*a^5*d*cosh(d*x + c)^6 + 35*(a^5 + 2*a^4*
b)*d*cosh(d*x + c)^4 + 5*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c)^2 +
(a^5 + 2*a^4*b)*d)*sinh(d*x + c)^3 + 4*(9*a^5*d*cosh(d*x + c)^7 + 21*(a^5 +
 2*a^4*b)*d*cosh(d*x + c)^5 + 5*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x +
c)^3 + 3*(a^5 + 2*a^4*b)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (9*a^5*d*cosh(d
*x + c)^8 + 28*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^6 + a^5*d + 10*(3*a^5 + 8*a^
4*b + 8*a^3*b^2)*d*cosh(d*x + c)^4 + 12*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^2)*
sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.45 $\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3} dx$

Optimal. Leaf size=154

$$\frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{5/2}d(a+b)^3}-\frac{b(7a+3b)\cosh(c+dx)}{8a^2d(a+b)^2\left(a\cosh^2(c+dx)+b\right)}-\frac{b\cosh^3(c+dx)}{4ad(a+b)\left(a\cosh^2(c+dx)+b\right)^2}-$$

```
[Out] (Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])
/(8*a^(5/2)*(a + b)^3*d) - ArcTanh[Cosh[c + d*x]]/((a + b)^3*d) - (b*Cosh[c
 + d*x]^3)/(4*a*(a + b)*d*(b + a*Cosh[c + d*x]^2)^2) - (b*(7*a + 3*b)*Cosh[
c + d*x])/(8*a^2*(a + b)^2*d*(b + a*Cosh[c + d*x]^2))
```

Rubi [A] time = 0.222861, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4133, 470, 578, 522, 206, 205}

$$\frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{5/2}d(a+b)^3}-\frac{b(7a+3b)\cosh(c+dx)}{8a^2d(a+b)^2\left(a\cosh^2(c+dx)+b\right)}-\frac{b\cosh^3(c+dx)}{4ad(a+b)\left(a\cosh^2(c+dx)+b\right)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```


```
[Out] (Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])
/(8*a^(5/2)*(a + b)^3*d) - ArcTanh[Cosh[c + d*x]]/((a + b)^3*d) - (b*Cosh[c
 + d*x]^3)/(4*a*(a + b)*d*(b + a*Cosh[c + d*x]^2)^2) - (b*(7*a + 3*b)*Cosh[
c + d*x])/(8*a^2*(a + b)^2*d*(b + a*Cosh[c + d*x]^2))
```


Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
```

```
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{x^6}{(1-x^2)(b+ax^2)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{b\cosh^3(c+dx)}{4a(a+b)d\left(b+a\cosh^2(c+dx)\right)^2} + \frac{\operatorname{Subst}\left(\int \frac{x^2(3b+(-4a-3b)x^2)}{(1-x^2)(b+ax^2)^2}\,dx, x, \cosh(c+dx)\right)}{4a(a+b)d}$$

$$= -\frac{b\cosh^3(c+dx)}{4a(a+b)d\left(b+a\cosh^2(c+dx)\right)^2} - \frac{b(7a+3b)\cosh(c+dx)}{8a^2(a+b)^2d\left(b+a\cosh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \right.}{}$$

$$= -\frac{b\cosh^3(c+dx)}{4a(a+b)d\left(b+a\cosh^2(c+dx)\right)^2} - \frac{b(7a+3b)\cosh(c+dx)}{8a^2(a+b)^2d\left(b+a\cosh^2(c+dx)\right)} - \frac{\operatorname{Subst}\left(\int \right.}{}$$

$$= \frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{5/2}(a+b)^3d} - \frac{\tanh^{-1}(\cosh(c+dx))}{(a+b)^3d} - \frac{b\cos}{4a(a+b)d\left(b\right.}$$

Mathematica [C] time = 2.36461, size = 440, normalized size = 2.86

$$\operatorname{sech}^5(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\operatorname{sech}(c+dx)(a\cosh(2(c+dx))+a+2b)^2\tan^{-1}\left(\frac{\sinh(c)\tanh\left(\frac{dx}{2}\right)\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\cosh}\right.}{}\right.}{a^{5/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^5*((8*b^2*(a + b)^2)/a^2 - (
2*b*(a + b)*(9*a + 5*b)*(a + 2*b + a*Cosh[2*(c + d*x)]))/a^2 + (Sqrt[b]*(15
*a^2 + 10*a*b + 3*b^2)*ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sin
h[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Co
sh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cosh[2*(c + d*x)]
)^2*Sech[c + d*x])/a^(5/2) + (Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[((Sq
rt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] +
Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2])
)/Sqrt[b]]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x])/a^(5/2) - 8*(a
+ 2*b + a*Cosh[2*(c + d*x)])^2*Log[Cosh[(c + d*x)/2]]*Sech[c + d*x] + 8*(a
+ 2*b + a*Cosh[2*(c + d*x)])^2*Log[Sinh[(c + d*x)/2]]*Sech[c + d*x]))/(64*(
a + b)^3*d*(a + b*Sech[c + d*x]^2)^3)
```

Maple [B] time = 0.082, size = 1476, normalized size = 9.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -9/4/d*b/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/
2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*a*tanh(1/2*d*x+1/2*c)^6+1
/4/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/
2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^6+13/
4/d*b^3/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2
*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a*tanh(1/2*d*x+1/2*c)^6+3/
4/d*b^4/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2
*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a^2*tanh(1/2*d*x+1/2*c)^6-
27/4/d*b/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/
2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*a*tanh(1/2*d*x+1/2*c)^4+9
/4/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/
2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^4-21/
4/d*b^3/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2
*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a*tanh(1/2*d*x+1/2*c)^4-9/
4/d*b^4/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2
*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a^2*tanh(1/2*d*x+1/2*c)^4-
27/4/d*b/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/
2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*a*tanh(1/2*d*x+1/2*c)^2-1
3/4/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1
/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^2+23
/4/d*b^3/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/
2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a*tanh(1/2*d*x+1/2*c)^2+9
/4/d*b^4/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/
2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a^2*tanh(1/2*d*x+1/2*c)^2
-9/4/d*b/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/
2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*a-21/4/d*b^2/(a+b)^3/(tan
h(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*ta
nh(1/2*d*x+1/2*c)^2*b+a+b)^2-15/4/d*b^3/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*
tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a
+b)^2/a-3/4/d*b^4/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+
2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a^2+15/8/d*b/(a+
b)^3/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(
1/2))+5/4/d*b^2/(a+b)^3/a/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*
c)^2+2*a-2*b)/(a*b)^(1/2))+3/8/d*b^3/(a+b)^3/a^2/(a*b)^(1/2)*arctan(1/4*(2*
(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))+1/d/(a+b)^3*ln(tanh(1/2*d
*x+1/2*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*((9*a^2*b*e^(7*c) + 5*a*b^2*e^(7*c))*e^(7*d*x) + (27*a^2*b*e^(5*c) + 4
3*a*b^2*e^(5*c) + 12*b^3*e^(5*c))*e^(5*d*x) + (27*a^2*b*e^(3*c) + 43*a*b^2*
e^(3*c) + 12*b^3*e^(3*c))*e^(3*d*x) + (9*a^2*b*e^c + 5*a*b^2*e^c)*e^(d*x))/
(a^6*d + 2*a^5*b*d + a^4*b^2*d + (a^6*d*e^(8*c) + 2*a^5*b*d*e^(8*c) + a^4*b
^2*d*e^(8*c))*e^(8*d*x) + 4*(a^6*d*e^(6*c) + 4*a^5*b*d*e^(6*c) + 5*a^4*b^2*
d*e^(6*c) + 2*a^3*b^3*d*e^(6*c))*e^(6*d*x) + 2*(3*a^6*d*e^(4*c) + 14*a^5*b*
d*e^(4*c) + 27*a^4*b^2*d*e^(4*c) + 24*a^3*b^3*d*e^(4*c) + 8*a^2*b^4*d*e^(4*
c))*e^(4*d*x) + 4*(a^6*d*e^(2*c) + 4*a^5*b*d*e^(2*c) + 5*a^4*b^2*d*e^(2*c)
+ 2*a^3*b^3*d*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c) + 1)*e^(-c))/(a^3*d +
3*a^2*b*d + 3*a*b^2*d + b^3*d) + log((e^(d*x + c) - 1)*e^(-c))/(a^3*d + 3*a
^2*b*d + 3*a*b^2*d + b^3*d) + 2*integrate(1/8*((15*a^2*b*e^(3*c) + 10*a*b^2
*e^(3*c) + 3*b^3*e^(3*c))*e^(3*d*x) - (15*a^2*b*e^c + 10*a*b^2*e^c + 3*b^3*
e^c)*e^(d*x))/(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3 + (a^6*e^(4*c) + 3*a^5*b
*e^(4*c) + 3*a^4*b^2*e^(4*c) + a^3*b^3*e^(4*c))*e^(4*d*x) + 2*(a^6*e^(2*c)
+ 5*a^5*b*e^(2*c) + 9*a^4*b^2*e^(2*c) + 7*a^3*b^3*e^(2*c) + 2*a^2*b^4*e^(2*
c))*e^(2*d*x)), x)
```

Fricas [B] time = 4.27551, size = 20882, normalized size = 135.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^7 + 28*(9*a^3*b +
14*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + 4*(9*a^3*b + 14*a^2*b
^2 + 5*a*b^3)*sinh(d*x + c)^7 + 4*(27*a^3*b + 70*a^2*b^2 + 55*a*b^3 + 12*b^
4)*cosh(d*x + c)^5 + 4*(27*a^3*b + 70*a^2*b^2 + 55*a*b^3 + 12*b^4 + 21*(9*a
^3*b + 14*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*(9*a^
3*b + 14*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^3 + (27*a^3*b + 70*a^2*b^2 + 55*a
*b^3 + 12*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(27*a^3*b + 70*a^2*b^2 +
55*a*b^3 + 12*b^4)*cosh(d*x + c)^3 + 4*(35*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)
*cosh(d*x + c)^4 + 27*a^3*b + 70*a^2*b^2 + 55*a*b^3 + 12*b^4 + 10*(27*a^3*b
 + 70*a^2*b^2 + 55*a*b^3 + 12*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 4*(21
*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^5 + 10*(27*a^3*b + 70*a^2*b
^2 + 55*a*b^3 + 12*b^4)*cosh(d*x + c)^3 + 3*(27*a^3*b + 70*a^2*b^2 + 55*a*b
^3 + 12*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 - ((15*a^4 + 10*a^3*b + 3*a^2*b
^2)*cosh(d*x + c)^8 + 8*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(d*x + c)*sinh(
d*x + c)^7 + (15*a^4 + 10*a^3*b + 3*a^2*b^2)*sinh(d*x + c)^8 + 4*(15*a^4 +
40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^6 + 4*(15*a^4 + 40*a^3*b + 2
3*a^2*b^2 + 6*a*b^3 + 7*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^2)*si
nh(d*x + c)^6 + 8*(7*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^3 + 3*(1
5*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2
*(45*a^4 + 150*a^3*b + 209*a^2*b^2 + 104*a*b^3 + 24*b^4)*cosh(d*x + c)^4 +
2*(35*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^4 + 45*a^4 + 150*a^3*b
+ 209*a^2*b^2 + 104*a*b^3 + 24*b^4 + 30*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6
```

*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 15*a^4 + 10*a^3*b + 3*a^2*b^2 +
8*(7*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^5 + 10*(15*a^4 + 40*a^3*
b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^3 + (45*a^4 + 150*a^3*b + 209*a^2*b
^2 + 104*a*b^3 + 24*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(15*a^4 + 40*a^
3*b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^2 + 4*(7*(15*a^4 + 10*a^3*b + 3*a
^2*b^2)*cosh(d*x + c)^6 + 15*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*cos
h(d*x + c)^4 + 15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3 + 3*(45*a^4 + 150*a
^3*b + 209*a^2*b^2 + 104*a*b^3 + 24*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
8*((15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^7 + 3*(15*a^4 + 40*a^3*b
+ 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^5 + (45*a^4 + 150*a^3*b + 209*a^2*b^2
+ 104*a*b^3 + 24*b^4)*cosh(d*x + c)^3 + (15*a^4 + 40*a^3*b + 23*a^2*b^2 +
6*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log((a*cosh(d*x + c)^4 +
4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a - 2*b)*cosh(d*
x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*
x + c)^3 + (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 +
3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + a*cosh(d*x + c) + (
3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))*sqrt(-b/a) + a)/(a*cosh(d*x + c)^4
+ 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(
d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(
d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 4*(9*a^3*b + 14
*a^2*b^2 + 5*a*b^3)*cosh(d*x + c) + 16*(a^4*cosh(d*x + c)^8 + 8*a^4*cosh(d*
x + c)*sinh(d*x + c)^7 + a^4*sinh(d*x + c)^8 + 4*(a^4 + 2*a^3*b)*cosh(d*x +
c)^6 + 4*(7*a^4*cosh(d*x + c)^2 + a^4 + 2*a^3*b)*sinh(d*x + c)^6 + 8*(7*a^
4*cosh(d*x + c)^3 + 3*(a^4 + 2*a^3*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3
*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + 2*(35*a^4*cosh(d*x + c)^4 + 3
*a^4 + 8*a^3*b + 8*a^2*b^2 + 30*(a^4 + 2*a^3*b)*cosh(d*x + c)^2)*sinh(d*x +
c)^4 + a^4 + 8*(7*a^4*cosh(d*x + c)^5 + 10*(a^4 + 2*a^3*b)*cosh(d*x + c)^3
+ (3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 +
2*a^3*b)*cosh(d*x + c)^2 + 4*(7*a^4*cosh(d*x + c)^6 + 15*(a^4 + 2*a^3*b)*co
sh(d*x + c)^4 + a^4 + 2*a^3*b + 3*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x +
c)^2)*sinh(d*x + c)^2 + 8*(a^4*cosh(d*x + c)^7 + 3*(a^4 + 2*a^3*b)*cosh(d*x
+ c)^5 + (3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 + (a^4 + 2*a^3*b)*c
osh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 16*(a
^4*cosh(d*x + c)^8 + 8*a^4*cosh(d*x + c)*sinh(d*x + c)^7 + a^4*sinh(d*x + c
)^8 + 4*(a^4 + 2*a^3*b)*cosh(d*x + c)^6 + 4*(7*a^4*cosh(d*x + c)^2 + a^4 +
2*a^3*b)*sinh(d*x + c)^6 + 8*(7*a^4*cosh(d*x + c)^3 + 3*(a^4 + 2*a^3*b)*cos
h(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)
^4 + 2*(35*a^4*cosh(d*x + c)^4 + 3*a^4 + 8*a^3*b + 8*a^2*b^2 + 30*(a^4 + 2*
a^3*b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + a^4 + 8*(7*a^4*cosh(d*x + c)^5 +
10*(a^4 + 2*a^3*b)*cosh(d*x + c)^3 + (3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x
+ c))*sinh(d*x + c)^3 + 4*(a^4 + 2*a^3*b)*cosh(d*x + c)^2 + 4*(7*a^4*cosh(
d*x + c)^6 + 15*(a^4 + 2*a^3*b)*cosh(d*x + c)^4 + a^4 + 2*a^3*b + 3*(3*a^4
+ 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(a^4*cosh(d*x +
c)^7 + 3*(a^4 + 2*a^3*b)*cosh(d*x + c)^5 + (3*a^4 + 8*a^3*b + 8*a^2*b^2)*c
osh(d*x + c)^3 + (a^4 + 2*a^3*b)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x
+ c) + sinh(d*x + c) - 1) + 4*(7*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*cosh(d*x
+ c)^6 + 5*(27*a^3*b + 70*a^2*b^2 + 55*a*b^3 + 12*b^4)*cosh(d*x + c)^4 + 9
*a^3*b + 14*a^2*b^2 + 5*a*b^3 + 3*(27*a^3*b + 70*a^2*b^2 + 55*a*b^3 + 12*b^
4)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d
*cosh(d*x + c)^8 + 8*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)*
sinh(d*x + c)^7 + (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*sinh(d*x + c)^8 +
4*(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^6 +
4*(7*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^2 + (a^7 + 5*a^6
*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d)*sinh(d*x + c)^6 + 2*(3*a^7 + 17*
a^6*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)^4
+ 8*(7*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^3 + 3*(a^7 +
5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)
^5 + 2*(35*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^4 + 30*(a^
7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^2 + (3*a^7

```
 + 17*a^6*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d)*sinh(d*x
 + c)^4 + 4*(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x
+ c)^2 + 8*(7*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^5 + 10*
(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^3 + (3*
a^7 + 17*a^6*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d*cosh(d
*x + c))*sinh(d*x + c)^3 + 4*(7*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cos
h(d*x + c)^6 + 15*(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cos
h(d*x + c)^4 + 3*(3*a^7 + 17*a^6*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 +
 8*a^2*b^5)*d*cosh(d*x + c)^2 + (a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*
a^3*b^4)*d)*sinh(d*x + c)^2 + (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d + 8*(
(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^7 + 3*(a^7 + 5*a^6*b
+ 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^5 + (3*a^7 + 17*a^6*b
+ 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)^3 + (a^
7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x
+ c)), -1/8*(2*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^7 + 14*(9*a^3
*b + 14*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + 2*(9*a^3*b + 14*
a^2*b^2 + 5*a*b^3)*sinh(d*x + c)^7 + 2*(27*a^3*b + 70*a^2*b^2 + 55*a*b^3 +
12*b^4)*cosh(d*x + c)^5 + 2*(27*a^3*b + 70*a^2*b^2 + 55*a*b^3 + 12*b^4 + 21
*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 10*(7*
(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^3 + (27*a^3*b + 70*a^2*b^2 +
 55*a*b^3 + 12*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 + 2*(27*a^3*b + 70*a^2*b
^2 + 55*a*b^3 + 12*b^4)*cosh(d*x + c)^3 + 2*(35*(9*a^3*b + 14*a^2*b^2 + 5*a
*b^3)*cosh(d*x + c)^4 + 27*a^3*b + 70*a^2*b^2 + 55*a*b^3 + 12*b^4 + 10*(27*
a^3*b + 70*a^2*b^2 + 55*a*b^3 + 12*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^3 +
2*(21*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^5 + 10*(27*a^3*b + 70*
a^2*b^2 + 55*a*b^3 + 12*b^4)*cosh(d*x + c)^3 + 3*(27*a^3*b + 70*a^2*b^2 + 5
5*a*b^3 + 12*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + ((15*a^4 + 10*a^3*b + 3*
a^2*b^2)*cosh(d*x + c)^8 + 8*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(d*x + c)*
sinh(d*x + c)^7 + (15*a^4 + 10*a^3*b + 3*a^2*b^2)*sinh(d*x + c)^8 + 4*(15*a
^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^6 + 4*(15*a^4 + 40*a^3*
b + 23*a^2*b^2 + 6*a*b^3 + 7*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^
2)*sinh(d*x + c)^6 + 8*(7*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^3 +
 3*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^
5 + 2*(45*a^4 + 150*a^3*b + 209*a^2*b^2 + 104*a*b^3 + 24*b^4)*cosh(d*x + c)
^4 + 2*(35*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^4 + 45*a^4 + 150*a
^3*b + 209*a^2*b^2 + 104*a*b^3 + 24*b^4 + 30*(15*a^4 + 40*a^3*b + 23*a^2*b^
2 + 6*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 15*a^4 + 10*a^3*b + 3*a^2*b
^2 + 8*(7*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^5 + 10*(15*a^4 + 40
*a^3*b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^3 + (45*a^4 + 150*a^3*b + 209*
a^2*b^2 + 104*a*b^3 + 24*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(15*a^4 +
40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^2 + 4*(7*(15*a^4 + 10*a^3*b
+ 3*a^2*b^2)*cosh(d*x + c)^6 + 15*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3
)*cosh(d*x + c)^4 + 15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3 + 3*(45*a^4 +
150*a^3*b + 209*a^2*b^2 + 104*a*b^3 + 24*b^4)*cosh(d*x + c)^2)*sinh(d*x + c
)^2 + 8*((15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^7 + 3*(15*a^4 + 40*a
^3*b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^5 + (45*a^4 + 150*a^3*b + 209*a^
2*b^2 + 104*a*b^3 + 24*b^4)*cosh(d*x + c)^3 + (15*a^4 + 40*a^3*b + 23*a^2*b
^2 + 6*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*(a*cosh(d*
x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (a + 4*b
)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a + 4*b)*sinh(d*x + c))*sqrt(b/a)/
b) - ((15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^8 + 8*(15*a^4 + 10*a^3*
b + 3*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (15*a^4 + 10*a^3*b + 3*a^2*b
^2)*sinh(d*x + c)^8 + 4*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x
 + c)^6 + 4*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3 + 7*(15*a^4 + 10*a^3*
b + 3*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(15*a^4 + 10*a^3*b +
 3*a^2*b^2)*cosh(d*x + c)^3 + 3*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*
cosh(d*x + c))*sinh(d*x + c)^5 + 2*(45*a^4 + 150*a^3*b + 209*a^2*b^2 + 104*
a*b^3 + 24*b^4)*cosh(d*x + c)^4 + 2*(35*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*cos
h(d*x + c)^4 + 45*a^4 + 150*a^3*b + 209*a^2*b^2 + 104*a*b^3 + 24*b^4 + 30*(
```

```
15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4
+ 15*a^4 + 10*a^3*b + 3*a^2*b^2 + 8*(7*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh
(d*x + c)^5 + 10*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^3
 + (45*a^4 + 150*a^3*b + 209*a^2*b^2 + 104*a*b^3 + 24*b^4)*cosh(d*x + c))*s
inh(d*x + c)^3 + 4*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)
^2 + 4*(7*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^6 + 15*(15*a^4 + 40
*a^3*b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^4 + 15*a^4 + 40*a^3*b + 23*a^2
*b^2 + 6*a*b^3 + 3*(45*a^4 + 150*a^3*b + 209*a^2*b^2 + 104*a*b^3 + 24*b^4)*
cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(
d*x + c)^7 + 3*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^5 +
 (45*a^4 + 150*a^3*b + 209*a^2*b^2 + 104*a*b^3 + 24*b^4)*cosh(d*x + c)^3 +
(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sq
rt(b/a)*arctan(1/2*(a*cosh(d*x + c) + a*sinh(d*x + c))*sqrt(b/a)/b) + 2*(9*
a^3*b + 14*a^2*b^2 + 5*a*b^3)*cosh(d*x + c) + 8*(a^4*cosh(d*x + c)^8 + 8*a^
4*cosh(d*x + c)*sinh(d*x + c)^7 + a^4*sinh(d*x + c)^8 + 4*(a^4 + 2*a^3*b)*c
osh(d*x + c)^6 + 4*(7*a^4*cosh(d*x + c)^2 + a^4 + 2*a^3*b)*sinh(d*x + c)^6
+ 8*(7*a^4*cosh(d*x + c)^3 + 3*(a^4 + 2*a^3*b)*cosh(d*x + c))*sinh(d*x + c)
^5 + 2*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + 2*(35*a^4*cosh(d*x +
 c)^4 + 3*a^4 + 8*a^3*b + 8*a^2*b^2 + 30*(a^4 + 2*a^3*b)*cosh(d*x + c)^2)*s
inh(d*x + c)^4 + a^4 + 8*(7*a^4*cosh(d*x + c)^5 + 10*(a^4 + 2*a^3*b)*cosh(d
*x + c)^3 + (3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 +
4*(a^4 + 2*a^3*b)*cosh(d*x + c)^2 + 4*(7*a^4*cosh(d*x + c)^6 + 15*(a^4 + 2*
a^3*b)*cosh(d*x + c)^4 + a^4 + 2*a^3*b + 3*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*co
sh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(a^4*cosh(d*x + c)^7 + 3*(a^4 + 2*a^3*b)
*cosh(d*x + c)^5 + (3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 + (a^4 + 2
*a^3*b)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1
) - 8*(a^4*cosh(d*x + c)^8 + 8*a^4*cosh(d*x + c)*sinh(d*x + c)^7 + a^4*sinh
(d*x + c)^8 + 4*(a^4 + 2*a^3*b)*cosh(d*x + c)^6 + 4*(7*a^4*cosh(d*x + c)^2
+ a^4 + 2*a^3*b)*sinh(d*x + c)^6 + 8*(7*a^4*cosh(d*x + c)^3 + 3*(a^4 + 2*a^
3*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(
d*x + c)^4 + 2*(35*a^4*cosh(d*x + c)^4 + 3*a^4 + 8*a^3*b + 8*a^2*b^2 + 30*(
a^4 + 2*a^3*b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + a^4 + 8*(7*a^4*cosh(d*x +
 c)^5 + 10*(a^4 + 2*a^3*b)*cosh(d*x + c)^3 + (3*a^4 + 8*a^3*b + 8*a^2*b^2)*
cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 + 2*a^3*b)*cosh(d*x + c)^2 + 4*(7*a
^4*cosh(d*x + c)^6 + 15*(a^4 + 2*a^3*b)*cosh(d*x + c)^4 + a^4 + 2*a^3*b + 3
*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(a^4*co
sh(d*x + c)^7 + 3*(a^4 + 2*a^3*b)*cosh(d*x + c)^5 + (3*a^4 + 8*a^3*b + 8*a^
2*b^2)*cosh(d*x + c)^3 + (a^4 + 2*a^3*b)*cosh(d*x + c))*sinh(d*x + c))*log(
cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(7*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*
cosh(d*x + c)^6 + 5*(27*a^3*b + 70*a^2*b^2 + 55*a*b^3 + 12*b^4)*cosh(d*x +
c)^4 + 9*a^3*b + 14*a^2*b^2 + 5*a*b^3 + 3*(27*a^3*b + 70*a^2*b^2 + 55*a*b^3
 + 12*b^4)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^
4*b^3)*d*cosh(d*x + c)^8 + 8*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d
*x + c)*sinh(d*x + c)^7 + (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*sinh(d*x
+ c)^8 + 4*(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x +
 c)^6 + 4*(7*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^2 + (a^7
 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d)*sinh(d*x + c)^6 + 2*(3*a
^7 + 17*a^6*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*
x + c)^4 + 8*(7*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^3 + 3
*(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c))*sinh(
d*x + c)^5 + 2*(35*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^4
+ 30*(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^2
+ (3*a^7 + 17*a^6*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d)*
sinh(d*x + c)^4 + 4*(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*c
osh(d*x + c)^2 + 8*(7*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)
^5 + 10*(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)
^3 + (3*a^7 + 17*a^6*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*
d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^
3)*d*cosh(d*x + c)^6 + 15*(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^
```

```
4)*d*cosh(d*x + c)^4 + 3*(3*a^7 + 17*a^6*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a
^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)^2 + (a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*
b^3 + 2*a^3*b^4)*d)*sinh(d*x + c)^2 + (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)
*d + 8*((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^7 + 3*(a^7 +
5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^5 + (3*a^7 + 1
7*a^6*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)
^3 + (a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c))*s
inh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral(csch(c + d*x)/(a + b*sech(c + d*x)**2)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.46 $\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=126

$$\frac{15\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8d(a+b)^{7/2}}-\frac{15\coth(c+dx)}{8d(a+b)^3}+\frac{5\coth(c+dx)}{8d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)}+\frac{\coth(c+dx)}{4d(a+b)\left(a-b\tanh^2(c+dx)\cdots\right.}$$

```
[Out] (15*Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*(a + b)^(7/2)*
d) - (15*Coth[c + d*x])/(8*(a + b)^3*d) + Coth[c + d*x]/(4*(a + b)*d*(a + b
 - b*Tanh[c + d*x]^2)^2) + (5*Coth[c + d*x])/(8*(a + b)^2*d*(a + b - b*Tanh
[c + d*x]^2))
```

Rubi [A] time = 0.10034, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4132, 290, 325, 208}

$$\frac{15\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8d(a+b)^{7/2}}-\frac{15\coth(c+dx)}{8d(a+b)^3}+\frac{5\coth(c+dx)}{8d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)}+\frac{\coth(c+dx)}{4d(a+b)\left(a-b\tanh^2(c+dx)\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (15*Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*(a + b)^(7/2)*
d) - (15*Coth[c + d*x])/(8*(a + b)^3*d) + Coth[c + d*x]/(4*(a + b)*d*(a + b
 - b*Tanh[c + d*x]^2)^2) + (5*Coth[c + d*x])/(8*(a + b)^2*d*(a + b - b*Tanh
[c + d*x]^2))
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{x^2\left(a+b-bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\coth(c+dx)}{4(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{5\operatorname{Subst}\left(\int \frac{1}{x^2\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{4(a+b)d}$$

$$= \frac{\coth(c+dx)}{4(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{5\coth(c+dx)}{8(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{15\operatorname{Sub}}{}$$

$$= -\frac{15\coth(c+dx)}{8(a+b)^3d} + \frac{\coth(c+dx)}{4(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{5\coth(c+dx)}{8(a+b)^2d\left(a+b-b\tanh\right.}$$

$$= \frac{15\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8(a+b)^{7/2}d} - \frac{15\coth(c+dx)}{8(a+b)^3d} + \frac{\coth(c+dx)}{4(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2}$$

Mathematica [C] time = 6.80161, size = 981, normalized size = 7.79

$$\frac{(\cosh(2c+2dx)a+a+2b)^3\left(\frac{15ib\tan^{-1}\left(\operatorname{sech}(dx)\left(\frac{i\sinh(2c)}{2\sqrt{a+b}\sqrt{b\cosh(4c)-b\sinh(4c)}}-\frac{i\cosh(2c)}{2\sqrt{a+b}\sqrt{b\cosh(4c)-b\sinh(4c)}}\right)(-a\sinh(dx)-2b\sinh(dx)+a\sinh(2c+\right.}{64\sqrt{a+b}d\sqrt{b\cosh(4c)-b\sinh(4c)}}\right.}{(a+b)^3\left(b\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6*((((-15*I)/64)*b*ArcTan[
Sech[d*x]*(((-I/2)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]])
 + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]))*(-(a*Si
nh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x])]*Cosh[2*c])/(Sqrt[a + b]*d*Sq
rt[b*Cosh[4*c] - b*Sinh[4*c]]) + (((15*I)/64)*b*ArcTan[Sech[d*x]*(((-I/2)*C
osh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) + ((I/2)*Sinh[2*c])
/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]))*(-(a*Sinh[d*x]) - 2*b*Sinh[
d*x] + a*Sinh[2*c + d*x])]*Sinh[2*c])/(Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] - b*S
inh[4*c]])))/((a + b)^3*(a + b*Sech[c + d*x]^2)^3) + ((a + 2*b + a*Cosh[2*c
 + 2*d*x])*Csch[c]*Csch[c + d*x]*Sech[2*c]*Sech[c + d*x]^6*(-32*a^4*Sinh[d*
x] - 64*a^3*b*Sinh[d*x] + 22*a^2*b^2*Sinh[d*x] + 80*a*b^3*Sinh[d*x] + 16*b^
4*Sinh[d*x] + 32*a^4*Sinh[3*d*x] + 46*a^3*b*Sinh[3*d*x] - 54*a^2*b^2*Sinh[3
*d*x] - 8*a*b^3*Sinh[3*d*x] - 48*a^4*Sinh[2*c - d*x] - 128*a^3*b*Sinh[2*c -
 d*x] - 106*a^2*b^2*Sinh[2*c - d*x] + 80*a*b^3*Sinh[2*c - d*x] + 16*b^4*Sin
h[2*c - d*x] + 48*a^4*Sinh[2*c + d*x] + 146*a^3*b*Sinh[2*c + d*x] + 182*a^2
*b^2*Sinh[2*c + d*x] + 80*a*b^3*Sinh[2*c + d*x] + 16*b^4*Sinh[2*c + d*x] -
32*a^4*Sinh[4*c + d*x] - 82*a^3*b*Sinh[4*c + d*x] - 54*a^2*b^2*Sinh[4*c + d
*x] - 80*a*b^3*Sinh[4*c + d*x] - 16*b^4*Sinh[4*c + d*x] - 8*a^4*Sinh[2*c +
```

```
3*d*x] + 18*a^3*b*Sinh[2*c + 3*d*x] + 54*a^2*b^2*Sinh[2*c + 3*d*x] + 8*a*b^
3*Sinh[2*c + 3*d*x] + 32*a^4*Sinh[4*c + 3*d*x] + 73*a^3*b*Sinh[4*c + 3*d*x]
 + 24*a^2*b^2*Sinh[4*c + 3*d*x] + 8*a*b^3*Sinh[4*c + 3*d*x] - 8*a^4*Sinh[6*
c + 3*d*x] - 9*a^3*b*Sinh[6*c + 3*d*x] - 24*a^2*b^2*Sinh[6*c + 3*d*x] - 8*a
*b^3*Sinh[6*c + 3*d*x] + 8*a^4*Sinh[2*c + 5*d*x] - 9*a^3*b*Sinh[2*c + 5*d*x
] - 2*a^2*b^2*Sinh[2*c + 5*d*x] + 9*a^3*b*Sinh[4*c + 5*d*x] + 2*a^2*b^2*Sin
h[4*c + 5*d*x] + 8*a^4*Sinh[6*c + 5*d*x]))/(512*a^2*(a + b)^3*d*(a + b*Sech
[c + d*x]^2)^3)
```

Maple [B] time = 0.102, size = 819, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -1/2/d/(a^3+3*a^2*b+3*a*b^2+b^3)*tanh(1/2*d*x+1/2*c)+9/4/d*b/(a+b)^3/(tanh(
1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh
(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7*a+9/4/d*b^2/(a+b)^3/(tanh(
1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh
(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7+27/4/d*b/(a+b)^3/(tanh(1/2
*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/
2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^5*a-1/4/d*b^2/(a+b)^3/(tanh(1/2
*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/
2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^5+27/4/d*b/(a+b)^3/(tanh(1/2*d*
x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d
*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^3*a-1/4/d*b^2/(a+b)^3/(tanh(1/2*d*
x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d
*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^3+9/4/d*b/(a+b)^3/(tanh(1/2*d*x+1/
2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1
/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)*a+9/4/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*
c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2
*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)+15/16/d*b^(1/2)/(a+b)^(7/2)*ln((a+b)^(1/
2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-15/16/d
*b^(1/2)/(a+b)^(7/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1
/2*c)*b^(1/2)-(a+b)^(1/2))-1/2/d/(a+b)^3/tanh(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.7032, size = 17042, normalized size = 135.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*cosh(d*x + c)^8 + 32*(8*
a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + 4*(8*
a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*sinh(d*x + c)^8 + 8*(16*a^4 + 41*a^3*
b + 27*a^2*b^2 + 40*a*b^3 + 8*b^4)*cosh(d*x + c)^6 + 8*(16*a^4 + 41*a^3*b +
 27*a^2*b^2 + 40*a*b^3 + 8*b^4 + 14*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3
)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 16*(14*(8*a^4 + 9*a^3*b + 24*a^2*b^2 +
 8*a*b^3)*cosh(d*x + c)^3 + 3*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 +
8*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 8*(24*a^4 + 64*a^3*b + 53*a^2*b^2 -
 40*a*b^3 - 8*b^4)*cosh(d*x + c)^4 + 8*(35*(8*a^4 + 9*a^3*b + 24*a^2*b^2 +
8*a*b^3)*cosh(d*x + c)^4 + 24*a^4 + 64*a^3*b + 53*a^2*b^2 - 40*a*b^3 - 8*b^
4 + 15*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 + 8*b^4)*cosh(d*x + c)^2)
*sinh(d*x + c)^4 + 32*a^4 - 36*a^3*b - 8*a^2*b^2 + 32*(7*(8*a^4 + 9*a^3*b +
 24*a^2*b^2 + 8*a*b^3)*cosh(d*x + c)^5 + 5*(16*a^4 + 41*a^3*b + 27*a^2*b^2
+ 40*a*b^3 + 8*b^4)*cosh(d*x + c)^3 + (24*a^4 + 64*a^3*b + 53*a^2*b^2 - 40*
a*b^3 - 8*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 8*(16*a^4 + 23*a^3*b - 27*a
^2*b^2 - 4*a*b^3)*cosh(d*x + c)^2 + 8*(14*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8
*a*b^3)*cosh(d*x + c)^6 + 15*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 + 8
*b^4)*cosh(d*x + c)^4 + 16*a^4 + 23*a^3*b - 27*a^2*b^2 - 4*a*b^3 + 6*(24*a^
4 + 64*a^3*b + 53*a^2*b^2 - 40*a*b^3 - 8*b^4)*cosh(d*x + c)^2)*sinh(d*x + c
)^2 - 15*(a^4*cosh(d*x + c)^10 + 10*a^4*cosh(d*x + c)*sinh(d*x + c)^9 + a^4
*sinh(d*x + c)^10 + (3*a^4 + 8*a^3*b)*cosh(d*x + c)^8 + (45*a^4*cosh(d*x +
c)^2 + 3*a^4 + 8*a^3*b)*sinh(d*x + c)^8 + 8*(15*a^4*cosh(d*x + c)^3 + (3*a^
4 + 8*a^3*b)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(a^4 + 4*a^3*b + 8*a^2*b^2)
*cosh(d*x + c)^6 + 2*(105*a^4*cosh(d*x + c)^4 + a^4 + 4*a^3*b + 8*a^2*b^2 +
 14*(3*a^4 + 8*a^3*b)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*a^4*cosh(d*x
 + c)^5 + 14*(3*a^4 + 8*a^3*b)*cosh(d*x + c)^3 + 3*(a^4 + 4*a^3*b + 8*a^2*b
^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(a^4 + 4*a^3*b + 8*a^2*b^2)*cosh(d*x
 + c)^4 + 2*(105*a^4*cosh(d*x + c)^6 + 35*(3*a^4 + 8*a^3*b)*cosh(d*x + c)^4
 - a^4 - 4*a^3*b - 8*a^2*b^2 + 15*(a^4 + 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)
^2)*sinh(d*x + c)^4 - a^4 + 8*(15*a^4*cosh(d*x + c)^7 + 7*(3*a^4 + 8*a^3*b)
*cosh(d*x + c)^5 + 5*(a^4 + 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 - (a^4 + 4
*a^3*b + 8*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - (3*a^4 + 8*a^3*b)*cosh
(d*x + c)^2 + (45*a^4*cosh(d*x + c)^8 + 28*(3*a^4 + 8*a^3*b)*cosh(d*x + c)^
6 + 30*(a^4 + 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 - 3*a^4 - 8*a^3*b - 12*(
a^4 + 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*a^4*cosh
(d*x + c)^9 + 4*(3*a^4 + 8*a^3*b)*cosh(d*x + c)^7 + 6*(a^4 + 4*a^3*b + 8*a^
2*b^2)*cosh(d*x + c)^5 - 4*(a^4 + 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 - (3
*a^4 + 8*a^3*b)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a + b))*log((a^2*cosh
(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2
*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*si
nh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b
)*cosh(d*x + c))*sinh(d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 +
a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*
b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x
 + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x
 + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(
d*x + c))*sinh(d*x + c) + a)) + 16*(2*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b
^3)*cosh(d*x + c)^7 + 3*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 + 8*b^4)
*cosh(d*x + c)^5 + 2*(24*a^4 + 64*a^3*b + 53*a^2*b^2 - 40*a*b^3 - 8*b^4)*co
sh(d*x + c)^3 + (16*a^4 + 23*a^3*b - 27*a^2*b^2 - 4*a*b^3)*cosh(d*x + c))*s
inh(d*x + c))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^10 + 1
0*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (
a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*sinh(d*x + c)^10 + (3*a^7 + 17*a^6*b
 + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)^8 + (45*(a^7 + 3*a^
6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^2 + (3*a^7 + 17*a^6*b + 33*a^5*b
^2 + 27*a^4*b^3 + 8*a^3*b^4)*d)*sinh(d*x + c)^8 + 2*(a^7 + 7*a^6*b + 23*a^5
*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)^6 + 8*(15*(a^7
```

+ 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^3 + (3*a^7 + 17*a^6*b + 33
*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(10
5*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^4 + 14*(3*a^7 + 17*
a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)^2 + (a^7 + 7*a
^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d)*sinh(d*x + c)^6
 - 2*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d*c
osh(d*x + c)^4 + 4*(63*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c
)^5 + 14*(3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*
x + c)^3 + 3*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*
b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^7 + 3*a^6*b + 3*a^5*b^2 +
 a^4*b^3)*d*cosh(d*x + c)^6 + 35*(3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^
3 + 8*a^3*b^4)*d*cosh(d*x + c)^4 + 15*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*
b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)^2 - (a^7 + 7*a^6*b + 23*a^5*b
^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d)*sinh(d*x + c)^4 - (3*a^7 + 17*
a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)^2 + 8*(15*(a^7
 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^7 + 7*(3*a^7 + 17*a^6*b +
 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)^5 + 5*(a^7 + 7*a^6*b
+ 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)^3 - (a^
7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x
+ c))*sinh(d*x + c)^3 + (45*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*
x + c)^8 + 28*(3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*co
sh(d*x + c)^6 + 30*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 +
8*a^2*b^5)*d*cosh(d*x + c)^4 - 12*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3
+ 28*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)^2 - (3*a^7 + 17*a^6*b + 33*a^5*b^
2 + 27*a^4*b^3 + 8*a^3*b^4)*d)*sinh(d*x + c)^2 - (a^7 + 3*a^6*b + 3*a^5*b^2
 + a^4*b^3)*d + 2*(5*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^
9 + 4*(3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x +
 c)^7 + 6*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5
)*d*cosh(d*x + c)^5 - 4*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b
^4 + 8*a^2*b^5)*d*cosh(d*x + c)^3 - (3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4
*b^3 + 8*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), -1/8*(2*(8*a^4 + 9*a^3*b
 + 24*a^2*b^2 + 8*a*b^3)*cosh(d*x + c)^8 + 16*(8*a^4 + 9*a^3*b + 24*a^2*b^2
 + 8*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(8*a^4 + 9*a^3*b + 24*a^2*b^2
 + 8*a*b^3)*sinh(d*x + c)^8 + 4*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3
+ 8*b^4)*cosh(d*x + c)^6 + 4*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 + 8
b^4 + 14(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*cosh(d*x + c)^2)*sinh(d*
x + c)^6 + 8*(14*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*cosh(d*x + c)^3 +
 3*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 + 8*b^4)*cosh(d*x + c))*sinh(
d*x + c)^5 + 4*(24*a^4 + 64*a^3*b + 53*a^2*b^2 - 40*a*b^3 - 8*b^4)*cosh(d*x
 + c)^4 + 4*(35*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*cosh(d*x + c)^4 +
24*a^4 + 64*a^3*b + 53*a^2*b^2 - 40*a*b^3 - 8*b^4 + 15*(16*a^4 + 41*a^3*b +
 27*a^2*b^2 + 40*a*b^3 + 8*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*a^4 -
 18*a^3*b - 4*a^2*b^2 + 16*(7*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*cosh
(d*x + c)^5 + 5*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 + 8*b^4)*cosh(d*
x + c)^3 + (24*a^4 + 64*a^3*b + 53*a^2*b^2 - 40*a*b^3 - 8*b^4)*cosh(d*x + c
))*sinh(d*x + c)^3 + 4*(16*a^4 + 23*a^3*b - 27*a^2*b^2 - 4*a*b^3)*cosh(d*x
+ c)^2 + 4*(14*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*cosh(d*x + c)^6 + 1
5*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 + 8*b^4)*cosh(d*x + c)^4 + 16*
a^4 + 23*a^3*b - 27*a^2*b^2 - 4*a*b^3 + 6*(24*a^4 + 64*a^3*b + 53*a^2*b^2 -
 40*a*b^3 - 8*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 15*(a^4*cosh(d*x + c)
^10 + 10*a^4*cosh(d*x + c)*sinh(d*x + c)^9 + a^4*sinh(d*x + c)^10 + (3*a^4
+ 8*a^3*b)*cosh(d*x + c)^8 + (45*a^4*cosh(d*x + c)^2 + 3*a^4 + 8*a^3*b)*sin
h(d*x + c)^8 + 8*(15*a^4*cosh(d*x + c)^3 + (3*a^4 + 8*a^3*b)*cosh(d*x + c))
*sinh(d*x + c)^7 + 2*(a^4 + 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^6 + 2*(105*a
^4*cosh(d*x + c)^4 + a^4 + 4*a^3*b + 8*a^2*b^2 + 14*(3*a^4 + 8*a^3*b)*cosh(
d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*a^4*cosh(d*x + c)^5 + 14*(3*a^4 + 8*a^3
*b)*cosh(d*x + c)^3 + 3*(a^4 + 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c))*sinh(d*x
 + c)^5 - 2*(a^4 + 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + 2*(105*a^4*cosh(d
x + c)^6 + 35(3*a^4 + 8*a^3*b)*cosh(d*x + c)^4 - a^4 - 4*a^3*b - 8*a^2*b^

```
2 + 15*(a^4 + 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - a^4 +
 8*(15*a^4*cosh(d*x + c)^7 + 7*(3*a^4 + 8*a^3*b)*cosh(d*x + c)^5 + 5*(a^4 +
 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 - (a^4 + 4*a^3*b + 8*a^2*b^2)*cosh(d*
x + c))*sinh(d*x + c)^3 - (3*a^4 + 8*a^3*b)*cosh(d*x + c)^2 + (45*a^4*cosh(
d*x + c)^8 + 28*(3*a^4 + 8*a^3*b)*cosh(d*x + c)^6 + 30*(a^4 + 4*a^3*b + 8*a
^2*b^2)*cosh(d*x + c)^4 - 3*a^4 - 8*a^3*b - 12*(a^4 + 4*a^3*b + 8*a^2*b^2)*
cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*a^4*cosh(d*x + c)^9 + 4*(3*a^4 + 8*
a^3*b)*cosh(d*x + c)^7 + 6*(a^4 + 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^5 - 4*
(a^4 + 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 - (3*a^4 + 8*a^3*b)*cosh(d*x +
c))*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cos
h(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b)
 + 8*(2*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*cosh(d*x + c)^7 + 3*(16*a^
4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 + 8*b^4)*cosh(d*x + c)^5 + 2*(24*a^4 +
 64*a^3*b + 53*a^2*b^2 - 40*a*b^3 - 8*b^4)*cosh(d*x + c)^3 + (16*a^4 + 23*a
^3*b - 27*a^2*b^2 - 4*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^7 + 3*a^6*b
+ 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^10 + 10*(a^7 + 3*a^6*b + 3*a^5*b^2 +
 a^4*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^7 + 3*a^6*b + 3*a^5*b^2 + a^
4*b^3)*d*sinh(d*x + c)^10 + (3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8
*a^3*b^4)*d*cosh(d*x + c)^8 + (45*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*c
osh(d*x + c)^2 + (3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d
)*sinh(d*x + c)^8 + 2*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4
 + 8*a^2*b^5)*d*cosh(d*x + c)^6 + 8*(15*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^
3)*d*cosh(d*x + c)^3 + (3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*
b^4)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^7 + 3*a^6*b + 3*a^5*b^2 +
 a^4*b^3)*d*cosh(d*x + c)^4 + 14*(3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^
3 + 8*a^3*b^4)*d*cosh(d*x + c)^2 + (a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3
 + 28*a^3*b^4 + 8*a^2*b^5)*d)*sinh(d*x + c)^6 - 2*(a^7 + 7*a^6*b + 23*a^5*b
^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)^4 + 4*(63*(a^7 +
3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^5 + 14*(3*a^7 + 17*a^6*b + 3
3*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)^3 + 3*(a^7 + 7*a^6*b +
23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c))*sinh(d*x
 + c)^5 + 2*(105*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^6 +
35*(3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)
^4 + 15*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*
d*cosh(d*x + c)^2 - (a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 +
 8*a^2*b^5)*d)*sinh(d*x + c)^4 - (3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^
3 + 8*a^3*b^4)*d*cosh(d*x + c)^2 + 8*(15*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b
^3)*d*cosh(d*x + c)^7 + 7*(3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a
^3*b^4)*d*cosh(d*x + c)^5 + 5*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28
*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)^3 - (a^7 + 7*a^6*b + 23*a^5*b^2 + 37*
a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (45*(a
^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^8 + 28*(3*a^7 + 17*a^6*
b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)^6 + 30*(a^7 + 7*a^
6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)^4 -
 12*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d*co
sh(d*x + c)^2 - (3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d)
*sinh(d*x + c)^2 - (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d + 2*(5*(a^7 + 3*
a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^9 + 4*(3*a^7 + 17*a^6*b + 33*a
^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)^7 + 6*(a^7 + 7*a^6*b + 23*
a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)^5 - 4*(a^7 +
 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c
)^3 - (3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x +
 c))*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral(csch(c + d*x)**2/(a + b*sech(c + d*x)**2)**3, x)
```

Giac [B] time = 1.41725, size = 485, normalized size = 3.85

$$\frac{15\,b\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{8\left(a^3d+3\,a^2bd+3\,ab^2d+b^3d\right)\sqrt{-ab-b^2}} - \frac{9\,a^3be^{(6\,dx+6\,c)}+24\,a^2b^2e^{(6\,dx+6\,c)}+8\,ab^3e^{(6\,dx+6\,c)}+27\,a^3be^{(4\,dx+4\,c)}+78\,a^2b^2}{4\left(a^5d+3\,a^4bd+3\,a^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 15/8*b*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/((a^3*d +
 3*a^2*b*d + 3*a*b^2*d + b^3*d)*sqrt(-a*b - b^2)) - 1/4*(9*a^3*b*e^(6*d*x +
 6*c) + 24*a^2*b^2*e^(6*d*x + 6*c) + 8*a*b^3*e^(6*d*x + 6*c) + 27*a^3*b*e^(
4*d*x + 4*c) + 78*a^2*b^2*e^(4*d*x + 4*c) + 88*a*b^3*e^(4*d*x + 4*c) + 16*b
^4*e^(4*d*x + 4*c) + 27*a^3*b*e^(2*d*x + 2*c) + 56*a^2*b^2*e^(2*d*x + 2*c)
+ 8*a*b^3*e^(2*d*x + 2*c) + 9*a^3*b + 2*a^2*b^2)/((a^5*d + 3*a^4*b*d + 3*a^
3*b^2*d + a^2*b^3*d)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*
x + 2*c) + a)^2) - 2/((a^3*d + 3*a^2*b*d + 3*a*b^2*d + b^3*d)*(e^(2*d*x + 2
*c) - 1))
```

3.47 $\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=213

$$-\frac{\left(4a^2-9ab-b^2\right)\cosh(c+dx)}{8ad(a+b)^3\left(a\cosh^2(c+dx)+b\right)}-\frac{\sqrt{b}\left(15a^2-10ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{3/2}d(a+b)^4}+\frac{b(2a-b)\cosh(c+dx)}{4ad(a+b)^2\left(a\cosh^2(c+dx)+b\right)}$$

```
[Out] -(Sqrt[b]*(15*a^2 - 10*a*b - b^2)*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/
(8*a^(3/2)*(a + b)^4*d) + ((a - 5*b)*ArcTanh[Cosh[c + d*x]])/(2*(a + b)^4*d
) + ((2*a - b)*b*Cosh[c + d*x])/(4*a*(a + b)^2*d*(b + a*Cosh[c + d*x]^2)^2)
 - ((4*a^2 - 9*a*b - b^2)*Cosh[c + d*x])/(8*a*(a + b)^3*d*(b + a*Cosh[c + d
*x]^2)) - (Cosh[c + d*x]*Coth[c + d*x]^2)/(2*(a + b)*d*(b + a*Cosh[c + d*x]
^2)^2)
```

Rubi [A] time = 0.342228, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4133, 470, 578, 527, 522, 206, 205}

$$-\frac{\left(4a^2-9ab-b^2\right)\cosh(c+dx)}{8ad(a+b)^3\left(a\cosh^2(c+dx)+b\right)}-\frac{\sqrt{b}\left(15a^2-10ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{3/2}d(a+b)^4}+\frac{b(2a-b)\cosh(c+dx)}{4ad(a+b)^2\left(a\cosh^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```


```
[Out] -(Sqrt[b]*(15*a^2 - 10*a*b - b^2)*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/
(8*a^(3/2)*(a + b)^4*d) + ((a - 5*b)*ArcTanh[Cosh[c + d*x]])/(2*(a + b)^4*d
) + ((2*a - b)*b*Cosh[c + d*x])/(4*a*(a + b)^2*d*(b + a*Cosh[c + d*x]^2)^2)
 - ((4*a^2 - 9*a*b - b^2)*Cosh[c + d*x])/(8*a*(a + b)^3*d*(b + a*Cosh[c + d
*x]^2)) - (Cosh[c + d*x]*Coth[c + d*x]^2)/(2*(a + b)*d*(b + a*Cosh[c + d*x]
^2)^2)
```


Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^6}{(1-x^2)^2(b+ax^2)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\cosh(c+dx)\coth^2(c+dx)}{2(a+b)d\left(b+a\cosh^2(c+dx)\right)^2} - \frac{\operatorname{Subst}\left(\int \frac{x^2(3b+(-a+2b)x^2)}{(1-x^2)(b+ax^2)^3}\,dx, x, \cosh(c+dx)\right)}{2(a+b)d}$$

$$= \frac{(2a-b)b\cosh(c+dx)}{4a(a+b)^2d\left(b+a\cosh^2(c+dx)\right)^2} - \frac{\cosh(c+dx)\coth^2(c+dx)}{2(a+b)d\left(b+a\cosh^2(c+dx)\right)^2} - \frac{\operatorname{Subst}\left(\int \frac{2(2a}{\ }\right.}{\ }$$

$$= \frac{(2a-b)b\cosh(c+dx)}{4a(a+b)^2d\left(b+a\cosh^2(c+dx)\right)^2} - \frac{\left(4a^2-9ab-b^2\right)\cosh(c+dx)}{8a(a+b)^3d\left(b+a\cosh^2(c+dx)\right)} - \frac{\cosh(c+}{2(a+b)d\,(b}$$

$$= \frac{(2a-b)b\cosh(c+dx)}{4a(a+b)^2d\left(b+a\cosh^2(c+dx)\right)^2} - \frac{\left(4a^2-9ab-b^2\right)\cosh(c+dx)}{8a(a+b)^3d\left(b+a\cosh^2(c+dx)\right)} - \frac{\cosh(c+}{2(a+b)d\,(b}$$

$$= -\frac{\sqrt{b}\left(15a^2-10ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{3/2}(a+b)^4d} + \frac{(a-5b)\tanh^{-1}(\cosh(c+dx))}{2(a+b)^4d} + \frac{\ }{4a(a}$$

Mathematica [C] time = 3.73653, size = 524, normalized size = 2.46

$$\frac{\operatorname{sech}^5(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\frac{\sqrt{b}(-15a^2+10ab+b^2)\operatorname{sech}(c+dx)(a\cosh(2(c+dx))+a+2b)^2\tan^{-1}\left(\frac{\sinh(c)\tanh\left(\frac{dx}{2}\right)\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos\right.}{\ }\right.}{a^{3/2}}\right.}{\ }$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^5*((-8*b^2*(a + b)^2)/a + (2
*b*(a + b)*(9*a + b)*(a + 2*b + a*Cosh[2*(c + d*x)]))/a + (Sqrt[b]*(-15*a^2
 + 10*a*b + b^2)*ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^
2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c]
- Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Se
ch[c + d*x])/a^(3/2) + (Sqrt[b]*(-15*a^2 + 10*a*b + b^2)*ArcTan[((Sqrt[a] +
 I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]
*(Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[
b]]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x])/a^(3/2) - (a + b)*(a +
 2*b + a*Cosh[2*(c + d*x)])^2*Csch[(c + d*x)/2]^2*Sech[c + d*x] + 4*(a - 5*
b)*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Log[Cosh[(c + d*x)/2]]*Sech[c + d*x] -
 4*(a - 5*b)*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Log[Sinh[(c + d*x)/2]]*Sech[
c + d*x] - (a + b)*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[(c + d*x)/2]^2*Se
ch[c + d*x]))/(64*(a + b)^4*d*(a + b*Sech[c + d*x]^2)^3)
```

Maple [B] time = 0.101, size = 1555, normalized size = 7.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/8/d*tanh(1/2*d*x+1/2*c)^2/(a^3+3*a^2*b+3*a*b^2+b^3)+9/4/d*b/(a+b)^4/(tanh
(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tan
h(1/2*d*x+1/2*c)^2*b+a+b)^2*a^2*tanh(1/2*d*x+1/2*c)^6-5/4/d*b^2/(a+b)^4/(ta
nh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*t
anh(1/2*d*x+1/2*c)^2*b+a+b)^2*a*tanh(1/2*d*x+1/2*c)^6-13/4/d*b^3/(a+b)^4/(t
anh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*
tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^6+1/4/d*b^4/(a+b)^4/(tan
h(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*ta
nh(1/2*d*x+1/2*c)^2*b+a+b)^2/a*tanh(1/2*d*x+1/2*c)^6+27/4/d*b/(a+b)^4/(tanh
(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tan
h(1/2*d*x+1/2*c)^2*b+a+b)^2*a^2*tanh(1/2*d*x+1/2*c)^4-21/4/d*b^2/(a+b)^4/(t
anh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*
tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*a*tanh(1/2*d*x+1/2*c)^4+29/4/d*b^3/(a+b)^4/(
tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2
*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^4-3/4/d*b^4/(a+b)^4/(ta
nh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*t
anh(1/2*d*x+1/2*c)^2*b+a+b)^2/a*tanh(1/2*d*x+1/2*c)^4+27/4/d*b/(a+b)^4/(tan
h(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*ta
nh(1/2*d*x+1/2*c)^2*b+a+b)^2*a^2*tanh(1/2*d*x+1/2*c)^2+1/4/d*b^2/(a+b)^4/(t
anh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*
tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*a*tanh(1/2*d*x+1/2*c)^2-23/4/d*b^3/(a+b)^4/(
tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2
*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^2+3/4/d*b^4/(a+b)^4/(ta
nh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*t
anh(1/2*d*x+1/2*c)^2*b+a+b)^2/a*tanh(1/2*d*x+1/2*c)^2+9/4/d*b/(a+b)^4/(tanh
(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tan
h(1/2*d*x+1/2*c)^2*b+a+b)^2*a^2+17/4/d*b^2/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a
+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*
b+a+b)^2*a+7/4/d*b^3/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)
^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2-1/4/d*b^4/(a+
b)^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)
^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a-15/8/d*b/(a+b)^4*a/(a*b)^(1/2)*arct
an(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))+5/4/d*b^2/(a+b)
^4/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/
2))+1/8/d*b^3/(a+b)^4/a/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)
^2+2*a-2*b)/(a*b)^(1/2))-1/8/d/(a+b)^3/tanh(1/2*d*x+1/2*c)^2-1/2/d/(a+b)^4*
ln(tanh(1/2*d*x+1/2*c))*a+5/2/d/(a+b)^4*ln(tanh(1/2*d*x+1/2*c))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(a - 5*b)*log((e^(d*x + c) + 1)*e^(-c))/(a^4*d + 4*a^3*b*d + 6*a^2*b^2*
d + 4*a*b^3*d + b^4*d) - 1/2*(a - 5*b)*log((e^(d*x + c) - 1)*e^(-c))/(a^4*d
 + 4*a^3*b*d + 6*a^2*b^2*d + 4*a*b^3*d + b^4*d) - 1/4*((4*a^3*e^(11*c) - 9*
a^2*b*e^(11*c) - a*b^2*e^(11*c))*e^(11*d*x) + (20*a^3*e^(9*c) + 23*a^2*b*e^
(9*c) - 29*a*b^2*e^(9*c) + 4*b^3*e^(9*c))*e^(9*d*x) + 2*(20*a^3*e^(7*c) + 5
7*a^2*b*e^(7*c) + 47*a*b^2*e^(7*c) - 2*b^3*e^(7*c))*e^(7*d*x) + 2*(20*a^3*e
^(5*c) + 57*a^2*b*e^(5*c) + 47*a*b^2*e^(5*c) - 2*b^3*e^(5*c))*e^(5*d*x) + (
```

```
20*a^3*e^(3*c) + 23*a^2*b*e^(3*c) - 29*a*b^2*e^(3*c) + 4*b^3*e^(3*c))*e^(3*
d*x) + (4*a^3*e^c - 9*a^2*b*e^c - a*b^2*e^c)*e^(d*x))/(a^6*d + 3*a^5*b*d +
3*a^4*b^2*d + a^3*b^3*d + (a^6*d*e^(12*c) + 3*a^5*b*d*e^(12*c) + 3*a^4*b^2*
d*e^(12*c) + a^3*b^3*d*e^(12*c))*e^(12*d*x) + 2*(a^6*d*e^(10*c) + 7*a^5*b*d
*e^(10*c) + 15*a^4*b^2*d*e^(10*c) + 13*a^3*b^3*d*e^(10*c) + 4*a^2*b^4*d*e^(
10*c))*e^(10*d*x) - (a^6*d*e^(8*c) + 3*a^5*b*d*e^(8*c) - 13*a^4*b^2*d*e^(8*
c) - 47*a^3*b^3*d*e^(8*c) - 48*a^2*b^4*d*e^(8*c) - 16*a*b^5*d*e^(8*c))*e^(8
*d*x) - 4*(a^6*d*e^(6*c) + 7*a^5*b*d*e^(6*c) + 23*a^4*b^2*d*e^(6*c) + 37*a^
3*b^3*d*e^(6*c) + 28*a^2*b^4*d*e^(6*c) + 8*a*b^5*d*e^(6*c))*e^(6*d*x) - (a^
6*d*e^(4*c) + 3*a^5*b*d*e^(4*c) - 13*a^4*b^2*d*e^(4*c) - 47*a^3*b^3*d*e^(4*
c) - 48*a^2*b^4*d*e^(4*c) - 16*a*b^5*d*e^(4*c))*e^(4*d*x) + 2*(a^6*d*e^(2*c
) + 7*a^5*b*d*e^(2*c) + 15*a^4*b^2*d*e^(2*c) + 13*a^3*b^3*d*e^(2*c) + 4*a^2
*b^4*d*e^(2*c))*e^(2*d*x)) - 8*integrate(1/32*((15*a^2*b*e^(3*c) - 10*a*b^2
*e^(3*c) - b^3*e^(3*c))*e^(3*d*x) - (15*a^2*b*e^c - 10*a*b^2*e^c - b^3*e^c)
*e^(d*x))/(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4 + (a^6*e^(4*c) +
 4*a^5*b*e^(4*c) + 6*a^4*b^2*e^(4*c) + 4*a^3*b^3*e^(4*c) + a^2*b^4*e^(4*c))
*e^(4*d*x) + 2*(a^6*e^(2*c) + 6*a^5*b*e^(2*c) + 14*a^4*b^2*e^(2*c) + 16*a^3
*b^3*e^(2*c) + 9*a^2*b^4*e^(2*c) + 2*a*b^5*e^(2*c))*e^(2*d*x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral(csch(c + d*x)**3/(a + b*sech(c + d*x)**2)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.48 $\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=165

$$-\frac{5\sqrt{b}(3a-4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8d(a+b)^{9/2}}-\frac{b(7a-4b)\tanh(c+dx)}{8d(a+b)^4\left(a-b\tanh^2(c+dx)+b\right)}-\frac{ab\tanh(c+dx)}{4d(a+b)^3\left(a-b\tanh^2(c+dx)+b\right)^2}-$$

```
[Out] (-5*(3*a - 4*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*(a
 + b)^(9/2)*d) + ((a - 2*b)*Coth[c + d*x])/((a + b)^4*d) - Coth[c + d*x]^3/
(3*(a + b)^3*d) - (a*b*Tanh[c + d*x])/(4*(a + b)^3*d*(a + b - b*Tanh[c + d*
x]^2)^2) - ((7*a - 4*b)*b*Tanh[c + d*x])/(8*(a + b)^4*d*(a + b - b*Tanh[c +
 d*x]^2))
```

Rubi [A] time = 0.272644, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4132, 456, 1259, 1261, 208}

$$-\frac{5\sqrt{b}(3a-4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8d(a+b)^{9/2}}-\frac{b(7a-4b)\tanh(c+dx)}{8d(a+b)^4\left(a-b\tanh^2(c+dx)+b\right)}-\frac{ab\tanh(c+dx)}{4d(a+b)^3\left(a-b\tanh^2(c+dx)+b\right)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```



```
[Out] (-5*(3*a - 4*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*(a
 + b)^(9/2)*d) + ((a - 2*b)*Coth[c + d*x])/((a + b)^4*d) - Coth[c + d*x]^3/
(3*(a + b)^3*d) - (a*b*Tanh[c + d*x])/(4*(a + b)^3*d*(a + b - b*Tanh[c + d*
x]^2)^2) - ((7*a - 4*b)*b*Tanh[c + d*x])/(8*(a + b)^4*d*(a + b - b*Tanh[c +
 d*x]^2))
```


Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
```

```
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{x^4\left(a+b-bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{ab\tanh(c+dx)}{4(a+b)^3 d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{b\,\text{Subst}\left(\int \frac{\frac{4}{b(a+b)}-\frac{4ax^2}{b(a+b)^2}-\frac{3ax^4}{(a+b)^3}}{x^4\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+\right.}{4d}$$

$$= -\frac{ab\tanh(c+dx)}{4(a+b)^3 d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{(7a-4b)b\tanh(c+dx)}{8(a+b)^4 d\left(a+b-b\tanh^2(c+dx)\right)} + \text{Sub}$$

$$= -\frac{ab\tanh(c+dx)}{4(a+b)^3 d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{(7a-4b)b\tanh(c+dx)}{8(a+b)^4 d\left(a+b-b\tanh^2(c+dx)\right)} + \text{Sub}$$

$$= \frac{(a-2b)\coth(c+dx)}{(a+b)^4 d} - \frac{\coth^3(c+dx)}{3(a+b)^3 d} - \frac{ab\tanh(c+dx)}{4(a+b)^3 d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{}{8(a}$$

$$= -\frac{5(3a-4b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8(a+b)^{9/2} d} + \frac{(a-2b)\coth(c+dx)}{(a+b)^4 d} - \frac{\coth^3(c+dx)}{3(a+b)^3 d} - \frac{}{4(a}$$

Mathematica [B] time = 5.3613, size = 985, normalized size = 5.97

$$-\frac{(\cosh(2(c+dx))a+a+2b)\operatorname{sech}^6(c+dx)\left(\frac{\operatorname{csch}(c)\operatorname{sech}(2c)\left(224\sinh(2c-dx)a^4-224\sinh(2c+dx)a^4+176\sinh(4c+dx)a^4+48\sinh(2c+3dx)a^4\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csch[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^6*((480*(3*a - 4*b)*b*ArcTa
nh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d
*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c
 + d*x)])^2*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c]
)^4]) + (Csch[c]*Csch[c + d*x]^3*Sech[2*c]*(4*(44*a^4 + 122*a^3*b + 63*a^2*
b^2 + 126*a*b^3 + 36*b^4)*Sinh[d*x] + (-96*a^4 - 71*a^3*b + 344*a^2*b^2 - 1
208*a*b^3 + 48*b^4)*Sinh[3*d*x] + 224*a^4*Sinh[2*c - d*x] + 576*a^3*b*Sinh[
2*c - d*x] + 124*a^2*b^2*Sinh[2*c - d*x] - 2184*a*b^3*Sinh[2*c - d*x] + 144
*b^4*Sinh[2*c - d*x] - 224*a^4*Sinh[2*c + d*x] - 657*a^3*b*Sinh[2*c + d*x]
- 538*a^2*b^2*Sinh[2*c + d*x] + 984*a*b^3*Sinh[2*c + d*x] + 144*b^4*Sinh[2*
c + d*x] + 176*a^4*Sinh[4*c + d*x] + 569*a^3*b*Sinh[4*c + d*x] + 666*a^2*b^
2*Sinh[4*c + d*x] + 1704*a*b^3*Sinh[4*c + d*x] - 144*b^4*Sinh[4*c + d*x] +
48*a^4*Sinh[2*c + 3*d*x] + 111*a^3*b*Sinh[2*c + 3*d*x] + 360*a^2*b^2*Sinh[2
*c + 3*d*x] + 312*a*b^3*Sinh[2*c + 3*d*x] - 48*b^4*Sinh[2*c + 3*d*x] - 96*a
^4*Sinh[4*c + 3*d*x] - 152*a^3*b*Sinh[4*c + 3*d*x] + 146*a^2*b^2*Sinh[4*c +
 3*d*x] - 728*a*b^3*Sinh[4*c + 3*d*x] - 48*b^4*Sinh[4*c + 3*d*x] + 48*a^4*S
inh[6*c + 3*d*x] + 192*a^3*b*Sinh[6*c + 3*d*x] + 558*a^2*b^2*Sinh[6*c + 3*d
*x] - 168*a*b^3*Sinh[6*c + 3*d*x] + 48*b^4*Sinh[6*c + 3*d*x] + 16*a^4*Sinh[
2*c + 5*d*x] - 598*a^2*b^2*Sinh[2*c + 5*d*x] + 48*a*b^3*Sinh[2*c + 5*d*x] +
 72*a^3*b*Sinh[4*c + 5*d*x] + 150*a^2*b^2*Sinh[4*c + 5*d*x] - 48*a*b^3*Sinh
[4*c + 5*d*x] + 16*a^4*Sinh[6*c + 5*d*x] + 27*a^3*b*Sinh[6*c + 5*d*x] - 388
*a^2*b^2*Sinh[6*c + 5*d*x] + 45*a^3*b*Sinh[8*c + 5*d*x] - 60*a^2*b^2*Sinh[8
*c + 5*d*x] + 16*a^4*Sinh[4*c + 7*d*x] - 83*a^3*b*Sinh[4*c + 7*d*x] + 6*a^2
*b^2*Sinh[4*c + 7*d*x] + 27*a^3*b*Sinh[6*c + 7*d*x] - 6*a^2*b^2*Sinh[6*c +
7*d*x] + 16*a^4*Sinh[8*c + 7*d*x] - 56*a^3*b*Sinh[8*c + 7*d*x]))/a))/(6144*
(a + b)^4*d*(a + b*Sech[c + d*x]^2)^3)
```

Maple [B] time = 0.121, size = 1443, normalized size = 8.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -1/24/d/(a+b)/(a^3+3*a^2*b+3*a*b^2+b^3)*a*tanh(1/2*d*x+1/2*c)^3-1/24/d/(a+b
)/(a^3+3*a^2*b+3*a*b^2+b^3)*b*tanh(1/2*d*x+1/2*c)^3+3/8/d/(a+b)/(a^3+3*a^2*
b+3*a*b^2+b^3)*a*tanh(1/2*d*x+1/2*c)-9/8/d/(a+b)/(a^3+3*a^2*b+3*a*b^2+b^3)*
tanh(1/2*d*x+1/2*c)*b-9/4/d*b/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d
*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh
(1/2*d*x+1/2*c)^7*a^2-5/4/d*b^2/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2
*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*ta
nh(1/2*d*x+1/2*c)^7*a+1/d*b^3/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d
*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh
(1/2*d*x+1/2*c)^7-27/4/d*b/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+
1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/
2*d*x+1/2*c)^5*a^2+13/4/d*b^2/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d
*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh
(1/2*d*x+1/2*c)^5*a-1/d*b^3/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x
+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1
/2*d*x+1/2*c)^5-27/4/d*b/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/
2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*
d*x+1/2*c)^3*a^2+13/4/d*b^2/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x
+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1
/2*d*x+1/2*c)^3*a-1/d*b^3/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1
/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2
*d*x+1/2*c)^3-9/4/d*b/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c
```

```
)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x
+1/2*c)*a^2-5/4/d*b^2/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c
)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x
+1/2*c)*a+1/d*b^3/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+
2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2
*c)-15/16/d*b^(1/2)/(a+b)^(9/2)*a*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*ta
nh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+15/16/d*b^(1/2)/(a+b)^(9/2)*a*ln(-(a
+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2))+
5/4/d*b^(3/2)/(a+b)^(9/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d
*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-5/4/d*b^(3/2)/(a+b)^(9/2)*ln(-(a+b)^(1/2)*ta
nh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2))-1/24/d/(a+b)
^3/tanh(1/2*d*x+1/2*c)^3+3/8/d/(a+b)^4/tanh(1/2*d*x+1/2*c)*a-9/8/d/(a+b)^4/
tanh(1/2*d*x+1/2*c)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral(csch(c + d*x)**4/(a + b*sech(c + d*x)**2)**3, x)
```

Giac [B] time = 1.52301, size = 568, normalized size = 3.44

$$-\frac{5\left(3\,ab-4\,b^2\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{8\left(a^4d+4\,a^3bd+6\,a^2b^2d+4\,ab^3d+b^4d\right)\sqrt{-ab-b^2}}+\frac{9\,a^3be^{(6\,dx+6\,c)}+20\,a^2b^2e^{(6\,dx+6\,c)}+27\,a^3be^{(4\,dx+4\,c)}+66\,a^2b^2e}{4\left(a^5d+4\,a^4bd+6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -5/8*(3*a*b - 4*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b
^2))/((a^4*d + 4*a^3*b*d + 6*a^2*b^2*d + 4*a*b^3*d + b^4*d)*sqrt(-a*b - b^2
)) + 1/4*(9*a^3*b*e^(6*d*x + 6*c) + 20*a^2*b^2*e^(6*d*x + 6*c) + 27*a^3*b*e
^(4*d*x + 4*c) + 66*a^2*b^2*e^(4*d*x + 4*c) + 56*a*b^3*e^(4*d*x + 4*c) - 16
*b^4*e^(4*d*x + 4*c) + 27*a^3*b*e^(2*d*x + 2*c) + 44*a^2*b^2*e^(2*d*x + 2*c
) - 16*a*b^3*e^(2*d*x + 2*c) + 9*a^3*b - 2*a^2*b^2)/((a^5*d + 4*a^4*b*d + 6
*a^3*b^2*d + 4*a^2*b^3*d + a*b^4*d)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c
) + 4*b*e^(2*d*x + 2*c) + a)^2) - 2/3*(9*b*e^(4*d*x + 4*c) + 6*a*e^(2*d*x +
 2*c) - 12*b*e^(2*d*x + 2*c) - 2*a + 7*b)/((a^4*d + 4*a^3*b*d + 6*a^2*b^2*d
 + 4*a*b^3*d + b^4*d)*(e^(2*d*x + 2*c) - 1)^3)
```

3.49 $\int \cosh^4(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=61

$$\frac{(3a+4b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{1}{8}x(3a+4b)+\frac{a\sinh(c+dx)\cosh^3(c+dx)}{4d}$$

```
[Out] ((3*a + 4*b)*x)/8 + ((3*a + 4*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (a*Co
sh[c + d*x]^3*Sinh[c + d*x])/(4*d)
```

Rubi [A] time = 0.0458518, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4045, 2635, 8}

$$\frac{(3a+4b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{1}{8}x(3a+4b)+\frac{a\sinh(c+dx)\cosh^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((3*a + 4*b)*x)/8 + ((3*a + 4*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (a*Co
sh[c + d*x]^3*Sinh[c + d*x])/(4*d)
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cosh^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx &= \frac{a\cosh^3(c+dx)\sinh(c+dx)}{4d}+\frac{1}{4}(3a+4b)\int \cosh^2(c+dx)\,dx\\ &= \frac{(3a+4b)\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{a\cosh^3(c+dx)\sinh(c+dx)}{4d}+\frac{1}{8}(\\ &= \frac{1}{8}(3a+4b)x+\frac{(3a+4b)\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{a\cosh^3(c+dx)\sin}{4d}\end{aligned}$$

Mathematica [A] time = 0.0924414, size = 45, normalized size = 0.74

$$\frac{4(3a+4b)(c+dx)+8(a+b)\sinh(2(c+dx))+a\sinh(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (4*(3*a + 4*b)*(c + d*x) + 8*(a + b)*Sinh[2*(c + d*x)] + a*Sinh[4*(c + d*x)
])/(32*d)
```

Maple [A] time = 0.04, size = 66, normalized size = 1.1

$$\frac{1}{d}\left(a\left(\left(\frac{(\cosh(dx+c))^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/d*(a*((1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)+3/8*d*x+3/8*c)+b*(1
/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 1.13573, size = 131, normalized size = 2.15

$$\frac{1}{64}a\left(24x+\frac{e^{(4dx+4c)}}{d}+\frac{8e^{(2dx+2c)}}{d}-\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)+\frac{1}{8}b\left(4x+\frac{e^{(2dx+2c)}}{d}-\frac{e^{(-2dx-2c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/64*a*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) + 1/8*b*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c
)/d)
```

Fricas [A] time = 2.06623, size = 163, normalized size = 2.67

$$\frac{a\cosh(dx+c)\sinh(dx+c)^3+(3a+4b)dx+\left(a\cosh(dx+c)^3+4(a+b)\cosh(dx+c)\right)\sinh(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/8*(a*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a + 4*b)*d*x + (a*cosh(d*x + c)^3
 + 4*(a + b)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21788, size = 157, normalized size = 2.57

$$\frac{8\,(dx+c)(3\,a+4\,b)+ae^{(4\,dx+4\,c)}+8\,ae^{(2\,dx+2\,c)}+8\,be^{(2\,dx+2\,c)}-\left(18\,ae^{(4\,dx+4\,c)}+24\,be^{(4\,dx+4\,c)}+8\,ae^{(2\,dx+2\,c)}+8\,be^{(2\,dx+\cdots}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/64*(8*(d*x + c)*(3*a + 4*b) + a*e^(4*d*x + 4*c) + 8*a*e^(2*d*x + 2*c) + 8
*b*e^(2*d*x + 2*c) - (18*a*e^(4*d*x + 4*c) + 24*b*e^(4*d*x + 4*c) + 8*a*e^(
2*d*x + 2*c) + 8*b*e^(2*d*x + 2*c) + a)*e^(-4*d*x - 4*c))/d
```

3.50 $\int \cosh^3(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)dx$

Optimal. Leaf size=30

$$\frac{(a+b)\sinh(c+dx)}{d}+\frac{a\sinh^3(c+dx)}{3d}$$

```
[Out] ((a + b)*Sinh[c + d*x])/d + (a*Sinh[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0502306, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4044, 3013}

$$\frac{(a+b)\sinh(c+dx)}{d}+\frac{a\sinh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + b)*Sinh[c + d*x])/d + (a*Sinh[c + d*x]^3)/(3*d)
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \cosh^3(c+dx)\left(a+b\,\text{sech}^2(c+dx)\right)dx &= \int \cosh(c+dx)\left(b+a\cosh^2(c+dx)\right)dx\\
&= \frac{i\,\text{Subst}\left(\int\left(a+b-ax^2\right)dx,x,-i\sinh(c+dx)\right)}{d}\\
&= \frac{(a+b)\sinh(c+dx)}{d}+\frac{a\sinh^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.0173721, size = 50, normalized size = 1.67

$$\frac{a\sinh^3(c+dx)}{3d}+\frac{a\sinh(c+dx)}{d}+\frac{b\sinh(c)\cosh(dx)}{d}+\frac{b\cosh(c)\sinh(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (b*Cosh[d*x]*Sinh[c])/d + (b*Cosh[c]*Sinh[d*x])/d + (a*Sinh[c + d*x])/d + (
a*Sinh[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.037, size = 34, normalized size = 1.1

$$\frac{1}{d}\left(a\left(\frac{2}{3}+\frac{(\cosh(dx+c))^{2}}{3}\right)\sinh(dx+c)+b\sinh(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/d*(a*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c)+b*sinh(d*x+c))
```

Maxima [B] time = 1.14902, size = 115, normalized size = 3.83

$$\frac{1}{24}\,a\left(\frac{e^{(3\,dx+3\,c)}}{d}+\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}-\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{1}{2}\,b\left(\frac{e^{(dx+c)}}{d}-\frac{e^{(-dx-c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/24*a*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x
- 3*c)/d) + 1/2*b*(e^(d*x + c)/d - e^(-d*x - c)/d)
```

Fricas [A] time = 2.099, size = 105, normalized size = 3.5

$$\frac{a\sinh(dx+c)^{3}+3\left(a\cosh(dx+c)^{2}+3\,a+4\,b\right)\sinh(dx+c)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/12*(a*sinh(d*x + c)^3 + 3*(a*cosh(d*x + c)^2 + 3*a + 4*b)*sinh(d*x + c))/
d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [B] time = 1.16692, size = 97, normalized size = 3.23

$$\frac{ae^{(3\,dx+3\,c)} + 9\,ae^{(dx+c)} + 12\,be^{(dx+c)} - \left(9\,ae^{(2\,dx+2\,c)} + 12\,be^{(2\,dx+2\,c)} + a\right)e^{(-3\,dx-3\,c)}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/24*(a*e^(3*d*x + 3*c) + 9*a*e^(d*x + c) + 12*b*e^(d*x + c) - (9*a*e^(2*d*
x + 2*c) + 12*b*e^(2*d*x + 2*c) + a)*e^(-3*d*x - 3*c))/d
```

3.51 $\int \cosh^2(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=31

$$\frac{1}{2}x(a+2b)+\frac{a\sinh(c+dx)\cosh(c+dx)}{2d}$$

```
[Out] ((a + 2*b)*x)/2 + (a*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

Rubi [A] time = 0.0314004, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4045, 8}

$$\frac{1}{2}x(a+2b)+\frac{a\sinh(c+dx)\cosh(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2*(a + b*Sech[c + d*x]^2),x]

[Out] ((a + 2*b)*x)/2 + (a*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cosh^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx&=\frac{a\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{1}{2}(a+2b)\int 1\,dx\\&=\frac{1}{2}(a+2b)x+\frac{a\cosh(c+dx)\sinh(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0306696, size = 33, normalized size = 1.06

$$\frac{a(c+dx)}{2d}+\frac{a\sinh(2(c+dx))}{4d}+bx$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2*(a + b*Sech[c + d*x]^2),x]

[Out] b*x + (a*(c + d*x))/(2*d) + (a*Sinh[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.035, size = 37, normalized size = 1.2

$$\frac{1}{d}\left(a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+(dx+c)\,b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/d*(a*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c)+(d*x+c)*b)
```

Maxima [A] time = 1.15954, size = 51, normalized size = 1.65

$$\frac{1}{8}\,a\left(4\,x+\frac{e^{(2\,dx+2\,c)}}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}\right)+bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/8*a*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) + b*x
```

Fricas [A] time = 1.97165, size = 74, normalized size = 2.39

$$\frac{(a+2\,b)dx+a\cosh(dx+c)\sinh(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*((a + 2*b)*d*x + a*cosh(d*x + c)*sinh(d*x + c))/d
```

Sympy [A] time = 47.1013, size = 60, normalized size = 1.94

$$a\left(\begin{cases}-\frac{x\sinh^2(c+dx)}{2}+\frac{x\cosh^2(c+dx)}{2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d} & \text{for } d\neq 0\\ x\cosh^2(c) & \text{otherwise}\end{cases}\right)+b\left(\begin{cases}x & \text{for } |x| <\\ G_{2,2}^{1,1}\left(\begin{matrix}1 & 2\\ 1 & 0\end{matrix}\middle|\,x\right)+G_{2,2}^{0,2}\left(\begin{matrix}2,1 & \\ & 1,0\end{matrix}\middle|\,x\right) & \text{otherwis}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*(a+b*sech(d*x+c)**2),x)
```

```
[Out] a*Piecewise((-x*sinh(c + d*x)**2/2 + x*cosh(c + d*x)**2/2 + sinh(c + d*x)*c
osh(c + d*x)/(2*d), Ne(d, 0)), (x*cosh(c)**2, True)) + b*Piecewise((x, Abs(
x) < 1), (meijerg(((1,), (2,)), ((1,), (0,)), x) + meijerg(((2, 1), ()), ((
), (1, 0)), x), True))
```

Giac [B] time = 1.21136, size = 89, normalized size = 2.87

$$\frac{4\,(dx+c)(a+2\,b)+ae^{(2\,dx+2\,c)}-\left(2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)e^{(-2\,dx-2\,c)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(4*(d*x + c)*(a + 2*b) + a*e^(2*d*x + 2*c) - (2*a*e^(2*d*x + 2*c) + 4*b
*e^(2*d*x + 2*c) + a)*e^(-2*d*x - 2*c))/d
```

3.52 $\int \cosh(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)dx$

Optimal. Leaf size=24

$$\frac{a\sinh(c+dx)}{d}+\frac{b\tan^{-1}(\sinh(c+dx))}{d}$$

```
[Out] (b*ArcTan[Sinh[c + d*x]])/d + (a*Sinh[c + d*x])/d
```

Rubi [A] time = 0.0298312, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4045, 3770}

$$\frac{a\sinh(c+dx)}{d}+\frac{b\tan^{-1}(\sinh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (b*ArcTan[Sinh[c + d*x]])/d + (a*Sinh[c + d*x])/d
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cosh(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)dx &= \frac{a\sinh(c+dx)}{d}+b\int \operatorname{sech}(c+dx)\,dx\\ &= \frac{b\tan^{-1}(\sinh(c+dx))}{d}+\frac{a\sinh(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0171176, size = 35, normalized size = 1.46

$$\frac{a\sinh(c)\cosh(dx)}{d}+\frac{a\cosh(c)\sinh(dx)}{d}+\frac{b\tan^{-1}(\sinh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (b*ArcTan[Sinh[c + d*x]])/d + (a*Cosh[d*x]*Sinh[c])/d + (a*Cosh[c]*Sinh[d*x
])/d
```

Maple [A] time = 0.031, size = 26, normalized size = 1.1

$$\frac{a\sinh(dx+c)}{d}+2\frac{b\arctan\left(e^{dx+c}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*(a+b*sech(d*x+c)^2),x)
```

```
[Out] a*sinh(d*x+c)/d+2/d*b*arctan(exp(d*x+c))
```

Maxima [A] time = 1.60006, size = 38, normalized size = 1.58

$$-\frac{2\,b\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{a\sinh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -2*b*arctan(e^(-d*x - c))/d + a*sinh(d*x + c)/d
```

Fricas [B] time = 2.15015, size = 266, normalized size = 11.08

$$\frac{a\cosh(dx+c)^2+2\,a\cosh(dx+c)\sinh(dx+c)+a\sinh(dx+c)^2+4\,(b\cosh(dx+c)+b\sinh(dx+c))\arctan(\cos}{2\,(d\cosh(dx+c)+d\sinh(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^
2 + 4*(b*cosh(d*x + c) + b*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x +
 c)) - a)/(d*cosh(d*x + c) + d*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\,\mathrm{sech}^2(c+dx)\right)\cosh(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*cosh(c + d*x), x)
```

Giac [A] time = 1.19392, size = 55, normalized size = 2.29

$$\frac{2\,b\arctan\left(e^{(dx+c)}\right)}{d}+\frac{ae^{(dx+c)}}{2\,d}-\frac{ae^{(-dx-c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 2*b*arctan(e^(d*x + c))/d + 1/2*a*e^(d*x + c)/d - 1/2*a*e^(-d*x - c)/d
```

3.53 $\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)dx$

Optimal. Leaf size=40

$$\frac{(2a+b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{b\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}$$

```
[Out] ((2*a + b)*ArcTan[Sinh[c + d*x]])/(2*d) + (b*Sech[c + d*x]*Tanh[c + d*x])/(
2*d)
```

Rubi [A] time = 0.0265989, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4046, 3770}

$$\frac{(2a+b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{b\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((2*a + b)*ArcTan[Sinh[c + d*x]])/(2*d) + (b*Sech[c + d*x]*Tanh[c + d*x])/(
2*d)
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)dx &= \frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}+\frac{1}{2}(2a+b)\int \operatorname{sech}(c+dx)\,dx\\ &= \frac{(2a+b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0207111, size = 48, normalized size = 1.2

$$\frac{a\tan^{-1}(\sinh(c+dx))}{d}+\frac{b\tan^{-1}(\sinh(c+dx))}{2d}+\frac{b\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (a*ArcTan[Sinh[c + d*x]])/d + (b*ArcTan[Sinh[c + d*x]])/(2*d) + (b*Sech[c +
 d*x]*Tanh[c + d*x])/(2*d)
```

Maple [A] time = 0.02, size = 45, normalized size = 1.1

$$2\,\frac{a\arctan\left(\mathrm{e}^{dx+c}\right)}{d}+\frac{b\mathrm{sech}\left(dx+c\right)\tanh\left(dx+c\right)}{2\,d}+\frac{b\arctan\left(\mathrm{e}^{dx+c}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)*(a+b*sech(d*x+c)^2),x)
```

```
[Out] 2/d*a*arctan(exp(d*x+c))+1/2*b*sech(d*x+c)*tanh(d*x+c)/d+1/d*b*arctan(exp(d
*x+c))
```

Maxima [B] time = 1.72274, size = 109, normalized size = 2.72

$$-b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1\right)}\right)+\frac{a\arctan\left(\sinh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*
d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a*arctan(sinh(d*x + c))/d
```

Fricas [B] time = 2.09404, size = 882, normalized size = 22.05

$$\frac{b\cosh\left(dx+c\right)^{3}+3\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}+b\sinh\left(dx+c\right)^{3}+\left(\left(2\,a+b\right)\cosh\left(dx+c\right)^{4}+4\,\left(2\,a+b\right)\cosh\left(dx+c\right.}{d\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] (b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3
+ ((2*a + b)*cosh(d*x + c)^4 + 4*(2*a + b)*cosh(d*x + c)*sinh(d*x + c)^3 +
(2*a + b)*sinh(d*x + c)^4 + 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*(2*a + b)*co
sh(d*x + c)^2 + 2*a + b)*sinh(d*x + c)^2 + 4*((2*a + b)*cosh(d*x + c)^3 + (
2*a + b)*cosh(d*x + c))*sinh(d*x + c) + 2*a + b)*arctan(cosh(d*x + c) + sin
h(d*x + c)) - b*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c))/(d
*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 +
2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 4*(d*co
sh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\,\mathrm{sech}^{2}\left(c+dx\right)\right)\mathrm{sech}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*sech(c + d*x), x)
```

Giac [B] time = 1.26995, size = 115, normalized size = 2.88

$$\frac{\left(\pi + 2 \arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)} - 1\right)e^{(-dx-c)}\right)\right)(2\,a + b)}{4\,d} + \frac{b\left(e^{(dx+c)} - e^{(-dx-c)}\right)}{\left(\left(e^{(dx+c)} - e^{(-dx-c)}\right)^2 + 4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/4*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(2*a + b)/d + b
*(e^(d*x + c) - e^(-d*x - c))/(((e^(d*x + c) - e^(-d*x - c))^2 + 4)*d)
```

3.54 $\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=30

$$\frac{(a+b)\tanh(c+dx)}{d}-\frac{b\tanh^3(c+dx)}{3d}$$

```
[Out] ((a + b)*Tanh[c + d*x])/d - (b*Tanh[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0402439, antiderivative size = 43, normalized size of antiderivative = 1.43, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4046, 3767, 8}

$$\frac{(3a+2b)\tanh(c+dx)}{3d}+\frac{b\tanh(c+dx)\operatorname{sech}^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((3*a + 2*b)*Tanh[c + d*x])/(3*d) + (b*Sech[c + d*x]^2*Tanh[c + d*x])/(3*d)
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)dx&=\frac{b\operatorname{sech}^2(c+dx)\tanh(c+dx)}{3d}+\frac{1}{3}(3a+2b)\int \operatorname{sech}^2(c+dx)\,dx\\&=\frac{b\operatorname{sech}^2(c+dx)\tanh(c+dx)}{3d}+\frac{(i(3a+2b))\operatorname{Subst}(\int 1\,dx,x,-i\tanh(c+dx)}{3d}\\&=\frac{(3a+2b)\tanh(c+dx)}{3d}+\frac{b\operatorname{sech}^2(c+dx)\tanh(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.0116299, size = 39, normalized size = 1.3

$$\frac{a\tanh(c+dx)}{d}-\frac{b\tanh^3(c+dx)}{3d}+\frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (a*Tanh[c + d*x])/d + (b*Tanh[c + d*x])/d - (b*Tanh[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.019, size = 34, normalized size = 1.1

$$\frac{1}{d}\left(a\tanh(dx+c)+b\left(\frac{2}{3}+\frac{(\mathrm{sech}(dx+c))^2}{3}\right)\tanh(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2*(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/d*(a*tanh(d*x+c)+b*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c))
```

Maxima [B] time = 1.04349, size = 151, normalized size = 5.03

$$\frac{4}{3}b\left(\frac{3e^{(-2dx-2c)}}{d\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1\right)}+\frac{1}{d\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1\right)}\right)+\frac{2a}{d\left(e^{(-2dx-2c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 4/3*b*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6
*d*x - 6*c) + 1))) + 2*a/(d*(e^(-2*d*x - 2*c) + 1))
```

Fricas [B] time = 1.92377, size = 427, normalized size = 14.23

$$-\frac{4\left((3a+b)\cosh(dx+c)^2-2b\cosh(dx+c)\sinh(dx+c)+(3a\right.}{3\left(d\cosh(dx+c)^4+4d\cosh(dx+c)\sinh(dx+c)^3+d\sinh(dx+c)^4+4d\cosh(dx+c)^2+2\left(3d\cosh(dx+c)^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -4/3*((3*a + b)*cosh(d*x + c)^2 - 2*b*cosh(d*x + c)*sinh(d*x + c) + (3*a +
b)*sinh(d*x + c)^2 + 3*a + 3*b)/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh
(d*x + c)^3 + d*sinh(d*x + c)^4 + 4*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c
)^2 + 2*d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d
*x + c) + 3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\,\mathrm{sech}^2(c+dx)\right)\mathrm{sech}^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*sech(c + d*x)**2, x)
```

Giac [B] time = 1.36054, size = 82, normalized size = 2.73

$$-\frac{2\left(3\,ae^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}+6\,be^{(2\,dx+2\,c)}+3\,a+2\,b\right)}{3\,d\left(e^{(2\,dx+2\,c)}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -2/3*(3*a*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c) + 6*b*e^(2*d*x + 2*c) + 3*a
 + 2*b)/(d*(e^(2*d*x + 2*c) + 1)^3)
```

3.55 $\int \mathbf{sech}^3(c+dx)\left(a+b\mathbf{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=70

$$\frac{(4a+3b)\tan^{-1}(\sinh(c+dx))}{8d}+\frac{(4a+3b)\tanh(c+dx)\text{sech}(c+dx)}{8d}+\frac{b\tanh(c+dx)\text{sech}^3(c+dx)}{4d}$$

```
[Out] ((4*a + 3*b)*ArcTan[Sinh[c + d*x]])/(8*d) + ((4*a + 3*b)*Sech[c + d*x]*Tanh
[c + d*x])/(8*d) + (b*Sech[c + d*x]^3*Tanh[c + d*x])/(4*d)
```

Rubi [A] time = 0.0502423, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4046, 3768, 3770}

$$\frac{(4a+3b)\tan^{-1}(\sinh(c+dx))}{8d}+\frac{(4a+3b)\tanh(c+dx)\text{sech}(c+dx)}{8d}+\frac{b\tanh(c+dx)\text{sech}^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((4*a + 3*b)*ArcTan[Sinh[c + d*x]])/(8*d) + ((4*a + 3*b)*Sech[c + d*x]*Tanh
[c + d*x])/(8*d) + (b*Sech[c + d*x]^3*Tanh[c + d*x])/(4*d)
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \text{sech}^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)dx&=\frac{b\text{sech}^3(c+dx)\tanh(c+dx)}{4d}+\frac{1}{4}(4a+3b)\int \text{sech}^3(c+dx)\,dx\\&=\frac{(4a+3b)\text{sech}(c+dx)\tanh(c+dx)}{8d}+\frac{b\text{sech}^3(c+dx)\tanh(c+dx)}{4d}+\frac{1}{8}\\&=\frac{(4a+3b)\tan^{-1}(\sinh(c+dx))}{8d}+\frac{(4a+3b)\text{sech}(c+dx)\tanh(c+dx)}{8d}+\end{aligned}$$

Mathematica [A] time = 0.104215, size = 60, normalized size = 0.86

$$\frac{(4a+3b)\tan^{-1}(\sinh(c+dx))+(4a+3b)\tanh(c+dx)\text{sech}(c+dx)+2b\tanh(c+dx)\text{sech}^3(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((4*a + 3*b)*ArcTan[Sinh[c + d*x]] + (4*a + 3*b)*Sech[c + d*x]*Tanh[c + d*x
] + 2*b*Sech[c + d*x]^3*Tanh[c + d*x])/(8*d)
```

Maple [A] time = 0.021, size = 83, normalized size = 1.2

$$\frac{a\mathrm{sech}\,(dx+c)\tanh\,(dx+c)}{2d}+\frac{a\arctan\left(\mathrm{e}^{dx+c}\right)}{d}+\frac{b\,(\mathrm{sech}\,(dx+c))^3\tanh\,(dx+c)}{4d}+\frac{3\,b\mathrm{sech}\,(dx+c)\tanh\,(dx+c)}{8d}+\frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3*(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/2/d*a*sech(d*x+c)*tanh(d*x+c)+1/d*a*arctan(exp(d*x+c))+1/4*b*sech(d*x+c)^
3*tanh(d*x+c)/d+3/8*b*sech(d*x+c)*tanh(d*x+c)/d+3/4/d*b*arctan(exp(d*x+c))
```

Maxima [B] time = 1.68479, size = 248, normalized size = 3.54

$$-\frac{1}{4}b\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{3\,e^{(-dx-c)}+11\,e^{(-3\,dx-3\,c)}-11\,e^{(-5\,dx-5\,c)}-3\,e^{(-7\,dx-7\,c)}}{d\left(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1\right)}\right)-a\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-}}{d\left(2\,e^{(-2}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/4*b*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3*d*x - 3*c) -
11*e^(-5*d*x - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*
d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - a*(arctan(e^(-d
*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(
-4*d*x - 4*c) + 1)))
```

Fricas [B] time = 2.15486, size = 2931, normalized size = 41.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*((4*a + 3*b)*cosh(d*x + c)^7 + 7*(4*a + 3*b)*cosh(d*x + c)*sinh(d*x + c
)^6 + (4*a + 3*b)*sinh(d*x + c)^7 + (4*a + 11*b)*cosh(d*x + c)^5 + (21*(4*a
 + 3*b)*cosh(d*x + c)^2 + 4*a + 11*b)*sinh(d*x + c)^5 + 5*(7*(4*a + 3*b)*co
sh(d*x + c)^3 + (4*a + 11*b)*cosh(d*x + c))*sinh(d*x + c)^4 - (4*a + 11*b)*
cosh(d*x + c)^3 + (35*(4*a + 3*b)*cosh(d*x + c)^4 + 10*(4*a + 11*b)*cosh(d*
x + c)^2 - 4*a - 11*b)*sinh(d*x + c)^3 + (21*(4*a + 3*b)*cosh(d*x + c)^5 +
10*(4*a + 11*b)*cosh(d*x + c)^3 - 3*(4*a + 11*b)*cosh(d*x + c))*sinh(d*x +
c)^2 + ((4*a + 3*b)*cosh(d*x + c)^8 + 8*(4*a + 3*b)*cosh(d*x + c)*sinh(d*x
+ c)^7 + (4*a + 3*b)*sinh(d*x + c)^8 + 4*(4*a + 3*b)*cosh(d*x + c)^6 + 4*(7
```

```
*(4*a + 3*b)*cosh(d*x + c)^2 + 4*a + 3*b)*sinh(d*x + c)^6 + 8*(7*(4*a + 3*b
)*cosh(d*x + c)^3 + 3*(4*a + 3*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(4*a +
 3*b)*cosh(d*x + c)^4 + 2*(35*(4*a + 3*b)*cosh(d*x + c)^4 + 30*(4*a + 3*b)*
cosh(d*x + c)^2 + 12*a + 9*b)*sinh(d*x + c)^4 + 8*(7*(4*a + 3*b)*cosh(d*x +
 c)^5 + 10*(4*a + 3*b)*cosh(d*x + c)^3 + 3*(4*a + 3*b)*cosh(d*x + c))*sinh(
d*x + c)^3 + 4*(4*a + 3*b)*cosh(d*x + c)^2 + 4*(7*(4*a + 3*b)*cosh(d*x + c)
^6 + 15*(4*a + 3*b)*cosh(d*x + c)^4 + 9*(4*a + 3*b)*cosh(d*x + c)^2 + 4*a +
 3*b)*sinh(d*x + c)^2 + 8*((4*a + 3*b)*cosh(d*x + c)^7 + 3*(4*a + 3*b)*cosh
(d*x + c)^5 + 3*(4*a + 3*b)*cosh(d*x + c)^3 + (4*a + 3*b)*cosh(d*x + c))*si
nh(d*x + c) + 4*a + 3*b)*arctan(cosh(d*x + c) + sinh(d*x + c)) - (4*a + 3*b
)*cosh(d*x + c) + (7*(4*a + 3*b)*cosh(d*x + c)^6 + 5*(4*a + 11*b)*cosh(d*x
+ c)^4 - 3*(4*a + 11*b)*cosh(d*x + c)^2 - 4*a - 3*b)*sinh(d*x + c))/(d*cosh
(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 + 4*d*c
osh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 8*(7*d*cosh(
d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(
35*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d
*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^
3 + 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 15*d*cosh(d*x + c)^4 + 9
*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 + 3*d*cosh(d
*x + c)^5 + 3*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\operatorname{sech}^2(c+dx)\right)\operatorname{sech}^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*sech(c + d*x)**3, x)
```

Giac [B] time = 1.20099, size = 213, normalized size = 3.04

$$\frac{\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)} - 1\right)e^{(-dx-c)}\right)\right)(4a+3b)}{16d} + \frac{4a\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 + 3b\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 + 16a\left(e^{(dx+c)} - \right.}{4\left(\left(e^{(dx+c)} - e^{(-dx-c)}\right)^2 + 4\right)^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/16*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(4*a + 3*b)/d
+ 1/4*(4*a*(e^(d*x + c) - e^(-d*x - c))^3 + 3*b*(e^(d*x + c) - e^(-d*x - c)
)^3 + 16*a*(e^(d*x + c) - e^(-d*x - c)) + 20*b*(e^(d*x + c) - e^(-d*x - c))
)/(((e^(d*x + c) - e^(-d*x - c))^2 + 4)^2*d)
```

3.56 $\int \operatorname{sech}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=50

$$-\frac{(a+2b)\tanh^3(c+dx)}{3d}+\frac{(a+b)\tanh(c+dx)}{d}+\frac{b\tanh^5(c+dx)}{5d}$$

```
[Out] ((a + b)*Tanh[c + d*x])/d - ((a + 2*b)*Tanh[c + d*x]^3)/(3*d) + (b*Tanh[c +
 d*x]^5)/(5*d)
```

Rubi [A] time = 0.0460788, antiderivative size = 65, normalized size of antiderivative = 1.3, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4046, 3767}

$$-\frac{(5a+4b)\tanh^3(c+dx)}{15d}+\frac{(5a+4b)\tanh(c+dx)}{5d}+\frac{b\tanh(c+dx)\operatorname{sech}^4(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((5*a + 4*b)*Tanh[c + d*x])/(5*d) + (b*Sech[c + d*x]^4*Tanh[c + d*x])/(5*d)
 - ((5*a + 4*b)*Tanh[c + d*x]^3)/(15*d)
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)dx&=\frac{b\operatorname{sech}^4(c+dx)\tanh(c+dx)}{5d}+\frac{1}{5}(5a+4b)\int \operatorname{sech}^4(c+dx)\,dx\\&=\frac{b\operatorname{sech}^4(c+dx)\tanh(c+dx)}{5d}+\frac{(i(5a+4b))\operatorname{Subst}\left(\int\left(1+x^2\right)dx,x,-i\tanh\right.}{5d}\\&=\frac{(5a+4b)\tanh(c+dx)}{5d}+\frac{b\operatorname{sech}^4(c+dx)\tanh(c+dx)}{5d}-\frac{(5a+4b)\tanh^3(c}{15d}\end{aligned}$$

Mathematica [A] time = 0.0129559, size = 71, normalized size = 1.42

$$-\frac{a\tanh^3(c+dx)}{3d}+\frac{a\tanh(c+dx)}{d}+\frac{b\tanh^5(c+dx)}{5d}-\frac{2b\tanh^3(c+dx)}{3d}+\frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (a*Tanh[c + d*x])/d + (b*Tanh[c + d*x])/d - (a*Tanh[c + d*x]^3)/(3*d) - (2*
b*Tanh[c + d*x]^3)/(3*d) + (b*Tanh[c + d*x]^5)/(5*d)
```

Maple [A] time = 0.02, size = 56, normalized size = 1.1

$$\frac{1}{d}\left(a\left(\frac{2}{3}+\frac{(\mathrm{sech}\,(dx+c))^2}{3}\right)\tanh(dx+c)+b\left(\frac{8}{15}+\frac{(\mathrm{sech}\,(dx+c))^4}{5}+\frac{4\,(\mathrm{sech}\,(dx+c))^2}{15}\right)\tanh(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4*(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/d*(a*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+b*(8/15+1/5*sech(d*x+c)^4+4/15*s
ech(d*x+c)^2)*tanh(d*x+c))
```

Maxima [B] time = 1.05755, size = 405, normalized size = 8.1

$$\frac{16}{15}\,b\left(\frac{5\,e^{(-2\,dx-2\,c)}}{d\left(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1\right)}+\frac{1}{d\left(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+1\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 16/15*b*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) +
10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 10*e^
(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
- 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x
- 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e
^(-10*d*x - 10*c) + 1))) + 4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c)
 + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) +
 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

Fricas [B] time = 2.06339, size = 938, normalized size = 18.76

$$-\frac{}{15\left(d\cosh(dx+c)^7+7\,d\cosh(dx+c)\sinh(dx+c)^6+d\sinh(dx+c)^7+5\,d\cosh(dx+c)^5+\left(21\,d\cosh(dx+c)^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -8/15*(2*(5*a + b)*cosh(d*x + c)^3 + 6*(5*a + b)*cosh(d*x + c)*sinh(d*x + c
)^2 + (5*a - 2*b)*sinh(d*x + c)^3 + 30*(a + b)*cosh(d*x + c) + (3*(5*a - 2*
b)*cosh(d*x + c)^2 + 5*a + 10*b)*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*co
sh(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 + 5*d*cosh(d*x + c)^5 + (21
*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^5 + 5*(7*d*cosh(d*x + c)^3 + 5*d*co
sh(d*x + c))*sinh(d*x + c)^4 + 11*d*cosh(d*x + c)^3 + (35*d*cosh(d*x + c)^4
```

```
 + 50*d*cosh(d*x + c)^2 + 9*d)*sinh(d*x + c)^3 + (21*d*cosh(d*x + c)^5 + 50
*d*cosh(d*x + c)^3 + 33*d*cosh(d*x + c))*sinh(d*x + c)^2 + 15*d*cosh(d*x +
c) + (7*d*cosh(d*x + c)^6 + 25*d*cosh(d*x + c)^4 + 27*d*cosh(d*x + c)^2 + 5
*d)*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\,\mathrm{sech}^2\left(c + dx\right)\right)\mathrm{sech}^4\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*sech(c + d*x)**4, x)
```

Giac [A] time = 1.1446, size = 115, normalized size = 2.3

$$-\frac{4\left(15\,ae^{(6\,dx+6\,c)} + 35\,ae^{(4\,dx+4\,c)} + 40\,be^{(4\,dx+4\,c)} + 25\,ae^{(2\,dx+2\,c)} + 20\,be^{(2\,dx+2\,c)} + 5\,a + 4\,b\right)}{15\,d\left(e^{(2\,dx+2\,c)} + 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -4/15*(15*a*e^(6*d*x + 6*c) + 35*a*e^(4*d*x + 4*c) + 40*b*e^(4*d*x + 4*c) +
 25*a*e^(2*d*x + 2*c) + 20*b*e^(2*d*x + 2*c) + 5*a + 4*b)/(d*(e^(2*d*x + 2*
c) + 1)^5)
```

3.57 $\int \cosh^4(c+dx)\left(a+b\mathbf{sech}^2(c+dx)\right)^2 dx$

Optimal. Leaf size=82

$$\frac{1}{8}x\left(3a^2+8ab+8b^2\right)+\frac{3a(a+2b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{a\sinh(c+dx)\cosh^3(c+dx)\left(a-b\tanh^2(c+dx)+b\right)}{4d}$$

```
[Out] ((3*a^2 + 8*a*b + 8*b^2)*x)/8 + (3*a*(a + 2*b)*Cosh[c + d*x]*Sinh[c + d*x])
/(8*d) + (a*Cosh[c + d*x]^3*Sinh[c + d*x]*(a + b - b*Tanh[c + d*x]^2))/(4*d
)
```

Rubi [A] time = 0.0929244, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4146, 413, 385, 206}

$$\frac{1}{8}x\left(3a^2+8ab+8b^2\right)+\frac{3a(a+2b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{a\sinh(c+dx)\cosh^3(c+dx)\left(a-b\tanh^2(c+dx)+b\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((3*a^2 + 8*a*b + 8*b^2)*x)/8 + (3*a*(a + 2*b)*Cosh[c + d*x]*Sinh[c + d*x])
/(8*d) + (a*Cosh[c + d*x]^3*Sinh[c + d*x]*(a + b - b*Tanh[c + d*x]^2))/(4*d
)
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cosh^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2 dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+b-bx^2\right)^2}{\left(1-x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{a\cosh^3(c+dx)\sinh(c+dx)\left(a+b-b\tanh^2(c+dx)\right)}{4d} - \frac{\operatorname{Subst}\left(\int \frac{-(a+b)(3a}{}\right.}{}$$

$$= \frac{3a(a+2b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{a\cosh^3(c+dx)\sinh(c+dx)\left(a+b\right.}{4d}$$

$$= \frac{1}{8}\left(3a^2+8ab+8b^2\right)x + \frac{3a(a+2b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{a\cosh^3(c}{}$$

Mathematica [A] time = 0.129441, size = 58, normalized size = 0.71

$$\frac{4\left(3a^2+8ab+8b^2\right)(c+dx)+a^2\sinh(4(c+dx))+8a(a+2b)\sinh(2(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (4*(3*a^2 + 8*a*b + 8*b^2)*(c + d*x) + 8*a*(a + 2*b)*Sinh[2*(c + d*x)] + a^
2*Sinh[4*(c + d*x)])/(32*d)
```

Maple [A] time = 0.041, size = 79, normalized size = 1.

$$\frac{1}{d}\left(a^2\left(\left(\frac{(\cosh(dx+c))^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(1/2\cosh(dx+c)\sinh(dx+c)+1/2dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*((1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)+3/8*d*x+3/8*c)+2*
a*b*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c)+b^2*(d*x+c))
```

Maxima [A] time = 1.04198, size = 142, normalized size = 1.73

$$\frac{1}{64}a^2\left(24x+\frac{e^{(4dx+4c)}}{d}+\frac{8e^{(2dx+2c)}}{d}-\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)+\frac{1}{4}ab\left(4x+\frac{e^{(2dx+2c)}}{d}-\frac{e^{(-2dx-2c)}}{d}\right)+b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/64*a^2*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 1/4*a*b*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x -
```

```
2*c)/d) + b^2*x
```

Fricas [A] time = 2.20434, size = 193, normalized size = 2.35

$$\frac{a^2 \cosh(dx+c)\sinh(dx+c)^3 + \left(3a^2 + 8ab + 8b^2\right)dx + \left(a^2\cosh(dx+c)^3 + 4\left(a^2 + 2ab\right)\cosh(dx+c)\right)\sinh(dx+}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(a^2*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 + 8*a*b + 8*b^2)*d*x + (a^2
*cosh(d*x + c)^3 + 4*(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15788, size = 204, normalized size = 2.49

$$\frac{a^2 e^{(4dx+4c)} + 8a^2 e^{(2dx+2c)} + 16abe^{(2dx+2c)} + 8\left(3a^2 + 8ab + 8b^2\right)(dx+c) - \left(18a^2 e^{(4dx+4c)} + 48abe^{(4dx+4c)} + 48b^2 e^{(4}\right.}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/64*(a^2*e^(4*d*x + 4*c) + 8*a^2*e^(2*d*x + 2*c) + 16*a*b*e^(2*d*x + 2*c)
+ 8*(3*a^2 + 8*a*b + 8*b^2)*(d*x + c) - (18*a^2*e^(4*d*x + 4*c) + 48*a*b*e^
(4*d*x + 4*c) + 48*b^2*e^(4*d*x + 4*c) + 8*a^2*e^(2*d*x + 2*c) + 16*a*b*e^(
2*d*x + 2*c) + a^2)*e^(-4*d*x - 4*c))/d
```

3.58 $\int \cosh^3(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=49

$$\frac{a^2\sinh^3(c+dx)}{3d}+\frac{a(a+2b)\sinh(c+dx)}{d}+\frac{b^2\tan^{-1}(\sinh(c+dx))}{d}$$

```
[Out] (b^2*ArcTan[Sinh[c + d*x]])/d + (a*(a + 2*b)*Sinh[c + d*x])/d + (a^2*Sinh[c
 + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0592035, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4147, 390, 203}

$$\frac{a^2\sinh^3(c+dx)}{3d}+\frac{a(a+2b)\sinh(c+dx)}{d}+\frac{b^2\tan^{-1}(\sinh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (b^2*ArcTan[Sinh[c + d*x]])/d + (a*(a + 2*b)*Sinh[c + d*x])/d + (a^2*Sinh[c
 + d*x]^3)/(3*d)
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cosh^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+ax^2\right)^2}{1+x^2} dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(a(a+2b)+a^2x^2+\frac{b^2}{1+x^2}\right) dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{a(a+2b)\sinh(c+dx)}{d}+\frac{a^2\sinh^3(c+dx)}{3d}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sin\right.}{d}$$
$$= \frac{b^2\tan^{-1}(\sinh(c+dx))}{d}+\frac{a(a+2b)\sinh(c+dx)}{d}+\frac{a^2\sinh^3(c+dx)}{3d}$$

Mathematica [A] time = 0.0260091, size = 72, normalized size = 1.47

$$\frac{a^2\sinh^3(c+dx)}{3d}+\frac{a^2\sinh(c+dx)}{d}+\frac{2ab\sinh(c)\cosh(dx)}{d}+\frac{2ab\cosh(c)\sinh(dx)}{d}+\frac{b^2\tan^{-1}(\sinh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (b^2*ArcTan[Sinh[c + d*x]])/d + (2*a*b*Cosh[d*x]*Sinh[c])/d + (2*a*b*Cosh[c
]*Sinh[d*x])/d + (a^2*Sinh[c + d*x])/d + (a^2*Sinh[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.041, size = 66, normalized size = 1.4

$$\frac{2\,a^2\sinh(dx+c)}{3\,d}+\frac{a^2\sinh(dx+c)(\cosh(dx+c))^2}{3\,d}+2\,\frac{ab\sinh(dx+c)}{d}+2\,\frac{b^2\arctan\left(e^{dx+c}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 2/3/d*a^2*sinh(d*x+c)+1/3/d*a^2*sinh(d*x+c)*cosh(d*x+c)^2+2*a*b*sinh(d*x+c)
/d+2/d*b^2*arctan(exp(d*x+c))
```

Maxima [B] time = 1.739, size = 142, normalized size = 2.9

$$\frac{1}{24}a^2\left(\frac{e^{(3\,dx+3\,c)}}{d}+\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}-\frac{e^{(-3\,dx-3\,c)}}{d}\right)+ab\left(\frac{e^{(dx+c)}}{d}-\frac{e^{(-dx-c)}}{d}\right)-\frac{2\,b^2\arctan\left(e^{(-dx-c)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/24*a^2*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*
x - 3*c)/d) + a*b*(e^(d*x + c)/d - e^(-d*x - c)/d) - 2*b^2*arctan(e^(-d*x -
 c))/d
```

Fricas [B] time = 2.2106, size = 1057, normalized size = 21.57

$$\frac{a^2\cosh(dx+c)^6 + 6a^2\cosh(dx+c)\sinh(dx+c)^5 + a^2\sinh(dx+c)^6 + 3\left(3a^2+8ab\right)\cosh(dx+c)^4 + 3\left(5a^2\cosh(d\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(a^2*cosh(d*x + c)^6 + 6*a^2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(
d*x + c)^6 + 3*(3*a^2 + 8*a*b)*cosh(d*x + c)^4 + 3*(5*a^2*cosh(d*x + c)^2 +
 3*a^2 + 8*a*b)*sinh(d*x + c)^4 + 4*(5*a^2*cosh(d*x + c)^3 + 3*(3*a^2 + 8*a
*b)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(3*a^2 + 8*a*b)*cosh(d*x + c)^2 + 3*
(5*a^2*cosh(d*x + c)^4 + 6*(3*a^2 + 8*a*b)*cosh(d*x + c)^2 - 3*a^2 - 8*a*b)
*sinh(d*x + c)^2 - a^2 + 48*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c)^2*si
nh(d*x + c) + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 + b^2*sinh(d*x + c)^3)*ar
ctan(cosh(d*x + c) + sinh(d*x + c)) + 6*(a^2*cosh(d*x + c)^5 + 2*(3*a^2 + 8
*a*b)*cosh(d*x + c)^3 - (3*a^2 + 8*a*b)*cosh(d*x + c))*sinh(d*x + c))/(d*co
sh(d*x + c)^3 + 3*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*d*cosh(d*x + c)*sinh(
d*x + c)^2 + d*sinh(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20217, size = 149, normalized size = 3.04

$$\frac{2\,b^2\arctan\left(e^{(dx+c)}\right)}{d} - \frac{\left(9\,a^2e^{(2\,dx+2\,c)} + 24\,abe^{(2\,dx+2\,c)} + a^2\right)e^{(-3\,dx-3\,c)}}{24\,d} + \frac{a^2d^2e^{(3\,dx+3\,c)} + 9\,a^2d^2e^{(dx+c)} + 24\,abd^2e^{(dx+c)}}{24\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 2*b^2*arctan(e^(d*x + c))/d - 1/24*(9*a^2*e^(2*d*x + 2*c) + 24*a*b*e^(2*d*x
 + 2*c) + a^2)*e^(-3*d*x - 3*c)/d + 1/24*(a^2*d^2*e^(3*d*x + 3*c) + 9*a^2*d
^2*e^(d*x + c) + 24*a*b*d^2*e^(d*x + c))/d^3
```

3.59 $\int \cosh^2(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=47

$$\frac{a^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{1}{2}ax(a+4b)+\frac{b^2\tanh(c+dx)}{d}$$

```
[Out] (a*(a + 4*b)*x)/2 + (a^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (b^2*Tanh[c +
 d*x])/d
```

Rubi [A] time = 0.0784786, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4146, 390, 385, 206}

$$\frac{a^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{1}{2}ax(a+4b)+\frac{b^2\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (a*(a + 4*b)*x)/2 + (a^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (b^2*Tanh[c +
 d*x])/d
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cosh^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b-bx^2\right)^2}{\left(1-x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(b^2+\frac{a(a+2b)-2abx^2}{\left(1-x^2\right)^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b^2\tanh(c+dx)}{d} + \frac{\text{Subst}\left(\int \frac{a(a+2b)-2abx^2}{\left(1-x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{a^2\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{b^2\tanh(c+dx)}{d} + \frac{(a(a+4b))\,\text{Subst}\left(\int \frac{1}{1-x^2}\right.}{2d}$$

$$= \frac{1}{2}a(a+4b)x + \frac{a^2\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{b^2\tanh(c+dx)}{d}$$

Mathematica [A] time = 0.141845, size = 52, normalized size = 1.11

$$\frac{a^2(c+dx)}{2d} + \frac{a^2\sinh(2(c+dx))}{4d} + 2abx + \frac{b^2\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] 2*a*b*x + (a^2*(c + d*x))/(2*d) + (a^2*Sinh[2*(c + d*x)])/(4*d) + (b^2*Tanh
[c + d*x])/d
```

Maple [A] time = 0.036, size = 51, normalized size = 1.1

$$\frac{1}{d}\left(a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2\,(dx+c)\,ab+b^2\tanh(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c)+2*(d*x+c)*a*b+b^2*tanh
(d*x+c))
```

Maxima [A] time = 1.13703, size = 85, normalized size = 1.81

$$\frac{1}{8}\,a^2\left(4\,x+\frac{e^{(2\,dx+2\,c)}}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}\right)+2\,abx+\frac{2\,b^2}{d\left(e^{(-2\,dx-2\,c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/8*a^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) + 2*a*b*x + 2*b^2/(d
*(e^(-2*d*x - 2*c) + 1))
```

Fricas [A] time = 2.14895, size = 196, normalized size = 4.17

$$\frac{a^2\sinh(dx+c)^3+4\left(\left(a^2+4\,ab\right)dx-2\,b^2\right)\cosh(dx+c)+\left(3\,a^2\cosh(dx+c)^2+a^2+8\,b^2\right)\sinh(dx+c)}{8\,d\cosh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(a^2*sinh(d*x + c)^3 + 4*((a^2 + 4*a*b)*d*x - 2*b^2)*cosh(d*x + c) + (3
*a^2*cosh(d*x + c)^2 + a^2 + 8*b^2)*sinh(d*x + c))/(d*cosh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16317, size = 180, normalized size = 3.83

$$\frac{a^2e^{(2\,dx+2\,c)}}{8\,d}+\frac{\left(a^2+4\,ab\right)(dx+c)}{2\,d}-\frac{a^2e^{(4\,dx+4\,c)}+4\,abe^{(4\,dx+4\,c)}+2\,a^2e^{(2\,dx+2\,c)}+4\,abe^{(2\,dx+2\,c)}+16\,b^2e^{(2\,dx+2\,c)}+a^2}{8\,d\left(e^{(4\,dx+4\,c)}+e^{(2\,dx+2\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*a^2*e^(2*d*x + 2*c)/d + 1/2*(a^2 + 4*a*b)*(d*x + c)/d - 1/8*(a^2*e^(4*d
*x + 4*c) + 4*a*b*e^(4*d*x + 4*c) + 2*a^2*e^(2*d*x + 2*c) + 4*a*b*e^(2*d*x
+ 2*c) + 16*b^2*e^(2*d*x + 2*c) + a^2)/(d*(e^(4*d*x + 4*c) + e^(2*d*x + 2*c
)))
```

3.60 $\int \cosh(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=56

$$\frac{a^2\sinh(c+dx)}{d}+\frac{b(4a+b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{b^2\tanh(c+dx)\mathrm{sech}(c+dx)}{2d}$$

```
[Out] (b*(4*a + b)*ArcTan[Sinh[c + d*x]])/(2*d) + (a^2*Sinh[c + d*x])/d + (b^2*Se
ch[c + d*x]*Tanh[c + d*x])/(2*d)
```

Rubi [A] time = 0.0678789, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4147, 390, 385, 203}

$$\frac{a^2\sinh(c+dx)}{d}+\frac{b(4a+b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{b^2\tanh(c+dx)\mathrm{sech}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (b*(4*a + b)*ArcTan[Sinh[c + d*x]])/(2*d) + (a^2*Sinh[c + d*x])/d + (b^2*Se
ch[c + d*x]*Tanh[c + d*x])/(2*d)
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cosh(c+dx)\left(a+b\,\text{sech}^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+ax^2\right)^2}{\left(1+x^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2+\frac{b(2a+b)+2abx^2}{\left(1+x^2\right)^2}\right)dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{a^2\sinh(c+dx)}{d} + \frac{\text{Subst}\left(\int \frac{b(2a+b)+2abx^2}{\left(1+x^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{a^2\sinh(c+dx)}{d} + \frac{b^2\text{sech}(c+dx)\tanh(c+dx)}{2d} + \frac{(b(4a+b))\,\text{Subst}\left(\int \frac{1}{1\ldots}\right.}{2\ldots}$$
$$= \frac{b(4a+b)\tan^{-1}(\sinh(c+dx))}{2d} + \frac{a^2\sinh(c+dx)}{d} + \frac{b^2\text{sech}(c+dx)\tanh(\ldots}{2d}$$

Mathematica [A] time = 0.0398912, size = 80, normalized size = 1.43

$$\frac{a^2\sinh(c)\cosh(dx)}{d} + \frac{a^2\cosh(c)\sinh(dx)}{d} + \frac{2ab\tan^{-1}(\sinh(c+dx))}{d} + \frac{b^2\tan^{-1}(\sinh(c+dx))}{2d} + \frac{b^2\tanh(c+dx)\text{se}\ldots}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (2*a*b*ArcTan[Sinh[c + d*x]])/d + (b^2*ArcTan[Sinh[c + d*x]])/(2*d) + (a^2*
Cosh[d*x]*Sinh[c])/d + (a^2*Cosh[c]*Sinh[d*x])/d + (b^2*Sech[c + d*x]*Tanh[
c + d*x])/(2*d)
```

Maple [A] time = 0.041, size = 63, normalized size = 1.1

$$\frac{a^2\sinh(dx+c)}{d} + 4\,\frac{ab\arctan\left(e^{dx+c}\right)}{d} + \frac{b^2\text{sech}(dx+c)\tanh(dx+c)}{2\,d} + \frac{b^2\arctan\left(e^{dx+c}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d*a^2*sinh(d*x+c)+4/d*a*b*arctan(exp(d*x+c))+1/2*b^2*sech(d*x+c)*tanh(d*x
+c)/d+1/d*b^2*arctan(exp(d*x+c))
```

Maxima [A] time = 1.59969, size = 136, normalized size = 2.43

$$-b^2\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)} - e^{(-3\,dx-3\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1\right)}\right) - \frac{4\,ab\arctan\left(e^{(-dx-c)}\right)}{d} + \frac{a^2\sinh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -b^2*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-
2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - 4*a*b*arctan(e^(-d*x - c))/d + a^2
```

```
*sinh(d*x + c)/d
```

Fricas [B] time = 2.22962, size = 1678, normalized size = 29.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*cosh(d*x + c)^6 + 6*a^2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d
*x + c)^6 + (a^2 + 2*b^2)*cosh(d*x + c)^4 + (15*a^2*cosh(d*x + c)^2 + a^2 +
 2*b^2)*sinh(d*x + c)^4 + 4*(5*a^2*cosh(d*x + c)^3 + (a^2 + 2*b^2)*cosh(d*x
 + c))*sinh(d*x + c)^3 - (a^2 + 2*b^2)*cosh(d*x + c)^2 + (15*a^2*cosh(d*x +
 c)^4 + 6*(a^2 + 2*b^2)*cosh(d*x + c)^2 - a^2 - 2*b^2)*sinh(d*x + c)^2 - a^
2 + 2*((4*a*b + b^2)*cosh(d*x + c)^5 + 5*(4*a*b + b^2)*cosh(d*x + c)*sinh(d
*x + c)^4 + (4*a*b + b^2)*sinh(d*x + c)^5 + 2*(4*a*b + b^2)*cosh(d*x + c)^3
 + 2*(5*(4*a*b + b^2)*cosh(d*x + c)^2 + 4*a*b + b^2)*sinh(d*x + c)^3 + 2*(5
*(4*a*b + b^2)*cosh(d*x + c)^3 + 3*(4*a*b + b^2)*cosh(d*x + c))*sinh(d*x +
c)^2 + (4*a*b + b^2)*cosh(d*x + c) + (5*(4*a*b + b^2)*cosh(d*x + c)^4 + 6*(
4*a*b + b^2)*cosh(d*x + c)^2 + 4*a*b + b^2)*sinh(d*x + c))*arctan(cosh(d*x
+ c) + sinh(d*x + c)) + 2*(3*a^2*cosh(d*x + c)^5 + 2*(a^2 + 2*b^2)*cosh(d*x
 + c)^3 - (a^2 + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^5 +
5*d*cosh(d*x + c)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5 + 2*d*cosh(d*x + c)^3
 + 2*(5*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 + 2*(5*d*cosh(d*x + c)^3 + 3
*d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) + (5*d*cosh(d*x + c)^4
+ 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17467, size = 157, normalized size = 2.8

$$\frac{a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{2\,d}+\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)\left(4\,ab+b^2\right)}{4\,d}+\frac{b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*a^2*(e^(d*x + c) - e^(-d*x - c))/d + 1/4*(pi + 2*arctan(1/2*(e^(2*d*x +
 2*c) - 1)*e^(-d*x - c)))*(4*a*b + b^2)/d + b^2*(e^(d*x + c) - e^(-d*x - c)
)/(((e^(d*x + c) - e^(-d*x - c))^2 + 4)*d)
```

3.61 $\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=90

$$\frac{\left(8a^2+8ab+3b^2\right)\tan^{-1}(\sinh(c+dx))}{8d}+\frac{3b(2a+b)\tanh(c+dx)\operatorname{sech}(c+dx)}{8d}+\frac{b\tanh(c+dx)\operatorname{sech}^3(c+dx)\left(a\sinh\right.}{4d}$$

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTan[Sinh[c + d*x]])/(8*d) + (3*b*(2*a + b)*Sech
[c + d*x]*Tanh[c + d*x])/(8*d) + (b*Sech[c + d*x]^3*(a + b + a*Sinh[c + d*x
]^2)*Tanh[c + d*x])/(4*d)
```

Rubi [A] time = 0.0838808, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4147, 413, 385, 203}

$$\frac{\left(8a^2+8ab+3b^2\right)\tan^{-1}(\sinh(c+dx))}{8d}+\frac{3b(2a+b)\tanh(c+dx)\operatorname{sech}(c+dx)}{8d}+\frac{b\tanh(c+dx)\operatorname{sech}^3(c+dx)\left(a\sinh\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTan[Sinh[c + d*x]])/(8*d) + (3*b*(2*a + b)*Sech
[c + d*x]*Tanh[c + d*x])/(8*d) + (b*Sech[c + d*x]^3*(a + b + a*Sinh[c + d*x
]^2)*Tanh[c + d*x])/(4*d)
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2 dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+b+ax^2\right)^2}{\left(1+x^2\right)^3} dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{b\operatorname{sech}^3(c+dx)\left(a+b+a\sinh^2(c+dx)\right)\tanh(c+dx)}{4d} + \frac{\operatorname{Subst}\left(\int \frac{(a+b)(4a+3}{(1}\right.}{}$$

$$= \frac{3b(2a+b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d} + \frac{b\operatorname{sech}^3(c+dx)\left(a+b+a\sinh^2(c+\right.}{4d}$$

$$= \frac{\left(8a^2+8ab+3b^2\right)\tan^{-1}(\sinh(c+dx))}{8d} + \frac{3b(2a+b)\operatorname{sech}(c+dx)\tanh(c+}{8d}$$

Mathematica [A] time = 0.131312, size = 71, normalized size = 0.79

$$\frac{\left(8a^2+8ab+3b^2\right)\tan^{-1}(\sinh(c+dx))+b(8a+3b)\tanh(c+dx)\operatorname{sech}(c+dx)+2b^2\tanh(c+dx)\operatorname{sech}^3(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTan[Sinh[c + d*x]] + b*(8*a + 3*b)*Sech[c + d*x
]*Tanh[c + d*x] + 2*b^2*Sech[c + d*x]^3*Tanh[c + d*x])/(8*d)
```

Maple [A] time = 0.027, size = 106, normalized size = 1.2

$$2\frac{a^2\arctan\left(e^{dx+c}\right)}{d} + \frac{ab\operatorname{sech}(dx+c)\tanh(dx+c)}{d} + 2\frac{ab\arctan\left(e^{dx+c}\right)}{d} + \frac{b^2\tanh(dx+c)(\operatorname{sech}(dx+c))^3}{4d} + \frac{3b^2\operatorname{sec}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 2/d*a^2*arctan(exp(d*x+c))+1/d*a*b*sech(d*x+c)*tanh(d*x+c)+2/d*a*b*arctan(e
xp(d*x+c))+1/4/d*b^2*tanh(d*x+c)*sech(d*x+c)^3+3/8*b^2*sech(d*x+c)*tanh(d*x
+c)/d+3/4/d*b^2*arctan(exp(d*x+c))
```

Maxima [B] time = 1.73507, size = 271, normalized size = 3.01

$$-\frac{1}{4}b^2\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{3e^{(-dx-c)}+11e^{(-3dx-3c)}-11e^{(-5dx-5c)}-3e^{(-7dx-7c)}}{d\left(4e^{(-2dx-2c)}+6e^{(-4dx-4c)}+4e^{(-6dx-6c)}+e^{(-8dx-8c)}+1\right)}\right) - 2ab\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{}{d(2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3*d*x - 3*c)
- 11*e^(-5*d*x - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-
4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 2*a*b*(arctan
(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c)
 + e^(-4*d*x - 4*c) + 1))) + a^2*arctan(sinh(d*x + c))/d
```

Fricas [B] time = 2.21768, size = 3445, normalized size = 38.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((8*a*b + 3*b^2)*cosh(d*x + c)^7 + 7*(8*a*b + 3*b^2)*cosh(d*x + c)*sinh
(d*x + c)^6 + (8*a*b + 3*b^2)*sinh(d*x + c)^7 + (8*a*b + 11*b^2)*cosh(d*x +
 c)^5 + (21*(8*a*b + 3*b^2)*cosh(d*x + c)^2 + 8*a*b + 11*b^2)*sinh(d*x + c)
^5 + 5*(7*(8*a*b + 3*b^2)*cosh(d*x + c)^3 + (8*a*b + 11*b^2)*cosh(d*x + c))
*sinh(d*x + c)^4 - (8*a*b + 11*b^2)*cosh(d*x + c)^3 + (35*(8*a*b + 3*b^2)*c
osh(d*x + c)^4 + 10*(8*a*b + 11*b^2)*cosh(d*x + c)^2 - 8*a*b - 11*b^2)*sinh
(d*x + c)^3 + (21*(8*a*b + 3*b^2)*cosh(d*x + c)^5 + 10*(8*a*b + 11*b^2)*cos
h(d*x + c)^3 - 3*(8*a*b + 11*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + ((8*a^2
+ 8*a*b + 3*b^2)*cosh(d*x + c)^8 + 8*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)*
sinh(d*x + c)^7 + (8*a^2 + 8*a*b + 3*b^2)*sinh(d*x + c)^8 + 4*(8*a^2 + 8*a*
b + 3*b^2)*cosh(d*x + c)^6 + 4*(7*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 +
 8*a^2 + 8*a*b + 3*b^2)*sinh(d*x + c)^6 + 8*(7*(8*a^2 + 8*a*b + 3*b^2)*cosh
(d*x + c)^3 + 3*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*
(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*(8*a^2 + 8*a*b + 3*b^2)*cos
h(d*x + c)^4 + 30*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 24*a^2 + 24*a*b
 + 9*b^2)*sinh(d*x + c)^4 + 8*(7*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^5 +
10*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^3 + 3*(8*a^2 + 8*a*b + 3*b^2)*cosh
(d*x + c))*sinh(d*x + c)^3 + 4*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 4*
(7*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^6 + 15*(8*a^2 + 8*a*b + 3*b^2)*cos
h(d*x + c)^4 + 9*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 8*a^2 + 8*a*b +
3*b^2)*sinh(d*x + c)^2 + 8*a^2 + 8*a*b + 3*b^2 + 8*((8*a^2 + 8*a*b + 3*b^2)
*cosh(d*x + c)^7 + 3*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^5 + 3*(8*a^2 + 8
*a*b + 3*b^2)*cosh(d*x + c)^3 + (8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c))*sinh
(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - (8*a*b + 3*b^2)*cosh(d*x
 + c) + (7*(8*a*b + 3*b^2)*cosh(d*x + c)^6 + 5*(8*a*b + 11*b^2)*cosh(d*x +
c)^4 - 3*(8*a*b + 11*b^2)*cosh(d*x + c)^2 - 8*a*b - 3*b^2)*sinh(d*x + c))/(
d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 +
 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 8*(7*d
*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4
 + 2*(35*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 +
8*(7*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x
 + c)^3 + 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 15*d*cosh(d*x + c)
^4 + 9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 + 3*d*
cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\operatorname{sech}^2(c + dx)\right)^2 \operatorname{sech}(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*sech(c + d*x), x)
```

Giac [B] time = 1.15525, size = 232, normalized size = 2.58

$$\frac{\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)} - 1\right)e^{(-dx-c)}\right)\right)\left(8\,a^2 + 8\,ab + 3\,b^2\right)}{16\,d} + \frac{8\,ab\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 + 3\,b^2\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 + 32\,ab\left(\right.}{4\left(\left(e^{(dx+c)} - e^{(-dx-c)}\right)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/16*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(8*a^2 + 8*a*b
 + 3*b^2)/d + 1/4*(8*a*b*(e^(d*x + c) - e^(-d*x - c))^3 + 3*b^2*(e^(d*x + c
) - e^(-d*x - c))^3 + 32*a*b*(e^(d*x + c) - e^(-d*x - c)) + 20*b^2*(e^(d*x
+ c) - e^(-d*x - c)))/(((e^(d*x + c) - e^(-d*x - c))^2 + 4)^2*d)
```

3.62 $\int \mathbf{sech}^2(c+dx)\left(a+b\mathbf{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=53

$$-\frac{2b(a+b)\tanh^3(c+dx)}{3d}+\frac{(a+b)^2\tanh(c+dx)}{d}+\frac{b^2\tanh^5(c+dx)}{5d}$$

```
[Out] ((a + b)^2*Tanh[c + d*x])/d - (2*b*(a + b)*Tanh[c + d*x]^3)/(3*d) + (b^2*Ta
nh[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0656931, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4146, 194}

$$-\frac{2b(a+b)\tanh^3(c+dx)}{3d}+\frac{(a+b)^2\tanh(c+dx)}{d}+\frac{b^2\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]
```


```
[Out] ((a + b)^2*Tanh[c + d*x])/d - (2*b*(a + b)*Tanh[c + d*x]^3)/(3*d) + (b^2*Ta
nh[c + d*x]^5)/(5*d)
```


Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx &= \frac{\operatorname{Subst}\left(\int \left(a+b-bx^2\right)^2\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(a^2\left(1+\frac{b(2a+b)}{a^2}\right)-2ab\left(1+\frac{b}{a}\right)x^2+b^2x^4\right)dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{(a+b)^2\tanh(c+dx)}{d}-\frac{2b(a+b)\tanh^3(c+dx)}{3d}+\frac{b^2\tanh^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.0295908, size = 93, normalized size = 1.75

$$\frac{a^2\tanh(c+dx)}{d}-\frac{2ab\tanh^3(c+dx)}{3d}+\frac{2ab\tanh(c+dx)}{d}+\frac{b^2\tanh^5(c+dx)}{5d}-\frac{2b^2\tanh^3(c+dx)}{3d}+\frac{b^2\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Tanh[c + d*x])/d + (2*a*b*Tanh[c + d*x])/d + (b^2*Tanh[c + d*x])/d - (
2*a*b*Tanh[c + d*x]^3)/(3*d) - (2*b^2*Tanh[c + d*x]^3)/(3*d) + (b^2*Tanh[c
+ d*x]^5)/(5*d)
```

Maple [A] time = 0.023, size = 70, normalized size = 1.3

$$\frac{1}{d}\left(a^2\tanh(dx+c)+2ab\left(2/3+1/3\,(\operatorname{sech}(dx+c))^2\right)\tanh(dx+c)+b^2\left(\frac{8}{15}+\frac{(\operatorname{sech}(dx+c))^4}{5}+\frac{4\,(\operatorname{sech}(dx+c))^2}{15}\right)\text{ta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*tanh(d*x+c)+2*a*b*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+b^2*(8/15+1/
5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c))
```

Maxima [B] time = 1.11952, size = 437, normalized size = 8.25

$$\frac{16}{15}b^2\left(\frac{5\,e^{(-2\,dx-2\,c)}}{d\left(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1\right)}+\frac{10}{d\left(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 16/15*b^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c)
+ 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 10*
e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*
x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*
x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) +
 e^(-10*d*x - 10*c) + 1))) + 8/3*a*b*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x -
2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*
c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 2*a^2/(d*(e^(-2*d*x - 2
*c) + 1))
```

Fricas [B] time = 2.07396, size = 1054, normalized size = 19.89

$$-\frac{4\left(\left(15\,a^2+10\,ab+4\,b^2\right)\cosh(dx+c)^4-8\left(5\,ab+2\,b^2\right)\cosh(dx+c)\sinh(dx+c)^3+\left(15\,a^2+10\,ab+4\,b^2\right.\right.}{15\left(d\cosh(dx+c)^6+6\,d\cosh(dx+c)\sinh(dx+c)^5+d\sinh(dx+c)^6+6\,d\cosh(dx+c)^4+3\left(5\,d\cosh(dx+c)^2+2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -4/15*((15*a^2 + 10*a*b + 4*b^2)*cosh(d*x + c)^4 - 8*(5*a*b + 2*b^2)*cosh(d
*x + c)*sinh(d*x + c)^3 + (15*a^2 + 10*a*b + 4*b^2)*sinh(d*x + c)^4 + 20*(3
*a^2 + 4*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(15*a^2 + 10*a*b + 4*b^2)*cosh(d
*x + c)^2 + 30*a^2 + 40*a*b + 10*b^2)*sinh(d*x + c)^2 + 45*a^2 + 70*a*b + 4
```

```
0*b^2 - 8*((5*a*b + 2*b^2)*cosh(d*x + c)^3 + 5*(a*b + b^2)*cosh(d*x + c))*s
inh(d*x + c))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*si
nh(d*x + c)^6 + 6*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 + 2*d)*sinh(d*
x + c)^4 + 4*(5*d*cosh(d*x + c)^3 + 4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 15
*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x + c)^4 + 12*d*cosh(d*x + c)^2 + 5*d)*s
inh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^5 + 8*d*cosh(d*x + c)^3 + 5*d*cosh(d*
x + c))*sinh(d*x + c) + 10*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\,\mathrm{sech}^2\left(c + dx\right)\right)^2 \mathrm{sech}^2\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*sech(c + d*x)**2, x)
```

Giac [B] time = 1.1662, size = 211, normalized size = 3.98

$$-\frac{2\left(15\,a^2e^{(8\,dx+8\,c)} + 60\,a^2e^{(6\,dx+6\,c)} + 60\,abe^{(6\,dx+6\,c)} + 90\,a^2e^{(4\,dx+4\,c)} + 140\,abe^{(4\,dx+4\,c)} + 80\,b^2e^{(4\,dx+4\,c)} + 60\,a^2e^{(2\,dx+2\,c)}\right.}{15\,d\left(e^{(2\,dx+2\,c)} + 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -2/15*(15*a^2*e^(8*d*x + 8*c) + 60*a^2*e^(6*d*x + 6*c) + 60*a*b*e^(6*d*x +
6*c) + 90*a^2*e^(4*d*x + 4*c) + 140*a*b*e^(4*d*x + 4*c) + 80*b^2*e^(4*d*x +
 4*c) + 60*a^2*e^(2*d*x + 2*c) + 100*a*b*e^(2*d*x + 2*c) + 40*b^2*e^(2*d*x
+ 2*c) + 15*a^2 + 20*a*b + 8*b^2)/(d*(e^(2*d*x + 2*c) + 1)^5)
```

3.63 $\int \mathbf{sech}^3(c+dx)\left(a+b\mathbf{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=128

$$\frac{\left(8a^2+12ab+5b^2\right)\tan^{-1}(\sinh(c+dx))}{16d}+\frac{\left(8a^2+12ab+5b^2\right)\tanh(c+dx)\text{sech}(c+dx)}{16d}+\frac{b(8a+5b)\tanh(c+dx)\text{sech}}{24d}$$

```
[Out] ((8*a^2 + 12*a*b + 5*b^2)*ArcTan[Sinh[c + d*x]])/(16*d) + ((8*a^2 + 12*a*b
+ 5*b^2)*Sech[c + d*x]*Tanh[c + d*x])/(16*d) + (b*(8*a + 5*b)*Sech[c + d*x]
^3*Tanh[c + d*x])/(24*d) + (b*Sech[c + d*x]^5*(a + b + a*Sinh[c + d*x]^2)*T
anh[c + d*x])/(6*d)
```

Rubi [A] time = 0.149448, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4147, 413, 385, 199, 203}

$$\frac{\left(8a^2+12ab+5b^2\right)\tan^{-1}(\sinh(c+dx))}{16d}+\frac{\left(8a^2+12ab+5b^2\right)\tanh(c+dx)\text{sech}(c+dx)}{16d}+\frac{b(8a+5b)\tanh(c+dx)\text{sech}}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((8*a^2 + 12*a*b + 5*b^2)*ArcTan[Sinh[c + d*x]])/(16*d) + ((8*a^2 + 12*a*b
+ 5*b^2)*Sech[c + d*x]*Tanh[c + d*x])/(16*d) + (b*(8*a + 5*b)*Sech[c + d*x]
^3*Tanh[c + d*x])/(24*d) + (b*Sech[c + d*x]^5*(a + b + a*Sinh[c + d*x]^2)*T
anh[c + d*x])/(6*d)
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
```

```
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \mathrm{sech}^3(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2 dx = \frac{\mathrm{Subst}\left(\int \frac{\left(a+b+ax^2\right)^2}{\left(1+x^2\right)^4}dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{b\,\mathrm{sech}^5(c+dx)\left(a+b+a\sinh^2(c+dx)\right)\tanh(c+dx)}{6d} + \frac{\mathrm{Subst}\left(\int \frac{(a+b)}{}\right.}{}$$

$$= \frac{b(8a+5b)\mathrm{sech}^3(c+dx)\tanh(c+dx)}{24d} + \frac{b\,\mathrm{sech}^5(c+dx)\left(a+b+a\sinh\right.}{6d}$$

$$= \frac{\left(8a^2+12ab+5b^2\right)\mathrm{sech}(c+dx)\tanh(c+dx)}{16d} + \frac{b(8a+5b)\mathrm{sech}^3(c+d}{24d}$$

$$= \frac{\left(8a^2+12ab+5b^2\right)\tan^{-1}(\sinh(c+dx))}{16d} + \frac{\left(8a^2+12ab+5b^2\right)\mathrm{sech}(c+}{16d}$$

Mathematica [A] time = 0.237709, size = 104, normalized size = 0.81

$$\frac{3\left(8a^2+12ab+5b^2\right)\tan^{-1}(\sinh(c+dx))+3\left(8a^2+12ab+5b^2\right)\tanh(c+dx)\mathrm{sech}(c+dx)+2b(12a+5b)\tanh(c+}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (3*(8*a^2 + 12*a*b + 5*b^2)*ArcTan[Sinh[c + d*x]] + 3*(8*a^2 + 12*a*b + 5*b
^2)*Sech[c + d*x]*Tanh[c + d*x] + 2*b*(12*a + 5*b)*Sech[c + d*x]^3*Tanh[c +
 d*x] + 8*b^2*Sech[c + d*x]^5*Tanh[c + d*x])/(48*d)
```

Maple [A] time = 0.026, size = 169, normalized size = 1.3

$$\frac{a^2\mathrm{sech}(dx+c)\tanh(dx+c)}{2d} + \frac{a^2\arctan\left(e^{dx+c}\right)}{d} + \frac{ab\tanh(dx+c)(\mathrm{sech}(dx+c))^3}{2d} + \frac{3ab\,\mathrm{sech}(dx+c)\tanh(dx+}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/2/d*a^2*sech(d*x+c)*tanh(d*x+c)+1/d*a^2*arctan(exp(d*x+c))+1/2/d*a*b*tanh
(d*x+c)*sech(d*x+c)^3+3/4/d*a*b*sech(d*x+c)*tanh(d*x+c)+3/2/d*a*b*arctan(ex
p(d*x+c))+1/6/d*b^2*tanh(d*x+c)*sech(d*x+c)^5+5/24/d*b^2*tanh(d*x+c)*sech(d
```

```
*x+c)^3+5/16*b^2*sech(d*x+c)*tanh(d*x+c)/d+5/8/d*b^2*arctan(exp(d*x+c))
```

Maxima [B] time = 1.70017, size = 470, normalized size = 3.67

$$-\frac{1}{24}\,b^2\left(\frac{15\,\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{15\,e^{(-dx-c)} + 85\,e^{(-3\,dx-3\,c)} + 198\,e^{(-5\,dx-5\,c)} - 198\,e^{(-7\,dx-7\,c)} - 85\,e^{(-9\,dx-9\,c)} - 15\,e^{(-11\,dx-11}}{d\left(6\,e^{(-2\,dx-2\,c)} + 15\,e^{(-4\,dx-4\,c)} + 20\,e^{(-6\,dx-6\,c)} + 15\,e^{(-8\,dx-8\,c)} + 6\,e^{(-10\,dx-10\,c)} + e^{(-12\,dx-12\,c)}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/24*b^2*(15*arctan(e^(-d*x - c))/d - (15*e^(-d*x - c) + 85*e^(-3*d*x - 3*
c) + 198*e^(-5*d*x - 5*c) - 198*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9*c) - 15
*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-
6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12
*c) + 1))) - 1/2*a*b*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3
*d*x - 3*c) - 11*e^(-5*d*x - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2
*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - a
^2*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*
d*x - 2*c) + e^(-4*d*x - 4*c) + 1)))
```

Fricas [B] time = 2.49081, size = 7609, normalized size = 59.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^11 + 33*(8*a^2 + 12*a*b + 5*
b^2)*cosh(d*x + c)*sinh(d*x + c)^10 + 3*(8*a^2 + 12*a*b + 5*b^2)*sinh(d*x +
 c)^11 + (72*a^2 + 204*a*b + 85*b^2)*cosh(d*x + c)^9 + (165*(8*a^2 + 12*a*b
 + 5*b^2)*cosh(d*x + c)^2 + 72*a^2 + 204*a*b + 85*b^2)*sinh(d*x + c)^9 + 9*
(55*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^3 + (72*a^2 + 204*a*b + 85*b^2)*
cosh(d*x + c))*sinh(d*x + c)^8 + 6*(8*a^2 + 28*a*b + 33*b^2)*cosh(d*x + c)^
7 + 6*(165*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^4 + 6*(72*a^2 + 204*a*b +
 85*b^2)*cosh(d*x + c)^2 + 8*a^2 + 28*a*b + 33*b^2)*sinh(d*x + c)^7 + 42*(3
3*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^5 + 2*(72*a^2 + 204*a*b + 85*b^2)*
cosh(d*x + c)^3 + (8*a^2 + 28*a*b + 33*b^2)*cosh(d*x + c))*sinh(d*x + c)^6
- 6*(8*a^2 + 28*a*b + 33*b^2)*cosh(d*x + c)^5 + 6*(231*(8*a^2 + 12*a*b + 5*
b^2)*cosh(d*x + c)^6 + 21*(72*a^2 + 204*a*b + 85*b^2)*cosh(d*x + c)^4 + 21*
(8*a^2 + 28*a*b + 33*b^2)*cosh(d*x + c)^2 - 8*a^2 - 28*a*b - 33*b^2)*sinh(d
*x + c)^5 + 6*(165*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^7 + 21*(72*a^2 +
204*a*b + 85*b^2)*cosh(d*x + c)^5 + 35*(8*a^2 + 28*a*b + 33*b^2)*cosh(d*x +
 c)^3 - 5*(8*a^2 + 28*a*b + 33*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 - (72*a^
2 + 204*a*b + 85*b^2)*cosh(d*x + c)^3 + (495*(8*a^2 + 12*a*b + 5*b^2)*cosh(
d*x + c)^8 + 84*(72*a^2 + 204*a*b + 85*b^2)*cosh(d*x + c)^6 + 210*(8*a^2 +
28*a*b + 33*b^2)*cosh(d*x + c)^4 - 60*(8*a^2 + 28*a*b + 33*b^2)*cosh(d*x +
c)^2 - 72*a^2 - 204*a*b - 85*b^2)*sinh(d*x + c)^3 + 3*(55*(8*a^2 + 12*a*b +
 5*b^2)*cosh(d*x + c)^9 + 12*(72*a^2 + 204*a*b + 85*b^2)*cosh(d*x + c)^7 +
42*(8*a^2 + 28*a*b + 33*b^2)*cosh(d*x + c)^5 - 20*(8*a^2 + 28*a*b + 33*b^2)
*cosh(d*x + c)^3 - (72*a^2 + 204*a*b + 85*b^2)*cosh(d*x + c))*sinh(d*x + c)
^2 + 3*((8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^12 + 12*(8*a^2 + 12*a*b + 5*
b^2)*cosh(d*x + c)*sinh(d*x + c)^11 + (8*a^2 + 12*a*b + 5*b^2)*sinh(d*x + c
```

```
)^12 + 6*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^10 + 6*(11*(8*a^2 + 12*a*b
+ 5*b^2)*cosh(d*x + c)^2 + 8*a^2 + 12*a*b + 5*b^2)*sinh(d*x + c)^10 + 20*(1
1*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^3 + 3*(8*a^2 + 12*a*b + 5*b^2)*cos
h(d*x + c))*sinh(d*x + c)^9 + 15*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^8 +
 15*(33*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^4 + 18*(8*a^2 + 12*a*b + 5*b
^2)*cosh(d*x + c)^2 + 8*a^2 + 12*a*b + 5*b^2)*sinh(d*x + c)^8 + 24*(33*(8*a
^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^5 + 30*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x
 + c)^3 + 5*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 20*(8
*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^6 + 4*(231*(8*a^2 + 12*a*b + 5*b^2)*co
sh(d*x + c)^6 + 315*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^4 + 105*(8*a^2 +
 12*a*b + 5*b^2)*cosh(d*x + c)^2 + 40*a^2 + 60*a*b + 25*b^2)*sinh(d*x + c)^
6 + 24*(33*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^7 + 63*(8*a^2 + 12*a*b +
5*b^2)*cosh(d*x + c)^5 + 35*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^3 + 5*(8
*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 15*(8*a^2 + 12*a*b
+ 5*b^2)*cosh(d*x + c)^4 + 15*(33*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^8
+ 84*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^6 + 70*(8*a^2 + 12*a*b + 5*b^2)
*cosh(d*x + c)^4 + 20*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^2 + 8*a^2 + 12
*a*b + 5*b^2)*sinh(d*x + c)^4 + 20*(11*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x +
c)^9 + 36*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^7 + 42*(8*a^2 + 12*a*b + 5
*b^2)*cosh(d*x + c)^5 + 20*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^3 + 3*(8*
a^2 + 12*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(8*a^2 + 12*a*b +
5*b^2)*cosh(d*x + c)^2 + 6*(11*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^10 +
45*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^8 + 70*(8*a^2 + 12*a*b + 5*b^2)*c
osh(d*x + c)^6 + 50*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^4 + 15*(8*a^2 +
12*a*b + 5*b^2)*cosh(d*x + c)^2 + 8*a^2 + 12*a*b + 5*b^2)*sinh(d*x + c)^2 +
 8*a^2 + 12*a*b + 5*b^2 + 12*((8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^11 + 5
*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^9 + 10*(8*a^2 + 12*a*b + 5*b^2)*cos
h(d*x + c)^7 + 10*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^5 + 5*(8*a^2 + 12*
a*b + 5*b^2)*cosh(d*x + c)^3 + (8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c))*sinh
(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 3*(8*a^2 + 12*a*b + 5*b^
2)*cosh(d*x + c) + 3*(11*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^10 + 3*(72*
a^2 + 204*a*b + 85*b^2)*cosh(d*x + c)^8 + 14*(8*a^2 + 28*a*b + 33*b^2)*cosh
(d*x + c)^6 - 10*(8*a^2 + 28*a*b + 33*b^2)*cosh(d*x + c)^4 - (72*a^2 + 204*
a*b + 85*b^2)*cosh(d*x + c)^2 - 8*a^2 - 12*a*b - 5*b^2)*sinh(d*x + c))/(d*c
osh(d*x + c)^12 + 12*d*cosh(d*x + c)*sinh(d*x + c)^11 + d*sinh(d*x + c)^12
+ 6*d*cosh(d*x + c)^10 + 6*(11*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^10 + 20
*(11*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^9 + 15*d*cosh(d*x
 + c)^8 + 15*(33*d*cosh(d*x + c)^4 + 18*d*cosh(d*x + c)^2 + d)*sinh(d*x + c
)^8 + 24*(33*d*cosh(d*x + c)^5 + 30*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*
sinh(d*x + c)^7 + 20*d*cosh(d*x + c)^6 + 4*(231*d*cosh(d*x + c)^6 + 315*d*c
osh(d*x + c)^4 + 105*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^6 + 24*(33*d*co
sh(d*x + c)^7 + 63*d*cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 + 5*d*cosh(d*x
+ c))*sinh(d*x + c)^5 + 15*d*cosh(d*x + c)^4 + 15*(33*d*cosh(d*x + c)^8 + 8
4*d*cosh(d*x + c)^6 + 70*d*cosh(d*x + c)^4 + 20*d*cosh(d*x + c)^2 + d)*sinh
(d*x + c)^4 + 20*(11*d*cosh(d*x + c)^9 + 36*d*cosh(d*x + c)^7 + 42*d*cosh(d
*x + c)^5 + 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 6*d
*cosh(d*x + c)^2 + 6*(11*d*cosh(d*x + c)^10 + 45*d*cosh(d*x + c)^8 + 70*d*c
osh(d*x + c)^6 + 50*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 + d)*sinh(d*x
+ c)^2 + 12*(d*cosh(d*x + c)^11 + 5*d*cosh(d*x + c)^9 + 10*d*cosh(d*x + c)^
7 + 10*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x
+ c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\,\mathrm{sech}^2\,(c + dx)\right)^2 \mathrm{sech}^3\,(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*sech(c + d*x)**3, x)
```

Giac [B] time = 1.13563, size = 397, normalized size = 3.1

$$\frac{\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)} - 1\right)e^{(-dx-c)}\right)\right)\left(8a^2 + 12ab + 5b^2\right)}{32d} + \frac{24a^2\left(e^{(dx+c)} - e^{(-dx-c)}\right)^5 + 36ab\left(e^{(dx+c)} - e^{(-dx-c)}\right)^5 + 15$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/32*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(8*a^2 + 12*a*
b + 5*b^2)/d + 1/24*(24*a^2*(e^(d*x + c) - e^(-d*x - c))^5 + 36*a*b*(e^(d*x
 + c) - e^(-d*x - c))^5 + 15*b^2*(e^(d*x + c) - e^(-d*x - c))^5 + 192*a^2*(
e^(d*x + c) - e^(-d*x - c))^3 + 384*a*b*(e^(d*x + c) - e^(-d*x - c))^3 + 16
0*b^2*(e^(d*x + c) - e^(-d*x - c))^3 + 384*a^2*(e^(d*x + c) - e^(-d*x - c))
 + 960*a*b*(e^(d*x + c) - e^(-d*x - c)) + 528*b^2*(e^(d*x + c) - e^(-d*x -
c)))/(((e^(d*x + c) - e^(-d*x - c))^2 + 4)^3*d)
```

3.64 $\int \mathbf{sech}^4(c+dx)\left(a+b\mathbf{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=80

$$\frac{b(2a+3b)\tanh^5(c+dx)}{5d}-\frac{(a+b)(a+3b)\tanh^3(c+dx)}{3d}+\frac{(a+b)^2\tanh(c+dx)}{d}-\frac{b^2\tanh^7(c+dx)}{7d}$$

```
[Out] ((a + b)^2*Tanh[c + d*x])/d - ((a + b)*(a + 3*b)*Tanh[c + d*x]^3)/(3*d) + (
b*(2*a + 3*b)*Tanh[c + d*x]^5)/(5*d) - (b^2*Tanh[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.076328, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4146, 373}

$$\frac{b(2a+3b)\tanh^5(c+dx)}{5d}-\frac{(a+b)(a+3b)\tanh^3(c+dx)}{3d}+\frac{(a+b)^2\tanh(c+dx)}{d}-\frac{b^2\tanh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + b)^2*Tanh[c + d*x])/d - ((a + b)*(a + 3*b)*Tanh[c + d*x]^3)/(3*d) + (
b*(2*a + 3*b)*Tanh[c + d*x]^5)/(5*d) - (b^2*Tanh[c + d*x]^7)/(7*d)
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \text{sech}^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int\left(1-x^2\right)\left(a+b-bx^2\right)^2\,dx,x,\tanh(c+dx)\right)}{d}$$

$$=\frac{\text{Subst}\left(\int\left((a+b)^2+(-a-3b)(a+b)x^2+b(2a+3b)x^4-b^2x^6\right)dx,x,\text{ta}\right.}{d}$$

$$=\frac{(a+b)^2\tanh(c+dx)}{d}-\frac{(a+b)(a+3b)\tanh^3(c+dx)}{3d}+\frac{b(2a+3b)\tan}{5d}$$

Mathematica [A] time = 0.0297176, size = 144, normalized size = 1.8

$$-\frac{a^2\tanh^3(c+dx)}{3d}+\frac{a^2\tanh(c+dx)}{d}+\frac{2ab\tanh^5(c+dx)}{5d}-\frac{4ab\tanh^3(c+dx)}{3d}+\frac{2ab\tanh(c+dx)}{d}-\frac{b^2\tanh^7(c+}{7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Tanh[c + d*x])/d + (2*a*b*Tanh[c + d*x])/d + (b^2*Tanh[c + d*x])/d - (
a^2*Tanh[c + d*x]^3)/(3*d) - (4*a*b*Tanh[c + d*x]^3)/(3*d) - (b^2*Tanh[c +
d*x]^3)/d + (2*a*b*Tanh[c + d*x]^5)/(5*d) + (3*b^2*Tanh[c + d*x]^5)/(5*d) -
 (b^2*Tanh[c + d*x]^7)/(7*d)
```

Maple [A] time = 0.026, size = 102, normalized size = 1.3

$$\frac{1}{d}\left(a^2\left(\frac{2}{3}+\frac{(\operatorname{sech}(dx+c))^2}{3}\right)\tanh(dx+c)+2\,ab\left(\frac{8}{15}+1/5\,(\operatorname{sech}(dx+c))^4+\frac{4\,(\operatorname{sech}(dx+c))^2}{15}\right)\tanh(dx+c)+b^2\left(\frac{1}{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+2*a*b*(8/15+1/5*sech(d*x+c)^4+
4/15*sech(d*x+c)^2)*tanh(d*x+c)+b^2*(16/35+1/7*sech(d*x+c)^6+6/35*sech(d*x+
c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c))
```

Maxima [B] time = 1.15371, size = 906, normalized size = 11.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 32/35*b^2*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c)
+ 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(
-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 21*e^(-4*d*x - 4*c)/(d*(7*e^(-
2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8
*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1
)) + 35*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*
e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d
*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4
*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 1
0*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 32/15*a*b*(5*e^(-
2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 10*e^(-4*d*x - 4*c)/
(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-
8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(
-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*
c) + 1))) + 4/3*a^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*
x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x
- 4*c) + e^(-6*d*x - 6*c) + 1)))
```

Fricas [B] time = 2.05501, size = 1828, normalized size = 22.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -8/105*(2*(35*a^2 + 14*a*b + 6*b^2)*cosh(d*x + c)^5 + 10*(35*a^2 + 14*a*b +
 6*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 + (35*a^2 - 28*a*b - 12*b^2)*sinh(d*x
 + c)^5 + 14*(25*a^2 + 34*a*b + 6*b^2)*cosh(d*x + c)^3 + (10*(35*a^2 - 28*a
*b - 12*b^2)*cosh(d*x + c)^2 + 105*a^2 + 84*a*b - 84*b^2)*sinh(d*x + c)^3 +
 2*(10*(35*a^2 + 14*a*b + 6*b^2)*cosh(d*x + c)^3 + 21*(25*a^2 + 34*a*b + 6*
b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 28*(25*a^2 + 46*a*b + 24*b^2)*cosh(d*
x + c) + (5*(35*a^2 - 28*a*b - 12*b^2)*cosh(d*x + c)^4 + 63*(5*a^2 + 4*a*b
- 4*b^2)*cosh(d*x + c)^2 + 70*a^2 + 112*a*b + 168*b^2)*sinh(d*x + c))/(d*co
sh(d*x + c)^9 + 9*d*cosh(d*x + c)*sinh(d*x + c)^8 + d*sinh(d*x + c)^9 + 7*d
*cosh(d*x + c)^7 + (36*d*cosh(d*x + c)^2 + 7*d)*sinh(d*x + c)^7 + 7*(12*d*c
osh(d*x + c)^3 + 7*d*cosh(d*x + c))*sinh(d*x + c)^6 + 22*d*cosh(d*x + c)^5
+ (126*d*cosh(d*x + c)^4 + 147*d*cosh(d*x + c)^2 + 20*d)*sinh(d*x + c)^5 +
(126*d*cosh(d*x + c)^5 + 245*d*cosh(d*x + c)^3 + 110*d*cosh(d*x + c))*sinh(
d*x + c)^4 + 42*d*cosh(d*x + c)^3 + (84*d*cosh(d*x + c)^6 + 245*d*cosh(d*x
+ c)^4 + 200*d*cosh(d*x + c)^2 + 28*d)*sinh(d*x + c)^3 + (36*d*cosh(d*x + c
)^7 + 147*d*cosh(d*x + c)^5 + 220*d*cosh(d*x + c)^3 + 126*d*cosh(d*x + c))*
sinh(d*x + c)^2 + 56*d*cosh(d*x + c) + (9*d*cosh(d*x + c)^8 + 49*d*cosh(d*x
 + c)^6 + 100*d*cosh(d*x + c)^4 + 84*d*cosh(d*x + c)^2 + 14*d)*sinh(d*x + c
))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\operatorname{sech}^2(c + dx)\right)^2 \operatorname{sech}^4(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*sech(c + d*x)**4, x)
```

Giac [B] time = 1.15939, size = 266, normalized size = 3.32

$$-\frac{4\left(105\,a^2e^{(10\,dx+10\,c)} + 455\,a^2e^{(8\,dx+8\,c)} + 560\,abe^{(8\,dx+8\,c)} + 770\,a^2e^{(6\,dx+6\,c)} + 1400\,abe^{(6\,dx+6\,c)} + 840\,b^2e^{(6\,dx+6\,c)} + 630\,\cdots\right.}{105\,d\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -4/105*(105*a^2*e^(10*d*x + 10*c) + 455*a^2*e^(8*d*x + 8*c) + 560*a*b*e^(8*
d*x + 8*c) + 770*a^2*e^(6*d*x + 6*c) + 1400*a*b*e^(6*d*x + 6*c) + 840*b^2*e
^(6*d*x + 6*c) + 630*a^2*e^(4*d*x + 4*c) + 1176*a*b*e^(4*d*x + 4*c) + 504*b
^2*e^(4*d*x + 4*c) + 245*a^2*e^(2*d*x + 2*c) + 392*a*b*e^(2*d*x + 2*c) + 16
8*b^2*e^(2*d*x + 2*c) + 35*a^2 + 56*a*b + 24*b^2)/(d*(e^(2*d*x + 2*c) + 1)^
7)
```

3.65 $\int \cosh^4(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=84

$$\frac{3}{8}ax\left(a^2+4ab+8b^2\right)+\frac{3a^2(a+4b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{a^3\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{b^3\tanh(c+dx)}{d}$$

```
[Out] (3*a*(a^2 + 4*a*b + 8*b^2)*x)/8 + (3*a^2*(a + 4*b)*Cosh[c + d*x]*Sinh[c + d
*x])/(8*d) + (a^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(4*d) + (b^3*Tanh[c + d*x]
)/d
```

Rubi [A] time = 0.111196, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4146, 390, 1157, 385, 206}

$$\frac{3}{8}ax\left(a^2+4ab+8b^2\right)+\frac{3a^2(a+4b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{a^3\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{b^3\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (3*a*(a^2 + 4*a*b + 8*b^2)*x)/8 + (3*a^2*(a + 4*b)*Cosh[c + d*x]*Sinh[c + d
*x])/(8*d) + (a^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(4*d) + (b^3*Tanh[c + d*x]
)/d
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
```

```
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cosh^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3 dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+b-bx^2\right)^3}{\left(1-x^2\right)^3}dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(b^3+\frac{a\left(a^2+3ab+3b^2\right)-3ab(a+2b)x^2+3ab^2x^4}{\left(1-x^2\right)^3}\right)dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b^3\tanh(c+dx)}{d} + \frac{\operatorname{Subst}\left(\int \frac{a\left(a^2+3ab+3b^2\right)-3ab(a+2b)x^2+3ab^2x^4}{\left(1-x^2\right)^3}dx, x, \tanh(c\right.}{d}$$

$$= \frac{a^3\cosh^3(c+dx)\sinh(c+dx)}{4d} + \frac{b^3\tanh(c+dx)}{d} - \frac{\operatorname{Subst}\left(\int \frac{-3a(a+2b)^2+}{\left(1-x^2\right.}\right.}{}$$

$$= \frac{3a^2(a+4b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{a^3\cosh^3(c+dx)\sinh(c+dx)}{4d}$$

$$= \frac{3}{8}a\left(a^2+4ab+8b^2\right)x + \frac{3a^2(a+4b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{a^3\cos}{}$$

Mathematica [A] time = 0.408999, size = 70, normalized size = 0.83

$$\frac{12a\left(a^2+4ab+8b^2\right)(c+dx)+8a^2(a+3b)\sinh(2(c+dx))+a^3\sinh(4(c+dx))+32b^3\tanh(c+dx)}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (12*a*(a^2 + 4*a*b + 8*b^2)*(c + d*x) + 8*a^2*(a + 3*b)*Sinh[2*(c + d*x)] +
 a^3*Sinh[4*(c + d*x)] + 32*b^3*Tanh[c + d*x])/(32*d)
```

Maple [A] time = 0.038, size = 93, normalized size = 1.1

$$\frac{1}{d}\left(a^3\left(\left(\frac{(\cosh(dx+c))^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+3a^2b\left(1/2\cosh(dx+c)\sinh(dx+c)+1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*((1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)+3/8*d*x+3/8*c)+3*
a^2*b*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c)+3*a*b^2*(d*x+c)+b^3*tanh(
```

```
d*x+c))
```

Maxima [A] time = 1.10589, size = 176, normalized size = 2.1

$$\frac{1}{64}a^3\left(24x+\frac{e^{(4dx+4c)}}{d}+\frac{8e^{(2dx+2c)}}{d}-\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)+\frac{3}{8}a^2b\left(4x+\frac{e^{(2dx+2c)}}{d}-\frac{e^{(-2dx-2c)}}{d}\right)+3ab^2x+\frac{2}{d\left(e^{(-2d}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/64*a^3*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 3/8*a^2*b*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x
 - 2*c)/d) + 3*a*b^2*x + 2*b^3/(d*(e^(-2*d*x - 2*c) + 1))
```

Fricas [A] time = 2.08693, size = 370, normalized size = 4.4

$$\frac{a^3\sinh(dx+c)^5+\left(10a^3\cosh(dx+c)^2+9a^3+24a^2b\right)\sinh(dx+c)^3-8\left(8b^3-3\left(a^3+4a^2b+8ab^2\right)dx\right)\cosh(dx+c}{64d\cosh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(a^3*sinh(d*x + c)^5 + (10*a^3*cosh(d*x + c)^2 + 9*a^3 + 24*a^2*b)*sin
h(d*x + c)^3 - 8*(8*b^3 - 3*(a^3 + 4*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c) +
(5*a^3*cosh(d*x + c)^4 + 8*a^3 + 24*a^2*b + 64*b^3 + 9*(3*a^3 + 8*a^2*b)*co
sh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19182, size = 257, normalized size = 3.06

$$\frac{3\left(a^3+4a^2b+8ab^2\right)(dx+c)}{8d}-\frac{2b^3}{d\left(e^{(2dx+2c)}+1\right)}-\frac{\left(18a^3e^{(4dx+4c)}+72a^2be^{(4dx+4c)}+144ab^2e^{(4dx+4c)}+8a^3e^{(2dx+2c)}+2\right.}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 3/8*(a^3 + 4*a^2*b + 8*a*b^2)*(d*x + c)/d - 2*b^3/(d*(e^(2*d*x + 2*c) + 1))
 - 1/64*(18*a^3*e^(4*d*x + 4*c) + 72*a^2*b*e^(4*d*x + 4*c) + 144*a*b^2*e^(4
*d*x + 4*c) + 8*a^3*e^(2*d*x + 2*c) + 24*a^2*b*e^(2*d*x + 2*c) + a^3)*e^(-4
*d*x - 4*c)/d + 1/64*(a^3*d*e^(4*d*x + 4*c) + 8*a^3*d*e^(2*d*x + 2*c) + 24*
a^2*b*d*e^(2*d*x + 2*c))/d^2
```

3.66 $\int \cosh^3(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=81

$$\frac{a^2(a+3b)\sinh(c+dx)}{d}+\frac{a^3\sinh^3(c+dx)}{3d}+\frac{b^2(6a+b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{b^3\tanh(c+dx)\mathbf{sech}(c+dx)}{2d}$$

```
[Out] (b^2*(6*a + b)*ArcTan[Sinh[c + d*x]])/(2*d) + (a^2*(a + 3*b)*Sinh[c + d*x])
/d + (a^3*Sinh[c + d*x]^3)/(3*d) + (b^3*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
```

Rubi [A] time = 0.0931799, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4147, 390, 385, 203}

$$\frac{a^2(a+3b)\sinh(c+dx)}{d}+\frac{a^3\sinh^3(c+dx)}{3d}+\frac{b^2(6a+b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{b^3\tanh(c+dx)\mathbf{sech}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (b^2*(6*a + b)*ArcTan[Sinh[c + d*x]])/(2*d) + (a^2*(a + 3*b)*Sinh[c + d*x])
/d + (a^3*Sinh[c + d*x]^3)/(3*d) + (b^3*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cosh^3(c+dx)\left(a+b\mathbf{sech}^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+ax^2\right)^3}{\left(1+x^2\right)^2}dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(a^2(a+3b)+a^3x^2+\frac{b^2(3a+b)+3ab^2x^2}{\left(1+x^2\right)^2}\right)dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{a^2(a+3b)\sinh(c+dx)}{d}+\frac{a^3\sinh^3(c+dx)}{3d}+\frac{\text{Subst}\left(\int \frac{b^2(3a+b)+3ab^2x^2}{\left(1+x^2\right)^2} d\ldots\right.}{d}$$

$$= \frac{a^2(a+3b)\sinh(c+dx)}{d}+\frac{a^3\sinh^3(c+dx)}{3d}+\frac{b^3\mathbf{sech}(c+dx)\tanh(c+\ldots}{2d}$$

$$= \frac{b^2(6a+b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{a^2(a+3b)\sinh(c+dx)}{d}+\frac{a^3\sinh^3(c\ldots}{3d}$$

Mathematica [C] time = 6.83592, size = 483, normalized size = 5.96

$$\frac{\coth^3(c+dx)\mathbf{csch}^2(c+dx)(a\cosh(c+dx)+b\mathbf{sech}(c+dx))^3\left(-256\sinh^8(c+dx)\left(a\sinh^2(c+dx)+a+b\right)^3\text{Hyperg}\ldots\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cosh[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (Coth[c + d*x]^3*Csch[c + d*x]^2*(a*Cosh[c + d*x] + b*Sech[c + d*x])^3*(-25
6*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*S
inh[c + d*x]^8*(a + b + a*Sinh[c + d*x]^2)^3 - (315*ArcTanh[Sqrt[-Sinh[c +
d*x]^2]]*(b^3*(2401 + 1875*Sinh[c + d*x]^2 + 243*Sinh[c + d*x]^4 - 47*Sinh[
c + d*x]^6) + 3*a^2*b*Cosh[c + d*x]^4*(2401 + 1875*Sinh[c + d*x]^2 + 243*Si
nh[c + d*x]^4 + Sinh[c + d*x]^6) + a^3*Cosh[c + d*x]^6*(2401 + 1875*Sinh[c
+ d*x]^2 + 243*Sinh[c + d*x]^4 + Sinh[c + d*x]^6) + 3*a*b^2*(2401 + 4276*Si
nh[c + d*x]^2 + 2118*Sinh[c + d*x]^4 + 148*Sinh[c + d*x]^6 + Sinh[c + d*x]^
8)))/Sqrt[-Sinh[c + d*x]^2] + 21*(b^3*(36015 + 16120*Sinh[c + d*x]^2 + 1473
*Sinh[c + d*x]^4) + 3*a*b^2*(36015 + 52135*Sinh[c + d*x]^2 + 17593*Sinh[c +
 d*x]^4 + 753*Sinh[c + d*x]^6) + 3*a^2*b*(36015 + 88150*Sinh[c + d*x]^2 + 6
9728*Sinh[c + d*x]^4 + 19786*Sinh[c + d*x]^6 + 753*Sinh[c + d*x]^8) + a^3*(
36015 + 124165*Sinh[c + d*x]^2 + 157878*Sinh[c + d*x]^4 + 89514*Sinh[c + d*
x]^6 + 19579*Sinh[c + d*x]^8 + 753*Sinh[c + d*x]^10))))/(3780*d*(a + 2*b +
a*Cosh[2*c + 2*d*x])^3)
```

Maple [A] time = 0.045, size = 103, normalized size = 1.3

$$\frac{2a^3\sinh(dx+c)}{3d}+\frac{a^3\sinh(dx+c)(\cosh(dx+c))^2}{3d}+3\frac{a^2b\sinh(dx+c)}{d}+6\frac{ab^2\arctan\left(e^{dx+c}\right)}{d}+\frac{b^3\mathrm{sech}(dx+c)\ldots}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 2/3/d*a^3*sinh(d*x+c)+1/3/d*a^3*sinh(d*x+c)*cosh(d*x+c)^2+3*a^2*b*sinh(d*x+
c)/d+6/d*a*b^2*arctan(exp(d*x+c))+1/2*b^3*sech(d*x+c)*tanh(d*x+c)/d+1/d*b^3
```

```
*arctan(exp(d*x+c))
```

Maxima [B] time = 1.7009, size = 242, normalized size = 2.99

$$-b^3\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1\right)}\right)+\frac{1}{24}\,a^3\left(\frac{e^{(3\,dx+3\,c)}}{d}+\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}-\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{3}{2}\,a^2b\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -b^3*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-
2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 1/24*a^3*(e^(3*d*x + 3*c)/d + 9*e^
(d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)/d) + 3/2*a^2*b*(e^(d*x +
c)/d - e^(-d*x - c)/d) - 6*a*b^2*arctan(e^(-d*x - c))/d
```

Fricas [B] time = 2.28865, size = 3555, normalized size = 43.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(a^3*cosh(d*x + c)^10 + 10*a^3*cosh(d*x + c)*sinh(d*x + c)^9 + a^3*sin
h(d*x + c)^10 + (11*a^3 + 36*a^2*b)*cosh(d*x + c)^8 + (45*a^3*cosh(d*x + c)
^2 + 11*a^3 + 36*a^2*b)*sinh(d*x + c)^8 + 8*(15*a^3*cosh(d*x + c)^3 + (11*a
^3 + 36*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(5*a^3 + 18*a^2*b + 12*b^
3)*cosh(d*x + c)^6 + 2*(105*a^3*cosh(d*x + c)^4 + 5*a^3 + 18*a^2*b + 12*b^3
 + 14*(11*a^3 + 36*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*a^3*cosh
(d*x + c)^5 + 14*(11*a^3 + 36*a^2*b)*cosh(d*x + c)^3 + 3*(5*a^3 + 18*a^2*b
+ 12*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(5*a^3 + 18*a^2*b + 12*b^3)*co
sh(d*x + c)^4 + 2*(105*a^3*cosh(d*x + c)^6 + 35*(11*a^3 + 36*a^2*b)*cosh(d*
x + c)^4 - 5*a^3 - 18*a^2*b - 12*b^3 + 15*(5*a^3 + 18*a^2*b + 12*b^3)*cosh(
d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*a^3*cosh(d*x + c)^7 + 7*(11*a^3 + 36*a^
2*b)*cosh(d*x + c)^5 + 5*(5*a^3 + 18*a^2*b + 12*b^3)*cosh(d*x + c)^3 - (5*a
^3 + 18*a^2*b + 12*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - a^3 - (11*a^3 + 36
*a^2*b)*cosh(d*x + c)^2 + (45*a^3*cosh(d*x + c)^8 + 28*(11*a^3 + 36*a^2*b)*
cosh(d*x + c)^6 + 30*(5*a^3 + 18*a^2*b + 12*b^3)*cosh(d*x + c)^4 - 11*a^3 -
 36*a^2*b - 12*(5*a^3 + 18*a^2*b + 12*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2
 + 24*((6*a*b^2 + b^3)*cosh(d*x + c)^7 + 7*(6*a*b^2 + b^3)*cosh(d*x + c)*si
nh(d*x + c)^6 + (6*a*b^2 + b^3)*sinh(d*x + c)^7 + 2*(6*a*b^2 + b^3)*cosh(d*
x + c)^5 + (12*a*b^2 + 2*b^3 + 21*(6*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x
 + c)^5 + 5*(7*(6*a*b^2 + b^3)*cosh(d*x + c)^3 + 2*(6*a*b^2 + b^3)*cosh(d*x
 + c))*sinh(d*x + c)^4 + (6*a*b^2 + b^3)*cosh(d*x + c)^3 + (35*(6*a*b^2 + b
^3)*cosh(d*x + c)^4 + 6*a*b^2 + b^3 + 20*(6*a*b^2 + b^3)*cosh(d*x + c)^2)*s
inh(d*x + c)^3 + (21*(6*a*b^2 + b^3)*cosh(d*x + c)^5 + 20*(6*a*b^2 + b^3)*c
osh(d*x + c)^3 + 3*(6*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + (7*(6*a
*b^2 + b^3)*cosh(d*x + c)^6 + 10*(6*a*b^2 + b^3)*cosh(d*x + c)^4 + 3*(6*a*b
^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x +
 c)) + 2*(5*a^3*cosh(d*x + c)^9 + 4*(11*a^3 + 36*a^2*b)*cosh(d*x + c)^7 + 6
*(5*a^3 + 18*a^2*b + 12*b^3)*cosh(d*x + c)^5 - 4*(5*a^3 + 18*a^2*b + 12*b^3
)*cosh(d*x + c)^3 - (11*a^3 + 36*a^2*b)*cosh(d*x + c))*sinh(d*x + c))/(d*co
sh(d*x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 + 2*d
```

```
*cosh(d*x + c)^5 + (21*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^5 + 5*(7*d*co
sh(d*x + c)^3 + 2*d*cosh(d*x + c))*sinh(d*x + c)^4 + d*cosh(d*x + c)^3 + (3
5*d*cosh(d*x + c)^4 + 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 + (21*d*cos
h(d*x + c)^5 + 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 +
(7*d*cosh(d*x + c)^6 + 10*d*cosh(d*x + c)^4 + 3*d*cosh(d*x + c)^2)*sinh(d*x
 + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19224, size = 240, normalized size = 2.96

$$\frac{b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)d}+\frac{\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)}-1\right)e^{(-dx-c)}\right)\right)\left(6ab^2+b^3\right)}{4d}+\frac{a^3d^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+12a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] b^3*(e^(d*x + c) - e^(-d*x - c))/(((e^(d*x + c) - e^(-d*x - c))^2 + 4)*d) +
 1/4*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(6*a*b^2 + b^3
)/d + 1/24*(a^3*d^2*(e^(d*x + c) - e^(-d*x - c))^3 + 12*a^3*d^2*(e^(d*x + c
) - e^(-d*x - c)) + 36*a^2*b*d^2*(e^(d*x + c) - e^(-d*x - c)))/d^3
```

3.67 $\int \cosh^2(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=72

$$\frac{1}{2}a^2x(a+6b)+\frac{a^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{b^2(3a+b)\tanh(c+dx)}{d}-\frac{b^3\tanh^3(c+dx)}{3d}$$

```
[Out] (a^2*(a + 6*b)*x)/2 + (a^3*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (b^2*(3*a +
 b)*Tanh[c + d*x])/d - (b^3*Tanh[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0923167, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4146, 390, 385, 206}

$$\frac{1}{2}a^2x(a+6b)+\frac{a^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{b^2(3a+b)\tanh(c+dx)}{d}-\frac{b^3\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]
```


```
[Out] (a^2*(a + 6*b)*x)/2 + (a^3*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (b^2*(3*a +
 b)*Tanh[c + d*x])/d - (b^3*Tanh[c + d*x]^3)/(3*d)
```


Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cosh^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int \frac{\left(a+b-bx^2\right)^3}{\left(1-x^2\right)^2}dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(b^2(3a+b)-b^3x^2+\frac{a^2(a+3b)-3a^2bx^2}{\left(1-x^2\right)^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{b^2(3a+b)\tanh(c+dx)}{d} - \frac{b^3\tanh^3(c+dx)}{3d} + \frac{\text{Subst}\left(\int \frac{a^2(a+3b)-3a^2bx^2}{\left(1-x^2\right)^2}c\right.}{d}$$
$$= \frac{a^3\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{b^2(3a+b)\tanh(c+dx)}{d} - \frac{b^3\tanh^3(c+}{3d}$$
$$= \frac{1}{2}a^2(a+6b)x + \frac{a^3\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{b^2(3a+b)\tanh(c+dx)}{d}$$

Mathematica [A] time = 0.479235, size = 64, normalized size = 0.89

$$\frac{6a^2(a+6b)(c+dx)+3a^3\sinh(2(c+dx))+4b^2\tanh(c+dx)\left(9a+b\text{sech}^2(c+dx)+2b\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (6*a^2*(a + 6*b)*(c + d*x) + 3*a^3*Sinh[2*(c + d*x)] + 4*b^2*(9*a + 2*b + b
*Sech[c + d*x]^2)*Tanh[c + d*x])/(12*d)
```

Maple [A] time = 0.044, size = 77, normalized size = 1.1

$$\frac{1}{d}\left(a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^2b(dx+c)+3ab^2\tanh(dx+c)+b^3\left(\frac{2}{3}+\frac{(\text{sech}(dx+c))^2}{3}\right)\tanh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c)+3*a^2*b*(d*x+c)+3*a*b^
2*tanh(d*x+c)+b^3*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c))
```

Maxima [B] time = 1.13754, size = 216, normalized size = 3.

$$\frac{1}{8}a^3\left(4x+\frac{e^{(2dx+2c)}}{d}-\frac{e^{(-2dx-2c)}}{d}\right)+3a^2bx+\frac{4}{3}b^3\left(\frac{3e^{(-2dx-2c)}}{d\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1\right)}+\frac{}{d\left(3e^{(-2dx-2c)}+3e\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a^3*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) + 3*a^2*b*x + 4/3*b^
3*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d
```

```
*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x
 - 6*c) + 1))) + 6*a*b^2/(d*(e^(-2*d*x - 2*c) + 1))
```

Fricas [B] time = 2.30101, size = 670, normalized size = 9.31

$$\frac{3\,a^3\sinh\left(dx+c\right)^5 - 4\left(18\,ab^2 + 4\,b^3 - 3\left(a^3 + 6\,a^2b\right)dx\right)\cosh\left(dx+c\right)^3 - 12\left(18\,ab^2 + 4\,b^3 - 3\left(a^3 + 6\,a^2b\right)dx\right)\cosh\left(dx + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*a^3*sinh(d*x + c)^5 - 4*(18*a*b^2 + 4*b^3 - 3*(a^3 + 6*a^2*b)*d*x)*
cosh(d*x + c)^3 - 12*(18*a*b^2 + 4*b^3 - 3*(a^3 + 6*a^2*b)*d*x)*cosh(d*x +
c)*sinh(d*x + c)^2 + (30*a^3*cosh(d*x + c)^2 + 9*a^3 + 72*a*b^2 + 16*b^3)*s
inh(d*x + c)^3 - 12*(18*a*b^2 + 4*b^3 - 3*(a^3 + 6*a^2*b)*d*x)*cosh(d*x + c
) + 3*(5*a^3*cosh(d*x + c)^4 + 2*a^3 + 24*a*b^2 + 16*b^3 + (9*a^3 + 72*a*b^
2 + 16*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d
*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18982, size = 215, normalized size = 2.99

$$\frac{a^3e^{(2\,dx+2\,c)}}{8\,d} + \frac{\left(a^3 + 6\,a^2b\right)(dx+c)}{2\,d} - \frac{\left(2\,a^3e^{(2\,dx+2\,c)} + 12\,a^2be^{(2\,dx+2\,c)} + a^3\right)e^{(-2\,dx-2\,c)}}{8\,d} - \frac{2\left(9\,ab^2e^{(4\,dx+4\,c)} + 18\,ab^2e^{(2\,dx+2\,c)}\right.}{3\,d\left(e^{(2\,dx+2\,c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*a^3*e^(2*d*x + 2*c)/d + 1/2*(a^3 + 6*a^2*b)*(d*x + c)/d - 1/8*(2*a^3*e^
(2*d*x + 2*c) + 12*a^2*b*e^(2*d*x + 2*c) + a^3)*e^(-2*d*x - 2*c)/d - 2/3*(9
*a*b^2*e^(4*d*x + 4*c) + 18*a*b^2*e^(2*d*x + 2*c) + 6*b^3*e^(2*d*x + 2*c) +
 9*a*b^2 + 2*b^3)/(d*(e^(2*d*x + 2*c) + 1)^3)
```

3.68 $\int \cosh(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=93

$$\frac{3b\left(8a^2+4ab+b^2\right)\tan^{-1}(\sinh(c+dx))}{8d}+\frac{a^3\sinh(c+dx)}{d}+\frac{3b^2(4a+b)\tanh(c+dx)\mathbf{sech}(c+dx)}{8d}+\frac{b^3\tanh(c+d}{4}$$

```
[Out] (3*b*(8*a^2 + 4*a*b + b^2)*ArcTan[Sinh[c + d*x]])/(8*d) + (a^3*Sinh[c + d*x
])/d + (3*b^2*(4*a + b)*Sech[c + d*x]*Tanh[c + d*x])/(8*d) + (b^3*Sech[c +
d*x]^3*Tanh[c + d*x])/(4*d)
```

Rubi [A] time = 0.102681, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4147, 390, 1157, 385, 203}

$$\frac{3b\left(8a^2+4ab+b^2\right)\tan^{-1}(\sinh(c+dx))}{8d}+\frac{a^3\sinh(c+dx)}{d}+\frac{3b^2(4a+b)\tanh(c+dx)\mathbf{sech}(c+dx)}{8d}+\frac{b^3\tanh(c+d}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]
```


```
[Out] (3*b*(8*a^2 + 4*a*b + b^2)*ArcTan[Sinh[c + d*x]])/(8*d) + (a^3*Sinh[c + d*x
])/d + (3*b^2*(4*a + b)*Sech[c + d*x]*Tanh[c + d*x])/(8*d) + (b^3*Sech[c +
d*x]^3*Tanh[c + d*x])/(4*d)
```


Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
```

```
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cosh(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+ax^2\right)^3}{\left(1+x^2\right)^3} dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(a^3 + \frac{b\left(3a^2+3ab+b^2\right)+3ab(2a+b)x^2+3a^2bx^4}{\left(1+x^2\right)^3}\right) dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{a^3\sinh(c+dx)}{d} + \frac{\text{Subst}\left(\int \frac{b\left(3a^2+3ab+b^2\right)+3ab(2a+b)x^2+3a^2bx^4}{\left(1+x^2\right)^3} dx, x, \sinh(c+dx\right.}{d}$$

$$= \frac{a^3\sinh(c+dx)}{d} + \frac{b^3\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4d} - \frac{\text{Subst}\left(\int \frac{-3b(2a+b)^2-12a^2b}{\left(1+x^2\right)^2}\right.}{4d}$$

$$= \frac{a^3\sinh(c+dx)}{d} + \frac{3b^2(4a+b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d} + \frac{b^3\operatorname{sech}^3(c+dx)}{4d}$$

$$= \frac{3b\left(8a^2+4ab+b^2\right)\tan^{-1}(\sinh(c+dx))}{8d} + \frac{a^3\sinh(c+dx)}{d} + \frac{3b^2(4a+b)\text{se}}{}$$

Mathematica [C] time = 7.90187, size = 575, normalized size = 6.18

$$-\frac{\cosh(c+dx)\coth^5(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\left(256\sinh^8(c+dx)\left(a\sinh^2(c+dx)+a+b\right)^3\text{HypergeometricPFQ}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cosh[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -(Cosh[c + d*x]*Coth[c + d*x]^5*(a + b*Sech[c + d*x]^2)^3*(256*Hypergeometr
icPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c +
d*x]^8*(a + b + a*Sinh[c + d*x]^2)^3 + 384*HypergeometricPFQ[{3/2, 2, 2, 2,
 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^8*(a + b + a*Sinh[c +
 d*x]^2)^2*(7*b + a*(7 + 5*Sinh[c + d*x]^2)) + (315*ArcTanh[Sqrt[-Sinh[c +
d*x]^2]]*(b^3*(16807 + 15000*Sinh[c + d*x]^2 + 2187*Sinh[c + d*x]^4 - 62*Si
nh[c + d*x]^6) + a^3*Cosh[c + d*x]^4*(16807 + 24604*Sinh[c + d*x]^2 + 11562
*Sinh[c + d*x]^4 + 1468*Sinh[c + d*x]^6 + 7*Sinh[c + d*x]^8) + 3*a*b^2*(168
07 + 29406*Sinh[c + d*x]^2 + 15312*Sinh[c + d*x]^4 + 1858*Sinh[c + d*x]^6 +
 9*Sinh[c + d*x]^8) + 3*a^2*b*(16807 + 43812*Sinh[c + d*x]^2 + 40442*Sinh[c
 + d*x]^4 + 14956*Sinh[c + d*x]^6 + 1719*Sinh[c + d*x]^8 + 8*Sinh[c + d*x]^
10)))/Sqrt[-Sinh[c + d*x]^2] - 21*(b^3*(252105 + 140965*Sinh[c + d*x]^2 + 8
226*Sinh[c + d*x]^4) + 3*a*b^2*(252105 + 357055*Sinh[c + d*x]^2 + 133071*Si
nh[c + d*x]^4 + 6393*Sinh[c + d*x]^6) + 3*a^2*b*(252105 + 573145*Sinh[c + d
*x]^2 + 437991*Sinh[c + d*x]^4 + 120431*Sinh[c + d*x]^6 + 5640*Sinh[c + d*x
```

```
]^8) + a^3*(252105 + 789235*Sinh[c + d*x]^2 + 922986*Sinh[c + d*x]^4 + 4915
74*Sinh[c + d*x]^6 + 107725*Sinh[c + d*x]^8 + 4887*Sinh[c + d*x]^10))))/(75
60*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^3)
```

Maple [A] time = 0.048, size = 125, normalized size = 1.3

$$\frac{a^3\sinh(dx+c)}{d}+6\,\frac{a^2b\arctan\left(e^{dx+c}\right)}{d}+\frac{3\,ab^2\mathrm{sech}\,(dx+c)\tanh(dx+c)}{2\,d}+3\,\frac{ab^2\arctan\left(e^{dx+c}\right)}{d}+\frac{b^3\,(\mathrm{sech}\,(dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d*a^3*sinh(d*x+c)+6/d*a^2*b*arctan(exp(d*x+c))+3/2/d*a*b^2*sech(d*x+c)*ta
nh(d*x+c)+3/d*a*b^2*arctan(exp(d*x+c))+1/4*b^3*sech(d*x+c)^3*tanh(d*x+c)/d+
3/8*b^3*sech(d*x+c)*tanh(d*x+c)/d+3/4/d*b^3*arctan(exp(d*x+c))
```

Maxima [B] time = 1.68732, size = 298, normalized size = 3.2

$$-\frac{1}{4}\,b^3\left(\frac{3\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{3\,e^{(-dx-c)}+11\,e^{(-3\,dx-3\,c)}-11\,e^{(-5\,dx-5\,c)}-3\,e^{(-7\,dx-7\,c)}}{d\left(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1\right)}\right)-3\,ab^2\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*b^3*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3*d*x - 3*c)
- 11*e^(-5*d*x - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-
4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 3*a*b^2*(arct
an(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*
c) + e^(-4*d*x - 4*c) + 1))) - 6*a^2*b*arctan(e^(-d*x - c))/d + a^3*sinh(d*
x + c)/d
```

Fricas [B] time = 2.67461, size = 4986, normalized size = 53.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^3*cosh(d*x + c)^10 + 20*a^3*cosh(d*x + c)*sinh(d*x + c)^9 + 2*a^3*
sinh(d*x + c)^10 + 3*(2*a^3 + 4*a*b^2 + b^3)*cosh(d*x + c)^8 + 3*(30*a^3*co
sh(d*x + c)^2 + 2*a^3 + 4*a*b^2 + b^3)*sinh(d*x + c)^8 + 24*(10*a^3*cosh(d*
x + c)^3 + (2*a^3 + 4*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + (4*a^3
+ 12*a*b^2 + 11*b^3)*cosh(d*x + c)^6 + (420*a^3*cosh(d*x + c)^4 + 4*a^3 + 1
2*a*b^2 + 11*b^3 + 84*(2*a^3 + 4*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c
)^6 + 6*(84*a^3*cosh(d*x + c)^5 + 28*(2*a^3 + 4*a*b^2 + b^3)*cosh(d*x + c)^
3 + (4*a^3 + 12*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - (4*a^3 + 1
2*a*b^2 + 11*b^3)*cosh(d*x + c)^4 + (420*a^3*cosh(d*x + c)^6 + 210*(2*a^3 +
 4*a*b^2 + b^3)*cosh(d*x + c)^4 - 4*a^3 - 12*a*b^2 - 11*b^3 + 15*(4*a^3 + 1
```

```
2*a*b^2 + 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(60*a^3*cosh(d*x + c
)^7 + 42*(2*a^3 + 4*a*b^2 + b^3)*cosh(d*x + c)^5 + 5*(4*a^3 + 12*a*b^2 + 11
*b^3)*cosh(d*x + c)^3 - (4*a^3 + 12*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d*x
 + c)^3 - 2*a^3 - 3*(2*a^3 + 4*a*b^2 + b^3)*cosh(d*x + c)^2 + 3*(30*a^3*cos
h(d*x + c)^8 + 28*(2*a^3 + 4*a*b^2 + b^3)*cosh(d*x + c)^6 + 5*(4*a^3 + 12*a
*b^2 + 11*b^3)*cosh(d*x + c)^4 - 2*a^3 - 4*a*b^2 - b^3 - 2*(4*a^3 + 12*a*b^
2 + 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*((8*a^2*b + 4*a*b^2 + b^3)
*cosh(d*x + c)^9 + 9*(8*a^2*b + 4*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^
8 + (8*a^2*b + 4*a*b^2 + b^3)*sinh(d*x + c)^9 + 4*(8*a^2*b + 4*a*b^2 + b^3)
*cosh(d*x + c)^7 + 4*(8*a^2*b + 4*a*b^2 + b^3 + 9*(8*a^2*b + 4*a*b^2 + b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 28*(3*(8*a^2*b + 4*a*b^2 + b^3)*cosh(d*
x + c)^3 + (8*a^2*b + 4*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 6*(8*
a^2*b + 4*a*b^2 + b^3)*cosh(d*x + c)^5 + 6*(21*(8*a^2*b + 4*a*b^2 + b^3)*co
sh(d*x + c)^4 + 8*a^2*b + 4*a*b^2 + b^3 + 14*(8*a^2*b + 4*a*b^2 + b^3)*cosh
(d*x + c)^2)*sinh(d*x + c)^5 + 2*(63*(8*a^2*b + 4*a*b^2 + b^3)*cosh(d*x + c
)^5 + 70*(8*a^2*b + 4*a*b^2 + b^3)*cosh(d*x + c)^3 + 15*(8*a^2*b + 4*a*b^2
+ b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(8*a^2*b + 4*a*b^2 + b^3)*cosh(d*
x + c)^3 + 4*(21*(8*a^2*b + 4*a*b^2 + b^3)*cosh(d*x + c)^6 + 35*(8*a^2*b +
4*a*b^2 + b^3)*cosh(d*x + c)^4 + 8*a^2*b + 4*a*b^2 + b^3 + 15*(8*a^2*b + 4*
a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 12*(3*(8*a^2*b + 4*a*b^2 +
b^3)*cosh(d*x + c)^7 + 7*(8*a^2*b + 4*a*b^2 + b^3)*cosh(d*x + c)^5 + 5*(8*a
^2*b + 4*a*b^2 + b^3)*cosh(d*x + c)^3 + (8*a^2*b + 4*a*b^2 + b^3)*cosh(d*x
+ c))*sinh(d*x + c)^2 + (8*a^2*b + 4*a*b^2 + b^3)*cosh(d*x + c) + (9*(8*a^2
*b + 4*a*b^2 + b^3)*cosh(d*x + c)^8 + 28*(8*a^2*b + 4*a*b^2 + b^3)*cosh(d*x
 + c)^6 + 30*(8*a^2*b + 4*a*b^2 + b^3)*cosh(d*x + c)^4 + 8*a^2*b + 4*a*b^2
+ b^3 + 12*(8*a^2*b + 4*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*arctan
(cosh(d*x + c) + sinh(d*x + c)) + 2*(10*a^3*cosh(d*x + c)^9 + 12*(2*a^3 + 4
*a*b^2 + b^3)*cosh(d*x + c)^7 + 3*(4*a^3 + 12*a*b^2 + 11*b^3)*cosh(d*x + c)
^5 - 2*(4*a^3 + 12*a*b^2 + 11*b^3)*cosh(d*x + c)^3 - 3*(2*a^3 + 4*a*b^2 + b
^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^9 + 9*d*cosh(d*x + c)*si
nh(d*x + c)^8 + d*sinh(d*x + c)^9 + 4*d*cosh(d*x + c)^7 + 4*(9*d*cosh(d*x +
 c)^2 + d)*sinh(d*x + c)^7 + 28*(3*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sin
h(d*x + c)^6 + 6*d*cosh(d*x + c)^5 + 6*(21*d*cosh(d*x + c)^4 + 14*d*cosh(d*
x + c)^2 + d)*sinh(d*x + c)^5 + 2*(63*d*cosh(d*x + c)^5 + 70*d*cosh(d*x + c
)^3 + 15*d*cosh(d*x + c))*sinh(d*x + c)^4 + 4*d*cosh(d*x + c)^3 + 4*(21*d*c
osh(d*x + c)^6 + 35*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 + d)*sinh(d*x
+ c)^3 + 12*(3*d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^
3 + d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) + (9*d*cosh(d*x + c)
^8 + 28*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4 + 12*d*cosh(d*x + c)^2 + d
)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)**2)**3,x)

[Out] Timed out
```

Giac [B] time = 1.18348, size = 274, normalized size = 2.95

$$\frac{a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{2d}+\frac{3\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)}-1\right)e^{(-dx-c)}\right)\right)\left(8a^2b+4ab^2+b^3\right)}{16d}+\frac{12ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+3b^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*a^3*(e^(d*x + c) - e^(-d*x - c))/d + 3/16*(pi + 2*arctan(1/2*(e^(2*d*x
+ 2*c) - 1)*e^(-d*x - c)))*(8*a^2*b + 4*a*b^2 + b^3)/d + 1/4*(12*a*b^2*(e^(
d*x + c) - e^(-d*x - c))^3 + 3*b^3*(e^(d*x + c) - e^(-d*x - c))^3 + 48*a*b^
2*(e^(d*x + c) - e^(-d*x - c)) + 20*b^3*(e^(d*x + c) - e^(-d*x - c)))/(((e^
(d*x + c) - e^(-d*x - c))^2 + 4)^2*d)
```

3.69 $\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=147

$$\frac{(2a+b)\left(8a^2+8ab+5b^2\right)\tan^{-1}(\sinh(c+dx))}{16d}+\frac{b\left(44a^2+44ab+15b^2\right)\tanh(c+dx)\operatorname{sech}(c+dx)}{48d}+\frac{b\tanh(c+dx)\operatorname{se}}{}$$

```
[Out] ((2*a + b)*(8*a^2 + 8*a*b + 5*b^2)*ArcTan[Sinh[c + d*x]])/(16*d) + (b*(44*a
^2 + 44*a*b + 15*b^2)*Sech[c + d*x]*Tanh[c + d*x])/(48*d) + (5*b*(2*a + b)*
Sech[c + d*x]^3*(a + b + a*Sinh[c + d*x]^2)*Tanh[c + d*x])/(24*d) + (b*Sech
[c + d*x]^5*(a + b + a*Sinh[c + d*x]^2)^2*Tanh[c + d*x])/(6*d)
```

Rubi [A] time = 0.14281, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4147, 413, 526, 385, 203}

$$\frac{(2a+b)\left(8a^2+8ab+5b^2\right)\tan^{-1}(\sinh(c+dx))}{16d}+\frac{b\left(44a^2+44ab+15b^2\right)\tanh(c+dx)\operatorname{sech}(c+dx)}{48d}+\frac{b\tanh(c+dx)\operatorname{se}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]
```


```
[Out] ((2*a + b)*(8*a^2 + 8*a*b + 5*b^2)*ArcTan[Sinh[c + d*x]])/(16*d) + (b*(44*a
^2 + 44*a*b + 15*b^2)*Sech[c + d*x]*Tanh[c + d*x])/(48*d) + (5*b*(2*a + b)*
Sech[c + d*x]^3*(a + b + a*Sinh[c + d*x]^2)*Tanh[c + d*x])/(24*d) + (b*Sech
[c + d*x]^5*(a + b + a*Sinh[c + d*x]^2)^2*Tanh[c + d*x])/(6*d)
```


Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int \frac{(a+b+ax^2)^3}{(1+x^2)^4}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{b\operatorname{sech}^5(c+dx)\left(a+b+a\sinh^2(c+dx)\right)^2\tanh(c+dx)}{6d} + \frac{\operatorname{Subst}\left(\int \frac{(a+b\cdot}{}\right.}{}$$

$$= \frac{5b(2a+b)\operatorname{sech}^3(c+dx)\left(a+b+a\sinh^2(c+dx)\right)\tanh(c+dx)}{24d} + \frac{b\operatorname{sech}}{}$$

$$= \frac{b\left(44a^2+44ab+15b^2\right)\operatorname{sech}(c+dx)\tanh(c+dx)}{48d} + \frac{5b(2a+b)\operatorname{sech}^3(c}{}$$

$$= \frac{(2a+b)\left(8a^2+8ab+5b^2\right)\tan^{-1}(\sinh(c+dx))}{16d} + \frac{b\left(44a^2+44ab+15b^2\right)}{}$$

Mathematica [C] time = 9.73498, size = 1430, normalized size = 9.73

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (Coth[c + d*x]^6*Csch[c + d*x]*(a + b*Sech[c + d*x]^2)^3*(117228825*(a + b)
^3*Sinh[c + d*x]^2 + 274542345*a*(a + b)^2*Sinh[c + d*x]^4 + 70189350*(a +
b)^3*Sinh[c + d*x]^4 + 215549775*a^2*(a + b)*Sinh[c + d*x]^6 + 168951510*a*
(a + b)^2*Sinh[c + d*x]^6 + 4093425*(a + b)^3*Sinh[c + d*x]^6 + 58009455*a^
3*Sinh[c + d*x]^8 + 135323370*a^2*(a + b)*Sinh[c + d*x]^8 + 9514449*a*(a +
b)^2*Sinh[c + d*x]^8 + 36772890*a^3*Sinh[c + d*x]^10 + 7808535*a^2*(a + b)*
Sinh[c + d*x]^10 - 75520*(a + b)^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1,
 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^10 - 13824*(a + b)^3*Hypergeo
metricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[
c + d*x]^10 - 1024*(a + b)^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1,
 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^10 + 2160711*a^3*Sinh[c
 + d*x]^12 - 189696*a*(a + b)^2*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1,
 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^12 - 38400*a*(a + b)^2*Hypergeom
etricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c
 + d*x]^12 - 3072*a*(a + b)^2*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1
, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^12 - 158976*a^2*(a + b
)*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*S
inh[c + d*x]^14 - 35328*a^2*(a + b)*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2},
```

```
{1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^14 - 3072*a^2*(a + b)*
HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[c +
 d*x]^2]*Sinh[c + d*x]^14 - 44800*a^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2},
{1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^16 - 10752*a^3*Hypergeomet
ricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c +
 d*x]^16 - 1024*a^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1,
 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^16 + 117228825*(a + b)^3*ArcTanh
[Sqrt[-Sinh[c + d*x]^2]]*Sqrt[-Sinh[c + d*x]^2] + 215549775*a^2*(a + b)*Arc
Tanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^4*Sqrt[-Sinh[c + d*x]^2] + 26046
5625*a*(a + b)^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^4*Sqrt[-Sinh
[c + d*x]^2] + 17069535*(a + b)^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c +
d*x]^4*Sqrt[-Sinh[c + d*x]^2] + 58009455*a^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]
]*Sinh[c + d*x]^6*Sqrt[-Sinh[c + d*x]^2] + 207173295*a^2*(a + b)*ArcTanh[Sq
rt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6*Sqrt[-Sinh[c + d*x]^2] + 41427855*a*(
a + b)^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6*Sqrt[-Sinh[c + d*x
]^2] + 56109375*a^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^8*Sqrt[-S
inh[c + d*x]^2] + 33756345*a^2*(a + b)*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh
[c + d*x]^8*Sqrt[-Sinh[c + d*x]^2] + 210735*a*(a + b)^2*ArcTanh[Sqrt[-Sinh[
c + d*x]^2]]*Sinh[c + d*x]^8*Sqrt[-Sinh[c + d*x]^2] + 9261945*a^3*ArcTanh[S
qrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^10*Sqrt[-Sinh[c + d*x]^2] + 174825*a^2
*(a + b)*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^10*Sqrt[-Sinh[c + d*
x]^2] + 48825*a^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^12*Sqrt[-Si
nh[c + d*x]^2] - 274542345*a*(a + b)^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*(-Si
nh[c + d*x]^2)^(3/2) - 109265625*(a + b)^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*
(-Sinh[c + d*x]^2)^(3/2) + 142065*(a + b)^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]
*Sinh[c + d*x]^4*(-Sinh[c + d*x]^2)^(3/2)))/(90720*d*(a + 2*b + a*Cosh[2*c
+ 2*d*x])^3)
```

Maple [A] time = 0.03, size = 193, normalized size = 1.3

$$2\,\frac{a^3\arctan\left(e^{dx+c}\right)}{d}+\frac{3\,a^2b\operatorname{sech}(dx+c)\tanh(dx+c)}{2\,d}+3\,\frac{a^2b\arctan\left(e^{dx+c}\right)}{d}+\frac{3\,ab^2\tanh(dx+c)\left(\operatorname{sech}(dx+c)\right)^3}{4\,d}+\dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 2/d*a^3*arctan(exp(d*x+c))+3/2*a^2*b*sech(d*x+c)*tanh(d*x+c)/d+3/d*a^2*b*ar
ctan(exp(d*x+c))+3/4/d*a*b^2*tanh(d*x+c)*sech(d*x+c)^3+9/8/d*a*b^2*sech(d*x
+c)*tanh(d*x+c)+9/4/d*a*b^2*arctan(exp(d*x+c))+1/6/d*b^3*tanh(d*x+c)*sech(d
*x+c)^5+5/24*b^3*sech(d*x+c)^3*tanh(d*x+c)/d+5/16*b^3*sech(d*x+c)*tanh(d*x+
c)/d+5/8/d*b^3*arctan(exp(d*x+c))
```

Maxima [B] time = 1.71562, size = 493, normalized size = 3.35

$$-\frac{1}{24}\,b^3\left(\frac{15\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{15\,e^{(-dx-c)}+85\,e^{(-3\,dx-3\,c)}+198\,e^{(-5\,dx-5\,c)}-198\,e^{(-7\,dx-7\,c)}-85\,e^{(-9\,dx-9\,c)}-15\,e^{(-11\,dx-11\dots}}{d\left(6\,e^{(-2\,dx-2\,c)}+15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}+15\,e^{(-8\,dx-8\,c)}+6\,e^{(-10\,dx-10\,c)}+e^{(-12\,dx-12\,c)}\dots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/24*b^3*(15*arctan(e^(-d*x - c))/d - (15*e^(-d*x - c) + 85*e^(-3*d*x - 3*
c) + 198*e^(-5*d*x - 5*c) - 198*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9*c) - 15
```

```
*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-
6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12
*c) + 1))) - 3/4*a*b^2*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(
-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x -
 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) -
 3*a^2*b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*
e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a^3*arctan(sinh(d*x + c))/d
```

Fricas [B] time = 2.4899, size = 8805, normalized size = 59.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^11 + 33*(24*a^2*b + 18*
a*b^2 + 5*b^3)*cosh(d*x + c)*sinh(d*x + c)^10 + 3*(24*a^2*b + 18*a*b^2 + 5*
b^3)*sinh(d*x + c)^11 + (216*a^2*b + 306*a*b^2 + 85*b^3)*cosh(d*x + c)^9 +
(216*a^2*b + 306*a*b^2 + 85*b^3 + 165*(24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*
x + c)^2)*sinh(d*x + c)^9 + 9*(55*(24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x +
c)^3 + (216*a^2*b + 306*a*b^2 + 85*b^3)*cosh(d*x + c))*sinh(d*x + c)^8 + 18
*(8*a^2*b + 14*a*b^2 + 11*b^3)*cosh(d*x + c)^7 + 18*(55*(24*a^2*b + 18*a*b^
2 + 5*b^3)*cosh(d*x + c)^4 + 8*a^2*b + 14*a*b^2 + 11*b^3 + 2*(216*a^2*b + 3
06*a*b^2 + 85*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 42*(33*(24*a^2*b + 18
*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 2*(216*a^2*b + 306*a*b^2 + 85*b^3)*cosh(d
*x + c)^3 + 3*(8*a^2*b + 14*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^6
- 18*(8*a^2*b + 14*a*b^2 + 11*b^3)*cosh(d*x + c)^5 + 18*(77*(24*a^2*b + 18*
a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 7*(216*a^2*b + 306*a*b^2 + 85*b^3)*cosh(d*
x + c)^4 - 8*a^2*b - 14*a*b^2 - 11*b^3 + 21*(8*a^2*b + 14*a*b^2 + 11*b^3)*c
osh(d*x + c)^2)*sinh(d*x + c)^5 + 18*(55*(24*a^2*b + 18*a*b^2 + 5*b^3)*cosh
(d*x + c)^7 + 7*(216*a^2*b + 306*a*b^2 + 85*b^3)*cosh(d*x + c)^5 + 35*(8*a^
2*b + 14*a*b^2 + 11*b^3)*cosh(d*x + c)^3 - 5*(8*a^2*b + 14*a*b^2 + 11*b^3)*
cosh(d*x + c))*sinh(d*x + c)^4 - (216*a^2*b + 306*a*b^2 + 85*b^3)*cosh(d*x
+ c)^3 + (495*(24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 84*(216*a^2*b
 + 306*a*b^2 + 85*b^3)*cosh(d*x + c)^6 + 630*(8*a^2*b + 14*a*b^2 + 11*b^3)*
cosh(d*x + c)^4 - 216*a^2*b - 306*a*b^2 - 85*b^3 - 180*(8*a^2*b + 14*a*b^2
+ 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 3*(55*(24*a^2*b + 18*a*b^2 + 5
*b^3)*cosh(d*x + c)^9 + 12*(216*a^2*b + 306*a*b^2 + 85*b^3)*cosh(d*x + c)^7
 + 126*(8*a^2*b + 14*a*b^2 + 11*b^3)*cosh(d*x + c)^5 - 60*(8*a^2*b + 14*a*b
^2 + 11*b^3)*cosh(d*x + c)^3 - (216*a^2*b + 306*a*b^2 + 85*b^3)*cosh(d*x +
c))*sinh(d*x + c)^2 + 3*((16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x +
c)^12 + 12*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)*sinh(d*x +
c)^11 + (16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*sinh(d*x + c)^12 + 6*(16*a^3
 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^10 + 6*(16*a^3 + 24*a^2*b + 1
8*a*b^2 + 5*b^3 + 11*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^2
)*sinh(d*x + c)^10 + 20*(11*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x
 + c)^3 + 3*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x
+ c)^9 + 15*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 15*(33
*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 16*a^3 + 24*a^2*b
 + 18*a*b^2 + 5*b^3 + 18*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x +
c)^2)*sinh(d*x + c)^8 + 24*(33*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(
d*x + c)^5 + 30*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 5*
(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 20*
(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 4*(231*(16*a^3 + 2
4*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 315*(16*a^3 + 24*a^2*b + 18*a
*b^2 + 5*b^3)*cosh(d*x + c)^4 + 80*a^3 + 120*a^2*b + 90*a*b^2 + 25*b^3 + 10
```

```
5*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 +
 24*(33*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 63*(16*a^3
 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 35*(16*a^3 + 24*a^2*b + 1
8*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 5*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)
*cosh(d*x + c))*sinh(d*x + c)^5 + 15*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)
*cosh(d*x + c)^4 + 15*(33*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x +
 c)^8 + 84*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 70*(16*
a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 16*a^3 + 24*a^2*b + 18
*a*b^2 + 5*b^3 + 20*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^4 + 20*(11*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x +
 c)^9 + 36*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 42*(16*
a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 20*(16*a^3 + 24*a^2*b
+ 18*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 3*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b
^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3 +
 6*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^2 + 6*(11*(16*a^3 +
 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^10 + 45*(16*a^3 + 24*a^2*b + 18
*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 70*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)
*cosh(d*x + c)^6 + 50*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^
4 + 16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3 + 15*(16*a^3 + 24*a^2*b + 18*a*b^2
 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 12*((16*a^3 + 24*a^2*b + 18*a*
b^2 + 5*b^3)*cosh(d*x + c)^11 + 5*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*co
sh(d*x + c)^9 + 10*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^7 +
 10*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 5*(16*a^3 + 24
*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + (16*a^3 + 24*a^2*b + 18*a*b^2
+ 5*b^3)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)
) - 3*(24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c) + 3*(11*(24*a^2*b + 18*a*
b^2 + 5*b^3)*cosh(d*x + c)^10 + 3*(216*a^2*b + 306*a*b^2 + 85*b^3)*cosh(d*x
 + c)^8 + 42*(8*a^2*b + 14*a*b^2 + 11*b^3)*cosh(d*x + c)^6 - 30*(8*a^2*b +
14*a*b^2 + 11*b^3)*cosh(d*x + c)^4 - 24*a^2*b - 18*a*b^2 - 5*b^3 - (216*a^2
*b + 306*a*b^2 + 85*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^1
2 + 12*d*cosh(d*x + c)*sinh(d*x + c)^11 + d*sinh(d*x + c)^12 + 6*d*cosh(d*x
 + c)^10 + 6*(11*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^10 + 20*(11*d*cosh(d*
x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^9 + 15*d*cosh(d*x + c)^8 + 15*(
33*d*cosh(d*x + c)^4 + 18*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 24*(33*d
*cosh(d*x + c)^5 + 30*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^
7 + 20*d*cosh(d*x + c)^6 + 4*(231*d*cosh(d*x + c)^6 + 315*d*cosh(d*x + c)^4
 + 105*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^6 + 24*(33*d*cosh(d*x + c)^7
+ 63*d*cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x
 + c)^5 + 15*d*cosh(d*x + c)^4 + 15*(33*d*cosh(d*x + c)^8 + 84*d*cosh(d*x +
 c)^6 + 70*d*cosh(d*x + c)^4 + 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 +
20*(11*d*cosh(d*x + c)^9 + 36*d*cosh(d*x + c)^7 + 42*d*cosh(d*x + c)^5 + 20
*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 6*d*cosh(d*x + c)
^2 + 6*(11*d*cosh(d*x + c)^10 + 45*d*cosh(d*x + c)^8 + 70*d*cosh(d*x + c)^6
 + 50*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 12*(d
*cosh(d*x + c)^11 + 5*d*cosh(d*x + c)^9 + 10*d*cosh(d*x + c)^7 + 10*d*cosh(
d*x + c)^5 + 5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\operatorname{sech}^2(c + dx)\right)^3 \operatorname{sech}(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*sech(c + d*x), x)
```

Giac [B] time = 1.16502, size = 420, normalized size = 2.86

$$\frac{\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)} - 1\right)e^{(-dx-c)}\right)\right)\left(16\,a^3 + 24\,a^2b + 18\,ab^2 + 5\,b^3\right)}{32\,d} + \frac{72\,a^2b\left(e^{(dx+c)} - e^{(-dx-c)}\right)^5 + 54\,ab^2\left(e^{(dx+c)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/32*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(16*a^3 + 24*a
^2*b + 18*a*b^2 + 5*b^3)/d + 1/24*(72*a^2*b*(e^(d*x + c) - e^(-d*x - c))^5
+ 54*a*b^2*(e^(d*x + c) - e^(-d*x - c))^5 + 15*b^3*(e^(d*x + c) - e^(-d*x -
 c))^5 + 576*a^2*b*(e^(d*x + c) - e^(-d*x - c))^3 + 576*a*b^2*(e^(d*x + c)
- e^(-d*x - c))^3 + 160*b^3*(e^(d*x + c) - e^(-d*x - c))^3 + 1152*a^2*b*(e^
(d*x + c) - e^(-d*x - c)) + 1440*a*b^2*(e^(d*x + c) - e^(-d*x - c)) + 528*b
^3*(e^(d*x + c) - e^(-d*x - c)))/(((e^(d*x + c) - e^(-d*x - c))^2 + 4)^3*d)
```

3.70 $\int \mathbf{sech}^2(c+dx)\left(a+b\mathbf{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=74

$$\frac{3b^2(a+b)\tanh^5(c+dx)}{5d}-\frac{b(a+b)^2\tanh^3(c+dx)}{d}+\frac{(a+b)^3\tanh(c+dx)}{d}-\frac{b^3\tanh^7(c+dx)}{7d}$$

```
[Out] ((a + b)^3*Tanh[c + d*x])/d - (b*(a + b)^2*Tanh[c + d*x]^3)/d + (3*b^2*(a +
 b)*Tanh[c + d*x]^5)/(5*d) - (b^3*Tanh[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.079839, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4146, 194}

$$\frac{3b^2(a+b)\tanh^5(c+dx)}{5d}-\frac{b(a+b)^2\tanh^3(c+dx)}{d}+\frac{(a+b)^3\tanh(c+dx)}{d}-\frac{b^3\tanh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + b)^3*Tanh[c + d*x])/d - (b*(a + b)^2*Tanh[c + d*x]^3)/d + (3*b^2*(a +
 b)*Tanh[c + d*x]^5)/(5*d) - (b^3*Tanh[c + d*x]^7)/(7*d)
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \text{sech}^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx=\frac{\text{Subst}\left(\int \left(a+b-bx^2\right)^3\,dx,x,\tanh(c+dx)\right)}{d}$$
$$=\frac{\text{Subst}\left(\int \left(a^3\left(1+\frac{b\left(3a^2+3ab+b^2\right)}{a^3}\right)-3b(a+b)^2x^2+3b^2(a+b)x^4-b^3x^6\right)dx,\ldots\right)}{d}$$
$$=\frac{(a+b)^3\tanh(c+dx)}{d}-\frac{b(a+b)^2\tanh^3(c+dx)}{d}+\frac{3b^2(a+b)\tanh^5(c+dx)}{5d}\ldots$$

Mathematica [B] time = 1.41505, size = 319, normalized size = 4.31

$$\frac{\text{sech}(c)\text{sech}(c+dx)\left(-35a\left(15a^2+26ab+16b^2\right)\sinh(2c+dx)+140\left(11a^2b+5a^3+10ab^2+4b^3\right)\sinh(dx)+1260a^2b\,\text{si}\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (Sech[c]*Sech[c + d*x]*(a + b*Sech[c + d*x]^2)^3*(140*(5*a^3 + 11*a^2*b + 1
0*a*b^2 + 4*b^3)*Sinh[d*x] - 35*a*(15*a^2 + 26*a*b + 16*b^2)*Sinh[2*c + d*x
] + 525*a^3*Sinh[2*c + 3*d*x] + 1260*a^2*b*Sinh[2*c + 3*d*x] + 1176*a*b^2*S
inh[2*c + 3*d*x] + 336*b^3*Sinh[2*c + 3*d*x] - 210*a^3*Sinh[4*c + 3*d*x] -
210*a^2*b*Sinh[4*c + 3*d*x] + 210*a^3*Sinh[4*c + 5*d*x] + 490*a^2*b*Sinh[4*
c + 5*d*x] + 392*a*b^2*Sinh[4*c + 5*d*x] + 112*b^3*Sinh[4*c + 5*d*x] - 35*a
^3*Sinh[6*c + 5*d*x] + 35*a^3*Sinh[6*c + 7*d*x] + 70*a^2*b*Sinh[6*c + 7*d*x
] + 56*a*b^2*Sinh[6*c + 7*d*x] + 16*b^3*Sinh[6*c + 7*d*x]))/(280*d*(a + 2*b
 + a*Cosh[2*(c + d*x)])^3)
```

Maple [A] time = 0.026, size = 116, normalized size = 1.6

$$\frac{1}{d}\left(a^3\tanh(dx+c)+3\,a^2b\left(2/3+1/3\,(\mathrm{sech}\,(dx+c))^2\right)\tanh(dx+c)+3\,ab^2\left(\frac{8}{15}+1/5\,(\mathrm{sech}\,(dx+c))^4+\frac{4\,(\mathrm{sech}\,(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*tanh(d*x+c)+3*a^2*b*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+3*a*b^2*(8
/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c)+b^3*(16/35+1/7*sech(d
*x+c)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c))
```

Maxima [B] time = 1.23289, size = 938, normalized size = 12.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 32/35*b^3*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c)
+ 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(
-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 21*e^(-4*d*x - 4*c)/(d*(7*e^(-
2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8
*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1
)) + 35*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*
e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d
*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4
*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 1
0*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 16/5*a*b^2*(5*e^(
-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 10*e^(-4*d*x - 4*c)
/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(
-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^
(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10
*c) + 1))) + 4*a^2*b*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d
*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x
 - 4*c) + e^(-6*d*x - 6*c) + 1))) + 2*a^3/(d*(e^(-2*d*x - 2*c) + 1))
```

Fricas [B] time = 2.05481, size = 2103, normalized size = 28.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -4/35*((35*a^3 + 35*a^2*b + 28*a*b^2 + 8*b^3)*cosh(d*x + c)^6 - 6*(35*a^2*b
 + 28*a*b^2 + 8*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (35*a^3 + 35*a^2*b + 2
8*a*b^2 + 8*b^3)*sinh(d*x + c)^6 + 14*(15*a^3 + 25*a^2*b + 14*a*b^2 + 4*b^3
)*cosh(d*x + c)^4 + (210*a^3 + 350*a^2*b + 196*a*b^2 + 56*b^3 + 15*(35*a^3
+ 35*a^2*b + 28*a*b^2 + 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 4*(5*(35*
a^2*b + 28*a*b^2 + 8*b^3)*cosh(d*x + c)^3 + 28*(5*a^2*b + 7*a*b^2 + 2*b^3)*
cosh(d*x + c))*sinh(d*x + c)^3 + 350*a^3 + 770*a^2*b + 700*a*b^2 + 280*b^3
+ 7*(75*a^3 + 155*a^2*b + 124*a*b^2 + 24*b^3)*cosh(d*x + c)^2 + (15*(35*a^3
 + 35*a^2*b + 28*a*b^2 + 8*b^3)*cosh(d*x + c)^4 + 525*a^3 + 1085*a^2*b + 86
8*a*b^2 + 168*b^3 + 84*(15*a^3 + 25*a^2*b + 14*a*b^2 + 4*b^3)*cosh(d*x + c)
^2)*sinh(d*x + c)^2 - 2*(3*(35*a^2*b + 28*a*b^2 + 8*b^3)*cosh(d*x + c)^5 +
56*(5*a^2*b + 7*a*b^2 + 2*b^3)*cosh(d*x + c)^3 + 7*(25*a^2*b + 44*a*b^2 + 2
4*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)
*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 + 8*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*
x + c)^2 + 2*d)*sinh(d*x + c)^6 + 4*(14*d*cosh(d*x + c)^3 + 9*d*cosh(d*x +
c))*sinh(d*x + c)^5 + 28*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 + 60*d
*cosh(d*x + c)^2 + 14*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 + 15*d*co
sh(d*x + c)^3 + 7*d*cosh(d*x + c))*sinh(d*x + c)^3 + 56*d*cosh(d*x + c)^2 +
 4*(7*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4 + 42*d*cosh(d*x + c)^2 + 14*
d)*sinh(d*x + c)^2 + 4*(2*d*cosh(d*x + c)^7 + 9*d*cosh(d*x + c)^5 + 14*d*co
sh(d*x + c)^3 + 7*d*cosh(d*x + c))*sinh(d*x + c) + 35*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\operatorname{sech}^2(c + dx)\right)^3 \operatorname{sech}^2(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*sech(c + d*x)**2, x)
```

Giac [B] time = 1.16792, size = 408, normalized size = 5.51

$$-\frac{2\left(35\,a^3e^{(12\,dx+12\,c)} + 210\,a^3e^{(10\,dx+10\,c)} + 210\,a^2be^{(10\,dx+10\,c)} + 525\,a^3e^{(8\,dx+8\,c)} + 910\,a^2be^{(8\,dx+8\,c)} + 560\,ab^2e^{(8\,dx+8\,c)} + 700\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -2/35*(35*a^3*e^(12*d*x + 12*c) + 210*a^3*e^(10*d*x + 10*c) + 210*a^2*b*e^(
10*d*x + 10*c) + 525*a^3*e^(8*d*x + 8*c) + 910*a^2*b*e^(8*d*x + 8*c) + 560*
a*b^2*e^(8*d*x + 8*c) + 700*a^3*e^(6*d*x + 6*c) + 1540*a^2*b*e^(6*d*x + 6*c
) + 1400*a*b^2*e^(6*d*x + 6*c) + 560*b^3*e^(6*d*x + 6*c) + 525*a^3*e^(4*d*x
```

```
 + 4*c) + 1260*a^2*b*e^(4*d*x + 4*c) + 1176*a*b^2*e^(4*d*x + 4*c) + 336*b^3
*e^(4*d*x + 4*c) + 210*a^3*e^(2*d*x + 2*c) + 490*a^2*b*e^(2*d*x + 2*c) + 39
2*a*b^2*e^(2*d*x + 2*c) + 112*b^3*e^(2*d*x + 2*c) + 35*a^3 + 70*a^2*b + 56*
a*b^2 + 16*b^3)/(d*(e^(2*d*x + 2*c) + 1)^7)
```

3.71 $\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=196

$$\frac{(144a^2b+64a^3+120ab^2+35b^3)\tan^{-1}(\sinh(c+dx))}{128d}+\frac{b(72a^2+92ab+35b^2)\tanh(c+dx)\operatorname{sech}^3(c+dx)}{192d}+\frac{(144a^2b\cdots}{}$$

```
[Out] ((64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*ArcTan[Sinh[c + d*x]])/(128*d) +
 ((64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*Sech[c + d*x]*Tanh[c + d*x])/(1
28*d) + (b*(72*a^2 + 92*a*b + 35*b^2)*Sech[c + d*x]^3*Tanh[c + d*x])/(192*d
) + (b*(12*a + 7*b)*Sech[c + d*x]^5*(a + b + a*Sinh[c + d*x]^2)*Tanh[c + d*
x])/(48*d) + (b*Sech[c + d*x]^7*(a + b + a*Sinh[c + d*x]^2)^2*Tanh[c + d*x]
)/(8*d)
```

Rubi [A] time = 0.232029, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4147, 413, 526, 385, 199, 203}

$$\frac{(144a^2b+64a^3+120ab^2+35b^3)\tan^{-1}(\sinh(c+dx))}{128d}+\frac{b(72a^2+92ab+35b^2)\tanh(c+dx)\operatorname{sech}^3(c+dx)}{192d}+\frac{(144a^2b\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]
```



```
[Out] ((64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*ArcTan[Sinh[c + d*x]])/(128*d) +
 ((64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*Sech[c + d*x]*Tanh[c + d*x])/(1
28*d) + (b*(72*a^2 + 92*a*b + 35*b^2)*Sech[c + d*x]^3*Tanh[c + d*x])/(192*d
) + (b*(12*a + 7*b)*Sech[c + d*x]^5*(a + b + a*Sinh[c + d*x]^2)*Tanh[c + d*
x])/(48*d) + (b*Sech[c + d*x]^7*(a + b + a*Sinh[c + d*x]^2)^2*Tanh[c + d*x]
)/(8*d)
```


Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
```

```
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+ax^2\right)^3}{\left(1+x^2\right)^5}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{b\operatorname{sech}^7(c+dx)\left(a+b+a\sinh^2(c+dx)\right)^2\tanh(c+dx)}{8d} + \frac{\text{Subst}\left(\int \frac{(a+}{}\right.}{}$$

$$= \frac{b(12a+7b)\operatorname{sech}^5(c+dx)\left(a+b+a\sinh^2(c+dx)\right)\tanh(c+dx)}{48d} + \frac{b\operatorname{se}}{}$$

$$= \frac{b\left(72a^2+92ab+35b^2\right)\operatorname{sech}^3(c+dx)\tanh(c+dx)}{192d} + \frac{b(12a+7b)\operatorname{sech}^5}{}$$

$$= \frac{\left(64a^3+144a^2b+120ab^2+35b^3\right)\operatorname{sech}(c+dx)\tanh(c+dx)}{128d} + \frac{b\left(72a^2\right.}{}$$

$$= \frac{\left(64a^3+144a^2b+120ab^2+35b^3\right)\tan^{-1}(\sinh(c+dx))}{128d} + \frac{\left(64a^3+144a^2\right.}{}$$

Mathematica [A] time = 9.32406, size = 297, normalized size = 1.52

$$\frac{\operatorname{sech}^8(c+dx)\left(a\cosh^2(c+dx)+b\right)^3\left(3\left(144a^2b+64a^3+120ab^2+35b^3\right)\tanh(c)\cosh^7(c+dx)+2b\left(144a^2+120ab\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((b + a*Cosh[c + d*x]^2)^3*Sech[c + d*x]^8*(6*(64*a^3 + 144*a^2*b + 120*a*b
^2 + 35*b^3)*ArcTan[Tanh[(c + d*x)/2]]*Cosh[c + d*x]^8 + 48*b^3*Sech[c]*Sin
```

```
h[d*x] + 8*b^2*(24*a + 7*b)*Cosh[c + d*x]^2*Sech[c]*Sinh[d*x] + 2*b*(144*a^
2 + 120*a*b + 35*b^2)*Cosh[c + d*x]^4*Sech[c]*Sinh[d*x] + 3*(64*a^3 + 144*a
^2*b + 120*a*b^2 + 35*b^3)*Cosh[c + d*x]^6*Sech[c]*Sinh[d*x] + 48*b^3*Cosh[
c + d*x]*Tanh[c] + 8*b^2*(24*a + 7*b)*Cosh[c + d*x]^3*Tanh[c] + 2*b*(144*a^
2 + 120*a*b + 35*b^2)*Cosh[c + d*x]^5*Tanh[c] + 3*(64*a^3 + 144*a^2*b + 120
*a*b^2 + 35*b^3)*Cosh[c + d*x]^7*Tanh[c]))/(48*d*(a + 2*b + a*Cosh[2*(c + d
*x)])^3)
```

Maple [A] time = 0.03, size = 280, normalized size = 1.4

$$\frac{a^3\operatorname{sech}(dx+c)\tanh(dx+c)}{2d}+\frac{a^3\arctan\left(e^{dx+c}\right)}{d}+\frac{3a^2b\tanh(dx+c)(\operatorname{sech}(dx+c))^3}{4d}+\frac{9a^2b\operatorname{sech}(dx+c)\tanh(dx\cdot}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/2/d*a^3*sech(d*x+c)*tanh(d*x+c)+1/d*a^3*arctan(exp(d*x+c))+3/4/d*a^2*b*ta
nh(d*x+c)*sech(d*x+c)^3+9/8*a^2*b*sech(d*x+c)*tanh(d*x+c)/d+9/4/d*a^2*b*arc
tan(exp(d*x+c))+1/2/d*a*b^2*tanh(d*x+c)*sech(d*x+c)^5+5/8/d*a*b^2*tanh(d*x+
c)*sech(d*x+c)^3+15/16/d*a*b^2*sech(d*x+c)*tanh(d*x+c)+15/8/d*a*b^2*arctan(
exp(d*x+c))+1/8/d*b^3*tanh(d*x+c)*sech(d*x+c)^7+7/48/d*b^3*tanh(d*x+c)*sech
(d*x+c)^5+35/192*b^3*sech(d*x+c)^3*tanh(d*x+c)/d+35/128*b^3*sech(d*x+c)*tan
h(d*x+c)/d+35/64/d*b^3*arctan(exp(d*x+c))
```

Maxima [B] time = 1.70378, size = 751, normalized size = 3.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/192*b^3*(105*arctan(e^(-d*x - c))/d - (105*e^(-d*x - c) + 805*e^(-3*d*x
- 3*c) + 2681*e^(-5*d*x - 5*c) + 5053*e^(-7*d*x - 7*c) - 5053*e^(-9*d*x - 9
*c) - 2681*e^(-11*d*x - 11*c) - 805*e^(-13*d*x - 13*c) - 105*e^(-15*d*x - 1
5*c))/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) +
70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(
-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1))) - 1/8*a*b^2*(15*arctan(e^(-d*x
- c))/d - (15*e^(-d*x - c) + 85*e^(-3*d*x - 3*c) + 198*e^(-5*d*x - 5*c) - 1
98*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9*c) - 15*e^(-11*d*x - 11*c))/(d*(6*e^
(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x -
 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c) + 1))) - 3/4*a^2*b*(3*arc
tan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x
- 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) +
4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - a^3*(arctan(e^(-d*x - c))/d
- (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*
c) + 1)))
```

Fricas [B] time = 2.73281, size = 16578, normalized size = 84.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/192*(3*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^15 + 45*(6
4*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)*sinh(d*x + c)^14 + 3*
(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*sinh(d*x + c)^15 + (960*a^3 + 331
2*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)^13 + (960*a^3 + 3312*a^2*b +
2760*a*b^2 + 805*b^3 + 315*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d
*x + c)^2)*sinh(d*x + c)^13 + 13*(105*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*
b^3)*cosh(d*x + c)^3 + (960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d
*x + c))*sinh(d*x + c)^12 + (1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)
*cosh(d*x + c)^11 + (4095*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*
x + c)^4 + 1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3 + 78*(960*a^3 + 33
12*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^11 + 11*(81
9*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^5 + 26*(960*a^3 +
 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)^3 + (1728*a^3 + 7344*a^2*
b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c))*sinh(d*x + c)^10 + (960*a^3 + 446
4*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c)^9 + (15015*(64*a^3 + 144*a^2
*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^6 + 715*(960*a^3 + 3312*a^2*b + 2760
*a*b^2 + 805*b^3)*cosh(d*x + c)^4 + 960*a^3 + 4464*a^2*b + 6792*a*b^2 + 505
3*b^3 + 55*(1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^9 + 3*(6435*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d
*x + c)^7 + 429*(960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)
^5 + 55*(1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c)^3 + 3
*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c))*sinh(d*x + c
)^8 - (960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c)^7 + (193
05*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^8 + 1716*(960*a^
3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)^6 + 330*(1728*a^3 + 73
44*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c)^4 - 960*a^3 - 4464*a^2*b -
6792*a*b^2 - 5053*b^3 + 36*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*c
osh(d*x + c)^2)*sinh(d*x + c)^7 + (15015*(64*a^3 + 144*a^2*b + 120*a*b^2 +
35*b^3)*cosh(d*x + c)^9 + 1716*(960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3
)*cosh(d*x + c)^7 + 462*(1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cos
h(d*x + c)^5 + 84*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x +
 c)^3 - 7*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c))*sin
h(d*x + c)^6 - (1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c
)^5 + (9009*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^10 + 12
87*(960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)^8 + 462*(172
8*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c)^6 + 126*(960*a^3
+ 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c)^4 - 1728*a^3 - 7344*a^2
*b - 9192*a*b^2 - 2681*b^3 - 21*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b
^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + (4095*(64*a^3 + 144*a^2*b + 120*a*b^
2 + 35*b^3)*cosh(d*x + c)^11 + 715*(960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805
*b^3)*cosh(d*x + c)^9 + 330*(1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)
*cosh(d*x + c)^7 + 126*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(
d*x + c)^5 - 35*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c
)^3 - 5*(1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c))*sinh
(d*x + c)^4 - (960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)^3
 + (1365*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^12 + 286*(
960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)^10 + 165*(1728*a
^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c)^8 + 84*(960*a^3 + 44
64*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c)^6 - 35*(960*a^3 + 4464*a^2*
b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c)^4 - 960*a^3 - 3312*a^2*b - 2760*a*
b^2 - 805*b^3 - 10*(1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x
 + c)^2)*sinh(d*x + c)^3 + (315*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*c
osh(d*x + c)^13 + 78*(960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x
 + c)^11 + 55*(1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c)
^9 + 36*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c)^7 - 21
```

```
*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c)^5 - 10*(1728*
a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c)^3 - 3*(960*a^3 + 33
12*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 3*((64*a^
3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^16 + 16*(64*a^3 + 144*a^2
*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)*sinh(d*x + c)^15 + (64*a^3 + 144*a^2
*b + 120*a*b^2 + 35*b^3)*sinh(d*x + c)^16 + 8*(64*a^3 + 144*a^2*b + 120*a*b
^2 + 35*b^3)*cosh(d*x + c)^14 + 8*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3
+ 15*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^2)*sinh(d*x +
c)^14 + 112*(5*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^3 +
(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c))*sinh(d*x + c)^13 +
 28*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^12 + 28*(65*(64
*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^4 + 64*a^3 + 144*a^2*b
 + 120*a*b^2 + 35*b^3 + 26*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d
*x + c)^2)*sinh(d*x + c)^12 + 112*(39*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*
b^3)*cosh(d*x + c)^5 + 26*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*
x + c)^3 + 3*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c))*sinh(
d*x + c)^11 + 56*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^10
 + 56*(143*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^6 + 143*
(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^4 + 64*a^3 + 144*a^
2*b + 120*a*b^2 + 35*b^3 + 33*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cos
h(d*x + c)^2)*sinh(d*x + c)^10 + 16*(715*(64*a^3 + 144*a^2*b + 120*a*b^2 +
35*b^3)*cosh(d*x + c)^7 + 1001*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*co
sh(d*x + c)^5 + 385*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)
^3 + 35*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c))*sinh(d*x +
 c)^9 + 70*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^8 + 2*(6
435*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^8 + 12012*(64*a
^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^6 + 6930*(64*a^3 + 144*a
^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^4 + 2240*a^3 + 5040*a^2*b + 4200*a
*b^2 + 1225*b^3 + 1260*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x +
 c)^2)*sinh(d*x + c)^8 + 16*(715*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*
cosh(d*x + c)^9 + 1716*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x +
 c)^7 + 1386*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^5 + 42
0*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^3 + 35*(64*a^3 +
144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 56*(64*a^3
 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^6 + 56*(143*(64*a^3 + 144*
a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^10 + 429*(64*a^3 + 144*a^2*b + 12
0*a*b^2 + 35*b^3)*cosh(d*x + c)^8 + 462*(64*a^3 + 144*a^2*b + 120*a*b^2 + 3
5*b^3)*cosh(d*x + c)^6 + 210*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh
(d*x + c)^4 + 64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3 + 35*(64*a^3 + 144*a^
2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 112*(39*(64*a^
3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^11 + 143*(64*a^3 + 144*a^
2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^9 + 198*(64*a^3 + 144*a^2*b + 120*a
*b^2 + 35*b^3)*cosh(d*x + c)^7 + 126*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b
^3)*cosh(d*x + c)^5 + 35*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x
 + c)^3 + 3*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c))*sinh(d
*x + c)^5 + 28*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^4 +
28*(65*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^12 + 286*(64
*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^10 + 495*(64*a^3 + 144
*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^8 + 420*(64*a^3 + 144*a^2*b + 12
0*a*b^2 + 35*b^3)*cosh(d*x + c)^6 + 175*(64*a^3 + 144*a^2*b + 120*a*b^2 + 3
5*b^3)*cosh(d*x + c)^4 + 64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3 + 30*(64*a
^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 112
*(5*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^13 + 26*(64*a^3
 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^11 + 55*(64*a^3 + 144*a^2*
b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^9 + 60*(64*a^3 + 144*a^2*b + 120*a*b^
2 + 35*b^3)*cosh(d*x + c)^7 + 35*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*
cosh(d*x + c)^5 + 10*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c
)^3 + (64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c))*sinh(d*x + c
)^3 + 64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3 + 8*(64*a^3 + 144*a^2*b + 120
```

```
*a*b^2 + 35*b^3)*cosh(d*x + c)^2 + 8*(15*(64*a^3 + 144*a^2*b + 120*a*b^2 +
35*b^3)*cosh(d*x + c)^14 + 91*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cos
h(d*x + c)^12 + 231*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)
^10 + 315*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^8 + 245*(
64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^6 + 105*(64*a^3 + 14
4*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^4 + 64*a^3 + 144*a^2*b + 120*a*
b^2 + 35*b^3 + 21*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^2
)*sinh(d*x + c)^2 + 16*((64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x
+ c)^15 + 7*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^13 + 21
*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^11 + 35*(64*a^3 +
144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^9 + 35*(64*a^3 + 144*a^2*b +
120*a*b^2 + 35*b^3)*cosh(d*x + c)^7 + 21*(64*a^3 + 144*a^2*b + 120*a*b^2 +
35*b^3)*cosh(d*x + c)^5 + 7*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(
d*x + c)^3 + (64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c))*sinh(
d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 3*(64*a^3 + 144*a^2*b + 1
20*a*b^2 + 35*b^3)*cosh(d*x + c) + (45*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35
*b^3)*cosh(d*x + c)^14 + 13*(960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*c
osh(d*x + c)^12 + 11*(1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d
*x + c)^10 + 9*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c)
^8 - 7*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c)^6 - 5*(
1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c)^4 - 192*a^3 -
432*a^2*b - 360*a*b^2 - 105*b^3 - 3*(960*a^3 + 3312*a^2*b + 2760*a*b^2 + 80
5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^16 + 16*d*cosh(d*x
+ c)*sinh(d*x + c)^15 + d*sinh(d*x + c)^16 + 8*d*cosh(d*x + c)^14 + 8*(15*d
*cosh(d*x + c)^2 + d)*sinh(d*x + c)^14 + 112*(5*d*cosh(d*x + c)^3 + d*cosh(
d*x + c))*sinh(d*x + c)^13 + 28*d*cosh(d*x + c)^12 + 28*(65*d*cosh(d*x + c)
^4 + 26*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^12 + 112*(39*d*cosh(d*x + c)^5
 + 26*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^11 + 56*d*cosh(d
*x + c)^10 + 56*(143*d*cosh(d*x + c)^6 + 143*d*cosh(d*x + c)^4 + 33*d*cosh(
d*x + c)^2 + d)*sinh(d*x + c)^10 + 16*(715*d*cosh(d*x + c)^7 + 1001*d*cosh(
d*x + c)^5 + 385*d*cosh(d*x + c)^3 + 35*d*cosh(d*x + c))*sinh(d*x + c)^9 +
70*d*cosh(d*x + c)^8 + 2*(6435*d*cosh(d*x + c)^8 + 12012*d*cosh(d*x + c)^6
+ 6930*d*cosh(d*x + c)^4 + 1260*d*cosh(d*x + c)^2 + 35*d)*sinh(d*x + c)^8 +
 16*(715*d*cosh(d*x + c)^9 + 1716*d*cosh(d*x + c)^7 + 1386*d*cosh(d*x + c)^
5 + 420*d*cosh(d*x + c)^3 + 35*d*cosh(d*x + c))*sinh(d*x + c)^7 + 56*d*cosh
(d*x + c)^6 + 56*(143*d*cosh(d*x + c)^10 + 429*d*cosh(d*x + c)^8 + 462*d*co
sh(d*x + c)^6 + 210*d*cosh(d*x + c)^4 + 35*d*cosh(d*x + c)^2 + d)*sinh(d*x
+ c)^6 + 112*(39*d*cosh(d*x + c)^11 + 143*d*cosh(d*x + c)^9 + 198*d*cosh(d*
x + c)^7 + 126*d*cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)
)*sinh(d*x + c)^5 + 28*d*cosh(d*x + c)^4 + 28*(65*d*cosh(d*x + c)^12 + 286*
d*cosh(d*x + c)^10 + 495*d*cosh(d*x + c)^8 + 420*d*cosh(d*x + c)^6 + 175*d*
cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 112*(5*d*cosh
(d*x + c)^13 + 26*d*cosh(d*x + c)^11 + 55*d*cosh(d*x + c)^9 + 60*d*cosh(d*x
 + c)^7 + 35*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*si
nh(d*x + c)^3 + 8*d*cosh(d*x + c)^2 + 8*(15*d*cosh(d*x + c)^14 + 91*d*cosh(
d*x + c)^12 + 231*d*cosh(d*x + c)^10 + 315*d*cosh(d*x + c)^8 + 245*d*cosh(d
*x + c)^6 + 105*d*cosh(d*x + c)^4 + 21*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)
^2 + 16*(d*cosh(d*x + c)^15 + 7*d*cosh(d*x + c)^13 + 21*d*cosh(d*x + c)^11
+ 35*d*cosh(d*x + c)^9 + 35*d*cosh(d*x + c)^7 + 21*d*cosh(d*x + c)^5 + 7*d*
cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\operatorname{sech}^2(c + dx)\right)^3 \operatorname{sech}^3(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*sech(c + d*x)**3, x)
```

Giac [B] time = 1.17047, size = 656, normalized size = 3.35

$$\frac{\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)\left(64\,a^3+144\,a^2b+120\,ab^2+35\,b^3\right)}{256\,d} + \frac{192\,a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^7+432\,a^2b\left(e^{(dx+ \right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/256*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(64*a^3 + 144
*a^2*b + 120*a*b^2 + 35*b^3)/d + 1/192*(192*a^3*(e^(d*x + c) - e^(-d*x - c)
)^7 + 432*a^2*b*(e^(d*x + c) - e^(-d*x - c))^7 + 360*a*b^2*(e^(d*x + c) - e
^(-d*x - c))^7 + 105*b^3*(e^(d*x + c) - e^(-d*x - c))^7 + 2304*a^3*(e^(d*x
+ c) - e^(-d*x - c))^5 + 6336*a^2*b*(e^(d*x + c) - e^(-d*x - c))^5 + 5280*a
*b^2*(e^(d*x + c) - e^(-d*x - c))^5 + 1540*b^3*(e^(d*x + c) - e^(-d*x - c))
^5 + 9216*a^3*(e^(d*x + c) - e^(-d*x - c))^3 + 29952*a^2*b*(e^(d*x + c) - e
^(-d*x - c))^3 + 28032*a*b^2*(e^(d*x + c) - e^(-d*x - c))^3 + 8176*b^3*(e^(
d*x + c) - e^(-d*x - c))^3 + 12288*a^3*(e^(d*x + c) - e^(-d*x - c)) + 46080
*a^2*b*(e^(d*x + c) - e^(-d*x - c)) + 50688*a*b^2*(e^(d*x + c) - e^(-d*x -
c)) + 17856*b^3*(e^(d*x + c) - e^(-d*x - c)))/(((e^(d*x + c) - e^(-d*x - c)
)^2 + 4)^4*d)
```

3.72 $\int \mathrm{sech}^4(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=108

$$-\frac{b^2(3a+4b)\tanh^7(c+dx)}{7d}+\frac{3b(a+b)(a+2b)\tanh^5(c+dx)}{5d}-\frac{(a+b)^2(a+4b)\tanh^3(c+dx)}{3d}+\frac{(a+b)^3\tanh(c+$$

```
[Out] ((a + b)^3*Tanh[c + d*x])/d - ((a + b)^2*(a + 4*b)*Tanh[c + d*x]^3)/(3*d) +
 (3*b*(a + b)*(a + 2*b)*Tanh[c + d*x]^5)/(5*d) - (b^2*(3*a + 4*b)*Tanh[c +
d*x]^7)/(7*d) + (b^3*Tanh[c + d*x]^9)/(9*d)
```

Rubi [A] time = 0.0937486, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4146, 373}

$$-\frac{b^2(3a+4b)\tanh^7(c+dx)}{7d}+\frac{3b(a+b)(a+2b)\tanh^5(c+dx)}{5d}-\frac{(a+b)^2(a+4b)\tanh^3(c+dx)}{3d}+\frac{(a+b)^3\tanh(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + b)^3*Tanh[c + d*x])/d - ((a + b)^2*(a + 4*b)*Tanh[c + d*x]^3)/(3*d) +
 (3*b*(a + b)*(a + 2*b)*Tanh[c + d*x]^5)/(5*d) - (b^2*(3*a + 4*b)*Tanh[c +
d*x]^7)/(7*d) + (b^3*Tanh[c + d*x]^9)/(9*d)
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \mathrm{sech}^4(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3\,dx &= \frac{\mathrm{Subst}\left(\int\left(1-x^2\right)\left(a+b-bx^2\right)^3\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\mathrm{Subst}\left(\int\left((a+b)^3-(a+b)^2(a+4b)x^2+3b(a+b)(a+2b)x^4-b^2(3a+\right.\right.}{d}\\ &= \frac{(a+b)^3\tanh(c+dx)}{d}-\frac{(a+b)^2(a+4b)\tanh^3(c+dx)}{3d}+\frac{3b(a+b)(a+\right.}{}\end{aligned}$$

Mathematica [B] time = 1.70855, size = 348, normalized size = 3.22

$$\mathrm{sech}(c)\mathrm{sech}^9(c+dx)\left(-315a\left(17a^2+36ab+24b^2\right)\sinh(2c+dx)+63\left(324a^2b+125a^3+312ab^2+128b^3\right)\sinh(dx)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (Sech[c]*Sech[c + d*x]^9*(63*(125*a^3 + 324*a^2*b + 312*a*b^2 + 128*b^3)*Si
nh[d*x] - 315*a*(17*a^2 + 36*a*b + 24*b^2)*Sinh[2*c + d*x] + 6825*a^3*Sinh[
2*c + 3*d*x] + 18648*a^2*b*Sinh[2*c + 3*d*x] + 18144*a*b^2*Sinh[2*c + 3*d*x
] + 5376*b^3*Sinh[2*c + 3*d*x] - 1995*a^3*Sinh[4*c + 3*d*x] - 2520*a^2*b*Si
nh[4*c + 3*d*x] + 3465*a^3*Sinh[4*c + 5*d*x] + 9072*a^2*b*Sinh[4*c + 5*d*x]
 + 7776*a*b^2*Sinh[4*c + 5*d*x] + 2304*b^3*Sinh[4*c + 5*d*x] - 315*a^3*Sinh
[6*c + 5*d*x] + 945*a^3*Sinh[6*c + 7*d*x] + 2268*a^2*b*Sinh[6*c + 7*d*x] +
1944*a*b^2*Sinh[6*c + 7*d*x] + 576*b^3*Sinh[6*c + 7*d*x] + 105*a^3*Sinh[8*c
 + 9*d*x] + 252*a^2*b*Sinh[8*c + 9*d*x] + 216*a*b^2*Sinh[8*c + 9*d*x] + 64*
b^3*Sinh[8*c + 9*d*x]))/(40320*d)
```

Maple [A] time = 0.029, size = 158, normalized size = 1.5

$$\frac{1}{d}\left(a^3\left(\frac{2}{3}+\frac{(\mathrm{sech}\,(dx+c))^2}{3}\right)\tanh(dx+c)+3\,a^2b\left(\frac{8}{15}+1/5\,(\mathrm{sech}\,(dx+c))^4+\frac{4\,(\mathrm{sech}\,(dx+c))^2}{15}\right)\tanh(dx+c)+3\,ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+3*a^2*b*(8/15+1/5*sech(d*x+c)^
4+4/15*sech(d*x+c)^2)*tanh(d*x+c)+3*a*b^2*(16/35+1/7*sech(d*x+c)^6+6/35*sec
h(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c)+b^3*(128/315+1/9*sech(d*x+c)^8+8
/63*sech(d*x+c)^6+16/105*sech(d*x+c)^4+64/315*sech(d*x+c)^2)*tanh(d*x+c))
```

Maxima [B] time = 1.1259, size = 1681, normalized size = 15.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 256/315*b^3*(9*e^(-2*d*x - 2*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c
) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 8
4*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-1
8*d*x - 18*c) + 1)) + 36*e^(-4*d*x - 4*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4
*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x -
 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*
c) + e^(-18*d*x - 18*c) + 1)) + 84*e^(-6*d*x - 6*c)/(d*(9*e^(-2*d*x - 2*c)
+ 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^
(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16
*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) + 126*e^(-8*d*x - 8*c)/(d*(9*e^(-2*
d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*
c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c)
 + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) + 1/(d*(9*e^(-2*d*x - 2*
c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126
*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(
-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1))) + 96/35*a*b^2*(7*e^(-2*d*x - 2*
c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*
```

```
e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*
x - 14*c) + 1)) + 21*e^(-4*d*x - 4*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x
 - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c)
 + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 35*e^(-6*d*x - 6*c)/(d
*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8
*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 1
4*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x
- 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c)
 + e^(-14*d*x - 14*c) + 1))) + 16/5*a^2*b*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d
*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c)
+ e^(-10*d*x - 10*c) + 1)) + 10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 1
0*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x
- 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d
*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 4/3*a^3*(3*e^(
-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c
) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c)
+ 1)))
```

Fricas [B] time = 2.39069, size = 3222, normalized size = 29.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -8/315*(2*(105*a^3 + 63*a^2*b + 54*a*b^2 + 16*b^3)*cosh(d*x + c)^7 + 14*(10
5*a^3 + 63*a^2*b + 54*a*b^2 + 16*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + (105*
a^3 - 126*a^2*b - 108*a*b^2 - 32*b^3)*sinh(d*x + c)^7 + 6*(245*a^3 + 399*a^
2*b + 162*a*b^2 + 48*b^3)*cosh(d*x + c)^5 + 3*(175*a^3 + 42*a^2*b - 324*a*b
^2 - 96*b^3 + 7*(105*a^3 - 126*a^2*b - 108*a*b^2 - 32*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^5 + 10*(7*(105*a^3 + 63*a^2*b + 54*a*b^2 + 16*b^3)*cosh(d*x
+ c)^3 + 3*(245*a^3 + 399*a^2*b + 162*a*b^2 + 48*b^3)*cosh(d*x + c))*sinh(d
*x + c)^4 + 18*(245*a^3 + 567*a^2*b + 426*a*b^2 + 64*b^3)*cosh(d*x + c)^3 +
 (35*(105*a^3 - 126*a^2*b - 108*a*b^2 - 32*b^3)*cosh(d*x + c)^4 + 945*a^3 +
 1134*a^2*b - 108*a*b^2 - 1152*b^3 + 30*(175*a^3 + 42*a^2*b - 324*a*b^2 - 9
6*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 6*(7*(105*a^3 + 63*a^2*b + 54*a*b
^2 + 16*b^3)*cosh(d*x + c)^5 + 10*(245*a^3 + 399*a^2*b + 162*a*b^2 + 48*b^3
)*cosh(d*x + c)^3 + 9*(245*a^3 + 567*a^2*b + 426*a*b^2 + 64*b^3)*cosh(d*x +
 c))*sinh(d*x + c)^2 + 210*(35*a^3 + 93*a^2*b + 90*a*b^2 + 32*b^3)*cosh(d*x
 + c) + (7*(105*a^3 - 126*a^2*b - 108*a*b^2 - 32*b^3)*cosh(d*x + c)^6 + 15*
(175*a^3 + 42*a^2*b - 324*a*b^2 - 96*b^3)*cosh(d*x + c)^4 + 525*a^3 + 882*a
^2*b + 756*a*b^2 + 1344*b^3 + 27*(105*a^3 + 126*a^2*b - 12*a*b^2 - 128*b^3)
*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^11 + 11*d*cosh(d*x + c)*s
inh(d*x + c)^10 + d*sinh(d*x + c)^11 + 9*d*cosh(d*x + c)^9 + (55*d*cosh(d*x
 + c)^2 + 9*d)*sinh(d*x + c)^9 + 3*(55*d*cosh(d*x + c)^3 + 27*d*cosh(d*x +
c))*sinh(d*x + c)^8 + 37*d*cosh(d*x + c)^7 + (330*d*cosh(d*x + c)^4 + 324*d
*cosh(d*x + c)^2 + 35*d)*sinh(d*x + c)^7 + 7*(66*d*cosh(d*x + c)^5 + 108*d*
cosh(d*x + c)^3 + 37*d*cosh(d*x + c))*sinh(d*x + c)^6 + 93*d*cosh(d*x + c)^
5 + 3*(154*d*cosh(d*x + c)^6 + 378*d*cosh(d*x + c)^4 + 245*d*cosh(d*x + c)^
2 + 25*d)*sinh(d*x + c)^5 + (330*d*cosh(d*x + c)^7 + 1134*d*cosh(d*x + c)^5
 + 1295*d*cosh(d*x + c)^3 + 465*d*cosh(d*x + c))*sinh(d*x + c)^4 + 162*d*co
sh(d*x + c)^3 + (165*d*cosh(d*x + c)^8 + 756*d*cosh(d*x + c)^6 + 1225*d*cos
h(d*x + c)^4 + 750*d*cosh(d*x + c)^2 + 90*d)*sinh(d*x + c)^3 + (55*d*cosh(d
*x + c)^9 + 324*d*cosh(d*x + c)^7 + 777*d*cosh(d*x + c)^5 + 930*d*cosh(d*x
+ c)^3 + 486*d*cosh(d*x + c))*sinh(d*x + c)^2 + 210*d*cosh(d*x + c) + (11*d
*cosh(d*x + c)^10 + 81*d*cosh(d*x + c)^8 + 245*d*cosh(d*x + c)^6 + 375*d*co
```

```
sh(d*x + c)^4 + 270*d*cosh(d*x + c)^2 + 42*d)*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\,\mathrm{sech}^2\left(c + dx\right)\right)^3 \mathrm{sech}^4\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*sech(c + d*x)**4, x)
```

Giac [B] time = 1.17785, size = 486, normalized size = 4.5

$$-\frac{4\left(315\,a^3e^{(14\,dx+14\,c)} + 1995\,a^3e^{(12\,dx+12\,c)} + 2520\,a^2be^{(12\,dx+12\,c)} + 5355\,a^3e^{(10\,dx+10\,c)} + 11340\,a^2be^{(10\,dx+10\,c)} + 7560\,ab^2e^{(1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -4/315*(315*a^3*e^(14*d*x + 14*c) + 1995*a^3*e^(12*d*x + 12*c) + 2520*a^2*b
*e^(12*d*x + 12*c) + 5355*a^3*e^(10*d*x + 10*c) + 11340*a^2*b*e^(10*d*x + 1
0*c) + 7560*a*b^2*e^(10*d*x + 10*c) + 7875*a^3*e^(8*d*x + 8*c) + 20412*a^2*
b*e^(8*d*x + 8*c) + 19656*a*b^2*e^(8*d*x + 8*c) + 8064*b^3*e^(8*d*x + 8*c)
+ 6825*a^3*e^(6*d*x + 6*c) + 18648*a^2*b*e^(6*d*x + 6*c) + 18144*a*b^2*e^(6
*d*x + 6*c) + 5376*b^3*e^(6*d*x + 6*c) + 3465*a^3*e^(4*d*x + 4*c) + 9072*a^
2*b*e^(4*d*x + 4*c) + 7776*a*b^2*e^(4*d*x + 4*c) + 2304*b^3*e^(4*d*x + 4*c)
 + 945*a^3*e^(2*d*x + 2*c) + 2268*a^2*b*e^(2*d*x + 2*c) + 1944*a*b^2*e^(2*d
*x + 2*c) + 576*b^3*e^(2*d*x + 2*c) + 105*a^3 + 252*a^2*b + 216*a*b^2 + 64*
b^3)/(d*(e^(2*d*x + 2*c) + 1)^9)
```

3.73 $\int \frac{\cosh^4(c+dx)}{a+b\mathbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=117

$$-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a+b}}+\frac{x\left(3a^2-4ab+8b^2\right)}{8a^3}+\frac{(3a-4b)\sinh(c+dx)\cosh(c+dx)}{8a^2d}+\frac{\sinh(c+dx)\cosh^3(c+d}{4ad}$$

```
[Out] ((3*a^2 - 4*a*b + 8*b^2)*x)/(8*a^3) - (b^(5/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*
x])/Sqrt[a + b]])/(a^3*Sqrt[a + b]*d) + ((3*a - 4*b)*Cosh[c + d*x]*Sinh[c +
 d*x])/(8*a^2*d) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*a*d)
```

Rubi [A] time = 0.189282, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4146, 414, 527, 522, 206, 208}

$$-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a+b}}+\frac{x\left(3a^2-4ab+8b^2\right)}{8a^3}+\frac{(3a-4b)\sinh(c+dx)\cosh(c+dx)}{8a^2d}+\frac{\sinh(c+dx)\cosh^3(c+d}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((3*a^2 - 4*a*b + 8*b^2)*x)/(8*a^3) - (b^(5/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*
x])/Sqrt[a + b]])/(a^3*Sqrt[a + b]*d) + ((3*a - 4*b)*Cosh[c + d*x]*Sinh[c +
 d*x])/(8*a^2*d) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*a*d)
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^4(c+dx)}{a+b\,\text{sech}^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad} + \frac{\text{Subst}\left(\int \frac{3a-b-3bx^2}{(1-x^2)^2(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{4ad}$$

$$= \frac{(3a-4b)\cosh(c+dx)\sinh(c+dx)}{8a^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad} + \frac{\text{Subst}\left(\int \frac{3a^2-ab+4b^2-(3a}{(1-x^2)(a+b-}\right.}{8}$$

$$= \frac{(3a-4b)\cosh(c+dx)\sinh(c+dx)}{8a^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad} - \frac{b^3\,\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx,\right.}{a^3d}$$

$$= \frac{(3a^2-4ab+8b^2)x}{8a^3} - \frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^3\sqrt{a+b}\,d} + \frac{(3a-4b)\cosh(c+dx)\sinh(c+dx)}{8a^2d} + \text{c}$$

Mathematica [A] time = 0.521615, size = 95, normalized size = 0.81

$$\frac{4\left(3a^2-4ab+8b^2\right)(c+dx)+a^2\sinh(4(c+dx))-\frac{32b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}}+8a(a-b)\sinh(2(c+dx))}{32a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (4*(3*a^2 - 4*a*b + 8*b^2)*(c + d*x) - (32*b^(5/2)*ArcTanh[(Sqrt[b]*Tanh[c
+ d*x])/Sqrt[a + b]])/Sqrt[a + b] + 8*a*(a - b)*Sinh[2*(c + d*x)] + a^2*Sin
h[4*(c + d*x)])/(32*a^3*d)
```

Maple [B] time = 0.084, size = 493, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/4/d/a/(tanh(1/2*d*x+1/2*c)+1)^4+1/2/d/a/(tanh(1/2*d*x+1/2*c)+1)^3+5/8/d/
a/(tanh(1/2*d*x+1/2*c)+1)-1/2/d/a^2/(tanh(1/2*d*x+1/2*c)+1)*b-7/8/d/a/(tanh
(1/2*d*x+1/2*c)+1)^2+1/2/d/a^2/(tanh(1/2*d*x+1/2*c)+1)^2*b+3/8/d/a*ln(tanh(
1/2*d*x+1/2*c)+1)-1/2/d/a^2*ln(tanh(1/2*d*x+1/2*c)+1)*b+1/d/a^3*ln(tanh(1/2
*d*x+1/2*c)+1)*b^2-1/2/d*b^(5/2)/a^3/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*
x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/2/d*b^(5/2)/a^3/(a+
b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)
+(a+b)^(1/2))+1/4/d/a/(tanh(1/2*d*x+1/2*c)-1)^4+1/2/d/a/(tanh(1/2*d*x+1/2*c
)-1)^3+7/8/d/a/(tanh(1/2*d*x+1/2*c)-1)^2-1/2/d/a^2/(tanh(1/2*d*x+1/2*c)-1)^
2*b+5/8/d/a/(tanh(1/2*d*x+1/2*c)-1)-1/2/d/a^2/(tanh(1/2*d*x+1/2*c)-1)*b-3/8
/d/a*ln(tanh(1/2*d*x+1/2*c)-1)+1/2/d/a^2*ln(tanh(1/2*d*x+1/2*c)-1)*b-1/d/a^
3*ln(tanh(1/2*d*x+1/2*c)-1)*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.75175, size = 4313, normalized size = 36.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/64*(a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh
(d*x + c)^8 + 8*(3*a^2 - 4*a*b + 8*b^2)*d*x*cosh(d*x + c)^4 + 8*(a^2 - a*b)
*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 + 2*a^2 - 2*a*b)*sinh(d*x + c)^
6 + 8*(7*a^2*cosh(d*x + c)^3 + 6*(a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c)^5
 + 2*(35*a^2*cosh(d*x + c)^4 + 4*(3*a^2 - 4*a*b + 8*b^2)*d*x + 60*(a^2 - a*
b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^5 + 4*(3*a^2 -
 4*a*b + 8*b^2)*d*x*cosh(d*x + c) + 20*(a^2 - a*b)*cosh(d*x + c)^3)*sinh(d*
x + c)^3 - 8*(a^2 - a*b)*cosh(d*x + c)^2 + 4*(7*a^2*cosh(d*x + c)^6 + 12*(3
*a^2 - 4*a*b + 8*b^2)*d*x*cosh(d*x + c)^2 + 30*(a^2 - a*b)*cosh(d*x + c)^4
- 2*a^2 + 2*a*b)*sinh(d*x + c)^2 + 32*(b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x
 + c)^3*sinh(d*x + c) + 6*b^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^2*cosh(
d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4)*sqrt(b/(a + b))*log((a^2*co
sh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 +
 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*
sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a
*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2
+ a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*
a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d
*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d
*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cos
h(d*x + c))*sinh(d*x + c) + a)) - a^2 + 8*(a^2*cosh(d*x + c)^7 + 4*(3*a^2 -
```

```
 4*a*b + 8*b^2)*d*x*cosh(d*x + c)^3 + 6*(a^2 - a*b)*cosh(d*x + c)^5 - 2*(a^
2 - a*b)*cosh(d*x + c))*sinh(d*x + c))/(a^3*d*cosh(d*x + c)^4 + 4*a^3*d*cos
h(d*x + c)^3*sinh(d*x + c) + 6*a^3*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a^
3*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*d*sinh(d*x + c)^4), 1/64*(a^2*cosh(
d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 + 8*
(3*a^2 - 4*a*b + 8*b^2)*d*x*cosh(d*x + c)^4 + 8*(a^2 - a*b)*cosh(d*x + c)^6
 + 4*(7*a^2*cosh(d*x + c)^2 + 2*a^2 - 2*a*b)*sinh(d*x + c)^6 + 8*(7*a^2*cos
h(d*x + c)^3 + 6*(a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^2*cos
h(d*x + c)^4 + 4*(3*a^2 - 4*a*b + 8*b^2)*d*x + 60*(a^2 - a*b)*cosh(d*x + c)
^2)*sinh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^5 + 4*(3*a^2 - 4*a*b + 8*b^2)*
d*x*cosh(d*x + c) + 20*(a^2 - a*b)*cosh(d*x + c)^3)*sinh(d*x + c)^3 - 8*(a^
2 - a*b)*cosh(d*x + c)^2 + 4*(7*a^2*cosh(d*x + c)^6 + 12*(3*a^2 - 4*a*b + 8
*b^2)*d*x*cosh(d*x + c)^2 + 30*(a^2 - a*b)*cosh(d*x + c)^4 - 2*a^2 + 2*a*b)
*sinh(d*x + c)^2 - 64*(b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)^3*sinh(d*x
 + c) + 6*b^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^2*cosh(d*x + c)*sinh(d*
x + c)^3 + b^2*sinh(d*x + c)^4)*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c
)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-
b/(a + b))/b) - a^2 + 8*(a^2*cosh(d*x + c)^7 + 4*(3*a^2 - 4*a*b + 8*b^2)*d*
x*cosh(d*x + c)^3 + 6*(a^2 - a*b)*cosh(d*x + c)^5 - 2*(a^2 - a*b)*cosh(d*x
+ c))*sinh(d*x + c))/(a^3*d*cosh(d*x + c)^4 + 4*a^3*d*cosh(d*x + c)^3*sinh(
d*x + c) + 6*a^3*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a^3*d*cosh(d*x + c)*
sinh(d*x + c)^3 + a^3*d*sinh(d*x + c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18464, size = 296, normalized size = 2.53

$$-\frac{b^3 \arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}a^3d}+\frac{\left(3\,a^2-4\,ab+8\,b^2\right)(dx+c)}{8\,a^3d}-\frac{\left(18\,a^2e^{(4\,dx+4\,c)}-24\,abe^{(4\,dx+4\,c)}+48\,b^2e^{(4\,dx+4\,c)}+8\,a^2e^{(2\,dx+}\right.}{64\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -b^3*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b
- b^2)*a^3*d) + 1/8*(3*a^2 - 4*a*b + 8*b^2)*(d*x + c)/(a^3*d) - 1/64*(18*a^
2*e^(4*d*x + 4*c) - 24*a*b*e^(4*d*x + 4*c) + 48*b^2*e^(4*d*x + 4*c) + 8*a^2
*e^(2*d*x + 2*c) - 8*a*b*e^(2*d*x + 2*c) + a^2)*e^(-4*d*x - 4*c)/(a^3*d) +
1/64*(a*d*e^(4*d*x + 4*c) + 8*a*d*e^(2*d*x + 2*c) - 8*b*d*e^(2*d*x + 2*c))/
(a^2*d^2)
```

3.74 $\int \frac{\cosh^3(c+dx)}{a+b\,\mathbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=76

$$\frac{b^2\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{a^{5/2}d\sqrt{a+b}}+\frac{(a-b)\sinh(c+dx)}{a^2d}+\frac{\sinh^3(c+dx)}{3ad}$$

```
[Out] (b^2*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(a^(5/2)*Sqrt[a + b]*d) +
 ((a - b)*Sinh[c + d*x])/(a^2*d) + Sinh[c + d*x]^3/(3*a*d)
```

Rubi [A] time = 0.09035, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4147, 390, 205}

$$\frac{b^2\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{a^{5/2}d\sqrt{a+b}}+\frac{(a-b)\sinh(c+dx)}{a^2d}+\frac{\sinh^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (b^2*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(a^(5/2)*Sqrt[a + b]*d) +
 ((a - b)*Sinh[c + d*x])/(a^2*d) + Sinh[c + d*x]^3/(3*a*d)
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{(1+x^2)^2}{a+b+ax^2}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(\frac{a-b}{a^2}+\frac{x^2}{a}+\frac{b^2}{a^2(a+b+ax^2)}\right)dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{(a-b)\sinh(c+dx)}{a^2d}+\frac{\sinh^3(c+dx)}{3ad}+\frac{b^2\operatorname{Subst}\left(\int \frac{1}{a+b+ax^2}\,dx, x, \sinh(c+dx)\right)}{a^2d}$$
$$= \frac{b^2\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{a^{5/2}\sqrt{a+b}d}+\frac{(a-b)\sinh(c+dx)}{a^2d}+\frac{\sinh^3(c+dx)}{3ad}$$

Mathematica [A] time = 0.287139, size = 79, normalized size = 1.04

$$\frac{a^{3/2}\sinh(3(c+dx))-\frac{12b^2\tan^{-1}\left(\frac{\sqrt{a+b}\operatorname{csch}(c+dx)}{\sqrt{a}}\right)}{\sqrt{a+b}}+3\sqrt{a}(3a-4b)\sinh(c+dx)}{12a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((-12*b^2*ArcTan[(Sqrt[a + b]*Csch[c + d*x])/Sqrt[a]])/Sqrt[a + b] + 3*Sqrt
[a]*(3*a - 4*b)*Sinh[c + d*x] + a^(3/2)*Sinh[3*(c + d*x)])/(12*a^(5/2)*d)
```

Maple [B] time = 0.083, size = 256, normalized size = 3.4

$$-\frac{1}{3da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}+\frac{1}{2da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}-\frac{1}{da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}+\frac{b}{da^2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/3/d/a/(tanh(1/2*d*x+1/2*c)+1)^3+1/2/d/a/(tanh(1/2*d*x+1/2*c)+1)^2-1/d/a/
(tanh(1/2*d*x+1/2*c)+1)+1/d/a^2/(tanh(1/2*d*x+1/2*c)+1)*b+1/d/a^(5/2)*b^2/(
a+b)^(1/2)*arctan(1/2*(2*tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)+2*b^(1/2))/a^(1/2)
)+1/d/a^(5/2)*b^2/(a+b)^(1/2)*arctan(1/2*(2*tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)
-2*b^(1/2))/a^(1/2))-1/3/d/a/(tanh(1/2*d*x+1/2*c)-1)^3-1/2/d/a/(tanh(1/2*d*
x+1/2*c)-1)^2-1/d/a/(tanh(1/2*d*x+1/2*c)-1)+1/d/a^2/(tanh(1/2*d*x+1/2*c)-1)
*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(3\left(3\,ae^{(4\,c)}-4\,be^{(4\,c)}\right)e^{(4\,dx)}-3\left(3\,ae^{(2\,c)}-4\,be^{(2\,c)}\right)e^{(2\,dx)}+ae^{(6\,dx+6\,c)}-a\right)e^{(-3\,dx-3\,c)}}{24\,a^2d}+\frac{1}{8}\int\frac{16\left(b^2e^{(3\,dx+3\,c)}+b^2e\right.}{a^3e^{(4\,dx+4\,c)}+a^3+2\left(a^3e^{(2\,c)}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/24*(3*(3*a*e^(4*c) - 4*b*e^(4*c))*e^(4*d*x) - 3*(3*a*e^(2*c) - 4*b*e^(2*c
))*e^(2*d*x) + a*e^(6*d*x + 6*c) - a)*e^(-3*d*x - 3*c)/(a^2*d) + 1/8*integr
ate(16*(b^2*e^(3*d*x + 3*c) + b^2*e^(d*x + c))/(a^3*e^(4*d*x + 4*c) + a^3 +
 2*(a^3*e^(2*c) + 2*a^2*b*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.79247, size = 3991, normalized size = 52.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/24*((a^3 + a^2*b)*cosh(d*x + c)^6 + 6*(a^3 + a^2*b)*cosh(d*x + c)*sinh(d
*x + c)^5 + (a^3 + a^2*b)*sinh(d*x + c)^6 + 3*(3*a^3 - a^2*b - 4*a*b^2)*cos
h(d*x + c)^4 + 3*(3*a^3 - a^2*b - 4*a*b^2 + 5*(a^3 + a^2*b)*cosh(d*x + c)^2
)*sinh(d*x + c)^4 + 4*(5*(a^3 + a^2*b)*cosh(d*x + c)^3 + 3*(3*a^3 - a^2*b -
 4*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - a^3 - a^2*b - 3*(3*a^3 - a^2*b -
 4*a*b^2)*cosh(d*x + c)^2 + 3*(5*(a^3 + a^2*b)*cosh(d*x + c)^4 - 3*a^3 + a^
2*b + 4*a*b^2 + 6*(3*a^3 - a^2*b - 4*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^
2 - 12*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c)^2*sinh(d*x + c) + 3*b^2*c
osh(d*x + c)*sinh(d*x + c)^2 + b^2*sinh(d*x + c)^3)*sqrt(-a^2 - a*b)*log((a
*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 -
2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*
x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c)
- 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 +
(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a
)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^
4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*
x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) +
a)) + 6*((a^3 + a^2*b)*cosh(d*x + c)^5 + 2*(3*a^3 - a^2*b - 4*a*b^2)*cosh(d
*x + c)^3 - (3*a^3 - a^2*b - 4*a*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 +
 a^3*b)*d*cosh(d*x + c)^3 + 3*(a^4 + a^3*b)*d*cosh(d*x + c)^2*sinh(d*x + c)
 + 3*(a^4 + a^3*b)*d*cosh(d*x + c)*sinh(d*x + c)^2 + (a^4 + a^3*b)*d*sinh(d
*x + c)^3), 1/24*((a^3 + a^2*b)*cosh(d*x + c)^6 + 6*(a^3 + a^2*b)*cosh(d*x
+ c)*sinh(d*x + c)^5 + (a^3 + a^2*b)*sinh(d*x + c)^6 + 3*(3*a^3 - a^2*b - 4
*a*b^2)*cosh(d*x + c)^4 + 3*(3*a^3 - a^2*b - 4*a*b^2 + 5*(a^3 + a^2*b)*cosh
(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(a^3 + a^2*b)*cosh(d*x + c)^3 + 3*(3*a^
3 - a^2*b - 4*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - a^3 - a^2*b - 3*(3*a^
3 - a^2*b - 4*a*b^2)*cosh(d*x + c)^2 + 3*(5*(a^3 + a^2*b)*cosh(d*x + c)^4 -
 3*a^3 + a^2*b + 4*a*b^2 + 6*(3*a^3 - a^2*b - 4*a*b^2)*cosh(d*x + c)^2)*sin
h(d*x + c)^2 + 24*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c)^2*sinh(d*x + c
) + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 + b^2*sinh(d*x + c)^3)*sqrt(a^2 + a
*b)*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*s
inh(d*x + c)^3 + (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*a + 4
*b)*sinh(d*x + c))/sqrt(a^2 + a*b)) + 24*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(
d*x + c)^2*sinh(d*x + c) + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 + b^2*sinh(d
*x + c)^3)*sqrt(a^2 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh
(d*x + c))/(a + b)) + 6*((a^3 + a^2*b)*cosh(d*x + c)^5 + 2*(3*a^3 - a^2*b -
 4*a*b^2)*cosh(d*x + c)^3 - (3*a^3 - a^2*b - 4*a*b^2)*cosh(d*x + c))*sinh(d
*x + c))/((a^4 + a^3*b)*d*cosh(d*x + c)^3 + 3*(a^4 + a^3*b)*d*cosh(d*x + c)
^2*sinh(d*x + c) + 3*(a^4 + a^3*b)*d*cosh(d*x + c)*sinh(d*x + c)^2 + (a^4 +
 a^3*b)*d*sinh(d*x + c)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(cosh(c + d*x)**3/(a + b*sech(c + d*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.75 $\int \frac{\cosh^2(c+dx)}{a+b\,\mathbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=75

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a+b}}+\frac{x(a-2b)}{2a^2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2ad}$$

```
[Out] ((a - 2*b)*x)/(2*a^2) + (b^(3/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b
]])/(a^2*Sqrt[a + b]*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.115501, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4146, 414, 522, 206, 208}

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a+b}}+\frac{x(a-2b)}{2a^2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a - 2*b)*x)/(2*a^2) + (b^(3/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b
]])/(a^2*Sqrt[a + b]*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*a*d)
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad} + \frac{\text{Subst}\left(\int \frac{a-b-bx^2}{(1-x^2)(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{2ad}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad} + \frac{(a-2b)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{2a^2d} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{a+b-\cdots}\right.}{\cdots}$$

$$= \frac{(a-2b)x}{2a^2} + \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^2\sqrt{a+b}\,d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2ad}$$

Mathematica [A] time = 0.227966, size = 67, normalized size = 0.89

$$\frac{\frac{4b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}} + 2(a-2b)(c+dx) + a\sinh(2(c+dx))}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (2*(a - 2*b)*(c + d*x) + (4*b^(3/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a
+ b]])/Sqrt[a + b] + a*Sinh[2*(c + d*x)])/(4*a^2*d)
```

Maple [B] time = 0.079, size = 278, normalized size = 3.7

$$-\frac{1}{2da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2} + \frac{1}{2da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1} + \frac{1}{2da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right) - \frac{b}{da^2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/2/d/a/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/a/(tanh(1/2*d*x+1/2*c)+1)+1/2/d/a*
ln(tanh(1/2*d*x+1/2*c)+1)-1/d/a^2*ln(tanh(1/2*d*x+1/2*c)+1)*b+1/2/d*b^(3/2)
/a^2/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)
*b^(1/2)+(a+b)^(1/2))-1/2/d/a^2*b^(3/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/
2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2))+1/2/d/a/(tanh(1/2
*d*x+1/2*c)-1)^2+1/2/d/a/(tanh(1/2*d*x+1/2*c)-1)-1/2/d/a*ln(tanh(1/2*d*x+1/
2*c)-1)+1/d/a^2*ln(tanh(1/2*d*x+1/2*c)-1)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.4233, size = 2188, normalized size = 29.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a - 2*b)*d*x*cosh(d*x + c)^2 + a*cosh(d*x + c)^4 + 4*a*cosh(d*x +
c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(2*(a - 2*b)*d*x + 3*a*cosh(d*x
+ c)^2)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x
 + c) + b*sinh(d*x + c)^2)*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2
*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh
(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2
 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*si
nh(d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*
sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(
a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*
x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*
sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x
+ c) + a)) + 4*(2*(a - 2*b)*d*x*cosh(d*x + c) + a*cosh(d*x + c)^3)*sinh(d*x
 + c) - a)/(a^2*d*cosh(d*x + c)^2 + 2*a^2*d*cosh(d*x + c)*sinh(d*x + c) + a
^2*d*sinh(d*x + c)^2), 1/8*(4*(a - 2*b)*d*x*cosh(d*x + c)^2 + a*cosh(d*x +
c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(2*(a - 2*
b)*d*x + 3*a*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(b*cosh(d*x + c)^2 + 2*b*
cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2)*sqrt(-b/(a + b))*arctan(1/
2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2
+ a + 2*b)*sqrt(-b/(a + b))/b) + 4*(2*(a - 2*b)*d*x*cosh(d*x + c) + a*cosh(
d*x + c)^3)*sinh(d*x + c) - a)/(a^2*d*cosh(d*x + c)^2 + 2*a^2*d*cosh(d*x +
c)*sinh(d*x + c) + a^2*d*sinh(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(cosh(c + d*x)**2/(a + b*sech(c + d*x)**2), x)
```

Giac [B] time = 1.18639, size = 178, normalized size = 2.37

$$\frac{b^2 \arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}a^2d} + \frac{(dx+c)(a-2\,b)}{2\,a^2d} + \frac{e^{(2\,dx+2\,c)}}{8\,ad} - \frac{\left(2\,ae^{(2\,dx+2\,c)} - 4\,be^{(2\,dx+2\,c)} + a\right)e^{(-2\,dx-2\,c)}}{8\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] b^2*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b -
 b^2)*a^2*d) + 1/2*(d*x + c)*(a - 2*b)/(a^2*d) + 1/8*e^(2*d*x + 2*c)/(a*d)
- 1/8*(2*a*e^(2*d*x + 2*c) - 4*b*e^(2*d*x + 2*c) + a)*e^(-2*d*x - 2*c)/(a^2
*d)
```

3.76 $\int \frac{\cosh(c+dx)}{a+b\,\mathbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=52

$$\frac{\sinh(c+dx)}{ad}-\frac{b\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{a^{3/2}d\sqrt{a+b}}$$

```
[Out] -((b*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(a^(3/2)*Sqrt[a + b]*d))
+ Sinh[c + d*x]/(a*d)
```

Rubi [A] time = 0.061165, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4147, 388, 205}

$$\frac{\sinh(c+dx)}{ad}-\frac{b\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{a^{3/2}d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((b*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(a^(3/2)*Sqrt[a + b]*d))
+ Sinh[c + d*x]/(a*d)
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{a+b\,\text{sech}^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{a+b+ax^2}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\sinh(c+dx)}{ad} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+b+ax^2}\,dx, x, \sinh(c+dx)\right)}{ad}$$
$$= -\frac{b\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{a^{3/2}\sqrt{a+b}d} + \frac{\sinh(c+dx)}{ad}$$

Mathematica [A] time = 0.12126, size = 52, normalized size = 1.

$$\frac{\sqrt{a}\sinh(c+dx) - \frac{b\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}}}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (-((b*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/Sqrt[a + b]) + Sqrt[a]*S
inh[c + d*x])/(a^(3/2)*d)
```

Maple [B] time = 0.069, size = 128, normalized size = 2.5

$$-\frac{b}{d}\arctan\left(\frac{1}{2}\left(2\tanh(1/2\,dx+c/2)\sqrt{a+b}+2\sqrt{b}\right)\frac{1}{\sqrt{a}}\right)a^{-\frac{3}{2}}\frac{1}{\sqrt{a+b}} - \frac{b}{d}\arctan\left(\frac{1}{2}\left(2\tanh(1/2\,dx+c/2)\sqrt{a+b}-2\sqrt{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/d/a^(3/2)*b/(a+b)^(1/2)*arctan(1/2*(2*tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)+2*
b^(1/2))/a^(1/2))-1/d/a^(3/2)*b/(a+b)^(1/2)*arctan(1/2*(2*tanh(1/2*d*x+1/2*
c)*(a+b)^(1/2)-2*b^(1/2))/a^(1/2))-1/d/a/(tanh(1/2*d*x+1/2*c)-1)-1/d/a/(tan
h(1/2*d*x+1/2*c)+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}}{2\,ad} - \frac{1}{2}\int \frac{4\left(be^{(3\,dx+3\,c)}+be^{(dx+c)}\right)}{a^2e^{(4\,dx+4\,c)}+a^2+2\left(a^2e^{(2\,c)}+2\,abe^{(2\,c)}\right)e^{(2\,dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)/(a*d) - 1/2*integrate(4*(b*e^(3*d*x
+ 3*c) + b*e^(d*x + c))/(a^2*e^(4*d*x + 4*c) + a^2 + 2*(a^2*e^(2*c) + 2*a*b
*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.37508, size = 1891, normalized size = 36.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x +
c) + (a^2 + a*b)*sinh(d*x + c)^2 - sqrt(-a^2 - a*b)*(b*cosh(d*x + c) + b*si
nh(d*x + c))*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a
*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 -
 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x +
 c))*sinh(d*x + c) + 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 +
 sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*s
qrt(-a^2 - a*b) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3
 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2
 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c
))*sinh(d*x + c) + a)) - a^2 - a*b)/((a^3 + a^2*b)*d*cosh(d*x + c) + (a^3 +
 a^2*b)*d*sinh(d*x + c)), 1/2*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*
cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 - 2*sqrt(a^2 + a*
b)*(b*cosh(d*x + c) + b*sinh(d*x + c))*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*
cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (3*a + 4*b)*cosh(d*x +
c) + (3*a*cosh(d*x + c)^2 + 3*a + 4*b)*sinh(d*x + c))/sqrt(a^2 + a*b)) - 2*
sqrt(a^2 + a*b)*(b*cosh(d*x + c) + b*sinh(d*x + c))*arctan(1/2*sqrt(a^2 + a
*b)*(cosh(d*x + c) + sinh(d*x + c))/(a + b)) - a^2 - a*b)/((a^3 + a^2*b)*d*
cosh(d*x + c) + (a^3 + a^2*b)*d*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(cosh(c + d*x)/(a + b*sech(c + d*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.77 $\int \frac{\mathbf{sech}(c+dx)}{a+b\mathbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=36

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}d\sqrt{a+b}}$$

```
[Out] ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b]*d)
```

Rubi [A] time = 0.0415728, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4147, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b]*d)
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{sech}(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+b+ax^2}\,dx, x, \sinh(c+dx)\right)}{d}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}\sqrt{a+b}d}\end{aligned}$$

Mathematica [A] time = 0.0652231, size = 36, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b]*d)
```

Maple [B] time = 0.047, size = 82, normalized size = 2.3

$$\frac{1}{d}\arctan\left(\frac{1}{2}\left(2\,\tanh\left(1/2\,dx+c/2\right)\sqrt{a+b}+2\,\sqrt{b}\right)\frac{1}{\sqrt{a}}\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{a+b}}+\frac{1}{d}\arctan\left(\frac{1}{2}\left(2\,\tanh\left(1/2\,dx+c/2\right)\sqrt{a+b}-2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/d/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)+2*b^(
1/2))/a^(1/2))+1/d/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*tanh(1/2*d*x+1/2*c)*(a
+b)^(1/2)-2*b^(1/2))/a^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}\left(dx+c\right)}{b\operatorname{sech}\left(dx+c\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate(sech(d*x + c)/(b*sech(d*x + c)^2 + a), x)
```

Fricas [B] time = 2.31261, size = 1291, normalized size = 35.86

$$\left[-\frac{\sqrt{-a^{2}-ab}\log\left(\frac{a\cosh(dx+c)^{4}+4\,a\cosh(dx+c)\sinh(dx+c)^{3}+a\sinh(dx+c)^{4}-2\,(3\,a+2\,b)\cosh(dx+c)^{2}+2\left(3\,a\cosh(dx+c)^{2}-3\,a-2\,b\right)\sinh(dx+c)^{2}+4\,(\ldots}{a\cosh(dx+c)^{4}+4\,a\cosh(dx+c)\sinh(dx+c)^{3}+a\sinh(dx+c)^{4}+2\,(a+2\,b)\,\mathrm{c}\ldots}\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a^2 - a*b)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x
+ c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*
x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*
cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*
x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d
*x + c))*sqrt(-a^2 - a*b) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(
d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(
d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*co
sh(d*x + c))*sinh(d*x + c) + a))/((a^2 + a*b)*d), (sqrt(a^2 + a*b)*arctan(1
/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)
^3 + (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*a + 4*b)*sinh(d*x
 + c))/sqrt(a^2 + a*b)) + sqrt(a^2 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh(
```

```
d*x + c) + sinh(d*x + c))/(a + b)))/((a^2 + a*b)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)**2),x)

[Out] Integral(sech(c + d*x)/(a + b*sech(c + d*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.78 $\int \frac{\operatorname{sech}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=36

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}d\sqrt{a+b}}$$

```
[Out] ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*d)
```

Rubi [A] time = 0.0605388, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4146, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*d)
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}d}\end{aligned}$$

Mathematica [A] time = 0.0785563, size = 36, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*d)
```

Maple [B] time = 0.043, size = 107, normalized size = 3.

$$\frac{1}{2d}\ln\left(\sqrt{a+b}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+2\tanh\left(1/2\,dx+c/2\right)\sqrt{b}+\sqrt{a+b}\right)\frac{1}{\sqrt{b}}\frac{1}{\sqrt{a+b}}-\frac{1}{2d}\ln\left(-\sqrt{a+b}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2/(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/2/d/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d
*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-1/2/d/b^(1/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*ta
nh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.18705, size = 1065, normalized size = 29.58

$$\left[\frac{\log\left(\frac{a^2\cosh(dx+c)^4+4a^2\cosh(dx+c)\sinh(dx+c)^3+a^2\sinh(dx+c)^4+2\left(a^2+2ab\right)\cosh(dx+c)^2+2\left(3a^2\cosh(dx+c)^2+a^2+2ab\right)\sinh(dx+c)^2+a^2+8ab+8b^2+4\left(\cdots\right.}{a\cosh(dx+c)^4+4a\cosh(dx+c)\sinh(dx+c)^3+a\sinh(dx+c)^4+2\left(a+2b\right)\cosh(dx+c)^2+2\left(3a\cosh(dx+c\cdots\right.}\right.}{2\sqrt{ab+b^2}d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*s
inh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2
 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c
)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(d*x + c)^2 + 2
*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^
2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c
)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(
d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c)
+ a))/(sqrt(a*b + b^2)*d), sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(d*x + c)^2 +
 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-a*b -
 b^2)/(a*b + b^2))/((a*b + b^2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)**2/(a + b*sech(c + d*x)**2), x)
```

Giac [A] time = 1.15064, size = 63, normalized size = 1.75

$$\frac{\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2
)*d)
```

3.79 $\int \frac{\mathbf{sech}^3(c+dx)}{a+b\mathbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=55

$$\frac{\tan^{-1}(\sinh(c+dx))}{bd}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{bd\sqrt{a+b}}$$

```
[Out] ArcTan[Sinh[c + d*x]]/(b*d) - (Sqrt[a]*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[
a + b]])/(b*Sqrt[a + b]*d)
```

Rubi [A] time = 0.0691455, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4147, 391, 203, 205}

$$\frac{\tan^{-1}(\sinh(c+dx))}{bd}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{bd\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ArcTan[Sinh[c + d*x]]/(b*d) - (Sqrt[a]*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[
a + b]])/(b*Sqrt[a + b]*d)
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)(a+b+ax^2)}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{bd} - \frac{a\operatorname{Subst}\left(\int \frac{1}{a+b+ax^2}\,dx, x, \sinh(c+dx)\right)}{bd}$$

$$= \frac{\tan^{-1}(\sinh(c+dx))}{bd} - \frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{b\sqrt{a+b}d}$$

Mathematica [B] time = 0.681698, size = 194, normalized size = 3.53

$$\frac{\operatorname{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b)\left(2\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+\sqrt{a}\cosh(c)\tan\right.}{2bd\sqrt{a+b}\sqrt{(\cosh(c)-\sinh}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(Sqrt[a]*ArcTan[(Sqrt[a +
b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Sinh[c]))/Sqrt[a]]*
Cosh[c] + 2*Sqrt[a + b]*ArcTan[Tanh[(c + d*x)/2]]*Sqrt[(Cosh[c] - Sinh[c])^
2] - Sqrt[a]*ArcTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*
(Cosh[c] + Sinh[c]))/Sqrt[a]]*Sinh[c]))/(2*b*Sqrt[a + b]*d*(a + b*Sech[c +
d*x]^2)*Sqrt[(Cosh[c] - Sinh[c])^2])
```

Maple [B] time = 0.048, size = 107, normalized size = 2.

$$-\frac{1}{bd}\sqrt{a}\arctan\left(\frac{1}{2}\left(2\tanh(1/2\,dx+c/2)\sqrt{a+b}+2\sqrt{b}\right)\frac{1}{\sqrt{a}}\right)\frac{1}{\sqrt{a+b}}+\frac{1}{bd}\sqrt{a}\arctan\left(\frac{1}{2}\left(-2\tanh(1/2\,dx+c/2)\sqrt{a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/d*a^(1/2)/b/(a+b)^(1/2)*arctan(1/2*(2*tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)+2*
b^(1/2))/a^(1/2))+1/d*a^(1/2)/b/(a+b)^(1/2)*arctan(1/2*(-2*tanh(1/2*d*x+1/2
*c)*(a+b)^(1/2)+2*b^(1/2))/a^(1/2))+2/d/b*arctan(tanh(1/2*d*x+1/2*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\arctan\left(e^{(dx+c)}\right)}{bd} - 8\int \frac{ae^{(3\,dx+3\,c)}+ae^{(dx+c)}}{4\left(abe^{(4\,dx+4\,c)}+ab+2\left(abe^{(2\,c)}+2\,b^2e^{(2\,c)}\right)e^{(2\,dx)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 2*arctan(e^(d*x + c))/(b*d) - 8*integrate(1/4*(a*e^(3*d*x + 3*c) + a*e^(d*x
 + c))/(a*b*e^(4*d*x + 4*c) + a*b + 2*(a*b*e^(2*c) + 2*b^2*e^(2*c))*e^(2*d*
```

```
x)), x)
```

Fricas [B] time = 2.30188, size = 1431, normalized size = 26.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a/(a + b))*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x
+ c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*
x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*
cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(
d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 - (a + b)*cosh(d*x + c)
+ (3*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*x + c))*sqrt(-a/(a + b)) + a)/
(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4
+ 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x
+ c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)
) + 4*arctan(cosh(d*x + c) + sinh(d*x + c)))/(b*d), -(sqrt(a/(a + b))*arcta
n(1/2*sqrt(a/(a + b))*(cosh(d*x + c) + sinh(d*x + c))) + sqrt(a/(a + b))*ar
ctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*
x + c)^3 + (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*a + 4*b)*si
nh(d*x + c))*sqrt(a/(a + b))/a) - 2*arctan(cosh(d*x + c) + sinh(d*x + c)))/
(b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)**3/(a + b*sech(c + d*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.80 $\int \frac{\mathbf{sech}^4(c+dx)}{a+b\mathbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=52

$$\frac{\tanh(c+dx)}{bd}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{b^{3/2}d\sqrt{a+b}}$$

```
[Out] -((a*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(b^(3/2)*Sqrt[a + b]*d))
 + Tanh[c + d*x]/(b*d)
```

Rubi [A] time = 0.0703809, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4146, 388, 208}

$$\frac{\tanh(c+dx)}{bd}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{b^{3/2}d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((a*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(b^(3/2)*Sqrt[a + b]*d))
 + Tanh[c + d*x]/(b*d)
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{a+b-bx^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\tanh(c+dx)}{bd} - \frac{a\,\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \tanh(c+dx)\right)}{bd}$$
$$= -\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}d} + \frac{\tanh(c+dx)}{bd}$$

Mathematica [B] time = 0.679913, size = 182, normalized size = 3.5

$$\frac{\operatorname{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\sqrt{a+b}\operatorname{sech}(c)\sinh(dx)\sqrt{b(\cosh(c)-\sinh(c))^4}\operatorname{sech}(c+dx)+a(\sinh(2c)-c\right.}{2bd\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}\left(a+b\operatorname{sech}^2(c+dx)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(a*ArcTanh[(Sech[d*x]*(Cos
h[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a +
b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(-Cosh[2*c] + Sinh[2*c]) + Sqrt[a + b]*S
ech[c]*Sech[c + d*x]*Sqrt[b*(Cosh[c] - Sinh[c])^4]*Sinh[d*x]))/(2*b*Sqrt[a
+ b]*d*(a + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])
```

Maple [B] time = 0.045, size = 141, normalized size = 2.7

$$-\frac{a}{2d}\ln\left(\sqrt{a+b}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+2\tanh(1/2\,dx+c/2)\sqrt{b}+\sqrt{a+b}\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{a+b}}+\frac{a}{2d}\ln\left(-\sqrt{a+b}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/2/d*a/b^(3/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/
2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/2/d*a/b^(3/2)/(a+b)^(1/2)*ln(-(a+b)^(1/
2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2))+2/d/b*t
anh(1/2*d*x+1/2*c)/(tanh(1/2*d*x+1/2*c)^2+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.27102, size = 1661, normalized size = 31.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c
)^2 + a)*sqrt(a*b + b^2)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sin
h(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3
*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 +
 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a
*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a
+ 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c
)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c
)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x
+ c))*sinh(d*x + c) + a)) - 4*a*b - 4*b^2)/((a*b^2 + b^3)*d*cosh(d*x + c)^2
 + 2*(a*b^2 + b^3)*d*cosh(d*x + c)*sinh(d*x + c) + (a*b^2 + b^3)*d*sinh(d*x
 + c)^2 + (a*b^2 + b^3)*d), -((a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d
*x + c) + a*sinh(d*x + c)^2 + a)*sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(d*x +
c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(
-a*b - b^2)/(a*b + b^2)) + 2*a*b + 2*b^2)/((a*b^2 + b^3)*d*cosh(d*x + c)^2
+ 2*(a*b^2 + b^3)*d*cosh(d*x + c)*sinh(d*x + c) + (a*b^2 + b^3)*d*sinh(d*x
+ c)^2 + (a*b^2 + b^3)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)**4/(a + b*sech(c + d*x)**2), x)
```

Giac [A] time = 1.16742, size = 100, normalized size = 1.92

$$-\frac{a\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}bd}-\frac{2}{bd\left(e^{(2\,dx+2\,c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -a*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b -
b^2)*b*d) - 2/(b*d*(e^(2*d*x + 2*c) + 1))
```

3.81 $\int \frac{\mathbf{sech}^5(c+dx)}{a+b\mathbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=86

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a+b}}-\frac{(2a-b)\tan^{-1}(\sinh(c+dx))}{2b^2d}+\frac{\tanh(c+dx)\mathrm{sech}(c+dx)}{2bd}$$

```
[Out] -((2*a - b)*ArcTan[Sinh[c + d*x]])/(2*b^2*d) + (a^(3/2)*ArcTan[(Sqrt[a]*Sin
h[c + d*x])/Sqrt[a + b]])/(b^2*Sqrt[a + b]*d) + (Sech[c + d*x]*Tanh[c + d*x
])/(2*b*d)
```

Rubi [A] time = 0.102759, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4147, 414, 522, 203, 205}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a+b}}-\frac{(2a-b)\tan^{-1}(\sinh(c+dx))}{2b^2d}+\frac{\tanh(c+dx)\mathrm{sech}(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^5/(a + b*Sech[c + d*x]^2),x]
```


```
[Out] -((2*a - b)*ArcTan[Sinh[c + d*x]])/(2*b^2*d) + (a^(3/2)*ArcTan[(Sqrt[a]*Sin
h[c + d*x])/Sqrt[a + b]])/(b^2*Sqrt[a + b]*d) + (Sech[c + d*x]*Tanh[c + d*x
])/(2*b*d)
```


Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\text{sech}^5(c+dx)}{a+b\,\text{sech}^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2(a+b+ax^2)}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\text{sech}(c+dx)\tanh(c+dx)}{2bd} - \frac{\text{Subst}\left(\int \frac{a-b-ax^2}{(1+x^2)(a+b+ax^2)}\,dx, x, \sinh(c+dx)\right)}{2bd}$$

$$= \frac{\text{sech}(c+dx)\tanh(c+dx)}{2bd} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{a+b+ax^2}\,dx, x, \sinh(c+dx)\right)}{b^2d} - \frac{(2a-b)\,\text{Subst}\left(\right.}{}$$

$$= -\frac{(2a-b)\tan^{-1}(\sinh(c+dx))}{2b^2d} + \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{b^2\sqrt{a+b}\,d} + \frac{\text{sech}(c+dx)\tanh(c+dx)}{2bd}$$

Mathematica [B] time = 1.86462, size = 213, normalized size = 2.48

$$\frac{\cosh(c)\text{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b)\left(2a^{3/2}(\tanh(c)-1)\tan^{-1}\left(\frac{\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}(\sinh(c)+\cosh(c))\text{csch}(c+}{\sqrt{a}}\right.\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^5/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (Cosh[c]*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(b*Sqrt[a + b]*Sec
h[c]^2*Sech[c + d*x]^2*Sqrt[(Cosh[c] - Sinh[c])^2]*Sinh[d*x] + 2*a^(3/2)*Ar
cTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Sinh
[c]))/Sqrt[a]]*(-1 + Tanh[c]) - Sqrt[a + b]*Sech[c]*Sqrt[(Cosh[c] - Sinh[c]
)^2]*(2*(2*a - b)*ArcTan[Tanh[(c + d*x)/2]] - b*Sech[c + d*x]*Tanh[c])))/(4
*b^2*Sqrt[a + b]*d*(a + b*Sech[c + d*x]^2)*Sqrt[(Cosh[c] - Sinh[c])^2])
```

Maple [B] time = 0.047, size = 189, normalized size = 2.2

$$\frac{1}{db^2}a^{\frac{3}{2}}\arctan\left(\frac{1}{2}\left(2\tanh(1/2\,dx+c/2)\sqrt{a+b}+2\sqrt{b}\right)\frac{1}{\sqrt{a}}\right)\frac{1}{\sqrt{a+b}} + \frac{1}{db^2}a^{\frac{3}{2}}\arctan\left(\frac{1}{2}\left(2\tanh(1/2\,dx+c/2)\sqrt{a+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^5/(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/d*a^(3/2)/b^2/(a+b)^(1/2)*arctan(1/2*(2*tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)+2
*b^(1/2))/a^(1/2))+1/d*a^(3/2)/b^2/(a+b)^(1/2)*arctan(1/2*(2*tanh(1/2*d*x+1
/2*c)*(a+b)^(1/2)-2*b^(1/2))/a^(1/2))-1/d/b/(tanh(1/2*d*x+1/2*c)^2+1)^2*tan
h(1/2*d*x+1/2*c)^3+1/d/b/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)+1/
```

```
d/b*arctan(tanh(1/2*d*x+1/2*c))-2/d/b^2*arctan(tanh(1/2*d*x+1/2*c))*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{e^{(3\,dx+3\,c)} - e^{(dx+c)}}{bde^{(4\,dx+4\,c)} + 2\,bde^{(2\,dx+2\,c)} + bd} - \frac{(2\,ae^{c} - be^{c})\arctan\left(e^{(dx+c)}\right)e^{(-c)}}{b^{2}d} + 32\int \frac{a^{2}e^{(3\,dx+3\,c)} + a^{2}e^{(dx+c)}}{16\left(ab^{2}e^{(4\,dx+4\,c)} + ab^{2} + 2\left(ab^{2}e^{(2\,c)} + 2\,b^{3}e^{(2\,c)}\right)e^{(2\,dx)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] (e^(3*d*x + 3*c) - e^(d*x + c))/(b*d*e^(4*d*x + 4*c) + 2*b*d*e^(2*d*x + 2*c
) + b*d) - (2*a*e^c - b*e^c)*arctan(e^(d*x + c))*e^(-c)/(b^2*d) + 32*integr
ate(1/16*(a^2*e^(3*d*x + 3*c) + a^2*e^(d*x + c))/(a*b^2*e^(4*d*x + 4*c) + a
*b^2 + 2*(a*b^2*e^(2*c) + 2*b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.52429, size = 3997, normalized size = 46.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*b*cosh(d*x + c)^3 + 6*b*cosh(d*x + c)*sinh(d*x + c)^2 + 2*b*sinh(d*
x + c)^3 + (a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(
d*x + c)^4 + 2*a*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c
)^2 + 4*(a*cosh(d*x + c)^3 + a*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(-a/(a
 + b))*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(
d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a -
 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*s
inh(d*x + c) + 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*
x + c)^2 + (a + b)*sinh(d*x + c)^3 - (a + b)*cosh(d*x + c) + (3*(a + b)*cos
h(d*x + c)^2 - a - b)*sinh(d*x + c))*sqrt(-a/(a + b)) + a)/(a*cosh(d*x + c)
^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*co
sh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*co
sh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - 2*((2*a - b)
*cosh(d*x + c)^4 + 4*(2*a - b)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a - b)*si
nh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*(2*a - b)*cosh(d*x + c)^
2 + 2*a - b)*sinh(d*x + c)^2 + 4*((2*a - b)*cosh(d*x + c)^3 + (2*a - b)*cos
h(d*x + c))*sinh(d*x + c) + 2*a - b)*arctan(cosh(d*x + c) + sinh(d*x + c))
 - 2*b*cosh(d*x + c) + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c))/(b^2*d*cos
h(d*x + c)^4 + 4*b^2*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*d*sinh(d*x + c)^
4 + 2*b^2*d*cosh(d*x + c)^2 + b^2*d + 2*(3*b^2*d*cosh(d*x + c)^2 + b^2*d)*s
inh(d*x + c)^2 + 4*(b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*sinh(d*x +
 c)), (b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x +
 c)^3 + (a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x
 + c)^4 + 2*a*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c)^2
 + 4*(a*cosh(d*x + c)^3 + a*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(a/(a + b
))*arctan(1/2*sqrt(a/(a + b))*(cosh(d*x + c) + sinh(d*x + c))) + (a*cosh(d*
x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*a*cosh
(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c)^2 + 4*(a*cosh(d*x +
 c)^3 + a*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(a/(a + b))*arctan(1/2*(a*c
osh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (3
```

```
*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*a + 4*b)*sinh(d*x + c))*
sqrt(a/(a + b))/a) - ((2*a - b)*cosh(d*x + c)^4 + 4*(2*a - b)*cosh(d*x + c)
*sinh(d*x + c)^3 + (2*a - b)*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2
+ 2*(3*(2*a - b)*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*((2*a - b)*
cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + 2*a - b)*arctan(
cosh(d*x + c) + sinh(d*x + c)) - b*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 - b
)*sinh(d*x + c))/(b^2*d*cosh(d*x + c)^4 + 4*b^2*d*cosh(d*x + c)*sinh(d*x +
c)^3 + b^2*d*sinh(d*x + c)^4 + 2*b^2*d*cosh(d*x + c)^2 + b^2*d + 2*(3*b^2*d
*cosh(d*x + c)^2 + b^2*d)*sinh(d*x + c)^2 + 4*(b^2*d*cosh(d*x + c)^3 + b^2*
d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**5/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)**5/(a + b*sech(c + d*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.82 $\int \frac{\mathbf{sech}^6(c+dx)}{a+b\mathbf{sech}^2(c+dx)} \, dx$

Optimal. Leaf size=77

$$\frac{a^2 \tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{b^{5/2}d\sqrt{a+b}} - \frac{(a-b)\tanh(c+dx)}{b^2 d} - \frac{\tanh^3(c+dx)}{3bd}$$

```
[Out] (a^2*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(b^(5/2)*Sqrt[a + b]*d)
- ((a - b)*Tanh[c + d*x])/(b^2*d) - Tanh[c + d*x]^3/(3*b*d)
```

Rubi [A] time = 0.0891387, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4146, 390, 208}

$$\frac{a^2 \tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{b^{5/2}d\sqrt{a+b}} - \frac{(a-b)\tanh(c+dx)}{b^2 d} - \frac{\tanh^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^6/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (a^2*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(b^(5/2)*Sqrt[a + b]*d)
- ((a - b)*Tanh[c + d*x])/(b^2*d) - Tanh[c + d*x]^3/(3*b*d)
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^2}{a+b-bx^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(-\frac{a-b}{b^2}-\frac{x^2}{b}+\frac{a^2}{b^2(a+b-bx^2)}\right)dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{(a-b)\tanh(c+dx)}{b^2d}-\frac{\tanh^3(c+dx)}{3bd}+\frac{a^2\operatorname{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \tanh(c+dx)\right)}{b^2d}$$

$$= \frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}d}-\frac{(a-b)\tanh(c+dx)}{b^2d}-\frac{\tanh^3(c+dx)}{3bd}$$

Mathematica [B] time = 2.23284, size = 214, normalized size = 2.78

$$\frac{\operatorname{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b)\left(3a^2(\cosh(2c)-\sinh(2c))\tanh^{-1}\left(\frac{(\cosh(2c)-\sinh(2c))\operatorname{sech}(dx)((a+2b)\sinh(dx)-a\sin}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right.\right.}{6b^2d\sqrt{a+b}\sqrt{b(\cosh(}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^6/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(3*a^2*ArcTanh[(Sech[d*x]*
(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[
a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]) + Sqrt[a + b
]*Sech[c + d*x]*Sqrt[b*(Cosh[c] - Sinh[c])^4]*(Sech[c]*(-3*a + 2*b + b*Sech
[c + d*x]^2)*Sinh[d*x] + b*Sech[c + d*x]*Tanh[c])))/(6*b^2*Sqrt[a + b]*d*(a
 + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])
```

Maple [B] time = 0.051, size = 313, normalized size = 4.1

$$\frac{a^2}{2d}\ln\left(\sqrt{a+b}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+2\tanh(1/2\,dx+c/2)\sqrt{b}+\sqrt{a+b}\right)b^{-\frac{5}{2}}\frac{1}{\sqrt{a+b}}-\frac{a^2}{2d}\ln\left(\sqrt{a+b}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^6/(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/2/d*a^2/b^(5/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1
/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-1/2/d*a^2/b^(5/2)/(a+b)^(1/2)*ln((a+b)^(
1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-2/d/b
^2/(tanh(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)^5*a+2/d/b/(tanh(1/2*d*x+
1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)^5-4/d/b^2/(tanh(1/2*d*x+1/2*c)^2+1)^3*tan
h(1/2*d*x+1/2*c)^3*a+4/3/d/b/(tanh(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c
)^3-2/d/b^2/(tanh(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)*a+2/d/b/(tanh(1
/2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.38192, size = 4709, normalized size = 61.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(12*(a^2*b + a*b^2)*cosh(d*x + c)^4 + 48*(a^2*b + a*b^2)*cosh(d*x + c)
*sinh(d*x + c)^3 + 12*(a^2*b + a*b^2)*sinh(d*x + c)^4 + 12*a^2*b + 4*a*b^2
- 8*b^3 + 24*(a^2*b - b^3)*cosh(d*x + c)^2 + 24*(a^2*b - b^3 + 3*(a^2*b + a
*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*(a^2*cosh(d*x + c)^6 + 6*a^2*cos
h(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d*x + c)^6 + 3*a^2*cosh(d*x + c)^4 +
3*(5*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 + 3*a^2*cosh(d*x + c)^2 + 4
*(5*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*a^2*c
osh(d*x + c)^4 + 6*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^2 + a^2 + 6*(a^
2*cosh(d*x + c)^5 + 2*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c
))*sqrt(a*b + b^2)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x
+ c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*c
osh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^
2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(
d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)
*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 +
a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 +
a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*
sinh(d*x + c) + a)) + 48*((a^2*b + a*b^2)*cosh(d*x + c)^3 + (a^2*b - b^3)*c
osh(d*x + c))*sinh(d*x + c))/((a*b^3 + b^4)*d*cosh(d*x + c)^6 + 6*(a*b^3 +
b^4)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a*b^3 + b^4)*d*sinh(d*x + c)^6 + 3*
(a*b^3 + b^4)*d*cosh(d*x + c)^4 + 3*(5*(a*b^3 + b^4)*d*cosh(d*x + c)^2 + (a
*b^3 + b^4)*d)*sinh(d*x + c)^4 + 3*(a*b^3 + b^4)*d*cosh(d*x + c)^2 + 4*(5*(
a*b^3 + b^4)*d*cosh(d*x + c)^3 + 3*(a*b^3 + b^4)*d*cosh(d*x + c))*sinh(d*x
+ c)^3 + 3*(5*(a*b^3 + b^4)*d*cosh(d*x + c)^4 + 6*(a*b^3 + b^4)*d*cosh(d*x
+ c)^2 + (a*b^3 + b^4)*d)*sinh(d*x + c)^2 + (a*b^3 + b^4)*d + 6*((a*b^3 + b
^4)*d*cosh(d*x + c)^5 + 2*(a*b^3 + b^4)*d*cosh(d*x + c)^3 + (a*b^3 + b^4)*d
*cosh(d*x + c))*sinh(d*x + c)), 1/3*(6*(a^2*b + a*b^2)*cosh(d*x + c)^4 + 24
*(a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 6*(a^2*b + a*b^2)*sinh(d*x
 + c)^4 + 6*a^2*b + 2*a*b^2 - 4*b^3 + 12*(a^2*b - b^3)*cosh(d*x + c)^2 + 12
*(a^2*b - b^3 + 3*(a^2*b + a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*(a^2
*cosh(d*x + c)^6 + 6*a^2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d*x + c)^
6 + 3*a^2*cosh(d*x + c)^4 + 3*(5*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4
 + 3*a^2*cosh(d*x + c)^2 + 4*(5*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*
sinh(d*x + c)^3 + 3*(5*a^2*cosh(d*x + c)^4 + 6*a^2*cosh(d*x + c)^2 + a^2)*s
inh(d*x + c)^2 + a^2 + 6*(a^2*cosh(d*x + c)^5 + 2*a^2*cosh(d*x + c)^3 + a^2
*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(d*x + c)
^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-a
*b - b^2)/(a*b + b^2)) + 24*((a^2*b + a*b^2)*cosh(d*x + c)^3 + (a^2*b - b^3
)*cosh(d*x + c))*sinh(d*x + c))/((a*b^3 + b^4)*d*cosh(d*x + c)^6 + 6*(a*b^3
 + b^4)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a*b^3 + b^4)*d*sinh(d*x + c)^6 +
 3*(a*b^3 + b^4)*d*cosh(d*x + c)^4 + 3*(5*(a*b^3 + b^4)*d*cosh(d*x + c)^2 +
 (a*b^3 + b^4)*d)*sinh(d*x + c)^4 + 3*(a*b^3 + b^4)*d*cosh(d*x + c)^2 + 4*(
5*(a*b^3 + b^4)*d*cosh(d*x + c)^3 + 3*(a*b^3 + b^4)*d*cosh(d*x + c))*sinh(d
```

```
*x + c)^3 + 3*(5*(a*b^3 + b^4)*d*cosh(d*x + c)^4 + 6*(a*b^3 + b^4)*d*cosh(d
*x + c)^2 + (a*b^3 + b^4)*d)*sinh(d*x + c)^2 + (a*b^3 + b^4)*d + 6*((a*b^3
+ b^4)*d*cosh(d*x + c)^5 + 2*(a*b^3 + b^4)*d*cosh(d*x + c)^3 + (a*b^3 + b^4
)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**6/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)**6/(a + b*sech(c + d*x)**2), x)
```

Giac [A] time = 1.25143, size = 159, normalized size = 2.06

$$\frac{a^2\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}b^2d}+\frac{2\left(3\,ae^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}-6\,be^{(2\,dx+2\,c)}+3\,a-2\,b\right)}{3\,b^2d\left(e^{(2\,dx+2\,c)}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] a^2*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b -
 b^2)*b^2*d) + 2/3*(3*a*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c) - 6*b*e^(2*d*
x + 2*c) + 3*a - 2*b)/(b^2*d*(e^(2*d*x + 2*c) + 1)^3)
```

3.83 $\int \frac{\cosh^3(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=125

$$-\frac{b^3\sinh(c+dx)}{2a^3d(a+b)\left(a\sinh^2(c+dx)+a+b\right)}+\frac{b^2(6a+5b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{7/2}d(a+b)^{3/2}}+\frac{(a-2b)\sinh(c+dx)}{a^3d}+\frac{\sinh^3(c+dx)}{3a^2d}$$

```
[Out] (b^2*(6*a + 5*b)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(2*a^(7/2)*(a
 + b)^(3/2)*d) + ((a - 2*b)*Sinh[c + d*x])/(a^3*d) + Sinh[c + d*x]^3/(3*a^2
*d) - (b^3*Sinh[c + d*x])/(2*a^3*(a + b)*d*(a + b + a*Sinh[c + d*x]^2))
```

Rubi [A] time = 0.153556, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4147, 390, 385, 205}

$$-\frac{b^3\sinh(c+dx)}{2a^3d(a+b)\left(a\sinh^2(c+dx)+a+b\right)}+\frac{b^2(6a+5b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{7/2}d(a+b)^{3/2}}+\frac{(a-2b)\sinh(c+dx)}{a^3d}+\frac{\sinh^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (b^2*(6*a + 5*b)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(2*a^(7/2)*(a
 + b)^(3/2)*d) + ((a - 2*b)*Sinh[c + d*x])/(a^3*d) + Sinh[c + d*x]^3/(3*a^2
*d) - (b^3*Sinh[c + d*x])/(2*a^3*(a + b)*d*(a + b + a*Sinh[c + d*x]^2))
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{(1+x^2)^3}{(a+b+ax^2)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{a-2b}{a^3}+\frac{x^2}{a^2}+\frac{b^2(3a+2b)+3ab^2x^2}{a^3(a+b+ax^2)^2}\right) dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{(a-2b)\sinh(c+dx)}{a^3d} + \frac{\sinh^3(c+dx)}{3a^2d} + \frac{\operatorname{Subst}\left(\int \frac{b^2(3a+2b)+3ab^2x^2}{(a+b+ax^2)^2}\,dx, x, \sinh(c+dx)\right)}{a^3d}$$

$$= \frac{(a-2b)\sinh(c+dx)}{a^3d} + \frac{\sinh^3(c+dx)}{3a^2d} - \frac{b^3\sinh(c+dx)}{2a^3(a+b)d\left(a+b+a\sinh^2(c+dx)\right)} + \frac{(b^2(6\ldots}{}$$

$$= \frac{b^2(6a+5b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{7/2}(a+b)^{3/2}d} + \frac{(a-2b)\sinh(c+dx)}{a^3d} + \frac{\sinh^3(c+dx)}{3a^2d} - \frac{}{2a^3(a+b\ldots}$$

Mathematica [A] time = 0.862626, size = 113, normalized size = 0.9

$$\frac{a^{3/2}\sinh(3(c+dx)) + 3\sqrt{a}\sinh(c+dx)\left(-\frac{4b^3}{(a+b)(a\cosh(2(c+dx))+a+2b)}+3a-8b\right) - \frac{6b^2(6a+5b)\tan^{-1}\left(\frac{\sqrt{a+b}\operatorname{csch}(c+dx)}{\sqrt{a}}\right)}{(a+b)^{3/2}}}{12a^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((-6*b^2*(6*a + 5*b)*ArcTan[(Sqrt[a + b]*Csch[c + d*x])/Sqrt[a]])/(a + b)^(
3/2) + 3*Sqrt[a]*(3*a - 8*b - (4*b^3)/((a + b)*(a + 2*b + a*Cosh[2*(c + d*x
)])))*Sinh[c + d*x] + a^(3/2)*Sinh[3*(c + d*x)])/(12*a^(7/2)*d)
```

Maple [B] time = 0.101, size = 517, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/3/d/a^2/(tanh(1/2*d*x+1/2*c)+1)^3+1/2/d/a^2/(tanh(1/2*d*x+1/2*c)+1)^2-1/
d/a^2/(tanh(1/2*d*x+1/2*c)+1)+2/d/a^3/(tanh(1/2*d*x+1/2*c)+1)*b+1/d*b^3/a^3
/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a
-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*tanh(1/2*d*x+1/2*c)^3-1/d*b^3/a^3/(ta
nh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*t
anh(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*tanh(1/2*d*x+1/2*c)+3/d*b^2/a^(5/2)/(a+b)
^(3/2)*arctan(1/2*(2*tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)+2*b^(1/2))/a^(1/2))+3/
d*b^2/a^(5/2)/(a+b)^(3/2)*arctan(1/2*(2*tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)-2*b
^(1/2))/a^(1/2))+5/2/d*b^3/a^(7/2)/(a+b)^(3/2)*arctan(1/2*(2*tanh(1/2*d*x+1
/2*c)*(a+b)^(1/2)+2*b^(1/2))/a^(1/2))+5/2/d*b^3/a^(7/2)/(a+b)^(3/2)*arctan(
1/2*(2*tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)-2*b^(1/2))/a^(1/2))-1/3/d/a^2/(tanh(
1/2*d*x+1/2*c)-1)^3-1/2/d/a^2/(tanh(1/2*d*x+1/2*c)-1)^2-1/d/a^2/(tanh(1/2*d
```

```
*x+1/2*c)-1)+2/d/a^3/(tanh(1/2*d*x+1/2*c)-1)*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^3 + a^2b - \left(a^3e^{(10\,c)} + a^2be^{(10\,c)}\right)e^{(10\,dx)} - \left(11\,a^3e^{(8\,c)} - 9\,a^2be^{(8\,c)} - 20\,ab^2e^{(8\,c)}\right)e^{(8\,dx)} - 2\left(5\,a^3e^{(6\,c)} + 11\,a^2be^{(6\,c)} - 42\,ab^2e^{(6}\right.}{24\left(\left(a^5de^{(7\,c)} + a^4bde^{(7\,c)}\right)e^{(7\,dx)} + 2\left(a^5de^{(5\,c)} + 3\,a^4bd\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/24*(a^3 + a^2*b - (a^3*e^(10*c) + a^2*b*e^(10*c))*e^(10*d*x) - (11*a^3*e
^(8*c) - 9*a^2*b*e^(8*c) - 20*a*b^2*e^(8*c))*e^(8*d*x) - 2*(5*a^3*e^(6*c) +
 11*a^2*b*e^(6*c) - 42*a*b^2*e^(6*c) - 60*b^3*e^(6*c))*e^(6*d*x) + 2*(5*a^3
*e^(4*c) + 11*a^2*b*e^(4*c) - 42*a*b^2*e^(4*c) - 60*b^3*e^(4*c))*e^(4*d*x)
+ (11*a^3*e^(2*c) - 9*a^2*b*e^(2*c) - 20*a*b^2*e^(2*c))*e^(2*d*x))/((a^5*d*
e^(7*c) + a^4*b*d*e^(7*c))*e^(7*d*x) + 2*(a^5*d*e^(5*c) + 3*a^4*b*d*e^(5*c)
 + 2*a^3*b^2*d*e^(5*c))*e^(5*d*x) + (a^5*d*e^(3*c) + a^4*b*d*e^(3*c))*e^(3*
d*x)) + 1/8*integrate(8*((6*a*b^2*e^(3*c) + 5*b^3*e^(3*c))*e^(3*d*x) + (6*a
*b^2*e^c + 5*b^3*e^c)*e^(d*x))/(a^5 + a^4*b + (a^5*e^(4*c) + a^4*b*e^(4*c))
*e^(4*d*x) + 2*(a^5*e^(2*c) + 3*a^4*b*e^(2*c) + 2*a^3*b^2*e^(2*c))*e^(2*d*x
)), x)
```

Fricas [B] time = 2.90541, size = 13824, normalized size = 110.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/24*((a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^10 + 10*(a^5 + 2*a^4*b + a^3
*b^2)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^5 + 2*a^4*b + a^3*b^2)*sinh(d*x +
c)^10 + (11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c)^8 + (11*
a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3 + 45*(a^5 + 2*a^4*b + a^3*b^2)*cosh
(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c
)^3 + (11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c))*sinh(d*x
+ c)^7 + 2*(5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*
x + c)^6 + 2*(5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4 + 105*
(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^4 + 14*(11*a^5 + 2*a^4*b - 29*a^3*b
^2 - 20*a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(a^5 + 2*a^4*b +
a^3*b^2)*cosh(d*x + c)^5 + 14*(11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*
cosh(d*x + c)^3 + 3*(5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4
)*cosh(d*x + c))*sinh(d*x + c)^5 - a^5 - 2*a^4*b - a^3*b^2 - 2*(5*a^5 + 16*
a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x + c)^4 + 2*(105*(a^5
+ 2*a^4*b + a^3*b^2)*cosh(d*x + c)^6 - 5*a^5 - 16*a^4*b + 31*a^3*b^2 + 102*
a^2*b^3 + 60*a*b^4 + 35*(11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d
*x + c)^4 + 15*(5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cos
h(d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x +
c)^7 + 7*(11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c)^5 + 5*(
5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x + c)^3 - (
5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x + c))*sinh
(d*x + c)^3 - (11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c)^2
+ (45*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^8 + 28*(11*a^5 + 2*a^4*b - 29
```

*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c)^6 - 11*a^5 - 2*a^4*b + 29*a^3*b^2 + 20
*a^2*b^3 + 30*(5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh
(d*x + c)^4 - 12*(5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*c
osh(d*x + c)^2)*sinh(d*x + c)^2 - 6*((6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^7
+ 7*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + (6*a^2*b^2 + 5*a*
b^3)*sinh(d*x + c)^7 + 2*(6*a^2*b^2 + 17*a*b^3 + 10*b^4)*cosh(d*x + c)^5 +
(12*a^2*b^2 + 34*a*b^3 + 20*b^4 + 21*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^5 + 5*(7*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^3 + 2*(6*a^2*b^
2 + 17*a*b^3 + 10*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 + (6*a^2*b^2 + 5*a*b^
3)*cosh(d*x + c)^3 + (35*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^4 + 6*a^2*b^2
+ 5*a*b^3 + 20*(6*a^2*b^2 + 17*a*b^3 + 10*b^4)*cosh(d*x + c)^2)*sinh(d*x +
c)^3 + (21*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^5 + 20*(6*a^2*b^2 + 17*a*b^3
 + 10*b^4)*cosh(d*x + c)^3 + 3*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c))*sinh(d*
x + c)^2 + (7*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^6 + 10*(6*a^2*b^2 + 17*a*
b^3 + 10*b^4)*cosh(d*x + c)^4 + 3*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^2)*si
nh(d*x + c))*sqrt(-a^2 - a*b)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*si
nh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*
cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a
+ 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*
sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) -
 cosh(d*x + c))*sqrt(-a^2 - a*b) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c
)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*
a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a +
2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 2*(5*(a^5 + 2*a^4*b + a^3*b^2)*co
sh(d*x + c)^9 + 4*(11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c
)^7 + 6*(5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x +
 c)^5 - 4*(5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x
 + c)^3 - (11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c))*sinh(
d*x + c))/((a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^7 + 7*(a^7 + 2*a^6*b +
 a^5*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^6 + (a^7 + 2*a^6*b + a^5*b^2)*d*sin
h(d*x + c)^7 + 2*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^5
+ (21*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^2 + 2*(a^7 + 4*a^6*b + 5*a^
5*b^2 + 2*a^4*b^3)*d)*sinh(d*x + c)^5 + (a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*
x + c)^3 + 5*(7*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^3 + 2*(a^7 + 4*a^
6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^4 + (35*(a^7 +
2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^4 + 20*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^
4*b^3)*d*cosh(d*x + c)^2 + (a^7 + 2*a^6*b + a^5*b^2)*d)*sinh(d*x + c)^3 + (
21*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^5 + 20*(a^7 + 4*a^6*b + 5*a^5*
b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^3 + 3*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x
 + c))*sinh(d*x + c)^2 + (7*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^6 + 1
0*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^4 + 3*(a^7 + 2*a^
6*b + a^5*b^2)*d*cosh(d*x + c)^2)*sinh(d*x + c)), 1/24*((a^5 + 2*a^4*b + a^
3*b^2)*cosh(d*x + c)^10 + 10*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)*sinh(d
*x + c)^9 + (a^5 + 2*a^4*b + a^3*b^2)*sinh(d*x + c)^10 + (11*a^5 + 2*a^4*b
- 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c)^8 + (11*a^5 + 2*a^4*b - 29*a^3*b^2
 - 20*a^2*b^3 + 45*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)
^8 + 8*(15*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^3 + (11*a^5 + 2*a^4*b -
29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(5*a^5 + 16*a^4
*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x + c)^6 + 2*(5*a^5 + 16*a
^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4 + 105*(a^5 + 2*a^4*b + a^3*b^2)*
cosh(d*x + c)^4 + 14*(11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x
+ c)^2)*sinh(d*x + c)^6 + 4*(63*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^5 +
 14*(11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c)^3 + 3*(5*a^5
 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x + c))*sinh(d*x
+ c)^5 - a^5 - 2*a^4*b - a^3*b^2 - 2*(5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a
^2*b^3 - 60*a*b^4)*cosh(d*x + c)^4 + 2*(105*(a^5 + 2*a^4*b + a^3*b^2)*cosh(
d*x + c)^6 - 5*a^5 - 16*a^4*b + 31*a^3*b^2 + 102*a^2*b^3 + 60*a*b^4 + 35*(1
1*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c)^4 + 15*(5*a^5 + 16
*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c

```
)^4 + 8*(15*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^7 + 7*(11*a^5 + 2*a^4*b
 - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c)^5 + 5*(5*a^5 + 16*a^4*b - 31*a^3*
b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x + c)^3 - (5*a^5 + 16*a^4*b - 31*a^3*
b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 - (11*a^5 + 2*
a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c)^2 + (45*(a^5 + 2*a^4*b + a^3
*b^2)*cosh(d*x + c)^8 + 28*(11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cos
h(d*x + c)^6 - 11*a^5 - 2*a^4*b + 29*a^3*b^2 + 20*a^2*b^3 + 30*(5*a^5 + 16*
a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x + c)^4 - 12*(5*a^5 +
16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x + c)^2)*sinh(d*x +
 c)^2 + 12*((6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^7 + 7*(6*a^2*b^2 + 5*a*b^3)
*cosh(d*x + c)*sinh(d*x + c)^6 + (6*a^2*b^2 + 5*a*b^3)*sinh(d*x + c)^7 + 2*
(6*a^2*b^2 + 17*a*b^3 + 10*b^4)*cosh(d*x + c)^5 + (12*a^2*b^2 + 34*a*b^3 +
20*b^4 + 21*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 5*(7*(
6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^3 + 2*(6*a^2*b^2 + 17*a*b^3 + 10*b^4)*co
sh(d*x + c))*sinh(d*x + c)^4 + (6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^3 + (35*
(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^4 + 6*a^2*b^2 + 5*a*b^3 + 20*(6*a^2*b^2
 + 17*a*b^3 + 10*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (21*(6*a^2*b^2 + 5
*a*b^3)*cosh(d*x + c)^5 + 20*(6*a^2*b^2 + 17*a*b^3 + 10*b^4)*cosh(d*x + c)^
3 + 3*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + (7*(6*a^2*b^2
+ 5*a*b^3)*cosh(d*x + c)^6 + 10*(6*a^2*b^2 + 17*a*b^3 + 10*b^4)*cosh(d*x +
c)^4 + 3*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(a^2 + a
*b)*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*s
inh(d*x + c)^3 + (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*a + 4
*b)*sinh(d*x + c))/sqrt(a^2 + a*b)) + 12*((6*a^2*b^2 + 5*a*b^3)*cosh(d*x +
c)^7 + 7*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + (6*a^2*b^2 +
 5*a*b^3)*sinh(d*x + c)^7 + 2*(6*a^2*b^2 + 17*a*b^3 + 10*b^4)*cosh(d*x + c)
^5 + (12*a^2*b^2 + 34*a*b^3 + 20*b^4 + 21*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x +
c)^2)*sinh(d*x + c)^5 + 5*(7*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^3 + 2*(6*a
^2*b^2 + 17*a*b^3 + 10*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 + (6*a^2*b^2 + 5
*a*b^3)*cosh(d*x + c)^3 + (35*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^4 + 6*a^2
*b^2 + 5*a*b^3 + 20*(6*a^2*b^2 + 17*a*b^3 + 10*b^4)*cosh(d*x + c)^2)*sinh(d
*x + c)^3 + (21*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^5 + 20*(6*a^2*b^2 + 17*
a*b^3 + 10*b^4)*cosh(d*x + c)^3 + 3*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c))*si
nh(d*x + c)^2 + (7*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^6 + 10*(6*a^2*b^2 +
17*a*b^3 + 10*b^4)*cosh(d*x + c)^4 + 3*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c))*sqrt(a^2 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c)
 + sinh(d*x + c))/(a + b)) + 2*(5*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^9
 + 4*(11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c)^7 + 6*(5*a^
5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x + c)^5 - 4*(5*
a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x + c)^3 - (11
*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a
^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^7 + 7*(a^7 + 2*a^6*b + a^5*b^2)*d*c
osh(d*x + c)*sinh(d*x + c)^6 + (a^7 + 2*a^6*b + a^5*b^2)*d*sinh(d*x + c)^7
+ 2*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^5 + (21*(a^7 +
2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^2 + 2*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4
*b^3)*d)*sinh(d*x + c)^5 + (a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^3 + 5*
(7*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^3 + 2*(a^7 + 4*a^6*b + 5*a^5*b
^2 + 2*a^4*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^4 + (35*(a^7 + 2*a^6*b + a^5
*b^2)*d*cosh(d*x + c)^4 + 20*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cosh
(d*x + c)^2 + (a^7 + 2*a^6*b + a^5*b^2)*d)*sinh(d*x + c)^3 + (21*(a^7 + 2*a
^6*b + a^5*b^2)*d*cosh(d*x + c)^5 + 20*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b
^3)*d*cosh(d*x + c)^3 + 3*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c))*sinh(d
*x + c)^2 + (7*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^6 + 10*(a^7 + 4*a^
6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^4 + 3*(a^7 + 2*a^6*b + a^5*b^2
)*d*cosh(d*x + c)^2)*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.84 $\int \frac{\cosh^2(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=144

$$\frac{b^{3/2}(5a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^3d(a+b)^{3/2}}+\frac{b(a+2b)\tanh(c+dx)}{2a^2d(a+b)\left(a-b\tanh^2(c+dx)+b\right)}+\frac{x(a-4b)}{2a^3}+\frac{\sinh(c+dx)\cosh(c+dx)}{2ad\left(a-b\tanh^2(c+dx)+b\right)}$$

```
[Out] ((a - 4*b)*x)/(2*a^3) + (b^(3/2)*(5*a + 4*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x]
)/Sqrt[a + b]])/(2*a^3*(a + b)^(3/2)*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*
a*d*(a + b - b*Tanh[c + d*x]^2)) + (b*(a + 2*b)*Tanh[c + d*x])/(2*a^2*(a +
b)*d*(a + b - b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.23507, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4146, 414, 527, 522, 206, 208}

$$\frac{b^{3/2}(5a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^3d(a+b)^{3/2}}+\frac{b(a+2b)\tanh(c+dx)}{2a^2d(a+b)\left(a-b\tanh^2(c+dx)+b\right)}+\frac{x(a-4b)}{2a^3}+\frac{\sinh(c+dx)\cosh(c+dx)}{2ad\left(a-b\tanh^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a - 4*b)*x)/(2*a^3) + (b^(3/2)*(5*a + 4*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x]
)/Sqrt[a + b]])/(2*a^3*(a + b)^(3/2)*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*
a*d*(a + b - b*Tanh[c + d*x]^2)) + (b*(a + 2*b)*Tanh[c + d*x])/(2*a^2*(a +
b)*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{a-b-3bx^2}{\left(1-x^2\right)\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{2ad}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)} + \frac{b(a+2b)\tanh(c+dx)}{2a^2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-2}{}\right.}{}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)} + \frac{b(a+2b)\tanh(c+dx)}{2a^2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{(a-4b)\,\text{Sub}}{}$$

$$= \frac{(a-4b)x}{2a^3} + \frac{b^{3/2}(5a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^3(a+b)^{3/2}d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)} + \frac{}{2a}$$

Mathematica [A] time = 1.31752, size = 103, normalized size = 0.72

$$\frac{\frac{2b^{3/2}(5a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} + \sinh(2(c+dx))\left(\frac{2ab^2}{(a+b)(a\cosh(2(c+dx))+a+2b)} + a\right) + 2(a-4b)(c+dx)}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (2*(a - 4*b)*(c + d*x) + (2*b^(3/2)*(5*a + 4*b)*ArcTanh[(Sqrt[b]*Tanh[c + d
*x])/Sqrt[a + b]])/(a + b)^(3/2) + (a + (2*a*b^2)/((a + b)*(a + 2*b + a*Cos
h[2*(c + d*x)])))*Sinh[2*(c + d*x)])/(4*a^3*d)
```

Maple [B] time = 0.102, size = 551, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/2/d/a^2/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/a^2/(tanh(1/2*d*x+1/2*c)+1)+1/2/
d/a^2*ln(tanh(1/2*d*x+1/2*c)+1)-2/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)*b+1/d*b^2
/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)
^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*tanh(1/2*d*x+1/2*c)^3+1/d*b^2/a^2
/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a
-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*tanh(1/2*d*x+1/2*c)+5/4/d*b^(3/2)/a^2
/(a+b)^(3/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(
1/2)+(a+b)^(1/2))-5/4/d*b^(3/2)/a^2/(a+b)^(3/2)*ln((a+b)^(1/2)*tanh(1/2*d*x
+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/d*b^(5/2)/a^3/(a+b)^
(3/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a
+b)^(1/2))-1/d*b^(5/2)/a^3/(a+b)^(3/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2
-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/2/d/a^2/(tanh(1/2*d*x+1/2*c)-
1)^2+1/2/d/a^2/(tanh(1/2*d*x+1/2*c)-1)-1/2/d/a^2*ln(tanh(1/2*d*x+1/2*c)-1)+
2/d/a^3*ln(tanh(1/2*d*x+1/2*c)-1)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.78313, size = 8946, normalized size = 62.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*((a^3 + a^2*b)*cosh(d*x + c)^8 + 8*(a^3 + a^2*b)*cosh(d*x + c)*sinh(d*
x + c)^7 + (a^3 + a^2*b)*sinh(d*x + c)^8 + 2*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(
a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c)^6 + 2*(a^3 + 3*a^2*b + 2*a*b^2
+ 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x + 14*(a^3 + a^2*b)*cosh(d*x + c)^2)*sinh(
d*x + c)^6 + 4*(14*(a^3 + a^2*b)*cosh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 2*a*b
^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 8*(a
*b^2 + 2*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*d*x)*cosh(d*x + c)^4 + 2*(3
5*(a^3 + a^2*b)*cosh(d*x + c)^4 - 4*a*b^2 - 8*b^3 + 4*(a^3 - a^2*b - 10*a*b
^2 - 8*b^3)*d*x + 15*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)
*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^3 + a^2*b)*cosh(d*x + c)^5
 + 5*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x +
 c)^3 - 4*(a*b^2 + 2*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*d*x)*cosh(d*x +
 c))*sinh(d*x + c)^3 - a^3 - a^2*b - 2*(a^3 + 3*a^2*b + 6*a*b^2 - 2*(a^3 -
```

3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c)^2 + 2*(14*(a^3 + a^2*b)*cosh(d*x + c)
^6 + 15*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*
x + c)^4 - a^3 - 3*a^2*b - 6*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x - 24*(
a*b^2 + 2*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*d*x)*cosh(d*x + c)^2)*sinh
(d*x + c)^2 + 2*((5*a^2*b + 4*a*b^2)*cosh(d*x + c)^6 + 6*(5*a^2*b + 4*a*b^2
)*cosh(d*x + c)*sinh(d*x + c)^5 + (5*a^2*b + 4*a*b^2)*sinh(d*x + c)^6 + 2*(
5*a^2*b + 14*a*b^2 + 8*b^3)*cosh(d*x + c)^4 + (10*a^2*b + 28*a*b^2 + 16*b^3
 + 15*(5*a^2*b + 4*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(5*a^2*b
 + 4*a*b^2)*cosh(d*x + c)^3 + 2*(5*a^2*b + 14*a*b^2 + 8*b^3)*cosh(d*x + c))*
sinh(d*x + c)^3 + (5*a^2*b + 4*a*b^2)*cosh(d*x + c)^2 + (15*(5*a^2*b + 4*a*
b^2)*cosh(d*x + c)^4 + 5*a^2*b + 4*a*b^2 + 12*(5*a^2*b + 14*a*b^2 + 8*b^3)*
cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(5*a^2*b + 4*a*b^2)*cosh(d*x + c)^5
 + 4*(5*a^2*b + 14*a*b^2 + 8*b^3)*cosh(d*x + c)^3 + (5*a^2*b + 4*a*b^2)*cos
h(d*x + c))*sinh(d*x + c))*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2
*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh
(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2
 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*si
nh(d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*
sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(
a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*
x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*
sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x
 + c) + a)) + 4*(2*(a^3 + a^2*b)*cosh(d*x + c)^7 + 3*(a^3 + 3*a^2*b + 2*a*b^
2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c)^5 - 8*(a*b^2 + 2*b^3 - (
a^3 - a^2*b - 10*a*b^2 - 8*b^3)*d*x)*cosh(d*x + c)^3 - (a^3 + 3*a^2*b + 6*a
b^2 - 2(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^5
 + a^4*b)*d*cosh(d*x + c)^6 + 6*(a^5 + a^4*b)*d*cosh(d*x + c)*sinh(d*x + c)
^5 + (a^5 + a^4*b)*d*sinh(d*x + c)^6 + 2*(a^5 + 3*a^4*b + 2*a^3*b^2)*d*cosh
(d*x + c)^4 + (15*(a^5 + a^4*b)*d*cosh(d*x + c)^2 + 2*(a^5 + 3*a^4*b + 2*a^
3*b^2)*d)*sinh(d*x + c)^4 + (a^5 + a^4*b)*d*cosh(d*x + c)^2 + 4*(5*(a^5 + a
^4*b)*d*cosh(d*x + c)^3 + 2*(a^5 + 3*a^4*b + 2*a^3*b^2)*d*cosh(d*x + c))*si
nh(d*x + c)^3 + (15*(a^5 + a^4*b)*d*cosh(d*x + c)^4 + 12*(a^5 + 3*a^4*b + 2
*a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 + a^4*b)*d)*sinh(d*x + c)^2 + 2*(3*(a^5
 + a^4*b)*d*cosh(d*x + c)^5 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2)*d*cosh(d*x + c)^
3 + (a^5 + a^4*b)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*((a^3 + a^2*b)*cosh(
d*x + c)^8 + 8*(a^3 + a^2*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 + a^2*b)*
sinh(d*x + c)^8 + 2*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*
d*x)*cosh(d*x + c)^6 + 2*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*
b^2)*d*x + 14*(a^3 + a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(14*(a^3 +
 a^2*b)*cosh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4
*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 8*(a*b^2 + 2*b^3 - (a^3 - a^2
*b - 10*a*b^2 - 8*b^3)*d*x)*cosh(d*x + c)^4 + 2*(35*(a^3 + a^2*b)*cosh(d*x
 + c)^4 - 4*a*b^2 - 8*b^3 + 4*(a^3 - a^2*b - 10*a*b^2 - 8*b^3)*d*x + 15*(a^3
 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c)^2)*si
nh(d*x + c)^4 + 8*(7*(a^3 + a^2*b)*cosh(d*x + c)^5 + 5*(a^3 + 3*a^2*b + 2*a
b^2 + 2(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c)^3 - 4*(a*b^2 + 2*b^3
 - (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - a^
3 - a^2*b - 2*(a^3 + 3*a^2*b + 6*a*b^2 - 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*c
osh(d*x + c)^2 + 2*(14*(a^3 + a^2*b)*cosh(d*x + c)^6 + 15*(a^3 + 3*a^2*b +
2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c)^4 - a^3 - 3*a^2*b
 - 6*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x - 24*(a*b^2 + 2*b^3 - (a^3 - a^
2*b - 10*a*b^2 - 8*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((5*a^2*b
 + 4*a*b^2)*cosh(d*x + c)^6 + 6*(5*a^2*b + 4*a*b^2)*cosh(d*x + c)*sinh(d*x
 + c)^5 + (5*a^2*b + 4*a*b^2)*sinh(d*x + c)^6 + 2*(5*a^2*b + 14*a*b^2 + 8*b^
3)*cosh(d*x + c)^4 + (10*a^2*b + 28*a*b^2 + 16*b^3 + 15*(5*a^2*b + 4*a*b^2)
*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(5*a^2*b + 4*a*b^2)*cosh(d*x + c)^
3 + 2*(5*a^2*b + 14*a*b^2 + 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + (5*a^2*
b + 4*a*b^2)*cosh(d*x + c)^2 + (15*(5*a^2*b + 4*a*b^2)*cosh(d*x + c)^4 + 5*
a^2*b + 4*a*b^2 + 12*(5*a^2*b + 14*a*b^2 + 8*b^3)*cosh(d*x + c)^2)*sinh(d*x

```
 + c)^2 + 2*(3*(5*a^2*b + 4*a*b^2)*cosh(d*x + c)^5 + 4*(5*a^2*b + 14*a*b^2
+ 8*b^3)*cosh(d*x + c)^3 + (5*a^2*b + 4*a*b^2)*cosh(d*x + c))*sinh(d*x + c)
)*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d
*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b) + 4*(2*(a^3 + a^
2*b)*cosh(d*x + c)^7 + 3*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*
b^2)*d*x)*cosh(d*x + c)^5 - 8*(a*b^2 + 2*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8*
b^3)*d*x)*cosh(d*x + c)^3 - (a^3 + 3*a^2*b + 6*a*b^2 - 2*(a^3 - 3*a^2*b - 4
*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + a^4*b)*d*cosh(d*x + c)^6
 + 6*(a^5 + a^4*b)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^5 + a^4*b)*d*sinh(d
*x + c)^6 + 2*(a^5 + 3*a^4*b + 2*a^3*b^2)*d*cosh(d*x + c)^4 + (15*(a^5 + a^
4*b)*d*cosh(d*x + c)^2 + 2*(a^5 + 3*a^4*b + 2*a^3*b^2)*d)*sinh(d*x + c)^4 +
 (a^5 + a^4*b)*d*cosh(d*x + c)^2 + 4*(5*(a^5 + a^4*b)*d*cosh(d*x + c)^3 + 2
*(a^5 + 3*a^4*b + 2*a^3*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (15*(a^5 +
a^4*b)*d*cosh(d*x + c)^4 + 12*(a^5 + 3*a^4*b + 2*a^3*b^2)*d*cosh(d*x + c)^2
 + (a^5 + a^4*b)*d)*sinh(d*x + c)^2 + 2*(3*(a^5 + a^4*b)*d*cosh(d*x + c)^5
+ 4*(a^5 + 3*a^4*b + 2*a^3*b^2)*d*cosh(d*x + c)^3 + (a^5 + a^4*b)*d*cosh(d*
x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.15766, size = 446, normalized size = 3.1

$$\frac{\left(5\,ab^2+4\,b^3\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{2\left(a^4d+a^3bd\right)\sqrt{-ab-b^2}} - \frac{2\,a^3e^{(6\,dx+6\,c)}-6\,a^2be^{(6\,dx+6\,c)}-8\,ab^2e^{(6\,dx+6\,c)}+7\,a^3e^{(4\,dx+4\,c)}-a^2be^{(4\,dx+4\,c)}-16\,\ldots}{24\left(a^4d+a^3bd\right)\left(ae^{(6\,dx+6\,c)}+\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*a*b^2 + 4*b^3)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b -
b^2))/((a^4*d + a^3*b*d)*sqrt(-a*b - b^2)) - 1/24*(2*a^3*e^(6*d*x + 6*c) -
6*a^2*b*e^(6*d*x + 6*c) - 8*a*b^2*e^(6*d*x + 6*c) + 7*a^3*e^(4*d*x + 4*c) -
 a^2*b*e^(4*d*x + 4*c) - 16*a*b^2*e^(4*d*x + 4*c) + 16*b^3*e^(4*d*x + 4*c)
+ 8*a^3*e^(2*d*x + 2*c) + 12*a^2*b*e^(2*d*x + 2*c) + 28*a*b^2*e^(2*d*x + 2*
c) + 3*a^3 + 3*a^2*b)/((a^4*d + a^3*b*d)*(a*e^(6*d*x + 6*c) + 2*a*e^(4*d*x
+ 4*c) + 4*b*e^(4*d*x + 4*c) + a*e^(2*d*x + 2*c))) + 1/2*(d*x + c)*(a - 4*b
)/(a^3*d) + 1/8*e^(2*d*x + 2*c)/(a^2*d)
```

3.85 $\int \frac{\cosh(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=100

$$\frac{b^2\sinh(c+dx)}{2a^2d(a+b)\left(a\sinh^2(c+dx)+a+b\right)}-\frac{b(4a+3b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{5/2}d(a+b)^{3/2}}+\frac{\sinh(c+dx)}{a^2d}$$

```
[Out] -(b*(4*a + 3*b)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(2*a^(5/2)*(a
+ b)^(3/2)*d) + Sinh[c + d*x]/(a^2*d) + (b^2*Sinh[c + d*x])/(2*a^2*(a + b)*
d*(a + b + a*Sinh[c + d*x]^2))
```

Rubi [A] time = 0.131521, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4147, 390, 385, 205}

$$\frac{b^2\sinh(c+dx)}{2a^2d(a+b)\left(a\sinh^2(c+dx)+a+b\right)}-\frac{b(4a+3b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{5/2}d(a+b)^{3/2}}+\frac{\sinh(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -(b*(4*a + 3*b)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(2*a^(5/2)*(a
+ b)^(3/2)*d) + Sinh[c + d*x]/(a^2*d) + (b^2*Sinh[c + d*x])/(2*a^2*(a + b)*
d*(a + b + a*Sinh[c + d*x]^2))
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{\left(a+b+ax^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2} - \frac{b(2a+b)+2abx^2}{a^2\left(a+b+ax^2\right)^2}\right)dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\sinh(c+dx)}{a^2d} - \frac{\text{Subst}\left(\int \frac{b(2a+b)+2abx^2}{\left(a+b+ax^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{a^2d}$$

$$= \frac{\sinh(c+dx)}{a^2d} + \frac{b^2\sinh(c+dx)}{2a^2(a+b)d\left(a+b+a\sinh^2(c+dx)\right)} - \frac{(b(4a+3b))\,\text{Subst}\left(\int \frac{1}{a+b+ax^2}\,dx, \right.}{2a^2(a+b)d}$$

$$= -\frac{b(4a+3b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{5/2}(a+b)^{3/2}d} + \frac{\sinh(c+dx)}{a^2d} + \frac{b^2\sinh(c+dx)}{2a^2(a+b)d\left(a+b+a\sinh^2(c+dx)\right)}$$

Mathematica [B] time = 1.83234, size = 234, normalized size = 2.34

$$\frac{\operatorname{sech}^3(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\frac{2\sqrt{a}b^2\tanh(c+dx)}{a+b} + 2\sqrt{a}\sinh(c)\cosh(dx)\operatorname{sech}(c+dx)(a\cosh(2(c+dx))+a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^3*((b*(4*a + 3*b)*ArcTan[(Sq
rt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Sinh[c]))/Sq
rt[a]]*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]*(Cosh[c] - Sinh[c]))/(
(a + b)^(3/2)*Sqrt[(Cosh[c] - Sinh[c])^2]) + 2*Sqrt[a]*Cosh[d*x]*(a + 2*b +
 a*Cosh[2*(c + d*x)])*Sech[c + d*x]*Sinh[c] + 2*Sqrt[a]*Cosh[c]*(a + 2*b +
a*Cosh[2*(c + d*x)])*Sech[c + d*x]*Sinh[d*x] + (2*Sqrt[a]*b^2*Tanh[c + d*x]
)/(a + b)))/(8*a^(5/2)*d*(a + b*Sech[c + d*x]^2)^2)
```

Maple [B] time = 0.091, size = 385, normalized size = 3.9

$$-\frac{1}{da^2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1} - \frac{b^2}{da^2(a+b)}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 a + b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 + 2\,(\tanh(1/\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/d/a^2/(tanh(1/2*d*x+1/2*c)+1)-1/d*b^2/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tan
h(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)
/(a+b)*tanh(1/2*d*x+1/2*c)^3+1/d*b^2/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/
2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)/(a+
```

```
b)*tanh(1/2*d*x+1/2*c)-2/d/a^(3/2)*b/(a+b)^(3/2)*arctan(1/2*(2*tanh(1/2*d*x
+1/2*c)*(a+b)^(1/2)+2*b^(1/2))/a^(1/2))+2/d/a^(3/2)*b/(a+b)^(3/2)*arctan(1/
2*(-2*tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)+2*b^(1/2))/a^(1/2))-3/2/d*b^2/a^(5/2)
/(a+b)^(3/2)*arctan(1/2*(2*tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)+2*b^(1/2))/a^(1/
2))+3/2/d/a^(5/2)*b^2/(a+b)^(3/2)*arctan(1/2*(-2*tanh(1/2*d*x+1/2*c)*(a+b)^
(1/2)+2*b^(1/2))/a^(1/2))-1/d/a^2/(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2+ab-\left(a^2e^{(6c)}+abe^{(6c)}\right)e^{(6dx)}-\left(a^2e^{(4c)}+5abe^{(4c)}+6b^2e^{(4c)}\right)e^{(4dx)}+\left(a^2e^{(2c)}+5abe^{(2c)}+6b^2e^{(2c)}\right)e^{(2dx)}}{2\left(\left(a^4de^{(5c)}+a^3bde^{(5c)}\right)e^{(5dx)}+2\left(a^4de^{(3c)}+3a^3bde^{(3c)}+2a^2b^2de^{(3c)}\right)e^{(3dx)}+\left(a^4de^{c}+a^3bde^{c}\right)e^{(dx)}\right)}-\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a^2 + a*b - (a^2*e^(6*c) + a*b*e^(6*c))*e^(6*d*x) - (a^2*e^(4*c) + 5*
a*b*e^(4*c) + 6*b^2*e^(4*c))*e^(4*d*x) + (a^2*e^(2*c) + 5*a*b*e^(2*c) + 6*b
^2*e^(2*c))*e^(2*d*x))/((a^4*d*e^(5*c) + a^3*b*d*e^(5*c))*e^(5*d*x) + 2*(a^
4*d*e^(3*c) + 3*a^3*b*d*e^(3*c) + 2*a^2*b^2*d*e^(3*c))*e^(3*d*x) + (a^4*d*e
^c + a^3*b*d*e^c)*e^(d*x)) - 1/2*integrate(2*((4*a*b*e^(3*c) + 3*b^2*e^(3*c
))*e^(3*d*x) + (4*a*b*e^c + 3*b^2*e^c)*e^(d*x))/(a^4 + a^3*b + (a^4*e^(4*c)
 + a^3*b*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) + 3*a^3*b*e^(2*c) + 2*a^2*b^2*
e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.62117, size = 7525, normalized size = 75.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^6 + 12*(a^4 + 2*a^3*b + a^2
*b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(a^4 + 2*a^3*b + a^2*b^2)*sinh(d*x
+ c)^6 + 2*(a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^4 + 2*(a^4
+ 6*a^3*b + 11*a^2*b^2 + 6*a*b^3 + 15*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x +
c)^2)*sinh(d*x + c)^4 - 2*a^4 - 4*a^3*b - 2*a^2*b^2 + 8*(5*(a^4 + 2*a^3*b +
 a^2*b^2)*cosh(d*x + c)^3 + (a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x
 + c))*sinh(d*x + c)^3 - 2*(a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x
+ c)^2 + 2*(15*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^4 - a^4 - 6*a^3*b -
11*a^2*b^2 - 6*a*b^3 + 6*(a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x +
c)^2)*sinh(d*x + c)^2 - ((4*a^2*b + 3*a*b^2)*cosh(d*x + c)^5 + 5*(4*a^2*b +
 3*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 + (4*a^2*b + 3*a*b^2)*sinh(d*x + c)
^5 + 2*(4*a^2*b + 11*a*b^2 + 6*b^3)*cosh(d*x + c)^3 + 2*(4*a^2*b + 11*a*b^2
 + 6*b^3 + 5*(4*a^2*b + 3*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(5*(4
*a^2*b + 3*a*b^2)*cosh(d*x + c)^3 + 3*(4*a^2*b + 11*a*b^2 + 6*b^3)*cosh(d*x
 + c))*sinh(d*x + c)^2 + (4*a^2*b + 3*a*b^2)*cosh(d*x + c) + (5*(4*a^2*b +
3*a*b^2)*cosh(d*x + c)^4 + 4*a^2*b + 3*a*b^2 + 6*(4*a^2*b + 11*a*b^2 + 6*b^
3)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(-a^2 - a*b)*log((a*cosh(d*x + c)^4
+ 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cos
h(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*c
osh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(cosh(d*x + c
)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^
```

```
2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a)/(a*cosh(d*x + c
)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*c
osh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*c
osh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 4*(3*(a^4 +
 2*a^3*b + a^2*b^2)*cosh(d*x + c)^5 + 2*(a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b
^3)*cosh(d*x + c)^3 - (a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x + c))
*sinh(d*x + c))/((a^6 + 2*a^5*b + a^4*b^2)*d*cosh(d*x + c)^5 + 5*(a^6 + 2*a
^5*b + a^4*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^4 + (a^6 + 2*a^5*b + a^4*b^2)
*d*sinh(d*x + c)^5 + 2*(a^6 + 4*a^5*b + 5*a^4*b^2 + 2*a^3*b^3)*d*cosh(d*x +
 c)^3 + 2*(5*(a^6 + 2*a^5*b + a^4*b^2)*d*cosh(d*x + c)^2 + (a^6 + 4*a^5*b +
 5*a^4*b^2 + 2*a^3*b^3)*d)*sinh(d*x + c)^3 + (a^6 + 2*a^5*b + a^4*b^2)*d*co
sh(d*x + c) + 2*(5*(a^6 + 2*a^5*b + a^4*b^2)*d*cosh(d*x + c)^3 + 3*(a^6 + 4
*a^5*b + 5*a^4*b^2 + 2*a^3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (5*(a^6
+ 2*a^5*b + a^4*b^2)*d*cosh(d*x + c)^4 + 6*(a^6 + 4*a^5*b + 5*a^4*b^2 + 2*a
^3*b^3)*d*cosh(d*x + c)^2 + (a^6 + 2*a^5*b + a^4*b^2)*d)*sinh(d*x + c)), 1/
2*((a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^6 + 6*(a^4 + 2*a^3*b + a^2*b^2)*
cosh(d*x + c)*sinh(d*x + c)^5 + (a^4 + 2*a^3*b + a^2*b^2)*sinh(d*x + c)^6 +
 (a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^4 + (a^4 + 6*a^3*b +
11*a^2*b^2 + 6*a*b^3 + 15*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^2)*sinh(d
*x + c)^4 - a^4 - 2*a^3*b - a^2*b^2 + 4*(5*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d
*x + c)^3 + (a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x + c))*sinh(d*x
+ c)^3 - (a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^2 + (15*(a^4
+ 2*a^3*b + a^2*b^2)*cosh(d*x + c)^4 - a^4 - 6*a^3*b - 11*a^2*b^2 - 6*a*b^3
 + 6*(a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^
2 - ((4*a^2*b + 3*a*b^2)*cosh(d*x + c)^5 + 5*(4*a^2*b + 3*a*b^2)*cosh(d*x +
 c)*sinh(d*x + c)^4 + (4*a^2*b + 3*a*b^2)*sinh(d*x + c)^5 + 2*(4*a^2*b + 11
*a*b^2 + 6*b^3)*cosh(d*x + c)^3 + 2*(4*a^2*b + 11*a*b^2 + 6*b^3 + 5*(4*a^2*
b + 3*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(5*(4*a^2*b + 3*a*b^2)*co
sh(d*x + c)^3 + 3*(4*a^2*b + 11*a*b^2 + 6*b^3)*cosh(d*x + c))*sinh(d*x + c)
^2 + (4*a^2*b + 3*a*b^2)*cosh(d*x + c) + (5*(4*a^2*b + 3*a*b^2)*cosh(d*x +
c)^4 + 4*a^2*b + 3*a*b^2 + 6*(4*a^2*b + 11*a*b^2 + 6*b^3)*cosh(d*x + c)^2)*
sinh(d*x + c))*sqrt(a^2 + a*b)*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x
 + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (3*a + 4*b)*cosh(d*x + c) + (3*
a*cosh(d*x + c)^2 + 3*a + 4*b)*sinh(d*x + c))/sqrt(a^2 + a*b)) - ((4*a^2*b
+ 3*a*b^2)*cosh(d*x + c)^5 + 5*(4*a^2*b + 3*a*b^2)*cosh(d*x + c)*sinh(d*x +
 c)^4 + (4*a^2*b + 3*a*b^2)*sinh(d*x + c)^5 + 2*(4*a^2*b + 11*a*b^2 + 6*b^3
)*cosh(d*x + c)^3 + 2*(4*a^2*b + 11*a*b^2 + 6*b^3 + 5*(4*a^2*b + 3*a*b^2)*c
osh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(5*(4*a^2*b + 3*a*b^2)*cosh(d*x + c)^3
+ 3*(4*a^2*b + 11*a*b^2 + 6*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + (4*a^2*b
+ 3*a*b^2)*cosh(d*x + c) + (5*(4*a^2*b + 3*a*b^2)*cosh(d*x + c)^4 + 4*a^2*b
 + 3*a*b^2 + 6*(4*a^2*b + 11*a*b^2 + 6*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))
*sqrt(a^2 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))
/(a + b)) + 2*(3*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^5 + 2*(a^4 + 6*a^3
*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^3 - (a^4 + 6*a^3*b + 11*a^2*b^2 +
6*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^6 + 2*a^5*b + a^4*b^2)*d*cosh(d*
x + c)^5 + 5*(a^6 + 2*a^5*b + a^4*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^4 + (a
^6 + 2*a^5*b + a^4*b^2)*d*sinh(d*x + c)^5 + 2*(a^6 + 4*a^5*b + 5*a^4*b^2 +
2*a^3*b^3)*d*cosh(d*x + c)^3 + 2*(5*(a^6 + 2*a^5*b + a^4*b^2)*d*cosh(d*x +
c)^2 + (a^6 + 4*a^5*b + 5*a^4*b^2 + 2*a^3*b^3)*d)*sinh(d*x + c)^3 + (a^6 +
2*a^5*b + a^4*b^2)*d*cosh(d*x + c) + 2*(5*(a^6 + 2*a^5*b + a^4*b^2)*d*cosh(
d*x + c)^3 + 3*(a^6 + 4*a^5*b + 5*a^4*b^2 + 2*a^3*b^3)*d*cosh(d*x + c))*sin
h(d*x + c)^2 + (5*(a^6 + 2*a^5*b + a^4*b^2)*d*cosh(d*x + c)^4 + 6*(a^6 + 4*
a^5*b + 5*a^4*b^2 + 2*a^3*b^3)*d*cosh(d*x + c)^2 + (a^6 + 2*a^5*b + a^4*b^2
)*d)*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.86 $\int \frac{\mathrm{sech}(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=82

$$\frac{(2a+b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{3/2}d(a+b)^{3/2}}-\frac{b\sinh(c+dx)}{2ad(a+b)\left(a\sinh^2(c+dx)+a+b\right)}$$

```
[Out] ((2*a + b)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(2*a^(3/2)*(a + b)^
(3/2)*d) - (b*Sinh[c + d*x])/(2*a*(a + b)*d*(a + b + a*Sinh[c + d*x]^2))
```

Rubi [A] time = 0.0655658, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4147, 385, 205}

$$\frac{(2a+b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{3/2}d(a+b)^{3/2}}-\frac{b\sinh(c+dx)}{2ad(a+b)\left(a\sinh^2(c+dx)+a+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((2*a + b)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(2*a^(3/2)*(a + b)^
(3/2)*d) - (b*Sinh[c + d*x])/(2*a*(a + b)*d*(a + b + a*Sinh[c + d*x]^2))
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1+x^2}{\left(a+b+ax^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{b\sinh(c+dx)}{2a(a+b)d\left(a+b+a\sinh^2(c+dx)\right)} + \frac{(2a+b)\operatorname{Subst}\left(\int \frac{1}{a+b+ax^2}\,dx, x, \sinh(c+dx)\right)}{2a(a+b)d}$$

$$= \frac{(2a+b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{3/2}(a+b)^{3/2}d} - \frac{b\sinh(c+dx)}{2a(a+b)d\left(a+b+a\sinh^2(c+dx)\right)}$$

Mathematica [A] time = 0.301814, size = 124, normalized size = 1.51

$$\frac{\left(2a^2+3ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right) - \sqrt{a}b\sqrt{a+b}\sinh(c+dx) + a(2a+b)\sinh^2(c+dx)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{a^{3/2}d(a+b)^{3/2}(a\cosh(2(c+dx))+a+2b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((2*a^2 + 3*a*b + b^2)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]] - Sqrt[a
]*b*Sqrt[a + b]*Sinh[c + d*x] + a*(2*a + b)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/
Sqrt[a + b]]*Sinh[c + d*x]^2)/(a^(3/2)*(a + b)^(3/2)*d*(a + 2*b + a*Cosh[2*
(c + d*x)]))
```

Maple [B] time = 0.068, size = 332, normalized size = 4.1

$$\frac{b}{da(a+b)}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 a + b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 + 2\,(\tanh(1/2\,dx+c/2))^2 a - 2\,(\tanh(1/2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^
2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*b/a/(a+b)*tanh(1/2*d*x+1/2*c)^3-1/d/(tan
h(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*ta
nh(1/2*d*x+1/2*c)^2*b+a+b)*b/a/(a+b)*tanh(1/2*d*x+1/2*c)+1/d/(a+b)^(3/2)/a^
(1/2)*arctan(1/2*(2*tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)+2*b^(1/2))/a^(1/2))-1/d
/(a+b)^(3/2)/a^(1/2)*arctan(1/2*(-2*tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)+2*b^(1/
2))/a^(1/2))+1/2/d/a^(3/2)*b/(a+b)^(3/2)*arctan(1/2*(2*tanh(1/2*d*x+1/2*c)*
(a+b)^(1/2)+2*b^(1/2))/a^(1/2))-1/2/d/a^(3/2)*b/(a+b)^(3/2)*arctan(1/2*(-2*
tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)+2*b^(1/2))/a^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{be^{(3\,dx+3\,c)} - be^{(dx+c)}}{a^3d + a^2bd + \left(a^3de^{(4\,c)} + a^2bde^{(4\,c)}\right)e^{(4\,dx)} + 2\left(a^3de^{(2\,c)} + 3\,a^2bde^{(2\,c)} + 2\,ab^2de^{(2\,c)}\right)e^{(2\,dx)}} + 2\int \frac{(2}{2\left(a^3 + a^2b + \left(a^3e^{(4\,c)}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -(b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^3*d + a^2*b*d + (a^3*d*e^(4*c) + a^
2*b*d*e^(4*c))*e^(4*d*x) + 2*(a^3*d*e^(2*c) + 3*a^2*b*d*e^(2*c) + 2*a*b^2*d
*e^(2*c))*e^(2*d*x)) + 2*integrate(1/2*((2*a*e^(3*c) + b*e^(3*c))*e^(3*d*x)
 + (2*a*e^c + b*e^c)*e^(d*x))/(a^3 + a^2*b + (a^3*e^(4*c) + a^2*b*e^(4*c))*
e^(4*d*x) + 2*(a^3*e^(2*c) + 3*a^2*b*e^(2*c) + 2*a*b^2*e^(2*c))*e^(2*d*x)),
 x)
```

Fricas [B] time = 2.39493, size = 4533, normalized size = 55.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a^2*b + a*b^2)*cosh(d*x + c)^3 + 12*(a^2*b + a*b^2)*cosh(d*x + c)
*sinh(d*x + c)^2 + 4*(a^2*b + a*b^2)*sinh(d*x + c)^3 + ((2*a^2 + a*b)*cosh(
d*x + c)^4 + 4*(2*a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 + a*b)*
sinh(d*x + c)^4 + 2*(2*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(2*a^2 +
 a*b)*cosh(d*x + c)^2 + 2*a^2 + 5*a*b + 2*b^2)*sinh(d*x + c)^2 + 2*a^2 + a*
b + 4*((2*a^2 + a*b)*cosh(d*x + c)^3 + (2*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c
))*sinh(d*x + c))*sqrt(-a^2 - a*b)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x +
c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*
(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 -
(3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x
+ c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x +
 c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*
x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 +
2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 +
(a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - 4*(a^2*b + a*b^2)*cosh(d*x +
 c) - 4*(a^2*b + a*b^2 - 3*(a^2*b + a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c))/
((a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^4 + 4*(a^5 + 2*a^4*b + a^3*b^2)*
d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^5 + 2*a^4*b + a^3*b^2)*d*sinh(d*x + c)
^4 + 2*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^
5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a
^2*b^3)*d)*sinh(d*x + c)^2 + (a^5 + 2*a^4*b + a^3*b^2)*d + 4*((a^5 + 2*a^4*
b + a^3*b^2)*d*cosh(d*x + c)^3 + (a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*d*
cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*(a^2*b + a*b^2)*cosh(d*x + c)^3 + 6*
(a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(a^2*b + a*b^2)*sinh(d*x
+ c)^3 - ((2*a^2 + a*b)*cosh(d*x + c)^4 + 4*(2*a^2 + a*b)*cosh(d*x + c)*sin
h(d*x + c)^3 + (2*a^2 + a*b)*sinh(d*x + c)^4 + 2*(2*a^2 + 5*a*b + 2*b^2)*co
sh(d*x + c)^2 + 2*(3*(2*a^2 + a*b)*cosh(d*x + c)^2 + 2*a^2 + 5*a*b + 2*b^2)
*sinh(d*x + c)^2 + 2*a^2 + a*b + 4*((2*a^2 + a*b)*cosh(d*x + c)^3 + (2*a^2
+ 5*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + a*b)*arctan(1/2*(
a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 +
 (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*a + 4*b)*sinh(d*x + c
))/sqrt(a^2 + a*b)) - ((2*a^2 + a*b)*cosh(d*x + c)^4 + 4*(2*a^2 + a*b)*cosh
(d*x + c)*sinh(d*x + c)^3 + (2*a^2 + a*b)*sinh(d*x + c)^4 + 2*(2*a^2 + 5*a*
b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(2*a^2 + a*b)*cosh(d*x + c)^2 + 2*a^2 + 5
*a*b + 2*b^2)*sinh(d*x + c)^2 + 2*a^2 + a*b + 4*((2*a^2 + a*b)*cosh(d*x + c
)^3 + (2*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + a*b)
*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/(a + b)) - 2*(a
^2*b + a*b^2)*cosh(d*x + c) - 2*(a^2*b + a*b^2 - 3*(a^2*b + a*b^2)*cosh(d*x
 + c)^2)*sinh(d*x + c))/((a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^4 + 4*(a
^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^5 + 2*a^4*b +
```

```
a^3*b^2)*d*sinh(d*x + c)^4 + 2*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*d*co
sh(d*x + c)^2 + 2*(3*(a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 + 4
*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*d)*sinh(d*x + c)^2 + (a^5 + 2*a^4*b + a^3*b
^2)*d + 4*((a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^3 + (a^5 + 4*a^4*b + 5
*a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(sech(c + d*x)/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.87 $\int \frac{\text{sech}^2(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=74

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{b}d(a+b)^{3/2}}+\frac{\tanh(c+dx)}{2d(a+b)\left(a-b\tanh^2(c+dx)+b\right)}$$

```
[Out] ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]]/(2*Sqrt[b]*(a + b)^(3/2)*d) +
Tanh[c + d*x]/(2*(a + b)*d*(a + b - b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.0735656, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4146, 199, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{b}d(a+b)^{3/2}}+\frac{\tanh(c+dx)}{2d(a+b)\left(a-b\tanh^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]]/(2*Sqrt[b]*(a + b)^(3/2)*d) +
Tanh[c + d*x]/(2*(a + b)*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{\left(a+b-bx^2\right)^2}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh(c+dx)}{2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx,x,\tanh(c+dx)\right)}{2(a+b)d}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{b}(a+b)^{3/2}d} + \frac{\tanh(c+dx)}{2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)}$$

Mathematica [B] time = 1.00121, size = 187, normalized size = 2.53

$$\frac{\operatorname{sech}^4(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\frac{(\cosh(2c)-\sinh(2c))(a\cosh(2(c+dx))+a+2b)\tanh^{-1}\left(\frac{(\cosh(2c)-\sinh(2c))\operatorname{sech}(dx)((a+2b)\sinh(dx)-a\sin}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right.}{\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right.}{8d(a+b)\left(a+b\operatorname{sech}^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*((ArcTanh[(Sech[d*x]*(Cosh
[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b
]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])*(Cosh[2*c
] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + Sech[2*c]*Sin
h[2*d*x] - ((a + 2*b)*Tanh[2*c])/a))/(8*(a + b)*d*(a + b*Sech[c + d*x]^2)^2
)
```

Maple [B] time = 0.059, size = 260, normalized size = 3.5

$$\frac{1}{d(a+b)}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 a+b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4+2\left(\tanh(1/2\,dx+c/2)\right)^2 a-2\left(\tanh(1/2\,d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^
2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*tanh(1/2*d*x+1/2*c)^3+1/d/(tanh(1/
2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1
/2*d*x+1/2*c)^2*b+a+b)/(a+b)*tanh(1/2*d*x+1/2*c)+1/4/d/(a+b)^(3/2)/b^(1/2)*
ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1
/2))-1/4/d/(a+b)^(3/2)/b^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(
1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.34458, size = 3563, normalized size = 48.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^2*b + 4*a*b^2 + 4*(a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)^2 + 8*
(a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)*sinh(d*x + c) + 4*(a^2*b + 3*a*b^2
+ 2*b^3)*sinh(d*x + c)^2 - (a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(
d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a
^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 4*(a^2*cosh(d*x +
 c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b + b^2)*log((a^
2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)
^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a
*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 +
 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x
+ c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*cosh(
d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a +
 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 +
 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)))/((a^4
*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^4 + 4*(a^4*b + 2*a^3*b^2 + a^2*b^
3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*d*sinh(d
*x + c)^4 + 2*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*cosh(d*x + c)^2 +
 2*(3*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (a^4*b + 4*a^3*b^2
+ 5*a^2*b^3 + 2*a*b^4)*d)*sinh(d*x + c)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*d
 + 4*((a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^3 + (a^4*b + 4*a^3*b^2
+ 5*a^2*b^3 + 2*a*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*a^2*b + 2*a
*b^2 + 2*(a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)^2 + 4*(a^2*b + 3*a*b^2 + 2
*b^3)*cosh(d*x + c)*sinh(d*x + c) + 2*(a^2*b + 3*a*b^2 + 2*b^3)*sinh(d*x +
c)^2 - (a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sin
h(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 +
 a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b
)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(d*x + c
)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-
a*b - b^2)/(a*b + b^2)))/((a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^4 +
 4*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b +
 2*a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3
+ 2*a*b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*
x + c)^2 + (a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d)*sinh(d*x + c)^2 + (
a^4*b + 2*a^3*b^2 + a^2*b^3)*d + 4*((a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*
x + c)^3 + (a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*cosh(d*x + c))*sinh(
d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(sech(c + d*x)**2/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [B] time = 1.26676, size = 180, normalized size = 2.43

$$\frac{\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{2\,\sqrt{-ab-b^2}(ad+bd)} - \frac{ae^{(2\,dx+2\,c)}+2\,be^{(2\,dx+2\,c)}+a}{(a^2d+abd)\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b -
 b^2)*(a*d + b*d)) - (a*e^(2*d*x + 2*c) + 2*b*e^(2*d*x + 2*c) + a)/((a^2*d
+ a*b*d)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a
))
```

3.88 $\int \frac{\mathbf{sech}^3(c+dx)}{\left(a+b\mathbf{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=73

$$\frac{\sinh(c+dx)}{2d(a+b)\left(a\sinh^2(c+dx)+a+b\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{a}d(a+b)^{3/2}}$$

```
[Out] ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]]/(2*Sqrt[a]*(a + b)^(3/2)*d) + S
inh[c + d*x]/(2*(a + b)*d*(a + b + a*Sinh[c + d*x]^2))
```

Rubi [A] time = 0.070761, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4147, 199, 205}

$$\frac{\sinh(c+dx)}{2d(a+b)\left(a\sinh^2(c+dx)+a+b\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{a}d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]]/(2*Sqrt[a]*(a + b)^(3/2)*d) + S
inh[c + d*x]/(2*(a + b)*d*(a + b + a*Sinh[c + d*x]^2))
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{\left(a+b+ax^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\sinh(c+dx)}{2(a+b)d\left(a+b+a\sinh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{a+b+ax^2}\,dx, x, \sinh(c+dx)\right)}{2(a+b)d}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{a}(a+b)^{3/2}d} + \frac{\sinh(c+dx)}{2(a+b)d\left(a+b+a\sinh^2(c+dx)\right)}$$

Mathematica [A] time = 0.216645, size = 108, normalized size = 1.48

$$\frac{\operatorname{sech}^4(c+dx)(a\cosh(2c+2dx)+a+2b)^2\left(\frac{\sinh(c+dx)}{(a+b)\left(a\sinh^2(c+dx)+a+b\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}(a+b)^{3/2}}\right)}{8d\left(a+b\operatorname{sech}^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*c + 2*d*x])^2*Sech[c + d*x]^4*(ArcTan[(Sqrt[a]*Sinh[c
+ d*x])/Sqrt[a + b]]/(Sqrt[a]*(a + b)^(3/2)) + Sinh[c + d*x]/((a + b)*(a +
b + a*Sinh[c + d*x]^2))))/(8*d*(a + b*Sech[c + d*x]^2)^2)
```

Maple [B] time = 0.059, size = 241, normalized size = 3.3

$$-\frac{1}{d(a+b)}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 a+b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4+2\left(\tanh(1/2\,dx+c/2)\right)^2 a-2\left(\tanh(1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)
^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*tanh(1/2*d*x+1/2*c)^3+1/d/(tanh(1
/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(
1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*tanh(1/2*d*x+1/2*c)+1/2/d/(a+b)^(3/2)/a^(1/2)
*arctan(1/2*(2*tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)+2*b^(1/2))/a^(1/2))+1/2/d/(a
+b)^(3/2)/a^(1/2)*arctan(1/2*(2*tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)-2*b^(1/2))/
a^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{e^{(3\,dx+3\,c)}-e^{(dx+c)}}{a^2d+abd+\left(a^2de^{(4\,c)}+abde^{(4\,c)}\right)e^{(4\,dx)}+2\left(a^2de^{(2\,c)}+3\,abde^{(2\,c)}+2\,b^2de^{(2\,c)}\right)e^{(2\,dx)}}+8\int\frac{}{8\left(a^2+ab+\left(a^2e^{(4\,c)}+abe^{(\cdot}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] (e^(3*d*x + 3*c) - e^(d*x + c))/(a^2*d + a*b*d + (a^2*d*e^(4*c) + a*b*d*e^(
4*c))*e^(4*d*x) + 2*(a^2*d*e^(2*c) + 3*a*b*d*e^(2*c) + 2*b^2*d*e^(2*c))*e^(
2*d*x)) + 8*integrate(1/8*(e^(3*d*x + 3*c) + e^(d*x + c))/(a^2 + a*b + (a^2
*e^(4*c) + a*b*e^(4*c))*e^(4*d*x) + 2*(a^2*e^(2*c) + 3*a*b*e^(2*c) + 2*b^2*
e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.42487, size = 3996, normalized size = 54.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^2 + a*b)*cosh(d*x + c)^3 + 12*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x
 + c)^2 + 4*(a^2 + a*b)*sinh(d*x + c)^3 - (a*cosh(d*x + c)^4 + 4*a*cosh(d*x
 + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2
*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (
a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(-a^2 - a*b)*log((a*cosh(d*x
 + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a +
2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2
+ 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh
(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d
*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a)/(a*cosh
(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a
+ 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2
+ 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - 4*(
a^2 + a*b)*cosh(d*x + c) + 4*(3*(a^2 + a*b)*cosh(d*x + c)^2 - a^2 - a*b)*si
nh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 4*(a^4 + 2*a^3*
b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^3*b + a^2*b^2)*d*
sinh(d*x + c)^4 + 2*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*cosh(d*x + c)^2
 + 2*(3*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (a^4 + 4*a^3*b + 5*a^
2*b^2 + 2*a*b^3)*d)*sinh(d*x + c)^2 + (a^4 + 2*a^3*b + a^2*b^2)*d + 4*((a^4
 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^3 + (a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*
b^3)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*(a^2 + a*b)*cosh(d*x + c)^3 +
6*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(a^2 + a*b)*sinh(d*x + c)^3
 + (a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)
^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d
*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) +
 a)*sqrt(a^2 + a*b)*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(
d*x + c)^2 + a*sinh(d*x + c)^3 + (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x
+ c)^2 + 3*a + 4*b)*sinh(d*x + c))/sqrt(a^2 + a*b)) + (a*cosh(d*x + c)^4 +
4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*
x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*
x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(a^2 + a*b)*arct
an(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/(a + b)) - 2*(a^2 +
a*b)*cosh(d*x + c) + 2*(3*(a^2 + a*b)*cosh(d*x + c)^2 - a^2 - a*b)*sinh(d*x
 + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 4*(a^4 + 2*a^3*b + a^
2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^3*b + a^2*b^2)*d*sinh(d
*x + c)^4 + 2*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*cosh(d*x + c)^2 + 2*(
3*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (a^4 + 4*a^3*b + 5*a^2*b^2
+ 2*a*b^3)*d)*sinh(d*x + c)^2 + (a^4 + 2*a^3*b + a^2*b^2)*d + 4*((a^4 + 2*a
^3*b + a^2*b^2)*d*cosh(d*x + c)^3 + (a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d
*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(sech(c + d*x)**3/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.89 $\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2} dx$

Optimal. Leaf size=83

$$\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2b^{3/2}d(a+b)^{3/2}}-\frac{a\tanh(c+dx)}{2bd(a+b)\left(a-b\tanh^2(c+dx)+b\right)}$$

```
[Out] ((a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(2*b^(3/2)*(a + b)
^(3/2)*d) - (a*Tanh[c + d*x])/(2*b*(a + b)*d*(a + b - b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.0904603, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4146, 385, 208}

$$\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2b^{3/2}d(a+b)^{3/2}}-\frac{a\tanh(c+dx)}{2bd(a+b)\left(a-b\tanh^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```


```
[Out] ((a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(2*b^(3/2)*(a + b)
^(3/2)*d) - (a*Tanh[c + d*x])/(2*b*(a + b)*d*(a + b - b*Tanh[c + d*x]^2))
```


Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{a\tanh(c+dx)}{2b(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{(a+2b)\operatorname{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \tanh(c+dx)\right)}{2b(a+b)d}$$

$$= \frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2b^{3/2}(a+b)^{3/2}d} - \frac{a\tanh(c+dx)}{2b(a+b)d\left(a+b-b\tanh^2(c+dx)\right)}$$

Mathematica [A] time = 0.207256, size = 88, normalized size = 1.06

$$4\left(\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8b^{3/2}d(a+b)^{3/2}} - \frac{a\sinh(2(c+dx))}{8bd(a+b)(a\cosh(2(c+dx))+a+2b)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] 4*(((a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*b^(3/2)*(a +
 b)^(3/2)*d) - (a*Sinh[2*(c + d*x)])/(8*b*(a + b)*d*(a + 2*b + a*Cosh[2*(c
+ d*x)])))
```

Maple [B] time = 0.058, size = 380, normalized size = 4.6

$$-\frac{a}{bd(a+b)}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 a + b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 + 2\left(\tanh(1/2\,dx+c/2)\right)^2 a - 2\left(\tanh(1/\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)
^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*a/b/(a+b)*tanh(1/2*d*x+1/2*c)^3-1/d/(ta
nh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*t
anh(1/2*d*x+1/2*c)^2*b+a+b)*a/b/(a+b)*tanh(1/2*d*x+1/2*c)+1/4/d/(a+b)^(3/2)
/b^(3/2)*a*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/
2)+(a+b)^(1/2))-1/4/d/(a+b)^(3/2)/b^(3/2)*a*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/
2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2))+1/2/d/(a+b)^(3/2)/b^(1/2)
*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(
1/2))-1/2/d/(a+b)^(3/2)/b^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tan
h(1/2*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.32172, size = 3765, normalized size = 45.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a^2*b + 4*a*b^2 + 4*(a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)^2 + 8*(
a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)*sinh(d*x + c) + 4*(a^2*b + 3*a*b^2 +
 2*b^3)*sinh(d*x + c)^2 + ((a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b)*
cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b)*sinh(d*x + c)^4 + 2*(a^2 + 4*
a*b + 4*b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b)*cosh(d*x + c)^2 + a^2 + 4
*a*b + 4*b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + 4*((a^2 + 2*a*b)*cosh(d*x + c
)^3 + (a^2 + 4*a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b + b^2)*l
og((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*
x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2
 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 +
(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(d*x + c)^2 + 2*a*cos
h(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a
*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 +
2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x +
c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)))
/((a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 4*(a^3*b^2 + 2*a^2*b^3
+ a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*
sinh(d*x + c)^4 + 2*(a^3*b^2 + 4*a^2*b^3 + 5*a*b^4 + 2*b^5)*d*cosh(d*x + c)
^2 + 2*(3*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (a^3*b^2 + 4*a^
2*b^3 + 5*a*b^4 + 2*b^5)*d)*sinh(d*x + c)^2 + (a^3*b^2 + 2*a^2*b^3 + a*b^4)
*d + 4*((a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^3 + (a^3*b^2 + 4*a^2*
b^3 + 5*a*b^4 + 2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*a^2*b + 2*a*
b^2 + 2*(a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)^2 + 4*(a^2*b + 3*a*b^2 + 2*
b^3)*cosh(d*x + c)*sinh(d*x + c) + 2*(a^2*b + 3*a*b^2 + 2*b^3)*sinh(d*x + c
)^2 + ((a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b)*cosh(d*x + c)*sinh(d
*x + c)^3 + (a^2 + 2*a*b)*sinh(d*x + c)^4 + 2*(a^2 + 4*a*b + 4*b^2)*cosh(d*
x + c)^2 + 2*(3*(a^2 + 2*a*b)*cosh(d*x + c)^2 + a^2 + 4*a*b + 4*b^2)*sinh(d
*x + c)^2 + a^2 + 2*a*b + 4*((a^2 + 2*a*b)*cosh(d*x + c)^3 + (a^2 + 4*a*b +
 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(d
*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*
sqrt(-a*b - b^2)/(a*b + b^2)))/((a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*cosh(d*x +
c)^4 + 4*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a
^3*b^2 + 2*a^2*b^3 + a*b^4)*d*sinh(d*x + c)^4 + 2*(a^3*b^2 + 4*a^2*b^3 + 5*
a*b^4 + 2*b^5)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*cos
h(d*x + c)^2 + (a^3*b^2 + 4*a^2*b^3 + 5*a*b^4 + 2*b^5)*d)*sinh(d*x + c)^2 +
 (a^3*b^2 + 2*a^2*b^3 + a*b^4)*d + 4*((a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*cosh(
d*x + c)^3 + (a^3*b^2 + 4*a^2*b^3 + 5*a*b^4 + 2*b^5)*d*cosh(d*x + c))*sinh(
d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(sech(c + d*x)**4/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [A] time = 1.17375, size = 189, normalized size = 2.28

$$\frac{(a+2\,b)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{2\left(abd+b^2d\right)\sqrt{-ab-b^2}}+\frac{ae^{(2\,dx+2\,c)}+2\,be^{(2\,dx+2\,c)}+a}{\left(abd+b^2d\right)\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a + 2*b)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/((
a*b*d + b^2*d)*sqrt(-a*b - b^2)) + (a*e^(2*d*x + 2*c) + 2*b*e^(2*d*x + 2*c)
 + a)/((a*b*d + b^2*d)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*
d*x + 2*c) + a))
```

3.90 $\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2} dx$

Optimal. Leaf size=101

$$-\frac{\sqrt{a}(2a+3b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2b^2d(a+b)^{3/2}}-\frac{a\sinh(c+dx)}{2bd(a+b)\left(a\sinh^2(c+dx)+a+b\right)}+\frac{\tan^{-1}(\sinh(c+dx))}{b^2d}$$

```
[Out] ArcTan[Sinh[c + d*x]]/(b^2*d) - (Sqrt[a]*(2*a + 3*b)*ArcTan[(Sqrt[a]*Sinh[c
 + d*x])/Sqrt[a + b]])/(2*b^2*(a + b)^(3/2)*d) - (a*Sinh[c + d*x])/(2*b*(a
+ b)*d*(a + b + a*Sinh[c + d*x]^2))
```

Rubi [A] time = 0.116827, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4147, 414, 522, 203, 205}

$$-\frac{\sqrt{a}(2a+3b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2b^2d(a+b)^{3/2}}-\frac{a\sinh(c+dx)}{2bd(a+b)\left(a\sinh^2(c+dx)+a+b\right)}+\frac{\tan^{-1}(\sinh(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^5/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ArcTan[Sinh[c + d*x]]/(b^2*d) - (Sqrt[a]*(2*a + 3*b)*ArcTan[(Sqrt[a]*Sinh[c
 + d*x])/Sqrt[a + b]])/(2*b^2*(a + b)^(3/2)*d) - (a*Sinh[c + d*x])/(2*b*(a
+ b)*d*(a + b + a*Sinh[c + d*x]^2))
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)\left(a+b+ax^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{a\sinh(c+dx)}{2b(a+b)d\left(a+b+a\sinh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{a+2b-ax^2}{(1+x^2)\left(a+b+ax^2\right)}\,dx, x, \sinh(c+dx)\right)}{2b(a+b)d}$$

$$= -\frac{a\sinh(c+dx)}{2b(a+b)d\left(a+b+a\sinh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{b^2d} - \frac{(a(2a+$$

$$= \frac{\tan^{-1}(\sinh(c+dx))}{b^2d} - \frac{\sqrt{a}(2a+3b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2b^2(a+b)^{3/2}d} - \frac{a\sinh(c+dx)}{2b(a+b)d\left(a+b+a\sinh^2\right.}$$

Mathematica [B] time = 2.46963, size = 282, normalized size = 2.79

$$\operatorname{sech}^3(c+dx)(a\cosh(2(c+dx))+a+2b)\left(-2ab\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\tanh(c+dx)+\sqrt{a}(2a+3b)\cosh(c)\mathrm{se}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^5/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^3*(Sqrt[a]*(2*a + 3*b)*ArcTa
n[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Sinh[c]
))/Sqrt[a]]*Cosh[c]*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x] - (a + 2*
b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]*(-4*(a + b)^(3/2)*ArcTan[Tanh[(c + d
*x)/2]]*Sqrt[(Cosh[c] - Sinh[c])^2] + Sqrt[a]*(2*a + 3*b)*ArcTan[(Sqrt[a +
b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Sinh[c]))/Sqrt[a]]*
Sinh[c]) - 2*a*b*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[c + d*x]))/(8
*b^2*(a + b)^(3/2)*d*(a + b*Sech[c + d*x]^2)^2*Sqrt[(Cosh[c] - Sinh[c])^2])
```

Maple [B] time = 0.064, size = 361, normalized size = 3.6

$$\frac{a}{bd\,(a+b)}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4a+b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4+2\,(\tanh(1/2\,dx+c/2))^2a-2\,(\tanh(1/2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^
2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*a/b/(a+b)*tanh(1/2*d*x+1/2*c)^3-1/d/(tan
h(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*ta
nh(1/2*d*x+1/2*c)^2*b+a+b)*a/b/(a+b)*tanh(1/2*d*x+1/2*c)-1/d*a^(3/2)/b^2/(a
+b)^(3/2)*arctan(1/2*(2*tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)+2*b^(1/2))/a^(1/2))
-1/d*a^(3/2)/b^2/(a+b)^(3/2)*arctan(1/2*(2*tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)-
2*b^(1/2))/a^(1/2))-3/2/d*a^(1/2)/b/(a+b)^(3/2)*arctan(1/2*(2*tanh(1/2*d*x+
1/2*c)*(a+b)^(1/2)+2*b^(1/2))/a^(1/2))-3/2/d*a^(1/2)/b/(a+b)^(3/2)*arctan(1
/2*(2*tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)-2*b^(1/2))/a^(1/2))+2/d/b^2*arctan(ta
nh(1/2*d*x+1/2*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{ae^{(3\,dx+3\,c)} - ae^{(dx+c)}}{a^2bd + ab^2d + \left(a^2bde^{(4\,c)} + ab^2de^{(4\,c)}\right)e^{(4\,dx)} + 2\left(a^2bde^{(2\,c)} + 3\,ab^2de^{(2\,c)} + 2\,b^3de^{(2\,c)}\right)e^{(2\,dx)}} + \frac{2\arctan\left(e^{(dx+c)}\right)}{b^2d} - 32\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -(a*e^(3*d*x + 3*c) - a*e^(d*x + c))/(a^2*b*d + a*b^2*d + (a^2*b*d*e^(4*c)
+ a*b^2*d*e^(4*c))*e^(4*d*x) + 2*(a^2*b*d*e^(2*c) + 3*a*b^2*d*e^(2*c) + 2*b
^3*d*e^(2*c))*e^(2*d*x)) + 2*arctan(e^(d*x + c))/(b^2*d) - 32*integrate(1/3
2*((2*a^2*e^(3*c) + 3*a*b*e^(3*c))*e^(3*d*x) + (2*a^2*e^c + 3*a*b*e^c)*e^(d
*x))/(a^2*b^2 + a*b^3 + (a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*e^(4*d*x) + 2*(a^
2*b^2*e^(2*c) + 3*a*b^3*e^(2*c) + 2*b^4*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.67491, size = 5195, normalized size = 51.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a*b*cosh(d*x + c)^3 + 12*a*b*cosh(d*x + c)*sinh(d*x + c)^2 + 4*a*b
*sinh(d*x + c)^3 - 4*a*b*cosh(d*x + c) - ((2*a^2 + 3*a*b)*cosh(d*x + c)^4 +
 4*(2*a^2 + 3*a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 + 3*a*b)*sinh(d*x
 + c)^4 + 2*(2*a^2 + 7*a*b + 6*b^2)*cosh(d*x + c)^2 + 2*(3*(2*a^2 + 3*a*b)*
cosh(d*x + c)^2 + 2*a^2 + 7*a*b + 6*b^2)*sinh(d*x + c)^2 + 2*a^2 + 3*a*b +
4*((2*a^2 + 3*a*b)*cosh(d*x + c)^3 + (2*a^2 + 7*a*b + 6*b^2)*cosh(d*x + c))
*sinh(d*x + c))*sqrt(-a/(a + b))*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)
*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3
*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3
*a + 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^3 + 3*(a
+ b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 - (a + b)*cosh
(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*x + c))*sqrt(-a/(a +
 b)) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d
*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)
*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x
 + c) + a)) - 8*((a^2 + a*b)*cosh(d*x + c)^4 + 4*(a^2 + a*b)*cosh(d*x + c)*
sinh(d*x + c)^3 + (a^2 + a*b)*sinh(d*x + c)^4 + 2*(a^2 + 3*a*b + 2*b^2)*cos
h(d*x + c)^2 + 2*(3*(a^2 + a*b)*cosh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sinh
(d*x + c)^2 + a^2 + a*b + 4*((a^2 + a*b)*cosh(d*x + c)^3 + (a^2 + 3*a*b + 2
```

```
*b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) +
 4*(3*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c))/((a^2*b^2 + a*b^3)*d*cosh(d
*x + c)^4 + 4*(a^2*b^2 + a*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b^2
+ a*b^3)*d*sinh(d*x + c)^4 + 2*(a^2*b^2 + 3*a*b^3 + 2*b^4)*d*cosh(d*x + c)^
2 + 2*(3*(a^2*b^2 + a*b^3)*d*cosh(d*x + c)^2 + (a^2*b^2 + 3*a*b^3 + 2*b^4)*
d)*sinh(d*x + c)^2 + (a^2*b^2 + a*b^3)*d + 4*((a^2*b^2 + a*b^3)*d*cosh(d*x
+ c)^3 + (a^2*b^2 + 3*a*b^3 + 2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*
(2*a*b*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)*sinh(d*x + c)^2 + 2*a*b*sinh(d
*x + c)^3 - 2*a*b*cosh(d*x + c) + ((2*a^2 + 3*a*b)*cosh(d*x + c)^4 + 4*(2*a
^2 + 3*a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 + 3*a*b)*sinh(d*x + c)^4
 + 2*(2*a^2 + 7*a*b + 6*b^2)*cosh(d*x + c)^2 + 2*(3*(2*a^2 + 3*a*b)*cosh(d*
x + c)^2 + 2*a^2 + 7*a*b + 6*b^2)*sinh(d*x + c)^2 + 2*a^2 + 3*a*b + 4*((2*a
^2 + 3*a*b)*cosh(d*x + c)^3 + (2*a^2 + 7*a*b + 6*b^2)*cosh(d*x + c))*sinh(d
*x + c))*sqrt(a/(a + b))*arctan(1/2*sqrt(a/(a + b))*(cosh(d*x + c) + sinh(d
*x + c))) + ((2*a^2 + 3*a*b)*cosh(d*x + c)^4 + 4*(2*a^2 + 3*a*b)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (2*a^2 + 3*a*b)*sinh(d*x + c)^4 + 2*(2*a^2 + 7*a*b +
6*b^2)*cosh(d*x + c)^2 + 2*(3*(2*a^2 + 3*a*b)*cosh(d*x + c)^2 + 2*a^2 + 7*a
*b + 6*b^2)*sinh(d*x + c)^2 + 2*a^2 + 3*a*b + 4*((2*a^2 + 3*a*b)*cosh(d*x +
 c)^3 + (2*a^2 + 7*a*b + 6*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a/(a + b
))*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*si
nh(d*x + c)^3 + (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*a + 4*
b)*sinh(d*x + c))*sqrt(a/(a + b))/a) - 4*((a^2 + a*b)*cosh(d*x + c)^4 + 4*(
a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + a*b)*sinh(d*x + c)^4 + 2*
(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + a*b)*cosh(d*x + c)^2 +
a^2 + 3*a*b + 2*b^2)*sinh(d*x + c)^2 + a^2 + a*b + 4*((a^2 + a*b)*cosh(d*x
+ c)^3 + (a^2 + 3*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*
x + c) + sinh(d*x + c)) + 2*(3*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c))/((
a^2*b^2 + a*b^3)*d*cosh(d*x + c)^4 + 4*(a^2*b^2 + a*b^3)*d*cosh(d*x + c)*si
nh(d*x + c)^3 + (a^2*b^2 + a*b^3)*d*sinh(d*x + c)^4 + 2*(a^2*b^2 + 3*a*b^3
+ 2*b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^2*b^2 + a*b^3)*d*cosh(d*x + c)^2 + (a^
2*b^2 + 3*a*b^3 + 2*b^4)*d)*sinh(d*x + c)^2 + (a^2*b^2 + a*b^3)*d + 4*((a^2
*b^2 + a*b^3)*d*cosh(d*x + c)^3 + (a^2*b^2 + 3*a*b^3 + 2*b^4)*d*cosh(d*x +
c))*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**5/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(sech(c + d*x)**5/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.91 $\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=101

$$\frac{a^2\tanh(c+dx)}{2b^2d(a+b)\left(a-b\tanh^2(c+dx)+b\right)}-\frac{a(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2b^{5/2}d(a+b)^{3/2}}+\frac{\tanh(c+dx)}{b^2d}$$

```
[Out] -(a*(3*a + 4*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(2*b^(5/2)*(a
 + b)^(3/2)*d) + Tanh[c + d*x]/(b^2*d) + (a^2*Tanh[c + d*x])/(2*b^2*(a + b)
*d*(a + b - b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.141507, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4146, 390, 385, 208}

$$\frac{a^2\tanh(c+dx)}{2b^2d(a+b)\left(a-b\tanh^2(c+dx)+b\right)}-\frac{a(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2b^{5/2}d(a+b)^{3/2}}+\frac{\tanh(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^6/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -(a*(3*a + 4*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(2*b^(5/2)*(a
 + b)^(3/2)*d) + Tanh[c + d*x]/(b^2*d) + (a^2*Tanh[c + d*x])/(2*b^2*(a + b)
*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\text{sech}^6(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{b^2} - \frac{a(a+2b)-2abx^2}{b^2\left(a+b-bx^2\right)^2}\right) dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh(c+dx)}{b^2d} - \frac{\text{Subst}\left(\int \frac{a(a+2b)-2abx^2}{\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{b^2d}$$

$$= \frac{\tanh(c+dx)}{b^2d} + \frac{a^2\tanh(c+dx)}{2b^2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} - \frac{(a(3a+4b))\,\text{Subst}\left(\int \frac{1}{a+b-bx^2}\right.}{2b^2(a+b}$$

$$= -\frac{a(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2b^{5/2}(a+b)^{3/2}d} + \frac{\tanh(c+dx)}{b^2d} + \frac{a^2\tanh(c+dx)}{2b^2(a+b)d\left(a+b-b\tanh^2(c+\right.}$$

Mathematica [B] time = 3.78992, size = 229, normalized size = 2.27

$$\frac{\text{sech}^4(c+dx)(a\cosh(2(c+dx))+a+2b)\left(2\text{sech}(c)\sinh(dx)\text{sech}(c+dx)(a\cosh(2(c+dx))+a+2b)+\frac{a(a\text{sech}(2c)\sin}{\cdots}\right.}{8b^2d\left(a+b\text{sech}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^6/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*(-((a*(3*a + 4*b)*ArcTanh[
(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]
))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c +
d*x)])*(Cosh[2*c] - Sinh[2*c]))/((a + b)^(3/2)*Sqrt[b*(Cosh[c] - Sinh[c])^4
])) + 2*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c]*Sech[c + d*x]*Sinh[d*x] + (
a*(a*Sech[2*c]*Sinh[2*d*x] - (a + 2*b)*Tanh[2*c]))/(a + b)))/(8*b^2*d*(a +
b*Sech[c + d*x]^2)^2)
```

Maple [B] time = 0.063, size = 413, normalized size = 4.1

$$\frac{a^2}{db^2(a+b)}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 a + b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 + 2\left(\tanh(1/2\,dx+c/2)\right)^2 a - 2\left(\tanh(1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d*a^2/b^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x
+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*tanh(1/2*d*x+1/2*c)^3+1/d*
a^2/b^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2
*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*tanh(1/2*d*x+1/2*c)-3/4/d*a^2/
```

```
b^(5/2)/(a+b)^(3/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2
*c)*b^(1/2)+(a+b)^(1/2))+3/4/d*a^2/b^(5/2)/(a+b)^(3/2)*ln((a+b)^(1/2)*tanh(
1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-1/d/(a+b)^(3/2)
/b^(3/2)*a*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/
2)+(a+b)^(1/2))+1/d*a/b^(3/2)/(a+b)^(3/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c
)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+2/d/b^2*tanh(1/2*d*x+1/2*c)/
(tanh(1/2*d*x+1/2*c)^2+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 2.49778, size = 6961, normalized size = 68.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [-1/4*(4*(3*a^3*b + 7*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^4 + 16*(3*a^3*b + 7*
a^2*b^2 + 4*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + 4*(3*a^3*b + 7*a^2*b^2 +
 4*a*b^3)*sinh(d*x + c)^4 + 12*a^3*b + 20*a^2*b^2 + 8*a*b^3 + 8*(3*a^3*b +
10*a^2*b^2 + 11*a*b^3 + 4*b^4)*cosh(d*x + c)^2 + 8*(3*a^3*b + 10*a^2*b^2 +
11*a*b^3 + 4*b^4 + 3*(3*a^3*b + 7*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^2)*sinh(
d*x + c)^2 - ((3*a^3 + 4*a^2*b)*cosh(d*x + c)^6 + 6*(3*a^3 + 4*a^2*b)*cosh(
d*x + c)*sinh(d*x + c)^5 + (3*a^3 + 4*a^2*b)*sinh(d*x + c)^6 + (9*a^3 + 24*
a^2*b + 16*a*b^2)*cosh(d*x + c)^4 + (9*a^3 + 24*a^2*b + 16*a*b^2 + 15*(3*a^
3 + 4*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(3*a^3 + 4*a^2*b)*cosh
(d*x + c)^3 + (9*a^3 + 24*a^2*b + 16*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3
+ 3*a^3 + 4*a^2*b + (9*a^3 + 24*a^2*b + 16*a*b^2)*cosh(d*x + c)^2 + (15*(3*
a^3 + 4*a^2*b)*cosh(d*x + c)^4 + 9*a^3 + 24*a^2*b + 16*a*b^2 + 6*(9*a^3 + 2
4*a^2*b + 16*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(3*a^3 + 4*a^2*
b)*cosh(d*x + c)^5 + 2*(9*a^3 + 24*a^2*b + 16*a*b^2)*cosh(d*x + c)^3 + (9*a
^3 + 24*a^2*b + 16*a*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b + b^2)*log
((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x
+ c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 +
 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a
^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^2 + 2*a*cosh(
d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*c
osh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*
(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)
^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) +
16*((3*a^3*b + 7*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^3 + (3*a^3*b + 10*a^2*b^2
 + 11*a*b^3 + 4*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^3*b^3 + 2*a^2*b^4 +
a*b^5)*d*cosh(d*x + c)^6 + 6*(a^3*b^3 + 2*a^2*b^4 + a*b^5)*d*cosh(d*x + c)*
sinh(d*x + c)^5 + (a^3*b^3 + 2*a^2*b^4 + a*b^5)*d*sinh(d*x + c)^6 + (3*a^3*
b^3 + 10*a^2*b^4 + 11*a*b^5 + 4*b^6)*d*cosh(d*x + c)^4 + (15*(a^3*b^3 + 2*a
^2*b^4 + a*b^5)*d*cosh(d*x + c)^2 + (3*a^3*b^3 + 10*a^2*b^4 + 11*a*b^5 + 4*
```

```
b^6)*d)*sinh(d*x + c)^4 + (3*a^3*b^3 + 10*a^2*b^4 + 11*a*b^5 + 4*b^6)*d*cos
h(d*x + c)^2 + 4*(5*(a^3*b^3 + 2*a^2*b^4 + a*b^5)*d*cosh(d*x + c)^3 + (3*a^
3*b^3 + 10*a^2*b^4 + 11*a*b^5 + 4*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (
15*(a^3*b^3 + 2*a^2*b^4 + a*b^5)*d*cosh(d*x + c)^4 + 6*(3*a^3*b^3 + 10*a^2*
b^4 + 11*a*b^5 + 4*b^6)*d*cosh(d*x + c)^2 + (3*a^3*b^3 + 10*a^2*b^4 + 11*a*
b^5 + 4*b^6)*d)*sinh(d*x + c)^2 + (a^3*b^3 + 2*a^2*b^4 + a*b^5)*d + 2*(3*(a
^3*b^3 + 2*a^2*b^4 + a*b^5)*d*cosh(d*x + c)^5 + 2*(3*a^3*b^3 + 10*a^2*b^4 +
 11*a*b^5 + 4*b^6)*d*cosh(d*x + c)^3 + (3*a^3*b^3 + 10*a^2*b^4 + 11*a*b^5 +
 4*b^6)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*(3*a^3*b + 7*a^2*b^2 + 4*a
*b^3)*cosh(d*x + c)^4 + 8*(3*a^3*b + 7*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)*sin
h(d*x + c)^3 + 2*(3*a^3*b + 7*a^2*b^2 + 4*a*b^3)*sinh(d*x + c)^4 + 6*a^3*b
+ 10*a^2*b^2 + 4*a*b^3 + 4*(3*a^3*b + 10*a^2*b^2 + 11*a*b^3 + 4*b^4)*cosh(d
*x + c)^2 + 4*(3*a^3*b + 10*a^2*b^2 + 11*a*b^3 + 4*b^4 + 3*(3*a^3*b + 7*a^2
*b^2 + 4*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((3*a^3 + 4*a^2*b)*cosh(
d*x + c)^6 + 6*(3*a^3 + 4*a^2*b)*cosh(d*x + c)*sinh(d*x + c)^5 + (3*a^3 + 4
*a^2*b)*sinh(d*x + c)^6 + (9*a^3 + 24*a^2*b + 16*a*b^2)*cosh(d*x + c)^4 + (
9*a^3 + 24*a^2*b + 16*a*b^2 + 15*(3*a^3 + 4*a^2*b)*cosh(d*x + c)^2)*sinh(d*
x + c)^4 + 4*(5*(3*a^3 + 4*a^2*b)*cosh(d*x + c)^3 + (9*a^3 + 24*a^2*b + 16*
a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*a^3 + 4*a^2*b + (9*a^3 + 24*a^2*b
 + 16*a*b^2)*cosh(d*x + c)^2 + (15*(3*a^3 + 4*a^2*b)*cosh(d*x + c)^4 + 9*a^
3 + 24*a^2*b + 16*a*b^2 + 6*(9*a^3 + 24*a^2*b + 16*a*b^2)*cosh(d*x + c)^2)*
sinh(d*x + c)^2 + 2*(3*(3*a^3 + 4*a^2*b)*cosh(d*x + c)^5 + 2*(9*a^3 + 24*a^
2*b + 16*a*b^2)*cosh(d*x + c)^3 + (9*a^3 + 24*a^2*b + 16*a*b^2)*cosh(d*x +
c))*sinh(d*x + c))*sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cos
h(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-a*b - b^2)/(a
*b + b^2)) + 8*((3*a^3*b + 7*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^3 + (3*a^3*b
+ 10*a^2*b^2 + 11*a*b^3 + 4*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^3*b^3 +
2*a^2*b^4 + a*b^5)*d*cosh(d*x + c)^6 + 6*(a^3*b^3 + 2*a^2*b^4 + a*b^5)*d*co
sh(d*x + c)*sinh(d*x + c)^5 + (a^3*b^3 + 2*a^2*b^4 + a*b^5)*d*sinh(d*x + c)
^6 + (3*a^3*b^3 + 10*a^2*b^4 + 11*a*b^5 + 4*b^6)*d*cosh(d*x + c)^4 + (15*(a
^3*b^3 + 2*a^2*b^4 + a*b^5)*d*cosh(d*x + c)^2 + (3*a^3*b^3 + 10*a^2*b^4 + 1
1*a*b^5 + 4*b^6)*d)*sinh(d*x + c)^4 + (3*a^3*b^3 + 10*a^2*b^4 + 11*a*b^5 +
4*b^6)*d*cosh(d*x + c)^2 + 4*(5*(a^3*b^3 + 2*a^2*b^4 + a*b^5)*d*cosh(d*x +
c)^3 + (3*a^3*b^3 + 10*a^2*b^4 + 11*a*b^5 + 4*b^6)*d*cosh(d*x + c))*sinh(d*
x + c)^3 + (15*(a^3*b^3 + 2*a^2*b^4 + a*b^5)*d*cosh(d*x + c)^4 + 6*(3*a^3*b
^3 + 10*a^2*b^4 + 11*a*b^5 + 4*b^6)*d*cosh(d*x + c)^2 + (3*a^3*b^3 + 10*a^2
*b^4 + 11*a*b^5 + 4*b^6)*d)*sinh(d*x + c)^2 + (a^3*b^3 + 2*a^2*b^4 + a*b^5)
*d + 2*(3*(a^3*b^3 + 2*a^2*b^4 + a*b^5)*d*cosh(d*x + c)^5 + 2*(3*a^3*b^3 +
10*a^2*b^4 + 11*a*b^5 + 4*b^6)*d*cosh(d*x + c)^3 + (3*a^3*b^3 + 10*a^2*b^4
+ 11*a*b^5 + 4*b^6)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**6/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(sech(c + d*x)**6/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [B] time = 1.2742, size = 306, normalized size = 3.03

$$-\frac{\left(3\,a^2+4\,ab\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{2\,\left(ab^2d+b^3d\right)\sqrt{-ab-b^2}}-\frac{3\,a^2e^{(4\,dx+4\,c)}+4\,abe^{(4\,dx+4\,c)}+6\,a^2e^{(2\,dx+2\,c)}+14\,abe^{(2\,dx+2\,c)}+8\,b^2e^{(2\,dx+2\,c)}+3\,a}{\left(ab^2d+b^3d\right)\left(ae^{(6\,dx+6\,c)}+3\,ae^{(4\,dx+4\,c)}+4\,be^{(4\,dx+4\,c)}+3\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*a^2 + 4*a*b)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b
^2))/((a*b^2*d + b^3*d)*sqrt(-a*b - b^2)) - (3*a^2*e^(4*d*x + 4*c) + 4*a*b*
e^(4*d*x + 4*c) + 6*a^2*e^(2*d*x + 2*c) + 14*a*b*e^(2*d*x + 2*c) + 8*b^2*e^
(2*d*x + 2*c) + 3*a^2 + 2*a*b)/((a*b^2*d + b^3*d)*(a*e^(6*d*x + 6*c) + 3*a*
e^(4*d*x + 4*c) + 4*b*e^(4*d*x + 4*c) + 3*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x
+ 2*c) + a))
```

3.92 $\int \frac{\text{sech}^7(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=153

$$\frac{a^{3/2}(4a+5b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2b^3d(a+b)^{3/2}}+\frac{a(2a+b)\sinh(c+dx)}{2b^2d(a+b)\left(a\sinh^2(c+dx)+a+b\right)}-\frac{(4a-b)\tan^{-1}(\sinh(c+dx))}{2b^3d}+\frac{\tanh(c}{2bd\left(a\sin\right.}$$

```
[Out] -((4*a - b)*ArcTan[Sinh[c + d*x]])/(2*b^3*d) + (a^(3/2)*(4*a + 5*b)*ArcTan[
(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(2*b^3*(a + b)^(3/2)*d) + (a*(2*a + b
)*Sinh[c + d*x])/(2*b^2*(a + b)*d*(a + b + a*Sinh[c + d*x]^2)) + (Sech[c +
d*x]*Tanh[c + d*x])/(2*b*d*(a + b + a*Sinh[c + d*x]^2))
```

Rubi [A] time = 0.199911, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4147, 414, 527, 522, 203, 205}

$$\frac{a^{3/2}(4a+5b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2b^3d(a+b)^{3/2}}+\frac{a(2a+b)\sinh(c+dx)}{2b^2d(a+b)\left(a\sinh^2(c+dx)+a+b\right)}-\frac{(4a-b)\tan^{-1}(\sinh(c+dx))}{2b^3d}+\frac{\tanh(c}{2bd\left(a\sin\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^7/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -((4*a - b)*ArcTan[Sinh[c + d*x]])/(2*b^3*d) + (a^(3/2)*(4*a + 5*b)*ArcTan[
(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(2*b^3*(a + b)^(3/2)*d) + (a*(2*a + b
)*Sinh[c + d*x])/(2*b^2*(a + b)*d*(a + b + a*Sinh[c + d*x]^2)) + (Sech[c +
d*x]*Tanh[c + d*x])/(2*b*d*(a + b + a*Sinh[c + d*x]^2))
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2\left(a+b+ax^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2bd\left(a+b+a\sinh^2(c+dx)\right)} - \frac{\operatorname{Subst}\left(\int \frac{a-b-3ax^2}{\left(1+x^2\right)\left(a+b+ax^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{2bd}$$

$$= \frac{a(2a+b)\sinh(c+dx)}{2b^2(a+b)d\left(a+b+a\sinh^2(c+dx)\right)} + \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2bd\left(a+b+a\sinh^2(c+dx)\right)} - \frac{\operatorname{Subst}\left(\int \frac{2\left(2a^2+2\right.}{(1+}\right.}{}$$

$$= \frac{a(2a+b)\sinh(c+dx)}{2b^2(a+b)d\left(a+b+a\sinh^2(c+dx)\right)} + \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2bd\left(a+b+a\sinh^2(c+dx)\right)} - \frac{(4a-b)\operatorname{Subst}\left(\right.}{}$$

$$= -\frac{(4a-b)\tan^{-1}(\sinh(c+dx))}{2b^3d} + \frac{a^{3/2}(4a+5b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2b^3(a+b)^{3/2}d} + \frac{a(2a+b)\sin}{2b^2(a+b)d\left(a+b\right.}$$

Mathematica [B] time = 5.04762, size = 489, normalized size = 3.2

$$\frac{\operatorname{sech}(c)\operatorname{sech}^3(c+dx)(a\cosh(2(c+dx))+a+2b)\left(a^{3/2}\sinh(2c)\operatorname{sech}(c+dx)\left(2a^2\cosh(2(c+dx))+2a^2+5b^2\right)\tan^{-1}\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^7/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c]*Sech[c + d*x]^3*(-(a^(3/2)*(4*a +
5*b)*ArcTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c]
 + Sinh[c]))/Sqrt[a]]*Cosh[c]^2*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*
x]) + b*(a + b)^(3/2)*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*Sqrt[
(Cosh[c] - Sinh[c])^2]*Sinh[c] - Cosh[c]*Sech[c + d*x]*(2*Sqrt[a + b]*(4*a^
```

```
2 + 3*a*b - b^2)*ArcTan[Tanh[(c + d*x)/2]]*(a + 2*b + a*Cosh[2*(c + d*x)])*
Sqrt[(Cosh[c] - Sinh[c])^2] - a^(5/2)*b*ArcTan[(Sqrt[a + b]*Csch[c + d*x]*S
qrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Sinh[c]))/Sqrt[a]]*(13 + 5*Cosh[2*(c
+ d*x)])*Sinh[c]) + a^(3/2)*ArcTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c]
 - Sinh[c])^2]*(Cosh[c] + Sinh[c]))/Sqrt[a]]*(2*a^2 + 5*b^2 + 2*a^2*Cosh[2*
(c + d*x)])*Sech[c + d*x]*Sinh[2*c] + b*(a + b)^(3/2)*(a + 2*b + a*Cosh[2*(
c + d*x)])*Sech[c + d*x]^3*Sqrt[(Cosh[c] - Sinh[c])^2]*Sinh[d*x] + 2*a^2*b*
Sqrt[a + b]*Cosh[c]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[c + d*x]))/(8*b^3*(a +
 b)^(3/2)*d*(a + b*Sech[c + d*x]^2)^2*Sqrt[(Cosh[c] - Sinh[c])^2])
```

Maple [B] time = 0.067, size = 448, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/d*a^2/b^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*
x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*tanh(1/2*d*x+1/2*c)^3+1/d
*a^2/b^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/
2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*tanh(1/2*d*x+1/2*c)+2/d/b^3*a
^(5/2)/(a+b)^(3/2)*arctan(1/2*(2*tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)+2*b^(1/2))
/a^(1/2))-2/d/b^3*a^(5/2)/(a+b)^(3/2)*arctan(1/2*(-2*tanh(1/2*d*x+1/2*c)*(a
+b)^(1/2)+2*b^(1/2))/a^(1/2))+5/2/d*a^(3/2)/b^2/(a+b)^(3/2)*arctan(1/2*(2*t
anh(1/2*d*x+1/2*c)*(a+b)^(1/2)+2*b^(1/2))/a^(1/2))-5/2/d/b^2*a^(3/2)/(a+b)^
(3/2)*arctan(1/2*(-2*tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)+2*b^(1/2))/a^(1/2))-1/
d/b^2/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)^3+1/d/b^2/(tanh(1/2*d
*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)+1/d/b^2*arctan(tanh(1/2*d*x+1/2*c))-4/
d/b^3*arctan(tanh(1/2*d*x+1/2*c))*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(2\,a^2e^{(7\,c)}+abe^{(7\,c)}\right)e^{(7\,dx)}+\left(2\,a^2e^{(5\,c)}+5\,abe^{(5\,c)}+4\,b^2e^{(5\,c)}\right)e^{(5\,dx)}-\left(2\,a^2e^{(3\,c)}+5\,abe^{(3\,c)}\right.}{a^2b^2d+ab^3d+\left(a^2b^2de^{(8\,c)}+ab^3de^{(8\,c)}\right)e^{(8\,dx)}+4\left(a^2b^2de^{(6\,c)}+2\,ab^3de^{(6\,c)}+b^4de^{(6\,c)}\right)e^{(6\,dx)}+2\left(3\,a^2b^2de^{(4\,c)}+7\,ab^3de^{(\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] ((2*a^2*e^(7*c) + a*b*e^(7*c))*e^(7*d*x) + (2*a^2*e^(5*c) + 5*a*b*e^(5*c) +
 4*b^2*e^(5*c))*e^(5*d*x) - (2*a^2*e^(3*c) + 5*a*b*e^(3*c) + 4*b^2*e^(3*c))
*e^(3*d*x) - (2*a^2*e^c + a*b*e^c)*e^(d*x))/(a^2*b^2*d + a*b^3*d + (a^2*b^2
*d*e^(8*c) + a*b^3*d*e^(8*c))*e^(8*d*x) + 4*(a^2*b^2*d*e^(6*c) + 2*a*b^3*d*
e^(6*c) + b^4*d*e^(6*c))*e^(6*d*x) + 2*(3*a^2*b^2*d*e^(4*c) + 7*a*b^3*d*e^(
4*c) + 4*b^4*d*e^(4*c))*e^(4*d*x) + 4*(a^2*b^2*d*e^(2*c) + 2*a*b^3*d*e^(2*c
) + b^4*d*e^(2*c))*e^(2*d*x)) - (4*a*e^c - b*e^c)*arctan(e^(d*x + c))*e^(-c
)/(b^3*d) + 128*integrate(1/128*((4*a^3*e^(3*c) + 5*a^2*b*e^(3*c))*e^(3*d*x
) + (4*a^3*e^c + 5*a^2*b*e^c)*e^(d*x))/(a^2*b^3 + a*b^4 + (a^2*b^3*e^(4*c)
+ a*b^4*e^(4*c))*e^(4*d*x) + 2*(a^2*b^3*e^(2*c) + 3*a*b^4*e^(2*c) + 2*b^5*e
^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 3.42556, size = 15336, normalized size = 100.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(2*a^2*b + a*b^2)*cosh(d*x + c)^7 + 28*(2*a^2*b + a*b^2)*cosh(d*x +
 c)*sinh(d*x + c)^6 + 4*(2*a^2*b + a*b^2)*sinh(d*x + c)^7 + 4*(2*a^2*b + 5*
a*b^2 + 4*b^3)*cosh(d*x + c)^5 + 4*(2*a^2*b + 5*a*b^2 + 4*b^3 + 21*(2*a^2*b
 + a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*(2*a^2*b + a*b^2)*cosh(d
*x + c)^3 + (2*a^2*b + 5*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*
(2*a^2*b + 5*a*b^2 + 4*b^3)*cosh(d*x + c)^3 + 4*(35*(2*a^2*b + a*b^2)*cosh(
d*x + c)^4 - 2*a^2*b - 5*a*b^2 - 4*b^3 + 10*(2*a^2*b + 5*a*b^2 + 4*b^3)*cos
h(d*x + c)^2)*sinh(d*x + c)^3 + 4*(21*(2*a^2*b + a*b^2)*cosh(d*x + c)^5 + 1
0*(2*a^2*b + 5*a*b^2 + 4*b^3)*cosh(d*x + c)^3 - 3*(2*a^2*b + 5*a*b^2 + 4*b^
3)*cosh(d*x + c))*sinh(d*x + c)^2 + ((4*a^3 + 5*a^2*b)*cosh(d*x + c)^8 + 8*
(4*a^3 + 5*a^2*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (4*a^3 + 5*a^2*b)*sinh(d*
x + c)^8 + 4*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^6 + 4*(4*a^3 + 9*a^2
*b + 5*a*b^2 + 7*(4*a^3 + 5*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*
(4*a^3 + 5*a^2*b)*cosh(d*x + c)^3 + 3*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x
+ c))*sinh(d*x + c)^5 + 2*(12*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^4 +
2*(35*(4*a^3 + 5*a^2*b)*cosh(d*x + c)^4 + 12*a^3 + 31*a^2*b + 20*a*b^2 + 30
*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(4*a^3
 + 5*a^2*b)*cosh(d*x + c)^5 + 10*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^
3 + (12*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*a^3 +
 5*a^2*b + 4*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^2 + 4*(7*(4*a^3 + 5*
a^2*b)*cosh(d*x + c)^6 + 15*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^4 + 4
*a^3 + 9*a^2*b + 5*a*b^2 + 3*(12*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^2
)*sinh(d*x + c)^2 + 8*((4*a^3 + 5*a^2*b)*cosh(d*x + c)^7 + 3*(4*a^3 + 9*a^2
*b + 5*a*b^2)*cosh(d*x + c)^5 + (12*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c
)^3 + (4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a/(a
+ b))*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d
*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a -
2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*si
nh(d*x + c) + 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x
 + c)^2 + (a + b)*sinh(d*x + c)^3 - (a + b)*cosh(d*x + c) + (3*(a + b)*cosh
(d*x + c)^2 - a - b)*sinh(d*x + c))*sqrt(-a/(a + b)) + a)/(a*cosh(d*x + c)^
4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cos
h(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cos
h(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - 4*((4*a^3 + 3
*a^2*b - a*b^2)*cosh(d*x + c)^8 + 8*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)
*sinh(d*x + c)^7 + (4*a^3 + 3*a^2*b - a*b^2)*sinh(d*x + c)^8 + 4*(4*a^3 + 7
*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^6 + 4*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^
3 + 7*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(4*
a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^3 + 3*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3
)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(12*a^3 + 25*a^2*b + 9*a*b^2 - 4*b^3)*
cosh(d*x + c)^4 + 2*(35*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^4 + 12*a^3
+ 25*a^2*b + 9*a*b^2 - 4*b^3 + 30*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*
x + c)^2)*sinh(d*x + c)^4 + 8*(7*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^5
+ 10*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^3 + (12*a^3 + 25*a^2*b
 + 9*a*b^2 - 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*a^3 + 3*a^2*b - a*b^
2 + 4*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^2 + 4*(7*(4*a^3 + 3*a
^2*b - a*b^2)*cosh(d*x + c)^6 + 15*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d
*x + c)^4 + 4*a^3 + 7*a^2*b + 2*a*b^2 - b^3 + 3*(12*a^3 + 25*a^2*b + 9*a*b^
2 - 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((4*a^3 + 3*a^2*b - a*b^2)*
cosh(d*x + c)^7 + 3*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^5 + (12
*a^3 + 25*a^2*b + 9*a*b^2 - 4*b^3)*cosh(d*x + c)^3 + (4*a^3 + 7*a^2*b + 2*a
```

```
*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x +
 c)) - 4*(2*a^2*b + a*b^2)*cosh(d*x + c) + 4*(7*(2*a^2*b + a*b^2)*cosh(d*x
+ c)^6 + 5*(2*a^2*b + 5*a*b^2 + 4*b^3)*cosh(d*x + c)^4 - 2*a^2*b - a*b^2 -
3*(2*a^2*b + 5*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^2*b^3 + a
*b^4)*d*cosh(d*x + c)^8 + 8*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)
^7 + (a^2*b^3 + a*b^4)*d*sinh(d*x + c)^8 + 4*(a^2*b^3 + 2*a*b^4 + b^5)*d*co
sh(d*x + c)^6 + 4*(7*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (a^2*b^3 + 2*a*b
^4 + b^5)*d)*sinh(d*x + c)^6 + 2*(3*a^2*b^3 + 7*a*b^4 + 4*b^5)*d*cosh(d*x +
 c)^4 + 8*(7*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)^3 + 3*(a^2*b^3 + 2*a*b^4 + b
^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^2*b^3 + a*b^4)*d*cosh(d*x +
 c)^4 + 30*(a^2*b^3 + 2*a*b^4 + b^5)*d*cosh(d*x + c)^2 + (3*a^2*b^3 + 7*a*b
^4 + 4*b^5)*d)*sinh(d*x + c)^4 + 4*(a^2*b^3 + 2*a*b^4 + b^5)*d*cosh(d*x + c
)^2 + 8*(7*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)^5 + 10*(a^2*b^3 + 2*a*b^4 + b^
5)*d*cosh(d*x + c)^3 + (3*a^2*b^3 + 7*a*b^4 + 4*b^5)*d*cosh(d*x + c))*sinh(
d*x + c)^3 + 4*(7*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)^6 + 15*(a^2*b^3 + 2*a*b
^4 + b^5)*d*cosh(d*x + c)^4 + 3*(3*a^2*b^3 + 7*a*b^4 + 4*b^5)*d*cosh(d*x +
c)^2 + (a^2*b^3 + 2*a*b^4 + b^5)*d)*sinh(d*x + c)^2 + (a^2*b^3 + a*b^4)*d +
 8*((a^2*b^3 + a*b^4)*d*cosh(d*x + c)^7 + 3*(a^2*b^3 + 2*a*b^4 + b^5)*d*cos
h(d*x + c)^5 + (3*a^2*b^3 + 7*a*b^4 + 4*b^5)*d*cosh(d*x + c)^3 + (a^2*b^3 +
 2*a*b^4 + b^5)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*(2*a^2*b + a*b^2)*c
osh(d*x + c)^7 + 14*(2*a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c)^6 + 2*(2*
a^2*b + a*b^2)*sinh(d*x + c)^7 + 2*(2*a^2*b + 5*a*b^2 + 4*b^3)*cosh(d*x + c
)^5 + 2*(2*a^2*b + 5*a*b^2 + 4*b^3 + 21*(2*a^2*b + a*b^2)*cosh(d*x + c)^2)*
sinh(d*x + c)^5 + 10*(7*(2*a^2*b + a*b^2)*cosh(d*x + c)^3 + (2*a^2*b + 5*a*
b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 - 2*(2*a^2*b + 5*a*b^2 + 4*b^3)
*cosh(d*x + c)^3 + 2*(35*(2*a^2*b + a*b^2)*cosh(d*x + c)^4 - 2*a^2*b - 5*a*
b^2 - 4*b^3 + 10*(2*a^2*b + 5*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)
^3 + 2*(21*(2*a^2*b + a*b^2)*cosh(d*x + c)^5 + 10*(2*a^2*b + 5*a*b^2 + 4*b^
3)*cosh(d*x + c)^3 - 3*(2*a^2*b + 5*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x
+ c)^2 + ((4*a^3 + 5*a^2*b)*cosh(d*x + c)^8 + 8*(4*a^3 + 5*a^2*b)*cosh(d*x
+ c)*sinh(d*x + c)^7 + (4*a^3 + 5*a^2*b)*sinh(d*x + c)^8 + 4*(4*a^3 + 9*a^2
*b + 5*a*b^2)*cosh(d*x + c)^6 + 4*(4*a^3 + 9*a^2*b + 5*a*b^2 + 7*(4*a^3 + 5
*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(4*a^3 + 5*a^2*b)*cosh(d*x
+ c)^3 + 3*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(
12*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^4 + 2*(35*(4*a^3 + 5*a^2*b)*cos
h(d*x + c)^4 + 12*a^3 + 31*a^2*b + 20*a*b^2 + 30*(4*a^3 + 9*a^2*b + 5*a*b^2
)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(4*a^3 + 5*a^2*b)*cosh(d*x + c)^5
 + 10*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^3 + (12*a^3 + 31*a^2*b + 20
*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*a^3 + 5*a^2*b + 4*(4*a^3 + 9*a^2
*b + 5*a*b^2)*cosh(d*x + c)^2 + 4*(7*(4*a^3 + 5*a^2*b)*cosh(d*x + c)^6 + 15
*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^4 + 4*a^3 + 9*a^2*b + 5*a*b^2 +
3*(12*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((4*a
^3 + 5*a^2*b)*cosh(d*x + c)^7 + 3*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)
^5 + (12*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^3 + (4*a^3 + 9*a^2*b + 5*
a*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a/(a + b))*arctan(1/2*sqrt(a/(a +
 b))*(cosh(d*x + c) + sinh(d*x + c))) + ((4*a^3 + 5*a^2*b)*cosh(d*x + c)^8
+ 8*(4*a^3 + 5*a^2*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (4*a^3 + 5*a^2*b)*sin
h(d*x + c)^8 + 4*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^6 + 4*(4*a^3 + 9
*a^2*b + 5*a*b^2 + 7*(4*a^3 + 5*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8
*(7*(4*a^3 + 5*a^2*b)*cosh(d*x + c)^3 + 3*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(
d*x + c))*sinh(d*x + c)^5 + 2*(12*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^
4 + 2*(35*(4*a^3 + 5*a^2*b)*cosh(d*x + c)^4 + 12*a^3 + 31*a^2*b + 20*a*b^2
+ 30*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(4
*a^3 + 5*a^2*b)*cosh(d*x + c)^5 + 10*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x +
 c)^3 + (12*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*a
^3 + 5*a^2*b + 4*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^2 + 4*(7*(4*a^3
+ 5*a^2*b)*cosh(d*x + c)^6 + 15*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^4
 + 4*a^3 + 9*a^2*b + 5*a*b^2 + 3*(12*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x +
c)^2)*sinh(d*x + c)^2 + 8*((4*a^3 + 5*a^2*b)*cosh(d*x + c)^7 + 3*(4*a^3 + 9
```

```
*a^2*b + 5*a*b^2)*cosh(d*x + c)^5 + (12*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x
 + c)^3 + (4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a/
(a + b))*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2
+ a*sinh(d*x + c)^3 + (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*
a + 4*b)*sinh(d*x + c))*sqrt(a/(a + b))/a) - 2*((4*a^3 + 3*a^2*b - a*b^2)*c
osh(d*x + c)^8 + 8*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)*sinh(d*x + c)^7
+ (4*a^3 + 3*a^2*b - a*b^2)*sinh(d*x + c)^8 + 4*(4*a^3 + 7*a^2*b + 2*a*b^2
- b^3)*cosh(d*x + c)^6 + 4*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3 + 7*(4*a^3 + 3*
a^2*b - a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(4*a^3 + 3*a^2*b - a
*b^2)*cosh(d*x + c)^3 + 3*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c))*
sinh(d*x + c)^5 + 2*(12*a^3 + 25*a^2*b + 9*a*b^2 - 4*b^3)*cosh(d*x + c)^4 +
 2*(35*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^4 + 12*a^3 + 25*a^2*b + 9*a*
b^2 - 4*b^3 + 30*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*
x + c)^4 + 8*(7*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^5 + 10*(4*a^3 + 7*a
^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^3 + (12*a^3 + 25*a^2*b + 9*a*b^2 - 4*b^
3)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*a^3 + 3*a^2*b - a*b^2 + 4*(4*a^3 + 7*
a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^2 + 4*(7*(4*a^3 + 3*a^2*b - a*b^2)*cos
h(d*x + c)^6 + 15*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^4 + 4*a^3
 + 7*a^2*b + 2*a*b^2 - b^3 + 3*(12*a^3 + 25*a^2*b + 9*a*b^2 - 4*b^3)*cosh(d
*x + c)^2)*sinh(d*x + c)^2 + 8*((4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^7 +
 3*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^5 + (12*a^3 + 25*a^2*b +
 9*a*b^2 - 4*b^3)*cosh(d*x + c)^3 + (4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(
d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 2*(2*a^2*b
 + a*b^2)*cosh(d*x + c) + 2*(7*(2*a^2*b + a*b^2)*cosh(d*x + c)^6 + 5*(2*a^2
*b + 5*a*b^2 + 4*b^3)*cosh(d*x + c)^4 - 2*a^2*b - a*b^2 - 3*(2*a^2*b + 5*a*
b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^2*b^3 + a*b^4)*d*cosh(d*x
+ c)^8 + 8*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b^3 + a
*b^4)*d*sinh(d*x + c)^8 + 4*(a^2*b^3 + 2*a*b^4 + b^5)*d*cosh(d*x + c)^6 + 4
*(7*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (a^2*b^3 + 2*a*b^4 + b^5)*d)*sinh
(d*x + c)^6 + 2*(3*a^2*b^3 + 7*a*b^4 + 4*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^2
*b^3 + a*b^4)*d*cosh(d*x + c)^3 + 3*(a^2*b^3 + 2*a*b^4 + b^5)*d*cosh(d*x +
c))*sinh(d*x + c)^5 + 2*(35*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 30*(a^2*b
^3 + 2*a*b^4 + b^5)*d*cosh(d*x + c)^2 + (3*a^2*b^3 + 7*a*b^4 + 4*b^5)*d)*si
nh(d*x + c)^4 + 4*(a^2*b^3 + 2*a*b^4 + b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^2*b
^3 + a*b^4)*d*cosh(d*x + c)^5 + 10*(a^2*b^3 + 2*a*b^4 + b^5)*d*cosh(d*x + c
)^3 + (3*a^2*b^3 + 7*a*b^4 + 4*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7
*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)^6 + 15*(a^2*b^3 + 2*a*b^4 + b^5)*d*cosh(
d*x + c)^4 + 3*(3*a^2*b^3 + 7*a*b^4 + 4*b^5)*d*cosh(d*x + c)^2 + (a^2*b^3 +
 2*a*b^4 + b^5)*d)*sinh(d*x + c)^2 + (a^2*b^3 + a*b^4)*d + 8*((a^2*b^3 + a*
b^4)*d*cosh(d*x + c)^7 + 3*(a^2*b^3 + 2*a*b^4 + b^5)*d*cosh(d*x + c)^5 + (3
*a^2*b^3 + 7*a*b^4 + 4*b^5)*d*cosh(d*x + c)^3 + (a^2*b^3 + 2*a*b^4 + b^5)*d
*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**7/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.93 $\int \frac{\cosh^2(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=204

$$\frac{b^{3/2}\left(35a^2+56ab+24b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^4d(a+b)^{5/2}}+\frac{b(4a+3b)(a+4b)\tanh(c+dx)}{8a^3d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)}+\frac{b(2a+3b)\tanh(c+}{4a^2d(a+b)\left(a-b\tanh^2(c\right.}$$

```
[Out] ((a - 6*b)*x)/(2*a^4) + (b^(3/2)*(35*a^2 + 56*a*b + 24*b^2)*ArcTanh[(Sqrt[b
]*Tanh[c + d*x])/Sqrt[a + b]])/(8*a^4*(a + b)^(5/2)*d) + (Cosh[c + d*x]*Sin
h[c + d*x])/(2*a*d*(a + b - b*Tanh[c + d*x]^2)^2) + (b*(2*a + 3*b)*Tanh[c +
 d*x])/(4*a^2*(a + b)*d*(a + b - b*Tanh[c + d*x]^2)^2) + (b*(4*a + 3*b)*(a
+ 4*b)*Tanh[c + d*x])/(8*a^3*(a + b)^2*d*(a + b - b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.377296, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4146, 414, 527, 522, 206, 208}

$$\frac{b^{3/2}\left(35a^2+56ab+24b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^4d(a+b)^{5/2}}+\frac{b(4a+3b)(a+4b)\tanh(c+dx)}{8a^3d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)}+\frac{b(2a+3b)\tanh(c+}{4a^2d(a+b)\left(a-b\tanh^2(c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a - 6*b)*x)/(2*a^4) + (b^(3/2)*(35*a^2 + 56*a*b + 24*b^2)*ArcTanh[(Sqrt[b
]*Tanh[c + d*x])/Sqrt[a + b]])/(8*a^4*(a + b)^(5/2)*d) + (Cosh[c + d*x]*Sin
h[c + d*x])/(2*a*d*(a + b - b*Tanh[c + d*x]^2)^2) + (b*(2*a + 3*b)*Tanh[c +
 d*x])/(4*a^2*(a + b)*d*(a + b - b*Tanh[c + d*x]^2)^2) + (b*(4*a + 3*b)*(a
+ 4*b)*Tanh[c + d*x])/(8*a^3*(a + b)^2*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)^2(a+b-bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{\operatorname{Subst}\left(\int \frac{a-b-5bx^2}{(1-x^2)(a+b-bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{2ad}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{b(2a+3b)\tanh(c+dx)}{4a^2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{\operatorname{Subst}\left(\int\right.}{}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{b(2a+3b)\tanh(c+dx)}{4a^2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{b(4a+}{8a^3(a+b)}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{b(2a+3b)\tanh(c+dx)}{4a^2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{b(4a+}{8a^3(a+b)}$$

$$= \frac{(a-6b)x}{2a^4} + \frac{b^{3/2}\left(35a^2+56ab+24b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^4(a+b)^{5/2}d} + \frac{\cosh(c+dx)\sinh(c}{2ad\left(a+b-b\tanh^2(c\right.}$$

Mathematica [A] time = 4.0039, size = 156, normalized size = 0.76

$$\frac{\frac{b^{3/2}\left(35a^2+56ab+24b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} + a\sinh(2(c+dx))\left(\frac{2b^3(5a\cosh(2(c+dx))+3a+8b)}{(a+b)^2(a\cosh(2(c+dx))+a+2b)^2} + \frac{13ab^2}{(a+b)^2(a\cosh(2(c+dx))+a+2b)} + 2\right) + 4}{8a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (4*(a - 6*b)*(c + d*x) + (b^(3/2)*(35*a^2 + 56*a*b + 24*b^2)*ArcTanh[(Sqrt[
b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/2) + a*(2 + (13*a*b^2)/((a + b)^
2*(a + 2*b + a*Cosh[2*(c + d*x)])) + (2*b^3*(3*a + 8*b + 5*a*Cosh[2*(c + d*
x)]))/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2))*Sinh[2*(c + d*x)])/(8*
a^4*d)
```

Maple [B] time = 0.128, size = 1435, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -1/2/d/a^3/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/a^3/(tanh(1/2*d*x+1/2*c)+1)+1/2/
d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)-3/d/a^4*ln(tanh(1/2*d*x+1/2*c)+1)*b+13/4/d*
b^2/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2
*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^7+2/d*b^
3/a^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c
)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^7+39/4/d*b
^2/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)
^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5+19/4/d*
b^3/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2
*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5-2/d*
b^4/a^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2
*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5+39/4
/d*b^2/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/
2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3+19/
4/d*b^3/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x
+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3-
2/d*b^4/a^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x
+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3+
13/4/d*b^2/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*
d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)+2
/d*b^3/a^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+
1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)+35/16
/d*b^(3/2)/a^2/(a^2+2*a*b+b^2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*
c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+7/2/d*b^(5/2)/a^3/(a^2+2*a*
b+b^2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*
c)*b^(1/2)+(a+b)^(1/2))+3/2/d*b^(7/2)/a^4/(a^2+2*a*b+b^2)/(a+b)^(1/2)*ln((a
+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-
35/16/d*b^(3/2)/a^2/(a^2+2*a*b+b^2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*
x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2))-7/2/d*b^(5/2)/a^3/(a^
2+2*a*b+b^2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d
*x+1/2*c)*b^(1/2)-(a+b)^(1/2))-3/2/d*b^(7/2)/a^4/(a^2+2*a*b+b^2)/(a+b)^(1/2
)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)-(a+b)
^(1/2))+1/2/d/a^3/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/a^3/(tanh(1/2*d*x+1/2*c)-
1)-1/2/d/a^3*ln(tanh(1/2*d*x+1/2*c)-1)+3/d/a^4*ln(tanh(1/2*d*x+1/2*c)-1)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.76583, size = 27738, normalized size = 135.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^12 + 24*(a^5 + 2*a^4*b + a
^3*b^2)*cosh(d*x + c)*sinh(d*x + c)^11 + 2*(a^5 + 2*a^4*b + a^3*b^2)*sinh(d
*x + c)^12 + 8*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11
*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)^10 + 4*(2*a^5 + 8*a^4*b + 10*a^3*b
^2 + 4*a^2*b^3 + 2*(a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x + 33*(a^5 +
 2*a^4*b + a^3*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 40*(11*(a^5 + 2*a^4
*b + a^3*b^2)*cosh(d*x + c)^3 + 2*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 +
(a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^
9 + 2*(5*a^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*
a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c)^8 + 2*(5*a^5
 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 495*(a^5 + 2*a^4*b + a^3
*b^2)*cosh(d*x + c)^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*
b^4)*d*x + 180*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11
*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 16*(99*(a^5 +
 2*a^4*b + a^3*b^2)*cosh(d*x + c)^5 + 60*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2
*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)^3 + (5*a
^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*a^4*b - 19
*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(
39*a^3*b^2 + 134*a^2*b^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*a^4*b - 57*a^3
*b^2 - 138*a^2*b^3 - 136*a*b^4 - 48*b^5)*d*x)*cosh(d*x + c)^6 + 4*(462*(a^5
 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^6 - 39*a^3*b^2 - 134*a^2*b^3 - 184*a*b^
4 - 80*b^5 + 420*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b -
11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)^4 + 4*(3*a^5 - 4*a^4*b - 57*a^3*
b^2 - 138*a^2*b^3 - 136*a*b^4 - 48*b^5)*d*x + 14*(5*a^5 + 26*a^4*b + 27*a^3
*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3
- 12*a*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(198*(a^5 + 2*a^4*b +
 a^3*b^2)*cosh(d*x + c)^7 + 252*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a
^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)^5 + 14*(5*a^5 + 2
6*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b
^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c)^3 - 3*(39*a^3*b^2 + 134*a^2*
b^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*a^4*b - 57*a^3*b^2 - 138*a^2*b^3 -
136*a*b^4 - 48*b^5)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*a^5 - 4*a^4*b -
 2*a^3*b^2 - 2*(5*a^5 + 26*a^4*b + 131*a^3*b^2 + 256*a^2*b^3 + 128*a*b^4 -
16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c)^
4 + 2*(495*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^8 + 840*(a^5 + 4*a^4*b +
 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh
(d*x + c)^6 - 5*a^5 - 26*a^4*b - 131*a^3*b^2 - 256*a^2*b^3 - 128*a*b^4 + 70
*(5*a^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*a^4*b
 - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c)^4 + 16*(a^5 - 2*a
^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x - 30*(39*a^3*b^2 + 134*a^2*b
^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*a^4*b - 57*a^3*b^2 - 138*a^2*b^3 - 1
36*a*b^4 - 48*b^5)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(55*(a^5 + 2*a
^4*b + a^3*b^2)*cosh(d*x + c)^9 + 120*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^
```

3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)^7 + 14*(5*a
^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*a^4*b - 19
*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c)^5 - 10*(39*a^3*b^2 + 1
34*a^2*b^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*a^4*b - 57*a^3*b^2 - 138*a^2
*b^3 - 136*a*b^4 - 48*b^5)*d*x)*cosh(d*x + c)^3 - (5*a^5 + 26*a^4*b + 131*a
^3*b^2 + 256*a^2*b^3 + 128*a*b^4 - 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*
b^3 - 12*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(2*a^5 + 8*a^4*b +
23*a^3*b^2 + 14*a^2*b^3 - 2*(a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*c
osh(d*x + c)^2 + 4*(33*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^10 + 90*(a^5
 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^
3)*d*x)*cosh(d*x + c)^8 + 14*(5*a^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 -
32*a*b^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cos
h(d*x + c)^6 - 2*a^5 - 8*a^4*b - 23*a^3*b^2 - 14*a^2*b^3 - 15*(39*a^3*b^2 +
 134*a^2*b^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*a^4*b - 57*a^3*b^2 - 138*a
^2*b^3 - 136*a*b^4 - 48*b^5)*d*x)*cosh(d*x + c)^4 + 2*(a^5 - 4*a^4*b - 11*a
^3*b^2 - 6*a^2*b^3)*d*x - 3*(5*a^5 + 26*a^4*b + 131*a^3*b^2 + 256*a^2*b^3 +
 128*a*b^4 - 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*c
osh(d*x + c)^2)*sinh(d*x + c)^2 + ((35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*cos
h(d*x + c)^10 + 10*(35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*cosh(d*x + c)*sinh(
d*x + c)^9 + (35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*sinh(d*x + c)^10 + 4*(35*
a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*cosh(d*x + c)^8 + (140*a^4*b
 + 504*a^3*b^2 + 544*a^2*b^3 + 192*a*b^4 + 45*(35*a^4*b + 56*a^3*b^2 + 24*a^
2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(35*a^4*b + 56*a^3*b^2 + 24
*a^2*b^3)*cosh(d*x + c)^3 + 4*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*
b^4)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*a^4*b + 448*a^3*b^2 + 800*a^2*
b^3 + 640*a*b^4 + 192*b^5)*cosh(d*x + c)^6 + 2*(105*a^4*b + 448*a^3*b^2 + 8
00*a^2*b^3 + 640*a*b^4 + 192*b^5 + 105*(35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)
*cosh(d*x + c)^4 + 56*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*cos
h(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*
cosh(d*x + c)^5 + 56*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*cosh
(d*x + c)^3 + 3*(105*a^4*b + 448*a^3*b^2 + 800*a^2*b^3 + 640*a*b^4 + 192*b^
5)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3
 + 48*a*b^4)*cosh(d*x + c)^4 + 2*(105*(35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*
cosh(d*x + c)^6 + 70*a^4*b + 252*a^3*b^2 + 272*a^2*b^3 + 96*a*b^4 + 140*(35
*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*cosh(d*x + c)^4 + 15*(105*a^
4*b + 448*a^3*b^2 + 800*a^2*b^3 + 640*a*b^4 + 192*b^5)*cosh(d*x + c)^2)*sin
h(d*x + c)^4 + 8*(15*(35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*cosh(d*x + c)^7 +
 28*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*cosh(d*x + c)^5 + 5*(
105*a^4*b + 448*a^3*b^2 + 800*a^2*b^3 + 640*a*b^4 + 192*b^5)*cosh(d*x + c)^
3 + 2*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*cosh(d*x + c))*sinh
(d*x + c)^3 + (35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*cosh(d*x + c)^2 + (45*(3
5*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*cosh(d*x + c)^8 + 112*(35*a^4*b + 126*a^
3*b^2 + 136*a^2*b^3 + 48*a*b^4)*cosh(d*x + c)^6 + 35*a^4*b + 56*a^3*b^2 + 2
4*a^2*b^3 + 30*(105*a^4*b + 448*a^3*b^2 + 800*a^2*b^3 + 640*a*b^4 + 192*b^5
)*cosh(d*x + c)^4 + 24*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*co
sh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*(35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*
cosh(d*x + c)^9 + 16*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*cosh
(d*x + c)^7 + 6*(105*a^4*b + 448*a^3*b^2 + 800*a^2*b^3 + 640*a*b^4 + 192*b^
5)*cosh(d*x + c)^5 + 8*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*co
sh(d*x + c)^3 + (35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*cosh(d*x + c))*sinh(d*
x + c))*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh
(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*
a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 +
4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a
^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^
2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*
x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2
*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4
*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 8*(3*(

```
a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^11 + 10*(a^5 + 4*a^4*b + 5*a^3*b^2 +
 2*a^2*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)^9
+ 2*(5*a^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*a^
4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c)^7 - 3*(39*a^3*
b^2 + 134*a^2*b^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*a^4*b - 57*a^3*b^2 -
138*a^2*b^3 - 136*a*b^4 - 48*b^5)*d*x)*cosh(d*x + c)^5 - (5*a^5 + 26*a^4*b
+ 131*a^3*b^2 + 256*a^2*b^3 + 128*a*b^4 - 16*(a^5 - 2*a^4*b - 19*a^3*b^2 -
28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c)^3 - (2*a^5 + 8*a^4*b + 23*a^3*b^2
 + 14*a^2*b^3 - 2*(a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x +
c))*sinh(d*x + c))/((a^8 + 2*a^7*b + a^6*b^2)*d*cosh(d*x + c)^10 + 10*(a^8
+ 2*a^7*b + a^6*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^8 + 2*a^7*b + a^6
*b^2)*d*sinh(d*x + c)^10 + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d*cosh
(d*x + c)^8 + (45*(a^8 + 2*a^7*b + a^6*b^2)*d*cosh(d*x + c)^2 + 4*(a^8 + 4*
a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d)*sinh(d*x + c)^8 + 2*(3*a^8 + 14*a^7*b + 2
7*a^6*b^2 + 24*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^6 + 8*(15*(a^8 + 2*a^7*
b + a^6*b^2)*d*cosh(d*x + c)^3 + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*
d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^8 + 2*a^7*b + a^6*b^2)*d*cosh(
d*x + c)^4 + 56*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d*cosh(d*x + c)^2 +
 (3*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*b^4)*d)*sinh(d*x + c)^
6 + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d*cosh(d*x + c)^4 + 4*(63*(a^
8 + 2*a^7*b + a^6*b^2)*d*cosh(d*x + c)^5 + 56*(a^8 + 4*a^7*b + 5*a^6*b^2 +
2*a^5*b^3)*d*cosh(d*x + c)^3 + 3*(3*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^
3 + 8*a^4*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^8 + 2*a^7*b + a
^6*b^2)*d*cosh(d*x + c)^6 + 140*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d*c
osh(d*x + c)^4 + 15*(3*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*b^4
)*d*cosh(d*x + c)^2 + 2*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d)*sinh(d*x
 + c)^4 + (a^8 + 2*a^7*b + a^6*b^2)*d*cosh(d*x + c)^2 + 8*(15*(a^8 + 2*a^7*
b + a^6*b^2)*d*cosh(d*x + c)^7 + 28*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)
*d*cosh(d*x + c)^5 + 5*(3*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*
b^4)*d*cosh(d*x + c)^3 + 2*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d*cosh(d
*x + c))*sinh(d*x + c)^3 + (45*(a^8 + 2*a^7*b + a^6*b^2)*d*cosh(d*x + c)^8
+ 112*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d*cosh(d*x + c)^6 + 30*(3*a^8
 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^4 + 24*(
a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d*cosh(d*x + c)^2 + (a^8 + 2*a^7*b +
 a^6*b^2)*d)*sinh(d*x + c)^2 + 2*(5*(a^8 + 2*a^7*b + a^6*b^2)*d*cosh(d*x +
c)^9 + 16*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d*cosh(d*x + c)^7 + 6*(3*
a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^5 + 8
*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d*cosh(d*x + c)^3 + (a^8 + 2*a^7*b
 + a^6*b^2)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*((a^5 + 2*a^4*b + a^3*b^2)
*cosh(d*x + c)^12 + 12*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)*sinh(d*x + c
)^11 + (a^5 + 2*a^4*b + a^3*b^2)*sinh(d*x + c)^12 + 4*(a^5 + 4*a^4*b + 5*a^
3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x
+ c)^10 + 2*(2*a^5 + 8*a^4*b + 10*a^3*b^2 + 4*a^2*b^3 + 2*(a^5 - 4*a^4*b -
11*a^3*b^2 - 6*a^2*b^3)*d*x + 33*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^2)
*sinh(d*x + c)^10 + 20*(11*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^3 + 2*(a
^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*
b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^9 + (5*a^5 + 26*a^4*b + 27*a^3*b^2 -
 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a
*b^4)*d*x)*cosh(d*x + c)^8 + (5*a^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 -
32*a*b^4 + 495*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^4 + 16*(a^5 - 2*a^4*
b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x + 180*(a^5 + 4*a^4*b + 5*a^3*b^
2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)
^2)*sinh(d*x + c)^8 + 8*(99*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^5 + 60*
(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^
2*b^3)*d*x)*cosh(d*x + c)^3 + (5*a^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 -
 32*a*b^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*co
sh(d*x + c))*sinh(d*x + c)^7 - 2*(39*a^3*b^2 + 134*a^2*b^3 + 184*a*b^4 + 80
*b^5 - 4*(3*a^5 - 4*a^4*b - 57*a^3*b^2 - 138*a^2*b^3 - 136*a*b^4 - 48*b^5)*
d*x)*cosh(d*x + c)^6 + 2*(462*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^6 - 3
```

9*a^3*b^2 - 134*a^2*b^3 - 184*a*b^4 - 80*b^5 + 420*(a^5 + 4*a^4*b + 5*a^3*b
^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c
)^4 + 4*(3*a^5 - 4*a^4*b - 57*a^3*b^2 - 138*a^2*b^3 - 136*a*b^4 - 48*b^5)*d
x + 14(5*a^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 -
2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*
x + c)^6 + 4*(198*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^7 + 252*(a^5 + 4*
a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*
x)*cosh(d*x + c)^5 + 14*(5*a^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*
b^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x
 + c)^3 - 3*(39*a^3*b^2 + 134*a^2*b^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*a
^4*b - 57*a^3*b^2 - 138*a^2*b^3 - 136*a*b^4 - 48*b^5)*d*x)*cosh(d*x + c))*s
inh(d*x + c)^5 - a^5 - 2*a^4*b - a^3*b^2 - (5*a^5 + 26*a^4*b + 131*a^3*b^2
 + 256*a^2*b^3 + 128*a*b^4 - 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 1
2*a*b^4)*d*x)*cosh(d*x + c)^4 + (495*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c
)^8 + 840*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11*a^3*
b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)^6 - 5*a^5 - 26*a^4*b - 131*a^3*b^2 - 25
6*a^2*b^3 - 128*a*b^4 + 70*(5*a^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32
*a*b^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(
d*x + c)^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x -
30*(39*a^3*b^2 + 134*a^2*b^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*a^4*b - 57
*a^3*b^2 - 138*a^2*b^3 - 136*a*b^4 - 48*b^5)*d*x)*cosh(d*x + c)^2)*sinh(d*x
 + c)^4 + 4*(55*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^9 + 120*(a^5 + 4*a^
4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)
*cosh(d*x + c)^7 + 14*(5*a^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^
4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x +
 c)^5 - 10*(39*a^3*b^2 + 134*a^2*b^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*a^
4*b - 57*a^3*b^2 - 138*a^2*b^3 - 136*a*b^4 - 48*b^5)*d*x)*cosh(d*x + c)^3 -
 (5*a^5 + 26*a^4*b + 131*a^3*b^2 + 256*a^2*b^3 + 128*a*b^4 - 16*(a^5 - 2*a^
4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)
^3 - 2*(2*a^5 + 8*a^4*b + 23*a^3*b^2 + 14*a^2*b^3 - 2*(a^5 - 4*a^4*b - 11*a
^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)^2 + 2*(33*(a^5 + 2*a^4*b + a^3*b^2)*
cosh(d*x + c)^10 + 90*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4
*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)^8 + 14*(5*a^5 + 26*a^4*b +
27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^
2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c)^6 - 2*a^5 - 8*a^4*b - 23*a^3*b^2 - 14*
a^2*b^3 - 15*(39*a^3*b^2 + 134*a^2*b^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*
a^4*b - 57*a^3*b^2 - 138*a^2*b^3 - 136*a*b^4 - 48*b^5)*d*x)*cosh(d*x + c)^4
 + 2*(a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x - 3*(5*a^5 + 26*a^4*b + 1
31*a^3*b^2 + 256*a^2*b^3 + 128*a*b^4 - 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*
a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((35*a^4*b + 56
*a^3*b^2 + 24*a^2*b^3)*cosh(d*x + c)^10 + 10*(35*a^4*b + 56*a^3*b^2 + 24*a^
2*b^3)*cosh(d*x + c)*sinh(d*x + c)^9 + (35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)
*sinh(d*x + c)^10 + 4*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*cos
h(d*x + c)^8 + (140*a^4*b + 504*a^3*b^2 + 544*a^2*b^3 + 192*a*b^4 + 45*(35*
a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(
35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*cosh(d*x + c)^3 + 4*(35*a^4*b + 126*a^3
*b^2 + 136*a^2*b^3 + 48*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*a^4*
b + 448*a^3*b^2 + 800*a^2*b^3 + 640*a*b^4 + 192*b^5)*cosh(d*x + c)^6 + 2*(1
05*a^4*b + 448*a^3*b^2 + 800*a^2*b^3 + 640*a*b^4 + 192*b^5 + 105*(35*a^4*b
 + 56*a^3*b^2 + 24*a^2*b^3)*cosh(d*x + c)^4 + 56*(35*a^4*b + 126*a^3*b^2 + 1
36*a^2*b^3 + 48*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(35*a^4*b +
 56*a^3*b^2 + 24*a^2*b^3)*cosh(d*x + c)^5 + 56*(35*a^4*b + 126*a^3*b^2 + 13
6*a^2*b^3 + 48*a*b^4)*cosh(d*x + c)^3 + 3*(105*a^4*b + 448*a^3*b^2 + 800*a^
2*b^3 + 640*a*b^4 + 192*b^5)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*(35*a^4*b +
 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*cosh(d*x + c)^4 + 2*(105*(35*a^4*b +
 56*a^3*b^2 + 24*a^2*b^3)*cosh(d*x + c)^6 + 70*a^4*b + 252*a^3*b^2 + 272*a^
2*b^3 + 96*a*b^4 + 140*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*co
sh(d*x + c)^4 + 15*(105*a^4*b + 448*a^3*b^2 + 800*a^2*b^3 + 640*a*b^4 + 192
*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*(35*a^4*b + 56*a^3*b^2 + 24*

```
a^2*b^3)*cosh(d*x + c)^7 + 28*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*
b^4)*cosh(d*x + c)^5 + 5*(105*a^4*b + 448*a^3*b^2 + 800*a^2*b^3 + 640*a*b^4
 + 192*b^5)*cosh(d*x + c)^3 + 2*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*
a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + (35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3
)*cosh(d*x + c)^2 + (45*(35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*cosh(d*x + c)^
8 + 112*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*cosh(d*x + c)^6 +
 35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3 + 30*(105*a^4*b + 448*a^3*b^2 + 800*a^2
*b^3 + 640*a*b^4 + 192*b^5)*cosh(d*x + c)^4 + 24*(35*a^4*b + 126*a^3*b^2 +
136*a^2*b^3 + 48*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*(35*a^4*b +
 56*a^3*b^2 + 24*a^2*b^3)*cosh(d*x + c)^9 + 16*(35*a^4*b + 126*a^3*b^2 + 13
6*a^2*b^3 + 48*a*b^4)*cosh(d*x + c)^7 + 6*(105*a^4*b + 448*a^3*b^2 + 800*a^
2*b^3 + 640*a*b^4 + 192*b^5)*cosh(d*x + c)^5 + 8*(35*a^4*b + 126*a^3*b^2 +
136*a^2*b^3 + 48*a*b^4)*cosh(d*x + c)^3 + (35*a^4*b + 56*a^3*b^2 + 24*a^2*b
^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x +
 c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt
(-b/(a + b))/b) + 4*(3*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^11 + 10*(a^5
 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^
3)*d*x)*cosh(d*x + c)^9 + 2*(5*a^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 3
2*a*b^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh
(d*x + c)^7 - 3*(39*a^3*b^2 + 134*a^2*b^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 -
 4*a^4*b - 57*a^3*b^2 - 138*a^2*b^3 - 136*a*b^4 - 48*b^5)*d*x)*cosh(d*x + c
)^5 - (5*a^5 + 26*a^4*b + 131*a^3*b^2 + 256*a^2*b^3 + 128*a*b^4 - 16*(a^5 -
 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c)^3 - (2*a^
5 + 8*a^4*b + 23*a^3*b^2 + 14*a^2*b^3 - 2*(a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a
^2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^8 + 2*a^7*b + a^6*b^2)*d*cos
h(d*x + c)^10 + 10*(a^8 + 2*a^7*b + a^6*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^
9 + (a^8 + 2*a^7*b + a^6*b^2)*d*sinh(d*x + c)^10 + 4*(a^8 + 4*a^7*b + 5*a^6
*b^2 + 2*a^5*b^3)*d*cosh(d*x + c)^8 + (45*(a^8 + 2*a^7*b + a^6*b^2)*d*cosh(
d*x + c)^2 + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d)*sinh(d*x + c)^8 +
 2*(3*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)
^6 + 8*(15*(a^8 + 2*a^7*b + a^6*b^2)*d*cosh(d*x + c)^3 + 4*(a^8 + 4*a^7*b +
 5*a^6*b^2 + 2*a^5*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^8 + 2*
a^7*b + a^6*b^2)*d*cosh(d*x + c)^4 + 56*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*
b^3)*d*cosh(d*x + c)^2 + (3*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^
4*b^4)*d)*sinh(d*x + c)^6 + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d*cos
h(d*x + c)^4 + 4*(63*(a^8 + 2*a^7*b + a^6*b^2)*d*cosh(d*x + c)^5 + 56*(a^8
+ 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d*cosh(d*x + c)^3 + 3*(3*a^8 + 14*a^7*b
+ 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2
*(105*(a^8 + 2*a^7*b + a^6*b^2)*d*cosh(d*x + c)^6 + 140*(a^8 + 4*a^7*b + 5*
a^6*b^2 + 2*a^5*b^3)*d*cosh(d*x + c)^4 + 15*(3*a^8 + 14*a^7*b + 27*a^6*b^2
+ 24*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^2 + 2*(a^8 + 4*a^7*b + 5*a^6*b^2
+ 2*a^5*b^3)*d)*sinh(d*x + c)^4 + (a^8 + 2*a^7*b + a^6*b^2)*d*cosh(d*x + c)
^2 + 8*(15*(a^8 + 2*a^7*b + a^6*b^2)*d*cosh(d*x + c)^7 + 28*(a^8 + 4*a^7*b
+ 5*a^6*b^2 + 2*a^5*b^3)*d*cosh(d*x + c)^5 + 5*(3*a^8 + 14*a^7*b + 27*a^6*b
^2 + 24*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^3 + 2*(a^8 + 4*a^7*b + 5*a^6*b
^2 + 2*a^5*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (45*(a^8 + 2*a^7*b + a^6
*b^2)*d*cosh(d*x + c)^8 + 112*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d*cos
h(d*x + c)^6 + 30*(3*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*b^4)*
d*cosh(d*x + c)^4 + 24*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d*cosh(d*x +
 c)^2 + (a^8 + 2*a^7*b + a^6*b^2)*d)*sinh(d*x + c)^2 + 2*(5*(a^8 + 2*a^7*b
+ a^6*b^2)*d*cosh(d*x + c)^9 + 16*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d
*cosh(d*x + c)^7 + 6*(3*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*b^
4)*d*cosh(d*x + c)^5 + 8*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d*cosh(d*x
 + c)^3 + (a^8 + 2*a^7*b + a^6*b^2)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2131, size = 552, normalized size = 2.71

$$\frac{\left(35\,a^2b^2 + 56\,ab^3 + 24\,b^4\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{8\,\left(a^6d + 2\,a^5bd + a^4b^2d\right)\sqrt{-ab-b^2}} - \frac{13\,a^3b^2e^{(6\,dx+6\,c)} + 40\,a^2b^3e^{(6\,dx+6\,c)} + 24\,ab^4e^{(6\,dx+6\,c)} + 39\,a^3b^2e^{(4\,dx+}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(35*a^2*b^2 + 56*a*b^3 + 24*b^4)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*
b)/sqrt(-a*b - b^2))/((a^6*d + 2*a^5*b*d + a^4*b^2*d)*sqrt(-a*b - b^2)) - 1
/4*(13*a^3*b^2*e^(6*d*x + 6*c) + 40*a^2*b^3*e^(6*d*x + 6*c) + 24*a*b^4*e^(6
*d*x + 6*c) + 39*a^3*b^2*e^(4*d*x + 4*c) + 134*a^2*b^3*e^(4*d*x + 4*c) + 18
4*a*b^4*e^(4*d*x + 4*c) + 80*b^5*e^(4*d*x + 4*c) + 39*a^3*b^2*e^(2*d*x + 2*
c) + 104*a^2*b^3*e^(2*d*x + 2*c) + 56*a*b^4*e^(2*d*x + 2*c) + 13*a^3*b^2 +
10*a^2*b^3)/((a^6*d + 2*a^5*b*d + a^4*b^2*d)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*
d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)^2) + 1/2*(d*x + c)*(a - 6*b)/(a^4*d)
+ 1/8*e^(2*d*x + 2*c)/(a^3*d) - 1/8*(2*a*e^(2*d*x + 2*c) - 12*b*e^(2*d*x +
2*c) + a)*e^(-2*d*x - 2*c)/(a^4*d)
```

3.94 $\int \frac{\cosh(c+dx)}{\left(a+b\textbf{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=154

$$-\frac{b^3\sinh(c+dx)}{4a^3d(a+b)\left(a\sinh^2(c+dx)+a+b\right)^2}+\frac{3b^2(4a+3b)\sinh(c+dx)}{8a^3d(a+b)^2\left(a\sinh^2(c+dx)+a+b\right)}-\frac{3b\left(4(a+b)^2+(2a+b)^2\right)\tan^{-1}\left(\right.}{8a^{7/2}d(a+b)^{5/2}}$$

```
[Out] (-3*b*(4*(a + b)^2 + (2*a + b)^2)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b
]])/(8*a^(7/2)*(a + b)^(5/2)*d) + Sinh[c + d*x]/(a^3*d) - (b^3*Sinh[c + d*x
])/(4*a^3*(a + b)*d*(a + b + a*Sinh[c + d*x]^2)^2) + (3*b^2*(4*a + 3*b)*Sin
h[c + d*x])/(8*a^3*(a + b)^2*d*(a + b + a*Sinh[c + d*x]^2))
```

Rubi [A] time = 0.184815, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4147, 390, 1157, 385, 205}

$$-\frac{b^3\sinh(c+dx)}{4a^3d(a+b)\left(a\sinh^2(c+dx)+a+b\right)^2}+\frac{3b^2(4a+3b)\sinh(c+dx)}{8a^3d(a+b)^2\left(a\sinh^2(c+dx)+a+b\right)}-\frac{3b\left(4(a+b)^2+(2a+b)^2\right)\tan^{-1}\left(\right.}{8a^{7/2}d(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (-3*b*(4*(a + b)^2 + (2*a + b)^2)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b
]])/(8*a^(7/2)*(a + b)^(5/2)*d) + Sinh[c + d*x]/(a^3*d) - (b^3*Sinh[c + d*x
])/(4*a^3*(a + b)*d*(a + b + a*Sinh[c + d*x]^2)^2) + (3*b^2*(4*a + 3*b)*Sin
h[c + d*x])/(8*a^3*(a + b)^2*d*(a + b + a*Sinh[c + d*x]^2))
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{\left(a+b+ax^2\right)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3}-\frac{b\left(3a^2+3ab+b^2\right)+3ab(2a+b)x^2+3a^2bx^4}{a^3\left(a+b+ax^2\right)^3}\right) dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\sinh(c+dx)}{a^3d}-\frac{\text{Subst}\left(\int \frac{b\left(3a^2+3ab+b^2\right)+3ab(2a+b)x^2+3a^2bx^4}{\left(a+b+ax^2\right)^3}\,dx, x, \sinh(c+dx)\right)}{a^3d}$$

$$= \frac{\sinh(c+dx)}{a^3d}-\frac{b^3\sinh(c+dx)}{4a^3(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2}+\frac{\text{Subst}\left(\int \frac{-3b(2a+b)^2-12ab(a+b)x^2}{\left(a+b+ax^2\right)^2}\,dx \ldots\right.}{4a^3(a+b)d}$$

$$= \frac{\sinh(c+dx)}{a^3d}-\frac{b^3\sinh(c+dx)}{4a^3(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2}+\frac{3b^2(4a+3b)\sinh(c+dx)}{8a^3(a+b)^2d\left(a+b+a\sinh^2(c \ldots\right.}$$

$$= -\frac{3b\left(4(a+b)^2+(2a+b)^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{7/2}(a+b)^{5/2}d}+\frac{\sinh(c+dx)}{a^3d}-\frac{b^3\sinh(c+ \ldots}{4a^3(a+b)d\left(a+b+a \ldots\right.}$$

Mathematica [A] time = 3.65207, size = 292, normalized size = 1.9

$$\frac{\text{sech}^5(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\frac{3b\left(8a^2+12ab+5b^2\right)(\cosh(c)-\sinh(c))\text{sech}(c+dx)(a\cosh(2(c+dx))+a+2b)^2\tan^{-1}\left(\frac{\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(\ldots}}{\ldots}\right)}{(a+b)^{5/2}\sqrt{(\cosh(c)-\sinh(c))^2}} \ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^5*((3*b*(8*a^2 + 12*a*b + 5*
b^2)*ArcTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c]
 + Sinh[c]))/Sqrt[a]]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]*(Cosh
[c] - Sinh[c]))/((a + b)^(5/2)*Sqrt[(Cosh[c] - Sinh[c])^2]) + 8*Sqrt[a]*Cos
h[d*x]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]*Sinh[c] + 8*Sqrt[a]*
Cosh[c]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]*Sinh[d*x] - (8*Sqrt
[a]*b^3*Tanh[c + d*x])/(a + b) + (6*Sqrt[a]*b^2*(4*a + 3*b)*(a + 2*b + a*Co
```

```
sh[2*(c + d*x)])*Tanh[c + d*x])/(a + b)^2))/(64*a^(7/2)*d*(a + b*Sech[c + d
*x]^2)^3)
```

Maple [B] time = 0.118, size = 1238, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -1/d/a^3/(tanh(1/2*d*x+1/2*c)+1)-3/d*b^2/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tan
h(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)
^2/(a+b)*tanh(1/2*d*x+1/2*c)^7-7/4/d*b^3/a^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tan
h(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)
^2/(a+b)*tanh(1/2*d*x+1/2*c)^7-3/d*b^2/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/
2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(
a+b)^2*tanh(1/2*d*x+1/2*c)^5+21/4/d*b^3/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh
(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^
2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5+21/4/d*b^4/a^3/(tanh(1/2*d*x+1/2*c)^4*a+b*t
anh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+
b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5+3/d*b^2/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tan
h(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)
^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3-21/4/d*b^3/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*
tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a
+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3-21/4/d*b^4/a^3/(tanh(1/2*d*x+1/2*c)^4*a
+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*
b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3+3/d*b^2/a^2/(tanh(1/2*d*x+1/2*c)^4*a
+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*
b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)+7/4/d*b^3/a^3/(tanh(1/2*d*x+1/2*c)^4*a+b
*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+
a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)-3/d*b/a^(3/2)/(a^2+2*a*b+b^2)/(a+b)^(1/2)*
arctan(1/2*(2*tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)+2*b^(1/2))/a^(1/2))-9/2/d*b^2
/a^(5/2)/(a^2+2*a*b+b^2)/(a+b)^(1/2)*arctan(1/2*(2*tanh(1/2*d*x+1/2*c)*(a+b
)^(1/2)+2*b^(1/2))/a^(1/2))-15/8/d*b^3/a^(7/2)/(a^2+2*a*b+b^2)/(a+b)^(1/2)*
arctan(1/2*(2*tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)+2*b^(1/2))/a^(1/2))-3/d*b/a^(
3/2)/(a^2+2*a*b+b^2)/(a+b)^(1/2)*arctan(1/2*(2*tanh(1/2*d*x+1/2*c)*(a+b)^(1
/2)-2*b^(1/2))/a^(1/2))-9/2/d*b^2/a^(5/2)/(a^2+2*a*b+b^2)/(a+b)^(1/2)*arcta
n(1/2*(2*tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)-2*b^(1/2))/a^(1/2))-15/8/d*b^3/a^(
7/2)/(a^2+2*a*b+b^2)/(a+b)^(1/2)*arctan(1/2*(2*tanh(1/2*d*x+1/2*c)*(a+b)^(1
/2)-2*b^(1/2))/a^(1/2))-1/d/a^3/(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*a^4 + 4*a^3*b + 2*a^2*b^2 - 2*(a^4*e^(10*c) + 2*a^3*b*e^(10*c) + a^
2*b^2*e^(10*c))*e^(10*d*x) - (6*a^4*e^(8*c) + 28*a^3*b*e^(8*c) + 50*a^2*b^2
*e^(8*c) + 25*a*b^3*e^(8*c))*e^(8*d*x) - (4*a^4*e^(6*c) + 24*a^3*b*e^(6*c)
+ 80*a^2*b^2*e^(6*c) + 129*a*b^3*e^(6*c) + 60*b^4*e^(6*c))*e^(6*d*x) + (4*a
```

```
^4*e^(4*c) + 24*a^3*b*e^(4*c) + 80*a^2*b^2*e^(4*c) + 129*a*b^3*e^(4*c) + 60
*b^4*e^(4*c))*e^(4*d*x) + (6*a^4*e^(2*c) + 28*a^3*b*e^(2*c) + 50*a^2*b^2*e^
(2*c) + 25*a*b^3*e^(2*c))*e^(2*d*x))/((a^7*d*e^(9*c) + 2*a^6*b*d*e^(9*c) +
a^5*b^2*d*e^(9*c))*e^(9*d*x) + 4*(a^7*d*e^(7*c) + 4*a^6*b*d*e^(7*c) + 5*a^5
*b^2*d*e^(7*c) + 2*a^4*b^3*d*e^(7*c))*e^(7*d*x) + 2*(3*a^7*d*e^(5*c) + 14*a
^6*b*d*e^(5*c) + 27*a^5*b^2*d*e^(5*c) + 24*a^4*b^3*d*e^(5*c) + 8*a^3*b^4*d*
e^(5*c))*e^(5*d*x) + 4*(a^7*d*e^(3*c) + 4*a^6*b*d*e^(3*c) + 5*a^5*b^2*d*e^(
3*c) + 2*a^4*b^3*d*e^(3*c))*e^(3*d*x) + (a^7*d*e^c + 2*a^6*b*d*e^c + a^5*b^
2*d*e^c)*e^(d*x)) - 1/2*integrate(3/2*((8*a^2*b*e^(3*c) + 12*a*b^2*e^(3*c)
+ 5*b^3*e^(3*c))*e^(3*d*x) + (8*a^2*b*e^c + 12*a*b^2*e^c + 5*b^3*e^c)*e^(d*
x))/(a^6 + 2*a^5*b + a^4*b^2 + (a^6*e^(4*c) + 2*a^5*b*e^(4*c) + a^4*b^2*e^(
4*c))*e^(4*d*x) + 2*(a^6*e^(2*c) + 4*a^5*b*e^(2*c) + 5*a^4*b^2*e^(2*c) + 2*
a^3*b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 3.49527, size = 23050, normalized size = 149.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(8*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^10 + 80*(a^6 +
 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)*sinh(d*x + c)^9 + 8*(a^6 + 3*
a^5*b + 3*a^4*b^2 + a^3*b^3)*sinh(d*x + c)^10 + 4*(6*a^6 + 34*a^5*b + 78*a^
4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c)^8 + 4*(6*a^6 + 34*a^5*b + 78
*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4 + 90*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^
3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 32*(30*(a^6 + 3*a^5*b + 3*a^4*b^2 + a
^3*b^3)*cosh(d*x + c)^3 + (6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*
a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^7 + 4*(4*a^6 + 28*a^5*b + 104*a^4*b^2
 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c)^6 + 4*(4*a^6 + 28*a^
5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5 + 420*(a^6 + 3*a^5
*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^4 + 28*(6*a^6 + 34*a^5*b + 78*a^4*b
^2 + 75*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 - 8*a^6 - 24
*a^5*b - 24*a^4*b^2 - 8*a^3*b^3 + 8*(252*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b
^3)*cosh(d*x + c)^5 + 28*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a
^2*b^4)*cosh(d*x + c)^3 + 3*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 +
 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^5 - 4*(4*a^6 + 28*a^5
*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c)^4 +
4*(420*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^6 - 4*a^6 - 28*a
^5*b - 104*a^4*b^2 - 209*a^3*b^3 - 189*a^2*b^4 - 60*a*b^5 + 70*(6*a^6 + 34*
a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c)^4 + 15*(4*a^6 +
 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x +
c)^2)*sinh(d*x + c)^4 + 16*(60*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d
*x + c)^7 + 14*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*co
sh(d*x + c)^5 + 5*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b
^4 + 60*a*b^5)*cosh(d*x + c)^3 - (4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*
b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(6*a^6 + 3
4*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c)^2 + 4*(90*(a^
6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^8 + 28*(6*a^6 + 34*a^5*b +
 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c)^6 - 6*a^6 - 34*a^5*b -
 78*a^4*b^2 - 75*a^3*b^3 - 25*a^2*b^4 + 15*(4*a^6 + 28*a^5*b + 104*a^4*b^2
+ 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c)^4 - 6*(4*a^6 + 28*a^5
*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c)^2)*s
inh(d*x + c)^2 - 3*((8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^9 + 9*
(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^8 + (8*a^4*b
 + 12*a^3*b^2 + 5*a^2*b^3)*sinh(d*x + c)^9 + 4*(8*a^4*b + 28*a^3*b^2 + 29*a
```

```
^2*b^3 + 10*a*b^4)*cosh(d*x + c)^7 + 4*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 +
 10*a*b^4 + 9*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x
+ c)^7 + 28*(3*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^3 + (8*a^4*
b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^6 + 2*
(24*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5)*cosh(d*x + c)^5
 + 2*(24*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5 + 63*(8*a^4
*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^4 + 42*(8*a^4*b + 28*a^3*b^2 + 2
9*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 2*(63*(8*a^4*b + 1
2*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^5 + 70*(8*a^4*b + 28*a^3*b^2 + 29*a^2*
b^3 + 10*a*b^4)*cosh(d*x + c)^3 + 5*(24*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 +
 136*a*b^4 + 40*b^5)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(8*a^4*b + 28*a^3*b
^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^3 + 4*(21*(8*a^4*b + 12*a^3*b^2 +
 5*a^2*b^3)*cosh(d*x + c)^6 + 8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4
+ 35*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^4 + 5*(24
*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5)*cosh(d*x + c)^2)*s
inh(d*x + c)^3 + 4*(9*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^7 +
21*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^5 + 5*(24*a
^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5)*cosh(d*x + c)^3 + 3*
(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c))*sinh(d*x + c)
^2 + (8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c) + (9*(8*a^4*b + 12*a^
3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^8 + 28*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3
+ 10*a*b^4)*cosh(d*x + c)^6 + 8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3 + 10*(24*a^4
*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5)*cosh(d*x + c)^4 + 12*(
8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c
))*sqrt(-a^2 - a*b)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c
)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x +
 c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*cos
h(d*x + c))*sinh(d*x + c) + 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x +
 c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x
+ c))*sqrt(-a^2 - a*b) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x
 + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x
 + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(
d*x + c))*sinh(d*x + c) + a)) + 8*(10*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)
*cosh(d*x + c)^9 + 4*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b
^4)*cosh(d*x + c)^7 + 3*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189
*a^2*b^4 + 60*a*b^5)*cosh(d*x + c)^5 - 2*(4*a^6 + 28*a^5*b + 104*a^4*b^2 +
209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c)^3 - (6*a^6 + 34*a^5*b +
 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^9
+ 3*a^8*b + 3*a^7*b^2 + a^6*b^3)*d*cosh(d*x + c)^9 + 9*(a^9 + 3*a^8*b + 3*a
^7*b^2 + a^6*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^8 + (a^9 + 3*a^8*b + 3*a^7*
b^2 + a^6*b^3)*d*sinh(d*x + c)^9 + 4*(a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^3
 + 2*a^5*b^4)*d*cosh(d*x + c)^7 + 4*(9*(a^9 + 3*a^8*b + 3*a^7*b^2 + a^6*b^3
)*d*cosh(d*x + c)^2 + (a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^3 + 2*a^5*b^4)*d
)*sinh(d*x + c)^7 + 2*(3*a^9 + 17*a^8*b + 41*a^7*b^2 + 51*a^6*b^3 + 32*a^5*
b^4 + 8*a^4*b^5)*d*cosh(d*x + c)^5 + 28*(3*(a^9 + 3*a^8*b + 3*a^7*b^2 + a^6
*b^3)*d*cosh(d*x + c)^3 + (a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^3 + 2*a^5*b^
4)*d*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(63*(a^9 + 3*a^8*b + 3*a^7*b^2 + a^
6*b^3)*d*cosh(d*x + c)^4 + 42*(a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^3 + 2*a^
5*b^4)*d*cosh(d*x + c)^2 + (3*a^9 + 17*a^8*b + 41*a^7*b^2 + 51*a^6*b^3 + 32
*a^5*b^4 + 8*a^4*b^5)*d)*sinh(d*x + c)^5 + 4*(a^9 + 5*a^8*b + 9*a^7*b^2 + 7
*a^6*b^3 + 2*a^5*b^4)*d*cosh(d*x + c)^3 + 2*(63*(a^9 + 3*a^8*b + 3*a^7*b^2
+ a^6*b^3)*d*cosh(d*x + c)^5 + 70*(a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^3 +
2*a^5*b^4)*d*cosh(d*x + c)^3 + 5*(3*a^9 + 17*a^8*b + 41*a^7*b^2 + 51*a^6*b^
3 + 32*a^5*b^4 + 8*a^4*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(21*(a^9 +
 3*a^8*b + 3*a^7*b^2 + a^6*b^3)*d*cosh(d*x + c)^6 + 35*(a^9 + 5*a^8*b + 9*a
^7*b^2 + 7*a^6*b^3 + 2*a^5*b^4)*d*cosh(d*x + c)^4 + 5*(3*a^9 + 17*a^8*b + 4
1*a^7*b^2 + 51*a^6*b^3 + 32*a^5*b^4 + 8*a^4*b^5)*d*cosh(d*x + c)^2 + (a^9 +
 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^3 + 2*a^5*b^4)*d)*sinh(d*x + c)^3 + (a^9 + 3
*a^8*b + 3*a^7*b^2 + a^6*b^3)*d*cosh(d*x + c) + 4*(9*(a^9 + 3*a^8*b + 3*a^7
```

*b^2 + a^6*b^3)*d*cosh(d*x + c)^7 + 21*(a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b
^3 + 2*a^5*b^4)*d*cosh(d*x + c)^5 + 5*(3*a^9 + 17*a^8*b + 41*a^7*b^2 + 51*a
^6*b^3 + 32*a^5*b^4 + 8*a^4*b^5)*d*cosh(d*x + c)^3 + 3*(a^9 + 5*a^8*b + 9*a
^7*b^2 + 7*a^6*b^3 + 2*a^5*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (9*(a^9
+ 3*a^8*b + 3*a^7*b^2 + a^6*b^3)*d*cosh(d*x + c)^8 + 28*(a^9 + 5*a^8*b + 9*
a^7*b^2 + 7*a^6*b^3 + 2*a^5*b^4)*d*cosh(d*x + c)^6 + 10*(3*a^9 + 17*a^8*b +
41*a^7*b^2 + 51*a^6*b^3 + 32*a^5*b^4 + 8*a^4*b^5)*d*cosh(d*x + c)^4 + 12*(
a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^3 + 2*a^5*b^4)*d*cosh(d*x + c)^2 + (a^9
+ 3*a^8*b + 3*a^7*b^2 + a^6*b^3)*d)*sinh(d*x + c)), 1/8*(4*(a^6 + 3*a^5*b
+ 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^10 + 40*(a^6 + 3*a^5*b + 3*a^4*b^2 + a
^3*b^3)*cosh(d*x + c)*sinh(d*x + c)^9 + 4*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*
b^3)*sinh(d*x + c)^10 + 2*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*
a^2*b^4)*cosh(d*x + c)^8 + 2*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 +
25*a^2*b^4 + 90*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^2)*sinh
(d*x + c)^8 + 16*(30*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^3
+ (6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c))*
sinh(d*x + c)^7 + 2*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2
*b^4 + 60*a*b^5)*cosh(d*x + c)^6 + 2*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*
a^3*b^3 + 189*a^2*b^4 + 60*a*b^5 + 420*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3
)*cosh(d*x + c)^4 + 28*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2
*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 - 4*a^6 - 12*a^5*b - 12*a^4*b^2 - 4*
a^3*b^3 + 4*(252*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^5 + 28
*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c)^3
+ 3*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)
*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a
^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c)^4 + 2*(420*(a^6 + 3*a^5*b +
3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^6 - 4*a^6 - 28*a^5*b - 104*a^4*b^2 - 209
*a^3*b^3 - 189*a^2*b^4 - 60*a*b^5 + 70*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*
a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c)^4 + 15*(4*a^6 + 28*a^5*b + 104*a^4*b^2
+ 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 +
8*(60*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^7 + 14*(6*a^6 + 3
4*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c)^5 + 5*(4*a^6
+ 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x +
c)^3 - (4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*
b^5)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75
*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c)^2 + 2*(90*(a^6 + 3*a^5*b + 3*a^4*b^2 +
a^3*b^3)*cosh(d*x + c)^8 + 28*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3
+ 25*a^2*b^4)*cosh(d*x + c)^6 - 6*a^6 - 34*a^5*b - 78*a^4*b^2 - 75*a^3*b^3
- 25*a^2*b^4 + 15*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b
^4 + 60*a*b^5)*cosh(d*x + c)^4 - 6*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^
3*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 3*((8*a^
4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^9 + 9*(8*a^4*b + 12*a^3*b^2 + 5
*a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^8 + (8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3
)*sinh(d*x + c)^9 + 4*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d
x + c)^7 + 4(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4 + 9*(8*a^4*b +
12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 28*(3*(8*a^4*b +
12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^3 + (8*a^4*b + 28*a^3*b^2 + 29*a^2*b
^3 + 10*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(24*a^4*b + 100*a^3*b^2 +
175*a^2*b^3 + 136*a*b^4 + 40*b^5)*cosh(d*x + c)^5 + 2*(24*a^4*b + 100*a^3*
b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5 + 63*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b
^3)*cosh(d*x + c)^4 + 42*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cos
h(d*x + c)^2)*sinh(d*x + c)^5 + 2*(63*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*co
sh(d*x + c)^5 + 70*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x
+ c)^3 + 5*(24*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5)*cosh
(d*x + c))*sinh(d*x + c)^4 + 4*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^
4)*cosh(d*x + c)^3 + 4*(21*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)
^6 + 8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4 + 35*(8*a^4*b + 28*a^3*b^
2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^4 + 5*(24*a^4*b + 100*a^3*b^2 + 17
5*a^2*b^3 + 136*a*b^4 + 40*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 4*(9*(8*

a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^7 + 21*(8*a^4*b + 28*a^3*b^2
+ 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^5 + 5*(24*a^4*b + 100*a^3*b^2 + 175*
a^2*b^3 + 136*a*b^4 + 40*b^5)*cosh(d*x + c)^3 + 3*(8*a^4*b + 28*a^3*b^2 + 2
9*a^2*b^3 + 10*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + (8*a^4*b + 12*a^3*b^
2 + 5*a^2*b^3)*cosh(d*x + c) + (9*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d
x + c)^8 + 28(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)
^6 + 8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3 + 10*(24*a^4*b + 100*a^3*b^2 + 175*a^
2*b^3 + 136*a*b^4 + 40*b^5)*cosh(d*x + c)^4 + 12*(8*a^4*b + 28*a^3*b^2 + 29
*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(a^2 + a*b)*arctan
(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x +
c)^3 + (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*a + 4*b)*sinh(d
*x + c))/sqrt(a^2 + a*b)) - 3*((8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x
+ c)^9 + 9*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^8
+ (8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*sinh(d*x + c)^9 + 4*(8*a^4*b + 28*a^3
*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^7 + 4*(8*a^4*b + 28*a^3*b^2 + 2
9*a^2*b^3 + 10*a*b^4 + 9*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^2
)*sinh(d*x + c)^7 + 28*(3*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^
3 + (8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c))*sinh(d*x
+ c)^6 + 2*(24*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5)*cosh
(d*x + c)^5 + 2*(24*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5
+ 63*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^4 + 42*(8*a^4*b + 28*
a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 2*(63*(
8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^5 + 70*(8*a^4*b + 28*a^3*b^
2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^3 + 5*(24*a^4*b + 100*a^3*b^2 + 17
5*a^2*b^3 + 136*a*b^4 + 40*b^5)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(8*a^4*b
+ 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^3 + 4*(21*(8*a^4*b + 1
2*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^6 + 8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3
+ 10*a*b^4 + 35*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c
)^4 + 5*(24*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5)*cosh(d*
x + c)^2)*sinh(d*x + c)^3 + 4*(9*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*
x + c)^7 + 21*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^
5 + 5*(24*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5)*cosh(d*x
+ c)^3 + 3*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c))*si
nh(d*x + c)^2 + (8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c) + (9*(8*a^
4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^8 + 28*(8*a^4*b + 28*a^3*b^2 +
29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^6 + 8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3 +
10*(24*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5)*cosh(d*x +
c)^4 + 12*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^2)*s
inh(d*x + c))*sqrt(a^2 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + s
inh(d*x + c))/(a + b)) + 4*(10*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d
x + c)^9 + 4(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*cos
h(d*x + c)^7 + 3*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^
4 + 60*a*b^5)*cosh(d*x + c)^5 - 2*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3
*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c)^3 - (6*a^6 + 34*a^5*b + 78*a^4
*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^9 + 3*a^8
*b + 3*a^7*b^2 + a^6*b^3)*d*cosh(d*x + c)^9 + 9*(a^9 + 3*a^8*b + 3*a^7*b^2
+ a^6*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^8 + (a^9 + 3*a^8*b + 3*a^7*b^2 + a
^6*b^3)*d*sinh(d*x + c)^9 + 4*(a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^3 + 2*a^
5*b^4)*d*cosh(d*x + c)^7 + 4*(9*(a^9 + 3*a^8*b + 3*a^7*b^2 + a^6*b^3)*d*cos
h(d*x + c)^2 + (a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^3 + 2*a^5*b^4)*d)*sinh(
d*x + c)^7 + 2*(3*a^9 + 17*a^8*b + 41*a^7*b^2 + 51*a^6*b^3 + 32*a^5*b^4 + 8
*a^4*b^5)*d*cosh(d*x + c)^5 + 28*(3*(a^9 + 3*a^8*b + 3*a^7*b^2 + a^6*b^3)*d
*cosh(d*x + c)^3 + (a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^3 + 2*a^5*b^4)*d*co
sh(d*x + c))*sinh(d*x + c)^6 + 2*(63*(a^9 + 3*a^8*b + 3*a^7*b^2 + a^6*b^3)*
d*cosh(d*x + c)^4 + 42*(a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^3 + 2*a^5*b^4)*
d*cosh(d*x + c)^2 + (3*a^9 + 17*a^8*b + 41*a^7*b^2 + 51*a^6*b^3 + 32*a^5*b^
4 + 8*a^4*b^5)*d)*sinh(d*x + c)^5 + 4*(a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^
3 + 2*a^5*b^4)*d*cosh(d*x + c)^3 + 2*(63*(a^9 + 3*a^8*b + 3*a^7*b^2 + a^6*b
^3)*d*cosh(d*x + c)^5 + 70*(a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^3 + 2*a^5*b

```
^4)*d*cosh(d*x + c)^3 + 5*(3*a^9 + 17*a^8*b + 41*a^7*b^2 + 51*a^6*b^3 + 32*
a^5*b^4 + 8*a^4*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(21*(a^9 + 3*a^8*
b + 3*a^7*b^2 + a^6*b^3)*d*cosh(d*x + c)^6 + 35*(a^9 + 5*a^8*b + 9*a^7*b^2
+ 7*a^6*b^3 + 2*a^5*b^4)*d*cosh(d*x + c)^4 + 5*(3*a^9 + 17*a^8*b + 41*a^7*b
^2 + 51*a^6*b^3 + 32*a^5*b^4 + 8*a^4*b^5)*d*cosh(d*x + c)^2 + (a^9 + 5*a^8*
b + 9*a^7*b^2 + 7*a^6*b^3 + 2*a^5*b^4)*d)*sinh(d*x + c)^3 + (a^9 + 3*a^8*b
+ 3*a^7*b^2 + a^6*b^3)*d*cosh(d*x + c) + 4*(9*(a^9 + 3*a^8*b + 3*a^7*b^2 +
a^6*b^3)*d*cosh(d*x + c)^7 + 21*(a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^3 + 2*
a^5*b^4)*d*cosh(d*x + c)^5 + 5*(3*a^9 + 17*a^8*b + 41*a^7*b^2 + 51*a^6*b^3
+ 32*a^5*b^4 + 8*a^4*b^5)*d*cosh(d*x + c)^3 + 3*(a^9 + 5*a^8*b + 9*a^7*b^2
+ 7*a^6*b^3 + 2*a^5*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (9*(a^9 + 3*a^8
*b + 3*a^7*b^2 + a^6*b^3)*d*cosh(d*x + c)^8 + 28*(a^9 + 5*a^8*b + 9*a^7*b^2
 + 7*a^6*b^3 + 2*a^5*b^4)*d*cosh(d*x + c)^6 + 10*(3*a^9 + 17*a^8*b + 41*a^7
*b^2 + 51*a^6*b^3 + 32*a^5*b^4 + 8*a^4*b^5)*d*cosh(d*x + c)^4 + 12*(a^9 + 5
*a^8*b + 9*a^7*b^2 + 7*a^6*b^3 + 2*a^5*b^4)*d*cosh(d*x + c)^2 + (a^9 + 3*a^
8*b + 3*a^7*b^2 + a^6*b^3)*d)*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.95 $\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=142

$$\frac{\left(8a^2+8ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{5/2}d(a+b)^{5/2}}-\frac{3b(2a+b)\sinh(c+dx)}{8a^2d(a+b)^2\left(a\sinh^2(c+dx)+a+b\right)}-\frac{b\sinh(c+dx)\cosh^2(c+dx)}{4ad(a+b)\left(a\sinh^2(c+dx)+a+b\right.}$$

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(8*a^
(5/2)*(a + b)^(5/2)*d) - (b*Cosh[c + d*x]^2*Sinh[c + d*x])/(4*a*(a + b)*d*(
a + b + a*Sinh[c + d*x]^2)^2) - (3*b*(2*a + b)*Sinh[c + d*x])/(8*a^2*(a + b
)^2*d*(a + b + a*Sinh[c + d*x]^2))
```

Rubi [A] time = 0.13668, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4147, 413, 385, 205}

$$\frac{\left(8a^2+8ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{5/2}d(a+b)^{5/2}}-\frac{3b(2a+b)\sinh(c+dx)}{8a^2d(a+b)^2\left(a\sinh^2(c+dx)+a+b\right)}-\frac{b\sinh(c+dx)\cosh^2(c+dx)}{4ad(a+b)\left(a\sinh^2(c+dx)+a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(8*a^
(5/2)*(a + b)^(5/2)*d) - (b*Cosh[c + d*x]^2*Sinh[c + d*x])/(4*a*(a + b)*d*(
a + b + a*Sinh[c + d*x]^2)^2) - (3*b*(2*a + b)*Sinh[c + d*x])/(8*a^2*(a + b
)^2*d*(a + b + a*Sinh[c + d*x]^2))
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(1+x^2\right)^2}{\left(a+b+ax^2\right)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{b\cosh^2(c+dx)\sinh(c+dx)}{4a(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} + \frac{\operatorname{Subst}\left(\int \frac{4a+b+(4a+3b)x^2}{\left(a+b+ax^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{4a(a+b)d}$$

$$= -\frac{b\cosh^2(c+dx)\sinh(c+dx)}{4a(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} - \frac{3b(2a+b)\sinh(c+dx)}{8a^2(a+b)^2d\left(a+b+a\sinh^2(c+dx)\right)} + \frac{\left(8a^2 +\right.}{}$$

$$= \frac{\left(8a^2+8ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{5/2}(a+b)^{5/2}d} - \frac{b\cosh^2(c+dx)\sinh(c+dx)}{4a(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} - \frac{}{8a^2(a+}$$

Mathematica [A] time = 2.85719, size = 214, normalized size = 1.51

$$\frac{\operatorname{sech}^5(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\frac{\left(8a^2+8ab+3b^2\right)(\sinh(c)-\cosh(c))\operatorname{sech}(c+dx)(a\cosh(2(c+dx))+a+2b)^2\tan^{-1}\left(\frac{\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}}{}\right.}{\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}}\right.}{64a^{5/2}d(a+b)^2\left(a+b\mathrm{se}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^5*(((8*a^2 + 8*a*b + 3*b^2)*
ArcTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Si
nh[c]))/Sqrt[a]]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]*(-Cosh[c]
+ Sinh[c]))/(Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]) + 8*Sqrt[a]*b^2*(a +
b)*Tanh[c + d*x] - 2*Sqrt[a]*b*(8*a + 5*b)*(a + 2*b + a*Cosh[2*(c + d*x)])*
Tanh[c + d*x]))/(64*a^(5/2)*(a + b)^2*d*(a + b*Sech[c + d*x]^2)^3)
```

Maple [B] time = 0.082, size = 1174, normalized size = 8.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 2/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^
2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*b/(a+b)/a*tanh(1/2*d*x+1/2*c)^7+3/4/d*
b^2/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2
```

```
*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^7+2/d/(t
anh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*
tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*b/(a+b)^2*tanh(1/2*d*x+1/2*c)^5-13/4/d*b^2/a
/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a
-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5-9/4/d*b^3/a
^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2
*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5-2/d/(tanh
(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tan
h(1/2*d*x+1/2*c)^2*b+a+b)^2*b/(a+b)^2*tanh(1/2*d*x+1/2*c)^3+13/4/d*b^2/a/(t
anh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*
tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3+9/4/d*b^3/a^2/
(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-
2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3-2/d/(tanh(1/
2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1
/2*d*x+1/2*c)^2*b+a+b)^2*b/(a+b)/a*tanh(1/2*d*x+1/2*c)-3/4/d*b^2/a^2/(tanh(
1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh
(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)+1/d/(a^2+2*a*b+b^2)/(a
+b)^(1/2)/a^(1/2)*arctan(1/2*(2*tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)+2*b^(1/2))/
a^(1/2))+1/d*b/a^(3/2)/(a^2+2*a*b+b^2)/(a+b)^(1/2)*arctan(1/2*(2*tanh(1/2*d
*x+1/2*c)*(a+b)^(1/2)+2*b^(1/2))/a^(1/2))+3/8/d*b^2/a^(5/2)/(a^2+2*a*b+b^2)
/(a+b)^(1/2)*arctan(1/2*(2*tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)+2*b^(1/2))/a^(1/
2))-1/d/(a^2+2*a*b+b^2)/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(-2*tanh(1/2*d*x+1/2
*c)*(a+b)^(1/2)+2*b^(1/2))/a^(1/2))-1/d/a^(3/2)/(a^2+2*a*b+b^2)/(a+b)^(1/2)
*arctan(1/2*(-2*tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)+2*b^(1/2))/a^(1/2))*b-3/8/d
/a^(5/2)/(a^2+2*a*b+b^2)/(a+b)^(1/2)*arctan(1/2*(-2*tanh(1/2*d*x+1/2*c)*(a+
b)^(1/2)+2*b^(1/2))/a^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*((8*a^2*b*e^(7*c) + 5*a*b^2*e^(7*c))*e^(7*d*x) + (8*a^2*b*e^(5*c) + 29
*a*b^2*e^(5*c) + 12*b^3*e^(5*c))*e^(5*d*x) - (8*a^2*b*e^(3*c) + 29*a*b^2*e^
(3*c) + 12*b^3*e^(3*c))*e^(3*d*x) - (8*a^2*b*e^c + 5*a*b^2*e^c)*e^(d*x))/(a
^6*d + 2*a^5*b*d + a^4*b^2*d + (a^6*d*e^(8*c) + 2*a^5*b*d*e^(8*c) + a^4*b^2
*d*e^(8*c))*e^(8*d*x) + 4*(a^6*d*e^(6*c) + 4*a^5*b*d*e^(6*c) + 5*a^4*b^2*d*
e^(6*c) + 2*a^3*b^3*d*e^(6*c))*e^(6*d*x) + 2*(3*a^6*d*e^(4*c) + 14*a^5*b*d*
e^(4*c) + 27*a^4*b^2*d*e^(4*c) + 24*a^3*b^3*d*e^(4*c) + 8*a^2*b^4*d*e^(4*c)
)*e^(4*d*x) + 4*(a^6*d*e^(2*c) + 4*a^5*b*d*e^(2*c) + 5*a^4*b^2*d*e^(2*c) +
2*a^3*b^3*d*e^(2*c))*e^(2*d*x)) + 2*integrate(1/8*((8*a^2*e^(3*c) + 8*a*b*e
^(3*c) + 3*b^2*e^(3*c))*e^(3*d*x) + (8*a^2*e^c + 8*a*b*e^c + 3*b^2*e^c)*e^(
d*x))/(a^5 + 2*a^4*b + a^3*b^2 + (a^5*e^(4*c) + 2*a^4*b*e^(4*c) + a^3*b^2*e
^(4*c))*e^(4*d*x) + 2*(a^5*e^(2*c) + 4*a^4*b*e^(2*c) + 5*a^3*b^2*e^(2*c) +
2*a^2*b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 3.13683, size = 15863, normalized size = 111.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^7 + 28*(8*a^4*b
+ 13*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + 4*(8*a^4*b + 13*a
^3*b^2 + 5*a^2*b^3)*sinh(d*x + c)^7 + 4*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3
+ 12*a*b^4)*cosh(d*x + c)^5 + 4*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b
^4 + 21*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5
 + 20*(7*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^3 + (8*a^4*b + 37
*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*(8*a^4
*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^3 - 4*(8*a^4*b + 37*
a^3*b^2 + 41*a^2*b^3 + 12*a*b^4 - 35*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*cos
h(d*x + c)^4 - 10*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*cosh(d*x +
 c)^2)*sinh(d*x + c)^3 + 4*(21*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*cosh(d*x
+ c)^5 + 10*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^3
- 3*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*cosh(d*x + c))*sinh(d*x
+ c)^2 + ((8*a^4 + 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^8 + 8*(8*a^4 + 8*a^3*
b + 3*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (8*a^4 + 8*a^3*b + 3*a^2*b^2
)*sinh(d*x + c)^8 + 4*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cosh(d*x +
c)^6 + 4*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3 + 7*(8*a^4 + 8*a^3*b + 3*
a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(8*a^4 + 8*a^3*b + 3*a^2*b
^2)*cosh(d*x + c)^3 + 3*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cosh(d*x
+ c))*sinh(d*x + c)^5 + 2*(24*a^4 + 88*a^3*b + 137*a^2*b^2 + 88*a*b^3 + 24*
b^4)*cosh(d*x + c)^4 + 2*(35*(8*a^4 + 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^4
+ 24*a^4 + 88*a^3*b + 137*a^2*b^2 + 88*a*b^3 + 24*b^4 + 30*(8*a^4 + 24*a^3*
b + 19*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*a^4 + 8*a^3*
b + 3*a^2*b^2 + 8*(7*(8*a^4 + 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^5 + 10*(8*
a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^3 + (24*a^4 + 88*a^3*b
 + 137*a^2*b^2 + 88*a*b^3 + 24*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(8*a
^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^2 + 4*(7*(8*a^4 + 8*a^3
*b + 3*a^2*b^2)*cosh(d*x + c)^6 + 15*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b
^3)*cosh(d*x + c)^4 + 8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3 + 3*(24*a^4 +
 88*a^3*b + 137*a^2*b^2 + 88*a*b^3 + 24*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)
^2 + 8*((8*a^4 + 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^7 + 3*(8*a^4 + 24*a^3*b
 + 19*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^5 + (24*a^4 + 88*a^3*b + 137*a^2*b^2
 + 88*a*b^3 + 24*b^4)*cosh(d*x + c)^3 + (8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*
a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 - a*b)*log((a*cosh(d*x + c)^
4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*c
osh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a
*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x +
 c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c
)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a)/(a*cosh(d*x +
 c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)
*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a
*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - 4*(8*a^4*
b + 13*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c) + 4*(7*(8*a^4*b + 13*a^3*b^2 + 5*
a^2*b^3)*cosh(d*x + c)^6 - 8*a^4*b - 13*a^3*b^2 - 5*a^2*b^3 + 5*(8*a^4*b +
37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^4 - 3*(8*a^4*b + 37*a^3*b
^2 + 41*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^8 + 3*a^7*b
 + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^8 + 8*(a^8 + 3*a^7*b + 3*a^6*b^2 +
a^5*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5
*b^3)*d*sinh(d*x + c)^8 + 4*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*
b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(
d*x + c)^2 + (a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d)*sinh(d*
x + c)^6 + 2*(3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b^3 + 32*a^4*b^4 + 8*a
^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*co
sh(d*x + c)^3 + 3*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*cos
h(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d
*cosh(d*x + c)^4 + 30*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d
*cosh(d*x + c)^2 + (3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b^3 + 32*a^4*b^4
```

```
 + 8*a^3*b^5)*d)*sinh(d*x + c)^4 + 4*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3
 + 2*a^4*b^4)*d*cosh(d*x + c)^2 + 8*(7*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3
)*d*cosh(d*x + c)^5 + 10*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4
)*d*cosh(d*x + c)^3 + (3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b^3 + 32*a^4*
b^4 + 8*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^8 + 3*a^7*b + 3
*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^6 + 15*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a
^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^4 + 3*(3*a^8 + 17*a^7*b + 41*a^6*b^2 +
51*a^5*b^3 + 32*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^2 + (a^8 + 5*a^7*b + 9
*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d)*sinh(d*x + c)^2 + (a^8 + 3*a^7*b + 3*a
^6*b^2 + a^5*b^3)*d + 8*((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x +
 c)^7 + 3*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x +
c)^5 + (3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b^3 + 32*a^4*b^4 + 8*a^3*b^5
)*d*cosh(d*x + c)^3 + (a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d
*cosh(d*x + c))*sinh(d*x + c)), -1/8*(2*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*
cosh(d*x + c)^7 + 14*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)*sinh(
d*x + c)^6 + 2*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*sinh(d*x + c)^7 + 2*(8*a^
4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^5 + 2*(8*a^4*b + 37
*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4 + 21*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*co
sh(d*x + c)^2)*sinh(d*x + c)^5 + 10*(7*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*c
osh(d*x + c)^3 + (8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*cosh(d*x +
c))*sinh(d*x + c)^4 - 2*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*cosh
(d*x + c)^3 - 2*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4 - 35*(8*a^4*b
 + 13*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^4 - 10*(8*a^4*b + 37*a^3*b^2 + 41*
a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(21*(8*a^4*b + 13*
a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^5 + 10*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^
3 + 12*a*b^4)*cosh(d*x + c)^3 - 3*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a
*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 - ((8*a^4 + 8*a^3*b + 3*a^2*b^2)*cosh(
d*x + c)^8 + 8*(8*a^4 + 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^7
+ (8*a^4 + 8*a^3*b + 3*a^2*b^2)*sinh(d*x + c)^8 + 4*(8*a^4 + 24*a^3*b + 19*
a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^6 + 4*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a
*b^3 + 7*(8*a^4 + 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8
*(7*(8*a^4 + 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^3 + 3*(8*a^4 + 24*a^3*b + 1
9*a^2*b^2 + 6*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(24*a^4 + 88*a^3*b
+ 137*a^2*b^2 + 88*a*b^3 + 24*b^4)*cosh(d*x + c)^4 + 2*(35*(8*a^4 + 8*a^3*b
 + 3*a^2*b^2)*cosh(d*x + c)^4 + 24*a^4 + 88*a^3*b + 137*a^2*b^2 + 88*a*b^3
+ 24*b^4 + 30*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^2)*si
nh(d*x + c)^4 + 8*a^4 + 8*a^3*b + 3*a^2*b^2 + 8*(7*(8*a^4 + 8*a^3*b + 3*a^2
*b^2)*cosh(d*x + c)^5 + 10*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cosh(d
*x + c)^3 + (24*a^4 + 88*a^3*b + 137*a^2*b^2 + 88*a*b^3 + 24*b^4)*cosh(d*x
+ c))*sinh(d*x + c)^3 + 4*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cosh(d*
x + c)^2 + 4*(7*(8*a^4 + 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^6 + 15*(8*a^4 +
 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^4 + 8*a^4 + 24*a^3*b + 19*a
^2*b^2 + 6*a*b^3 + 3*(24*a^4 + 88*a^3*b + 137*a^2*b^2 + 88*a*b^3 + 24*b^4)*
cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((8*a^4 + 8*a^3*b + 3*a^2*b^2)*cosh(d*
x + c)^7 + 3*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^5 + (2
4*a^4 + 88*a^3*b + 137*a^2*b^2 + 88*a*b^3 + 24*b^4)*cosh(d*x + c)^3 + (8*a^
4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2
 + a*b)*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 +
 a*sinh(d*x + c)^3 + (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*a
 + 4*b)*sinh(d*x + c))/sqrt(a^2 + a*b)) - ((8*a^4 + 8*a^3*b + 3*a^2*b^2)*co
sh(d*x + c)^8 + 8*(8*a^4 + 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)
^7 + (8*a^4 + 8*a^3*b + 3*a^2*b^2)*sinh(d*x + c)^8 + 4*(8*a^4 + 24*a^3*b +
19*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^6 + 4*(8*a^4 + 24*a^3*b + 19*a^2*b^2 +
6*a*b^3 + 7*(8*a^4 + 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6
+ 8*(7*(8*a^4 + 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^3 + 3*(8*a^4 + 24*a^3*b
+ 19*a^2*b^2 + 6*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(24*a^4 + 88*a^3
*b + 137*a^2*b^2 + 88*a*b^3 + 24*b^4)*cosh(d*x + c)^4 + 2*(35*(8*a^4 + 8*a^
3*b + 3*a^2*b^2)*cosh(d*x + c)^4 + 24*a^4 + 88*a^3*b + 137*a^2*b^2 + 88*a*b
^3 + 24*b^4 + 30*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^2)
```

```
*sinh(d*x + c)^4 + 8*a^4 + 8*a^3*b + 3*a^2*b^2 + 8*(7*(8*a^4 + 8*a^3*b + 3*
a^2*b^2)*cosh(d*x + c)^5 + 10*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cos
h(d*x + c)^3 + (24*a^4 + 88*a^3*b + 137*a^2*b^2 + 88*a*b^3 + 24*b^4)*cosh(d
*x + c))*sinh(d*x + c)^3 + 4*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cosh
(d*x + c)^2 + 4*(7*(8*a^4 + 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^6 + 15*(8*a^
4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^4 + 8*a^4 + 24*a^3*b + 1
9*a^2*b^2 + 6*a*b^3 + 3*(24*a^4 + 88*a^3*b + 137*a^2*b^2 + 88*a*b^3 + 24*b^
4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((8*a^4 + 8*a^3*b + 3*a^2*b^2)*cosh
(d*x + c)^7 + 3*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^5 +
 (24*a^4 + 88*a^3*b + 137*a^2*b^2 + 88*a*b^3 + 24*b^4)*cosh(d*x + c)^3 + (8
*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(
a^2 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/(a +
b)) - 2*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c) + 2*(7*(8*a^4*b +
13*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^6 - 8*a^4*b - 13*a^3*b^2 - 5*a^2*b^3
+ 5*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^4 - 3*(8*a
^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c))/
((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^8 + 8*(a^8 + 3*a^7*b
 + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^8 + 3*a^7*b +
3*a^6*b^2 + a^5*b^3)*d*sinh(d*x + c)^8 + 4*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a
^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^8 + 3*a^7*b + 3*a^6*b^2 + a
^5*b^3)*d*cosh(d*x + c)^2 + (a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*
b^4)*d)*sinh(d*x + c)^6 + 2*(3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b^3 + 3
2*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^8 + 3*a^7*b + 3*a^6*b^2
+ a^5*b^3)*d*cosh(d*x + c)^3 + 3*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2
*a^4*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^8 + 3*a^7*b + 3*a^6*b
^2 + a^5*b^3)*d*cosh(d*x + c)^4 + 30*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3
 + 2*a^4*b^4)*d*cosh(d*x + c)^2 + (3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b
^3 + 32*a^4*b^4 + 8*a^3*b^5)*d)*sinh(d*x + c)^4 + 4*(a^8 + 5*a^7*b + 9*a^6*
b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^2 + 8*(7*(a^8 + 3*a^7*b + 3*a^
6*b^2 + a^5*b^3)*d*cosh(d*x + c)^5 + 10*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*
b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^3 + (3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^
5*b^3 + 32*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^
8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^6 + 15*(a^8 + 5*a^7*b +
9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^4 + 3*(3*a^8 + 17*a^7*b
+ 41*a^6*b^2 + 51*a^5*b^3 + 32*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^2 + (a^
8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d)*sinh(d*x + c)^2 + (a^8
+ 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d + 8*((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^
3)*d*cosh(d*x + c)^7 + 3*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4
)*d*cosh(d*x + c)^5 + (3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b^3 + 32*a^4*
b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^3 + (a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3
 + 2*a^4*b^4)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)**2)**3,x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.96 $\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=108

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8\sqrt{b}d(a+b)^{5/2}}+\frac{3\tanh(c+dx)}{8d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)}+\frac{\tanh(c+dx)}{4d(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}$$

```
[Out] (3*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*Sqrt[b]*(a + b)^(5/2)*d
) + Tanh[c + d*x]/(4*(a + b)*d*(a + b - b*Tanh[c + d*x]^2)^2) + (3*Tanh[c +
 d*x])/(8*(a + b)^2*d*(a + b - b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.090436, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4146, 199, 208}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8\sqrt{b}d(a+b)^{5/2}}+\frac{3\tanh(c+dx)}{8d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)}+\frac{\tanh(c+dx)}{4d(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (3*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*Sqrt[b]*(a + b)^(5/2)*d
) + Tanh[c + d*x]/(4*(a + b)*d*(a + b - b*Tanh[c + d*x]^2)^2) + (3*Tanh[c +
 d*x])/(8*(a + b)^2*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{\left(a+b-bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh(c+dx)}{4(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{3\operatorname{Subst}\left(\int \frac{1}{\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{4(a+b)d}$$

$$= \frac{\tanh(c+dx)}{4(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{3\tanh(c+dx)}{8(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{3\operatorname{Subs}}{}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8\sqrt{b}(a+b)^{5/2}d} + \frac{\tanh(c+dx)}{4(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{3\tanh(c}{8(a+b)^2d\left(a+b-\right.}$$

Mathematica [B] time = 2.07828, size = 258, normalized size = 2.39

$$\frac{\operatorname{sech}^6(c+dx)(a\cosh(2(c+dx))+a+2b)\left(-\frac{\operatorname{sech}(2c)\left(\left(5a^2+16ab+8b^2\right)\sinh(2c)-a(5a+2b)\sinh(2dx)\right)(a\cosh(2(c+dx))+a+2b)}{a^2}+\frac{4b(a+b)\operatorname{se}}{}\right.}{64d(a+b)^2\left(a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^6*((3*ArcTanh[(Sech[d*x]*(Co
sh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a +
 b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*(Cosh
[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (4*b*(a +
 b)*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/a^2 - ((a + 2*b + a*Co
sh[2*(c + d*x)])*Sech[2*c]*((5*a^2 + 16*a*b + 8*b^2)*Sinh[2*c] - a*(5*a + 2
*b)*Sinh[2*d*x]))/a^2))/(64*(a + b)^2*d*(a + b*Sech[c + d*x]^2)^3)
```

Maple [B] time = 0.073, size = 612, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 5/4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c
)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^7+15/4/d/(
tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2
*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5*a+3/4/d/(tanh
(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tan
h(1/2*d*x+1/2*c)^2*b+a+b)^2*b/(a+b)^2*tanh(1/2*d*x+1/2*c)^5+15/4/d/(tanh(1/
2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1
/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3*a+3/4/d/(tanh(1/2*d*
x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d
```

```
*x+1/2*c)^2*b+a+b)^2*b/(a+b)^2*tanh(1/2*d*x+1/2*c)^3+5/4/d/(tanh(1/2*d*x+1/
2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1
/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)+3/16/d/(a^2+2*a*b+b^2)/b^(1/2)/(
a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/
2)+(a+b)^(1/2))-3/16/d/(a^2+2*a*b+b^2)/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*t
anh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.79423, size = 11784, normalized size = 109.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^6 + 2
4*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)*sinh(d*x + c)
^5 + 4*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a*b^4)*sinh(d*x + c)^6 + 20*a
^4*b + 28*a^3*b^2 + 8*a^2*b^3 + 4*(15*a^4*b + 61*a^3*b^2 + 102*a^2*b^3 + 72
*a*b^4 + 16*b^5)*cosh(d*x + c)^4 + 4*(15*a^4*b + 61*a^3*b^2 + 102*a^2*b^3 +
 72*a*b^4 + 16*b^5 + 15*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a*b^4)*cosh(
d*x + c)^2)*sinh(d*x + c)^4 + 16*(5*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*
a*b^4)*cosh(d*x + c)^3 + (15*a^4*b + 61*a^3*b^2 + 102*a^2*b^3 + 72*a*b^4 +
16*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(15*a^4*b + 47*a^3*b^2 + 40*a^2*
b^3 + 8*a*b^4)*cosh(d*x + c)^2 + 4*(15*a^4*b + 47*a^3*b^2 + 40*a^2*b^3 + 8*
a*b^4 + 15*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^4 +
6*(15*a^4*b + 61*a^3*b^2 + 102*a^2*b^3 + 72*a*b^4 + 16*b^5)*cosh(d*x + c)^2
)*sinh(d*x + c)^2 - 3*(a^4*cosh(d*x + c)^8 + 8*a^4*cosh(d*x + c)*sinh(d*x +
 c)^7 + a^4*sinh(d*x + c)^8 + 4*(a^4 + 2*a^3*b)*cosh(d*x + c)^6 + 4*(7*a^4*
cosh(d*x + c)^2 + a^4 + 2*a^3*b)*sinh(d*x + c)^6 + 8*(7*a^4*cosh(d*x + c)^3
 + 3*(a^4 + 2*a^3*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^4 + 8*a^3*b +
8*a^2*b^2)*cosh(d*x + c)^4 + 2*(35*a^4*cosh(d*x + c)^4 + 3*a^4 + 8*a^3*b +
8*a^2*b^2 + 30*(a^4 + 2*a^3*b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + a^4 + 8*(
7*a^4*cosh(d*x + c)^5 + 10*(a^4 + 2*a^3*b)*cosh(d*x + c)^3 + (3*a^4 + 8*a^3
*b + 8*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 + 2*a^3*b)*cosh(d*x
 + c)^2 + 4*(7*a^4*cosh(d*x + c)^6 + 15*(a^4 + 2*a^3*b)*cosh(d*x + c)^4 + a
^4 + 2*a^3*b + 3*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x +
c)^2 + 8*(a^4*cosh(d*x + c)^7 + 3*(a^4 + 2*a^3*b)*cosh(d*x + c)^5 + (3*a^4
+ 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 + (a^4 + 2*a^3*b)*cosh(d*x + c))*sin
h(d*x + c))*sqrt(a*b + b^2)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*
sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2
*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^
2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4
*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 +
 a + 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x
```

+ c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x
+ c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d
*x + c))*sinh(d*x + c) + a)) + 8*(3*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*
a*b^4)*cosh(d*x + c)^5 + 2*(15*a^4*b + 61*a^3*b^2 + 102*a^2*b^3 + 72*a*b^4
+ 16*b^5)*cosh(d*x + c)^3 + (15*a^4*b + 47*a^3*b^2 + 40*a^2*b^3 + 8*a*b^4)*
cosh(d*x + c))*sinh(d*x + c))/((a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*
cosh(d*x + c)^8 + 8*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*cosh(d*x +
c)*sinh(d*x + c)^7 + (a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*sinh(d*x +
c)^8 + 4*(a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh(d*
x + c)^6 + 4*(7*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*cosh(d*x + c)^2
+ (a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d)*sinh(d*x + c)
^6 + 2*(3*a^7*b + 17*a^6*b^2 + 41*a^5*b^3 + 51*a^4*b^4 + 32*a^3*b^5 + 8*a^2
*b^6)*d*cosh(d*x + c)^4 + 8*(7*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*
cosh(d*x + c)^3 + 3*(a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)
*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 +
a^4*b^4)*d*cosh(d*x + c)^4 + 30*(a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4
+ 2*a^3*b^5)*d*cosh(d*x + c)^2 + (3*a^7*b + 17*a^6*b^2 + 41*a^5*b^3 + 51*a^
4*b^4 + 32*a^3*b^5 + 8*a^2*b^6)*d)*sinh(d*x + c)^4 + 4*(a^7*b + 5*a^6*b^2 +
9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^7*b + 3*a^6
*b^2 + 3*a^5*b^3 + a^4*b^4)*d*cosh(d*x + c)^5 + 10*(a^7*b + 5*a^6*b^2 + 9*a
^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh(d*x + c)^3 + (3*a^7*b + 17*a^6*b^2 +
41*a^5*b^3 + 51*a^4*b^4 + 32*a^3*b^5 + 8*a^2*b^6)*d*cosh(d*x + c))*sinh(d*
x + c)^3 + 4*(7*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*cosh(d*x + c)^6
+ 15*(a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh(d*x +
c)^4 + 3*(3*a^7*b + 17*a^6*b^2 + 41*a^5*b^3 + 51*a^4*b^4 + 32*a^3*b^5 + 8*a
^2*b^6)*d*cosh(d*x + c)^2 + (a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*
a^3*b^5)*d)*sinh(d*x + c)^2 + (a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d +
8*((a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*cosh(d*x + c)^7 + 3*(a^7*b
+ 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh(d*x + c)^5 + (3*a^7
*b + 17*a^6*b^2 + 41*a^5*b^3 + 51*a^4*b^4 + 32*a^3*b^5 + 8*a^2*b^6)*d*cosh(
d*x + c)^3 + (a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh
(d*x + c))*sinh(d*x + c)), -1/8*(2*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a
*b^4)*cosh(d*x + c)^6 + 12*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a*b^4)*co
sh(d*x + c)*sinh(d*x + c)^5 + 2*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a*b^
4)*sinh(d*x + c)^6 + 10*a^4*b + 14*a^3*b^2 + 4*a^2*b^3 + 2*(15*a^4*b + 61*a
^3*b^2 + 102*a^2*b^3 + 72*a*b^4 + 16*b^5)*cosh(d*x + c)^4 + 2*(15*a^4*b + 6
1*a^3*b^2 + 102*a^2*b^3 + 72*a*b^4 + 16*b^5 + 15*(5*a^4*b + 21*a^3*b^2 + 24
*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(5*(5*a^4*b + 21*a
^3*b^2 + 24*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^3 + (15*a^4*b + 61*a^3*b^2 + 1
02*a^2*b^3 + 72*a*b^4 + 16*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(15*a^4*
b + 47*a^3*b^2 + 40*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^2 + 2*(15*a^4*b + 47*a
^3*b^2 + 40*a^2*b^3 + 8*a*b^4 + 15*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a
*b^4)*cosh(d*x + c)^4 + 6*(15*a^4*b + 61*a^3*b^2 + 102*a^2*b^3 + 72*a*b^4 +
16*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 3*(a^4*cosh(d*x + c)^8 + 8*a^4*
cosh(d*x + c)*sinh(d*x + c)^7 + a^4*sinh(d*x + c)^8 + 4*(a^4 + 2*a^3*b)*cos
h(d*x + c)^6 + 4*(7*a^4*cosh(d*x + c)^2 + a^4 + 2*a^3*b)*sinh(d*x + c)^6 +
8*(7*a^4*cosh(d*x + c)^3 + 3*(a^4 + 2*a^3*b)*cosh(d*x + c))*sinh(d*x + c)^5
+ 2*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + 2*(35*a^4*cosh(d*x + c
)^4 + 3*a^4 + 8*a^3*b + 8*a^2*b^2 + 30*(a^4 + 2*a^3*b)*cosh(d*x + c)^2)*sin
h(d*x + c)^4 + a^4 + 8*(7*a^4*cosh(d*x + c)^5 + 10*(a^4 + 2*a^3*b)*cosh(d*x
+ c)^3 + (3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*
(a^4 + 2*a^3*b)*cosh(d*x + c)^2 + 4*(7*a^4*cosh(d*x + c)^6 + 15*(a^4 + 2*a^
3*b)*cosh(d*x + c)^4 + a^4 + 2*a^3*b + 3*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh
(d*x + c)^2)*sinh(d*x + c)^2 + 8*(a^4*cosh(d*x + c)^7 + 3*(a^4 + 2*a^3*b)*c
osh(d*x + c)^5 + (3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 + (a^4 + 2*a
^3*b)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(d*x
+ c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sq
rt(-a*b - b^2)/(a*b + b^2)) + 4*(3*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a
*b^4)*cosh(d*x + c)^5 + 2*(15*a^4*b + 61*a^3*b^2 + 102*a^2*b^3 + 72*a*b^4 +

```
 16*b^5)*cosh(d*x + c)^3 + (15*a^4*b + 47*a^3*b^2 + 40*a^2*b^3 + 8*a*b^4)*c
osh(d*x + c))*sinh(d*x + c))/((a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*c
osh(d*x + c)^8 + 8*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*cosh(d*x + c
)*sinh(d*x + c)^7 + (a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*sinh(d*x +
c)^8 + 4*(a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh(d*x
 + c)^6 + 4*(7*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*cosh(d*x + c)^2
+ (a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d)*sinh(d*x + c)^
6 + 2*(3*a^7*b + 17*a^6*b^2 + 41*a^5*b^3 + 51*a^4*b^4 + 32*a^3*b^5 + 8*a^2*
b^6)*d*cosh(d*x + c)^4 + 8*(7*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*c
osh(d*x + c)^3 + 3*(a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*
d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a
^4*b^4)*d*cosh(d*x + c)^4 + 30*(a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 +
 2*a^3*b^5)*d*cosh(d*x + c)^2 + (3*a^7*b + 17*a^6*b^2 + 41*a^5*b^3 + 51*a^4
*b^4 + 32*a^3*b^5 + 8*a^2*b^6)*d)*sinh(d*x + c)^4 + 4*(a^7*b + 5*a^6*b^2 +
9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^7*b + 3*a^6*
b^2 + 3*a^5*b^3 + a^4*b^4)*d*cosh(d*x + c)^5 + 10*(a^7*b + 5*a^6*b^2 + 9*a^
5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh(d*x + c)^3 + (3*a^7*b + 17*a^6*b^2 +
41*a^5*b^3 + 51*a^4*b^4 + 32*a^3*b^5 + 8*a^2*b^6)*d*cosh(d*x + c))*sinh(d*x
 + c)^3 + 4*(7*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*cosh(d*x + c)^6
+ 15*(a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh(d*x + c
)^4 + 3*(3*a^7*b + 17*a^6*b^2 + 41*a^5*b^3 + 51*a^4*b^4 + 32*a^3*b^5 + 8*a^
2*b^6)*d*cosh(d*x + c)^2 + (a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a
^3*b^5)*d)*sinh(d*x + c)^2 + (a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d +
8*((a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*cosh(d*x + c)^7 + 3*(a^7*b +
 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh(d*x + c)^5 + (3*a^7*
b + 17*a^6*b^2 + 41*a^5*b^3 + 51*a^4*b^4 + 32*a^3*b^5 + 8*a^2*b^6)*d*cosh(d
*x + c)^3 + (a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh(
d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.35344, size = 386, normalized size = 3.57

$$\frac{3\arctan\left(\frac{ae^{(2dx+2c)}+a+2b}{2\sqrt{-ab-b^2}}\right)}{8\left(a^2d+2abd+b^2d\right)\sqrt{-ab-b^2}} - \frac{5a^3e^{(6dx+6c)}+16a^2be^{(6dx+6c)}+8ab^2e^{(6dx+6c)}+15a^3e^{(4dx+4c)}+46a^2be^{(4dx+4c)}+56}{4\left(a^4d+2a^3bd+a^2b^2d\right)\left(ae^{(4dx}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/((a^2*d + 2*
a*b*d + b^2*d)*sqrt(-a*b - b^2)) - 1/4*(5*a^3*e^(6*d*x + 6*c) + 16*a^2*b*e^
(6*d*x + 6*c) + 8*a*b^2*e^(6*d*x + 6*c) + 15*a^3*e^(4*d*x + 4*c) + 46*a^2*b
*e^(4*d*x + 4*c) + 56*a*b^2*e^(4*d*x + 4*c) + 16*b^3*e^(4*d*x + 4*c) + 15*a
^3*e^(2*d*x + 2*c) + 32*a^2*b*e^(2*d*x + 2*c) + 8*a*b^2*e^(2*d*x + 2*c) + 5
```

```
*a^3 + 2*a^2*b)/((a^4*d + 2*a^3*b*d + a^2*b^2*d)*(a*e^(4*d*x + 4*c) + 2*a*e
^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)^2)
```

3.97 $\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3} dx$

Optimal. Leaf size=123

$$\frac{(4a+b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{3/2}d(a+b)^{5/2}}+\frac{(4a+b)\sinh(c+dx)}{8ad(a+b)^2\left(a\sinh^2(c+dx)+a+b\right)}-\frac{b\sinh(c+dx)}{4ad(a+b)\left(a\sinh^2(c+dx)+a+b\right)^2}$$

```
[Out] ((4*a + b)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(8*a^(3/2)*(a + b)^
(5/2)*d) - (b*Sinh[c + d*x])/(4*a*(a + b)*d*(a + b + a*Sinh[c + d*x]^2)^2)
+ ((4*a + b)*Sinh[c + d*x])/(8*a*(a + b)^2*d*(a + b + a*Sinh[c + d*x]^2))
```

Rubi [A] time = 0.10837, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4147, 385, 199, 205}

$$\frac{(4a+b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{3/2}d(a+b)^{5/2}}+\frac{(4a+b)\sinh(c+dx)}{8ad(a+b)^2\left(a\sinh^2(c+dx)+a+b\right)}-\frac{b\sinh(c+dx)}{4ad(a+b)\left(a\sinh^2(c+dx)+a+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((4*a + b)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(8*a^(3/2)*(a + b)^
(5/2)*d) - (b*Sinh[c + d*x])/(4*a*(a + b)*d*(a + b + a*Sinh[c + d*x]^2)^2)
+ ((4*a + b)*Sinh[c + d*x])/(8*a*(a + b)^2*d*(a + b + a*Sinh[c + d*x]^2))
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{\left(a+b+ax^2\right)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{b\sinh(c+dx)}{4a(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} + \frac{(4a+b)\,\text{Subst}\left(\int \frac{1}{\left(a+b+ax^2\right)^2}\,dx, x, \sinh(c+dx\right.}{4a(a+b)d}$$

$$= -\frac{b\sinh(c+dx)}{4a(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} + \frac{(4a+b)\sinh(c+dx)}{8a(a+b)^2d\left(a+b+a\sinh^2(c+dx)\right)} + \frac{(4a}{}$$

$$= \frac{(4a+b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{3/2}(a+b)^{5/2}d} - \frac{b\sinh(c+dx)}{4a(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} + \frac{(4a+}{8a(a+b)^2d\,(a}$$

Mathematica [A] time = 0.811625, size = 159, normalized size = 1.29

$$-\frac{\operatorname{sech}^6(c+dx)(a\cosh(2(c+dx))+a+2b)^3\left(\frac{8\sinh(c+dx)}{\left(a\sinh^2(c+dx)+a+b\right)^2} - (4a+b)\left(\frac{5(a+b)\sinh(c+dx)+3a\sinh^3(c+dx)}{(a+b)^2\left(a\sinh^2(c+dx)+a+b\right)^2} + \frac{3\tan^{-1}\left(\frac{\sqrt{a}\sinh}{\sqrt{a}}\right.}{\sqrt{a}(a+b)^5}\right.\right.}{192ad\left(a+b\operatorname{sech}^2(c+dx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -((a + 2*b + a*Cosh[2*(c + d*x)])^3*Sech[c + d*x]^6*((8*Sinh[c + d*x])/(a +
 b + a*Sinh[c + d*x]^2)^2 - (4*a + b)*((3*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sq
rt[a + b]])/(Sqrt[a]*(a + b)^(5/2)) + (5*(a + b)*Sinh[c + d*x] + 3*a*Sinh[c
 + d*x]^3)/((a + b)^2*(a + b + a*Sinh[c + d*x]^2)^2))))/(192*a*d*(a + b*Sec
h[c + d*x]^2)^3)
```

Maple [B] time = 0.079, size = 1038, normalized size = 8.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -1/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)
^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^7+1/4/d/(ta
nh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*t
anh(1/2*d*x+1/2*c)^2*b+a+b)^2*b/(a+b)/a*tanh(1/2*d*x+1/2*c)^7-1/d/(tanh(1/2
*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/
2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5*a+5/4/d/(tanh(1/2*d*x
+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*
```

```
x+1/2*c)^2*b+a+b)^2*b/(a+b)^2*tanh(1/2*d*x+1/2*c)^5-3/4/d*b^2/a/(tanh(1/2*d
*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*
d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5+1/d/(tanh(1/2*d*x+1/2*c
)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*
c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3*a-5/4/d/(tanh(1/2*d*x+1/2*c)^4*
a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2
*b+a+b)^2*b/(a+b)^2*tanh(1/2*d*x+1/2*c)^3+3/4/d*b^2/a/(tanh(1/2*d*x+1/2*c)^
4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)
^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3+1/d/(tanh(1/2*d*x+1/2*c)^4*a+b*ta
nh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b
)^2/(a+b)*tanh(1/2*d*x+1/2*c)-1/4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x
+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*b/(a+b
)/a*tanh(1/2*d*x+1/2*c)+1/2/d/(a^2+2*a*b+b^2)/(a+b)^(1/2)/a^(1/2)*arctan(1/
2*(2*tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)+2*b^(1/2))/a^(1/2))-1/2/d/(a^2+2*a*b+b
^2)/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(-2*tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)+2*b^
(1/2))/a^(1/2))+1/8/d*b/a^(3/2)/(a^2+2*a*b+b^2)/(a+b)^(1/2)*arctan(1/2*(2*t
anh(1/2*d*x+1/2*c)*(a+b)^(1/2)+2*b^(1/2))/a^(1/2))-1/8/d/a^(3/2)/(a^2+2*a*b
+b^2)/(a+b)^(1/2)*arctan(1/2*(-2*tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)+2*b^(1/2))
/a^(1/2))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(4\,a^2e^{(7\,c)} + abe^{(7\,c)}\right)e^{(7\,dx)} + \left(4\,a^2e^{(5\,c)} + 9\,abe^{(5\,c)}\right.}{4\left(a^5d + 2\,a^4bd + a^3b^2d + \left(a^5de^{(8\,c)} + 2\,a^4bde^{(8\,c)} + a^3b^2de^{(8\,c)}\right)e^{(8\,dx)} + 4\left(a^5de^{(6\,c)} + 4\,a^4bde^{(6\,c)} + 5\,a^3b^2de^{(6\,c)} + 2\,a^2b^3de^{(6\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((4*a^2*e^(7*c) + a*b*e^(7*c))*e^(7*d*x) + (4*a^2*e^(5*c) + 9*a*b*e^(5*
c) - 4*b^2*e^(5*c))*e^(5*d*x) - (4*a^2*e^(3*c) + 9*a*b*e^(3*c) - 4*b^2*e^(3
*c))*e^(3*d*x) - (4*a^2*e^c + a*b*e^c)*e^(d*x))/(a^5*d + 2*a^4*b*d + a^3*b^
2*d + (a^5*d*e^(8*c) + 2*a^4*b*d*e^(8*c) + a^3*b^2*d*e^(8*c))*e^(8*d*x) + 4
*(a^5*d*e^(6*c) + 4*a^4*b*d*e^(6*c) + 5*a^3*b^2*d*e^(6*c) + 2*a^2*b^3*d*e^(
6*c))*e^(6*d*x) + 2*(3*a^5*d*e^(4*c) + 14*a^4*b*d*e^(4*c) + 27*a^3*b^2*d*e^
(4*c) + 24*a^2*b^3*d*e^(4*c) + 8*a*b^4*d*e^(4*c))*e^(4*d*x) + 4*(a^5*d*e^(2
*c) + 4*a^4*b*d*e^(2*c) + 5*a^3*b^2*d*e^(2*c) + 2*a^2*b^3*d*e^(2*c))*e^(2*d
*x)) + 8*integrate(1/32*((4*a*e^(3*c) + b*e^(3*c))*e^(3*d*x) + (4*a*e^c + b
*e^c)*e^(d*x))/(a^4 + 2*a^3*b + a^2*b^2 + (a^4*e^(4*c) + 2*a^3*b*e^(4*c) +
a^2*b^2*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) + 4*a^3*b*e^(2*c) + 5*a^2*b^2*e
^(2*c) + 2*a*b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.93106, size = 14071, normalized size = 114.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(4*a^4 + 5*a^3*b + a^2*b^2)*cosh(d*x + c)^7 + 28*(4*a^4 + 5*a^3*b
+ a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^6 + 4*(4*a^4 + 5*a^3*b + a^2*b^2)*si
nh(d*x + c)^7 + 4*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^5
+ 4*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 21*(4*a^4 + 5*a^3*b + a^2*b^2
```

```
)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*(4*a^4 + 5*a^3*b + a^2*b^2)*cosh
(d*x + c)^3 + (4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c))*sinh(
d*x + c)^4 - 4*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^3 + 4
*(35*(4*a^4 + 5*a^3*b + a^2*b^2)*cosh(d*x + c)^4 - 4*a^4 - 13*a^3*b - 5*a^2
*b^2 + 4*a*b^3 + 10*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^3 + 4*(21*(4*a^4 + 5*a^3*b + a^2*b^2)*cosh(d*x + c)^5 + 10
*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^3 - 3*(4*a^4 + 13*a
^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - ((4*a^3 + a^2*
b)*cosh(d*x + c)^8 + 8*(4*a^3 + a^2*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (4*a
^3 + a^2*b)*sinh(d*x + c)^8 + 4*(4*a^3 + 9*a^2*b + 2*a*b^2)*cosh(d*x + c)^6
 + 4*(4*a^3 + 9*a^2*b + 2*a*b^2 + 7*(4*a^3 + a^2*b)*cosh(d*x + c)^2)*sinh(d
*x + c)^6 + 8*(7*(4*a^3 + a^2*b)*cosh(d*x + c)^3 + 3*(4*a^3 + 9*a^2*b + 2*a
*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(12*a^3 + 35*a^2*b + 40*a*b^2 + 8*
b^3)*cosh(d*x + c)^4 + 2*(35*(4*a^3 + a^2*b)*cosh(d*x + c)^4 + 12*a^3 + 35*
a^2*b + 40*a*b^2 + 8*b^3 + 30*(4*a^3 + 9*a^2*b + 2*a*b^2)*cosh(d*x + c)^2)*
sinh(d*x + c)^4 + 8*(7*(4*a^3 + a^2*b)*cosh(d*x + c)^5 + 10*(4*a^3 + 9*a^2*
b + 2*a*b^2)*cosh(d*x + c)^3 + (12*a^3 + 35*a^2*b + 40*a*b^2 + 8*b^3)*cosh(
d*x + c))*sinh(d*x + c)^3 + 4*a^3 + a^2*b + 4*(4*a^3 + 9*a^2*b + 2*a*b^2)*c
osh(d*x + c)^2 + 4*(7*(4*a^3 + a^2*b)*cosh(d*x + c)^6 + 15*(4*a^3 + 9*a^2*b
 + 2*a*b^2)*cosh(d*x + c)^4 + 4*a^3 + 9*a^2*b + 2*a*b^2 + 3*(12*a^3 + 35*a^
2*b + 40*a*b^2 + 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((4*a^3 + a^2*
b)*cosh(d*x + c)^7 + 3*(4*a^3 + 9*a^2*b + 2*a*b^2)*cosh(d*x + c)^5 + (12*a^
3 + 35*a^2*b + 40*a*b^2 + 8*b^3)*cosh(d*x + c)^3 + (4*a^3 + 9*a^2*b + 2*a*b
^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 - a*b)*log((a*cosh(d*x + c)^4 +
 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh
(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*co
sh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)
^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2
 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a)/(a*cosh(d*x + c)
^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*co
sh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*co
sh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - 4*(4*a^4 + 5
*a^3*b + a^2*b^2)*cosh(d*x + c) + 4*(7*(4*a^4 + 5*a^3*b + a^2*b^2)*cosh(d*x
 + c)^6 + 5*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^4 - 4*a^
4 - 5*a^3*b - a^2*b^2 - 3*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x
 + c)^2)*sinh(d*x + c))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x +
 c)^8 + 8*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)*sinh(d*x +
c)^7 + (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*sinh(d*x + c)^8 + 4*(a^7 + 5
*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^7 +
 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^2 + (a^7 + 5*a^6*b + 9*a^5*
b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d)*sinh(d*x + c)^6 + 2*(3*a^7 + 17*a^6*b + 41*
a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^
7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^3 + 3*(a^7 + 5*a^6*b + 9
*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*
(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^4 + 30*(a^7 + 5*a^6*b
 + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^2 + (3*a^7 + 17*a^6*b
 + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d)*sinh(d*x + c)^4 + 4
*(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^2 + 8*
(7*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^5 + 10*(a^7 + 5*a^
6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^3 + (3*a^7 + 17*a^
6*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c))*si
nh(d*x + c)^3 + 4*(7*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^
6 + 15*(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^
4 + 3*(3*a^7 + 17*a^6*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)
*d*cosh(d*x + c)^2 + (a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d)
*sinh(d*x + c)^2 + (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d + 8*((a^7 + 3*a^
6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^7 + 3*(a^7 + 5*a^6*b + 9*a^5*b^2
 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^5 + (3*a^7 + 17*a^6*b + 41*a^5*b^
2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)^3 + (a^7 + 5*a^6*b
```

```
 + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*
(2*(4*a^4 + 5*a^3*b + a^2*b^2)*cosh(d*x + c)^7 + 14*(4*a^4 + 5*a^3*b + a^2*
b^2)*cosh(d*x + c)*sinh(d*x + c)^6 + 2*(4*a^4 + 5*a^3*b + a^2*b^2)*sinh(d*x
 + c)^7 + 2*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^5 + 2*(4
*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 21*(4*a^4 + 5*a^3*b + a^2*b^2)*cosh
(d*x + c)^2)*sinh(d*x + c)^5 + 10*(7*(4*a^4 + 5*a^3*b + a^2*b^2)*cosh(d*x +
 c)^3 + (4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c))*sinh(d*x +
c)^4 - 2*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^3 + 2*(35*(
4*a^4 + 5*a^3*b + a^2*b^2)*cosh(d*x + c)^4 - 4*a^4 - 13*a^3*b - 5*a^2*b^2 +
 4*a*b^3 + 10*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^2)*sin
h(d*x + c)^3 + 2*(21*(4*a^4 + 5*a^3*b + a^2*b^2)*cosh(d*x + c)^5 + 10*(4*a^
4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^3 - 3*(4*a^4 + 13*a^3*b +
 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + ((4*a^3 + a^2*b)*cos
h(d*x + c)^8 + 8*(4*a^3 + a^2*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (4*a^3 + a
^2*b)*sinh(d*x + c)^8 + 4*(4*a^3 + 9*a^2*b + 2*a*b^2)*cosh(d*x + c)^6 + 4*(
4*a^3 + 9*a^2*b + 2*a*b^2 + 7*(4*a^3 + a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c
)^6 + 8*(7*(4*a^3 + a^2*b)*cosh(d*x + c)^3 + 3*(4*a^3 + 9*a^2*b + 2*a*b^2)*
cosh(d*x + c))*sinh(d*x + c)^5 + 2*(12*a^3 + 35*a^2*b + 40*a*b^2 + 8*b^3)*c
osh(d*x + c)^4 + 2*(35*(4*a^3 + a^2*b)*cosh(d*x + c)^4 + 12*a^3 + 35*a^2*b
 + 40*a*b^2 + 8*b^3 + 30*(4*a^3 + 9*a^2*b + 2*a*b^2)*cosh(d*x + c)^2)*sinh(d
*x + c)^4 + 8*(7*(4*a^3 + a^2*b)*cosh(d*x + c)^5 + 10*(4*a^3 + 9*a^2*b + 2*
a*b^2)*cosh(d*x + c)^3 + (12*a^3 + 35*a^2*b + 40*a*b^2 + 8*b^3)*cosh(d*x +
c))*sinh(d*x + c)^3 + 4*a^3 + a^2*b + 4*(4*a^3 + 9*a^2*b + 2*a*b^2)*cosh(d*
x + c)^2 + 4*(7*(4*a^3 + a^2*b)*cosh(d*x + c)^6 + 15*(4*a^3 + 9*a^2*b + 2*a
*b^2)*cosh(d*x + c)^4 + 4*a^3 + 9*a^2*b + 2*a*b^2 + 3*(12*a^3 + 35*a^2*b +
40*a*b^2 + 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((4*a^3 + a^2*b)*cos
h(d*x + c)^7 + 3*(4*a^3 + 9*a^2*b + 2*a*b^2)*cosh(d*x + c)^5 + (12*a^3 + 35
*a^2*b + 40*a*b^2 + 8*b^3)*cosh(d*x + c)^3 + (4*a^3 + 9*a^2*b + 2*a*b^2)*co
sh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + a*b)*arctan(1/2*(a*cosh(d*x + c)^3 +
 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (3*a + 4*b)*cosh(d
*x + c) + (3*a*cosh(d*x + c)^2 + 3*a + 4*b)*sinh(d*x + c))/sqrt(a^2 + a*b))
 + ((4*a^3 + a^2*b)*cosh(d*x + c)^8 + 8*(4*a^3 + a^2*b)*cosh(d*x + c)*sinh(
d*x + c)^7 + (4*a^3 + a^2*b)*sinh(d*x + c)^8 + 4*(4*a^3 + 9*a^2*b + 2*a*b^2
)*cosh(d*x + c)^6 + 4*(4*a^3 + 9*a^2*b + 2*a*b^2 + 7*(4*a^3 + a^2*b)*cosh(d
*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(4*a^3 + a^2*b)*cosh(d*x + c)^3 + 3*(4*a^
3 + 9*a^2*b + 2*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(12*a^3 + 35*a^2*
b + 40*a*b^2 + 8*b^3)*cosh(d*x + c)^4 + 2*(35*(4*a^3 + a^2*b)*cosh(d*x + c)
^4 + 12*a^3 + 35*a^2*b + 40*a*b^2 + 8*b^3 + 30*(4*a^3 + 9*a^2*b + 2*a*b^2)*
cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(4*a^3 + a^2*b)*cosh(d*x + c)^5 + 1
0*(4*a^3 + 9*a^2*b + 2*a*b^2)*cosh(d*x + c)^3 + (12*a^3 + 35*a^2*b + 40*a*b
^2 + 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*a^3 + a^2*b + 4*(4*a^3 + 9*a
^2*b + 2*a*b^2)*cosh(d*x + c)^2 + 4*(7*(4*a^3 + a^2*b)*cosh(d*x + c)^6 + 15
*(4*a^3 + 9*a^2*b + 2*a*b^2)*cosh(d*x + c)^4 + 4*a^3 + 9*a^2*b + 2*a*b^2 +
3*(12*a^3 + 35*a^2*b + 40*a*b^2 + 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
 8*((4*a^3 + a^2*b)*cosh(d*x + c)^7 + 3*(4*a^3 + 9*a^2*b + 2*a*b^2)*cosh(d*
x + c)^5 + (12*a^3 + 35*a^2*b + 40*a*b^2 + 8*b^3)*cosh(d*x + c)^3 + (4*a^3
+ 9*a^2*b + 2*a*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + a*b)*arctan(1
/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/(a + b)) - 2*(4*a^4 + 5*
a^3*b + a^2*b^2)*cosh(d*x + c) + 2*(7*(4*a^4 + 5*a^3*b + a^2*b^2)*cosh(d*x
+ c)^6 + 5*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^4 - 4*a^4
 - 5*a^3*b - a^2*b^2 - 3*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x
+ c)^2)*sinh(d*x + c))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x +
c)^8 + 8*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)*sinh(d*x + c
)^7 + (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*sinh(d*x + c)^8 + 4*(a^7 + 5*
a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^7 +
3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^2 + (a^7 + 5*a^6*b + 9*a^5*b
^2 + 7*a^4*b^3 + 2*a^3*b^4)*d)*sinh(d*x + c)^6 + 2*(3*a^7 + 17*a^6*b + 41*a
^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^7
 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^3 + 3*(a^7 + 5*a^6*b + 9*
```

```
a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(
a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^4 + 30*(a^7 + 5*a^6*b
+ 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^2 + (3*a^7 + 17*a^6*b
+ 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d)*sinh(d*x + c)^4 + 4*
(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^2 + 8*(
7*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^5 + 10*(a^7 + 5*a^6
*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^3 + (3*a^7 + 17*a^6
*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c))*sin
h(d*x + c)^3 + 4*(7*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^6
 + 15*(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^4
 + 3*(3*a^7 + 17*a^6*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*
d*cosh(d*x + c)^2 + (a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d)*
sinh(d*x + c)^2 + (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d + 8*((a^7 + 3*a^6
*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^7 + 3*(a^7 + 5*a^6*b + 9*a^5*b^2
+ 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^5 + (3*a^7 + 17*a^6*b + 41*a^5*b^2
 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)^3 + (a^7 + 5*a^6*b
+ 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.98 $\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=125

$$\frac{(a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8b^{3/2}d(a+b)^{5/2}}+\frac{(a+4b)\tanh(c+dx)}{8bd(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)}-\frac{a\tanh(c+dx)}{4bd(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}$$

```
[Out] ((a + 4*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*b^(3/2)*(a + b)
^(5/2)*d) - (a*Tanh[c + d*x])/(4*b*(a + b)*d*(a + b - b*Tanh[c + d*x]^2)^2)
 + ((a + 4*b)*Tanh[c + d*x])/(8*b*(a + b)^2*d*(a + b - b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.107352, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4146, 385, 199, 208}

$$\frac{(a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8b^{3/2}d(a+b)^{5/2}}+\frac{(a+4b)\tanh(c+dx)}{8bd(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)}-\frac{a\tanh(c+dx)}{4bd(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 4*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*b^(3/2)*(a + b)
^(5/2)*d) - (a*Tanh[c + d*x])/(4*b*(a + b)*d*(a + b - b*Tanh[c + d*x]^2)^2)
 + ((a + 4*b)*Tanh[c + d*x])/(8*b*(a + b)^2*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a+b-bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{a\tanh(c+dx)}{4b(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{(a+4b)\,\text{Subst}\left(\int \frac{1}{\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+\right.}{4b(a+b)d}$$

$$= -\frac{a\tanh(c+dx)}{4b(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{(a+4b)\tanh(c+dx)}{8b(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{(a\ -}{}$$

$$= \frac{(a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8b^{3/2}(a+b)^{5/2}d} - \frac{a\tanh(c+dx)}{4b(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{(a+}{8b(a+b)^2d}$$

Mathematica [A] time = 3.42108, size = 250, normalized size = 2.

$$\frac{\operatorname{sech}^6(c+dx)(a\cosh(2(c+dx))+a+2b)\left(-\frac{4(a+b)\operatorname{sech}(2c)((a+2b)\sinh(2c)-a\sinh(2dx))}{a} + \frac{\operatorname{sech}(2c)((a+4b)\sinh(2c)-(a-2b)\sinh(2dx))(a}{b}\right.}{64d(a+b)^2\left(a+b\operatorname{sec}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^6*(((a + 4*b)*ArcTanh[(Sech[
d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*
Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])
^2*(Cosh[2*c] - Sinh[2*c]))/(b*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) -
 (4*(a + b)*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/a + ((a + 2*b
+ a*Cosh[2*(c + d*x)])*Sech[2*c]*((a + 4*b)*Sinh[2*c] - (a - 2*b)*Sinh[2*d*
x]))/b))/(64*(a + b)^2*d*(a + b*Sech[c + d*x]^2)^3)
```

Maple [B] time = 0.092, size = 1084, normalized size = 8.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -1/4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*
c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/b/(a+b)*tanh(1/2*d*x+1/2*c)^7*a+1/d
/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a
-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^7-3/4/d/(tanh(1
```

```
/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(
1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2/b*tanh(1/2*d*x+1/2*c)^5*a^2+5/4/d/(tanh(1
/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(
1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5*a-1/d/(tanh(1/2*d*x
+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*
x+1/2*c)^2*b+a+b)^2*b/(a+b)^2*tanh(1/2*d*x+1/2*c)^5-3/4/d/(tanh(1/2*d*x+1/2
*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/
2*c)^2*b+a+b)^2/(a+b)^2/b*tanh(1/2*d*x+1/2*c)^3*a^2+5/4/d/(tanh(1/2*d*x+1/2
*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/
2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3*a-1/d/(tanh(1/2*d*x+1/2*c)^4*
a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2
*b+a+b)^2*b/(a+b)^2*tanh(1/2*d*x+1/2*c)^3-1/4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*
tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a
+b)^2/b/(a+b)*tanh(1/2*d*x+1/2*c)*a+1/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2
*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a
+b)*tanh(1/2*d*x+1/2*c)+1/16/d/b^(3/2)/(a^2+2*a*b+b^2)*a/(a+b)^(1/2)*ln((a+
b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-1
/16/d/b^(3/2)/(a^2+2*a*b+b^2)*a/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/
2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2))+1/4/d/(a^2+2*a*b+b^2)/b^(
1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)
*b^(1/2)+(a+b)^(1/2))-1/4/d/(a^2+2*a*b+b^2)/b^(1/2)/(a+b)^(1/2)*ln(-(a+b)^(
1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.87447, size = 12354, normalized size = 98.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(a^4*b + 5*a^3*b^2 + 4*a^2*b^3)*cosh(d*x + c)^6 + 24*(a^4*b + 5*a^
3*b^2 + 4*a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + 4*(a^4*b + 5*a^3*b^2 + 4
*a^2*b^3)*sinh(d*x + c)^6 + 4*a^4*b - 4*a^3*b^2 - 8*a^2*b^3 + 4*(3*a^4*b +
5*a^3*b^2 - 6*a^2*b^3 - 24*a*b^4 - 16*b^5)*cosh(d*x + c)^4 + 4*(3*a^4*b + 5
*a^3*b^2 - 6*a^2*b^3 - 24*a*b^4 - 16*b^5 + 15*(a^4*b + 5*a^3*b^2 + 4*a^2*b^
3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(5*(a^4*b + 5*a^3*b^2 + 4*a^2*b^3)
*cosh(d*x + c)^3 + (3*a^4*b + 5*a^3*b^2 - 6*a^2*b^3 - 24*a*b^4 - 16*b^5)*co
sh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a^4*b - a^3*b^2 - 20*a^2*b^3 - 16*a*b^4
)*cosh(d*x + c)^2 + 4*(3*a^4*b - a^3*b^2 - 20*a^2*b^3 - 16*a*b^4 + 15*(a^4*
b + 5*a^3*b^2 + 4*a^2*b^3)*cosh(d*x + c)^4 + 6*(3*a^4*b + 5*a^3*b^2 - 6*a^2
*b^3 - 24*a*b^4 - 16*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((a^4 + 4*a^3*
b)*cosh(d*x + c)^8 + 8*(a^4 + 4*a^3*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^4
 + 4*a^3*b)*sinh(d*x + c)^8 + 4*(a^4 + 6*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^6
 + 4*(a^4 + 6*a^3*b + 8*a^2*b^2 + 7*(a^4 + 4*a^3*b)*cosh(d*x + c)^2)*sinh(d
```

```
*x + c)^6 + 8*(7*(a^4 + 4*a^3*b)*cosh(d*x + c)^3 + 3*(a^4 + 6*a^3*b + 8*a^2
*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^4 + 20*a^3*b + 40*a^2*b^2 + 3
2*a*b^3)*cosh(d*x + c)^4 + 2*(35*(a^4 + 4*a^3*b)*cosh(d*x + c)^4 + 3*a^4 +
20*a^3*b + 40*a^2*b^2 + 32*a*b^3 + 30*(a^4 + 6*a^3*b + 8*a^2*b^2)*cosh(d*x
+ c)^2)*sinh(d*x + c)^4 + a^4 + 4*a^3*b + 8*(7*(a^4 + 4*a^3*b)*cosh(d*x + c
)^5 + 10*(a^4 + 6*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 + (3*a^4 + 20*a^3*b +
40*a^2*b^2 + 32*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 + 6*a^3*b +
8*a^2*b^2)*cosh(d*x + c)^2 + 4*(7*(a^4 + 4*a^3*b)*cosh(d*x + c)^6 + 15*(a^4
 + 6*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + a^4 + 6*a^3*b + 8*a^2*b^2 + 3*(3*
a^4 + 20*a^3*b + 40*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
8*((a^4 + 4*a^3*b)*cosh(d*x + c)^7 + 3*(a^4 + 6*a^3*b + 8*a^2*b^2)*cosh(d*x
 + c)^5 + (3*a^4 + 20*a^3*b + 40*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^3 + (a^4
 + 6*a^3*b + 8*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b + b^2)*log((
a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x +
c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2
*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2
 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*
x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*cos
h(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a
 + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2
 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 8*
(3*(a^4*b + 5*a^3*b^2 + 4*a^2*b^3)*cosh(d*x + c)^5 + 2*(3*a^4*b + 5*a^3*b^2
 - 6*a^2*b^3 - 24*a*b^4 - 16*b^5)*cosh(d*x + c)^3 + (3*a^4*b - a^3*b^2 - 20
*a^2*b^3 - 16*a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^6*b^2 + 3*a^5*b^3 +
3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^8 + 8*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4
 + a^3*b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*
b^4 + a^3*b^5)*d*sinh(d*x + c)^8 + 4*(a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a
^3*b^5 + 2*a^2*b^6)*d*cosh(d*x + c)^6 + 4*(7*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b
^4 + a^3*b^5)*d*cosh(d*x + c)^2 + (a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a^3*
b^5 + 2*a^2*b^6)*d)*sinh(d*x + c)^6 + 2*(3*a^6*b^2 + 17*a^5*b^3 + 41*a^4*b^
4 + 51*a^3*b^5 + 32*a^2*b^6 + 8*a*b^7)*d*cosh(d*x + c)^4 + 8*(7*(a^6*b^2 +
3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^3 + 3*(a^6*b^2 + 5*a^5*b^3
 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*
(35*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^4 + 30*(a^6
*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6)*d*cosh(d*x + c)^2 + (
3*a^6*b^2 + 17*a^5*b^3 + 41*a^4*b^4 + 51*a^3*b^5 + 32*a^2*b^6 + 8*a*b^7)*d)
*sinh(d*x + c)^4 + 4*(a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b
^6)*d*cosh(d*x + c)^2 + 8*(7*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*
cosh(d*x + c)^5 + 10*(a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b
^6)*d*cosh(d*x + c)^3 + (3*a^6*b^2 + 17*a^5*b^3 + 41*a^4*b^4 + 51*a^3*b^5 +
 32*a^2*b^6 + 8*a*b^7)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^6*b^2 + 3
*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^6 + 15*(a^6*b^2 + 5*a^5*b^3
 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6)*d*cosh(d*x + c)^4 + 3*(3*a^6*b^2 + 17
*a^5*b^3 + 41*a^4*b^4 + 51*a^3*b^5 + 32*a^2*b^6 + 8*a*b^7)*d*cosh(d*x + c)^
2 + (a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6)*d)*sinh(d*x +
 c)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d + 8*((a^6*b^2 + 3*a^5
*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^7 + 3*(a^6*b^2 + 5*a^5*b^3 + 9*
a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6)*d*cosh(d*x + c)^5 + (3*a^6*b^2 + 17*a^5*b^
3 + 41*a^4*b^4 + 51*a^3*b^5 + 32*a^2*b^6 + 8*a*b^7)*d*cosh(d*x + c)^3 + (a^
6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6)*d*cosh(d*x + c))*sin
h(d*x + c)), 1/8*(2*(a^4*b + 5*a^3*b^2 + 4*a^2*b^3)*cosh(d*x + c)^6 + 12*(a
^4*b + 5*a^3*b^2 + 4*a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(a^4*b + 5*
a^3*b^2 + 4*a^2*b^3)*sinh(d*x + c)^6 + 2*a^4*b - 2*a^3*b^2 - 4*a^2*b^3 + 2*
(3*a^4*b + 5*a^3*b^2 - 6*a^2*b^3 - 24*a*b^4 - 16*b^5)*cosh(d*x + c)^4 + 2*(
3*a^4*b + 5*a^3*b^2 - 6*a^2*b^3 - 24*a*b^4 - 16*b^5 + 15*(a^4*b + 5*a^3*b^2
 + 4*a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(5*(a^4*b + 5*a^3*b^2 +
4*a^2*b^3)*cosh(d*x + c)^3 + (3*a^4*b + 5*a^3*b^2 - 6*a^2*b^3 - 24*a*b^4 -
16*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(3*a^4*b - a^3*b^2 - 20*a^2*b^3
- 16*a*b^4)*cosh(d*x + c)^2 + 2*(3*a^4*b - a^3*b^2 - 20*a^2*b^3 - 16*a*b^4
```

```
+ 15*(a^4*b + 5*a^3*b^2 + 4*a^2*b^3)*cosh(d*x + c)^4 + 6*(3*a^4*b + 5*a^3*b
^2 - 6*a^2*b^3 - 24*a*b^4 - 16*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((a^
4 + 4*a^3*b)*cosh(d*x + c)^8 + 8*(a^4 + 4*a^3*b)*cosh(d*x + c)*sinh(d*x + c
)^7 + (a^4 + 4*a^3*b)*sinh(d*x + c)^8 + 4*(a^4 + 6*a^3*b + 8*a^2*b^2)*cosh(
d*x + c)^6 + 4*(a^4 + 6*a^3*b + 8*a^2*b^2 + 7*(a^4 + 4*a^3*b)*cosh(d*x + c)
^2)*sinh(d*x + c)^6 + 8*(7*(a^4 + 4*a^3*b)*cosh(d*x + c)^3 + 3*(a^4 + 6*a^3
*b + 8*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^4 + 20*a^3*b + 40*a
^2*b^2 + 32*a*b^3)*cosh(d*x + c)^4 + 2*(35*(a^4 + 4*a^3*b)*cosh(d*x + c)^4
+ 3*a^4 + 20*a^3*b + 40*a^2*b^2 + 32*a*b^3 + 30*(a^4 + 6*a^3*b + 8*a^2*b^2)
*cosh(d*x + c)^2)*sinh(d*x + c)^4 + a^4 + 4*a^3*b + 8*(7*(a^4 + 4*a^3*b)*co
sh(d*x + c)^5 + 10*(a^4 + 6*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 + (3*a^4 + 2
0*a^3*b + 40*a^2*b^2 + 32*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 +
6*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^2 + 4*(7*(a^4 + 4*a^3*b)*cosh(d*x + c)^6
 + 15*(a^4 + 6*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + a^4 + 6*a^3*b + 8*a^2*b
^2 + 3*(3*a^4 + 20*a^3*b + 40*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^2)*sinh(d*x
 + c)^2 + 8*((a^4 + 4*a^3*b)*cosh(d*x + c)^7 + 3*(a^4 + 6*a^3*b + 8*a^2*b^2
)*cosh(d*x + c)^5 + (3*a^4 + 20*a^3*b + 40*a^2*b^2 + 32*a*b^3)*cosh(d*x + c
)^3 + (a^4 + 6*a^3*b + 8*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b -
 b^2)*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*s
inh(d*x + c)^2 + a + 2*b)*sqrt(-a*b - b^2)/(a*b + b^2)) + 4*(3*(a^4*b + 5*a
^3*b^2 + 4*a^2*b^3)*cosh(d*x + c)^5 + 2*(3*a^4*b + 5*a^3*b^2 - 6*a^2*b^3 -
24*a*b^4 - 16*b^5)*cosh(d*x + c)^3 + (3*a^4*b - a^3*b^2 - 20*a^2*b^3 - 16*a
*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3
*b^5)*d*cosh(d*x + c)^8 + 8*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*c
osh(d*x + c)*sinh(d*x + c)^7 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*
d*sinh(d*x + c)^8 + 4*(a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*
b^6)*d*cosh(d*x + c)^6 + 4*(7*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d
*cosh(d*x + c)^2 + (a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6
)*d)*sinh(d*x + c)^6 + 2*(3*a^6*b^2 + 17*a^5*b^3 + 41*a^4*b^4 + 51*a^3*b^5
+ 32*a^2*b^6 + 8*a*b^7)*d*cosh(d*x + c)^4 + 8*(7*(a^6*b^2 + 3*a^5*b^3 + 3*a
^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^3 + 3*(a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 +
7*a^3*b^5 + 2*a^2*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^6*b^2 +
3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^4 + 30*(a^6*b^2 + 5*a^5*b^
3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6)*d*cosh(d*x + c)^2 + (3*a^6*b^2 + 17*
a^5*b^3 + 41*a^4*b^4 + 51*a^3*b^5 + 32*a^2*b^6 + 8*a*b^7)*d)*sinh(d*x + c)^
4 + 4*(a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6)*d*cosh(d*x
+ c)^2 + 8*(7*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^5
 + 10*(a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6)*d*cosh(d*x
+ c)^3 + (3*a^6*b^2 + 17*a^5*b^3 + 41*a^4*b^4 + 51*a^3*b^5 + 32*a^2*b^6 + 8
*a*b^7)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^6*b^2 + 3*a^5*b^3 + 3*a^
4*b^4 + a^3*b^5)*d*cosh(d*x + c)^6 + 15*(a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 +
7*a^3*b^5 + 2*a^2*b^6)*d*cosh(d*x + c)^4 + 3*(3*a^6*b^2 + 17*a^5*b^3 + 41*a
^4*b^4 + 51*a^3*b^5 + 32*a^2*b^6 + 8*a*b^7)*d*cosh(d*x + c)^2 + (a^6*b^2 +
5*a^5*b^3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6)*d)*sinh(d*x + c)^2 + (a^6*b^
2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d + 8*((a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^
4 + a^3*b^5)*d*cosh(d*x + c)^7 + 3*(a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a^3
*b^5 + 2*a^2*b^6)*d*cosh(d*x + c)^5 + (3*a^6*b^2 + 17*a^5*b^3 + 41*a^4*b^4
+ 51*a^3*b^5 + 32*a^2*b^6 + 8*a*b^7)*d*cosh(d*x + c)^3 + (a^6*b^2 + 5*a^5*b
^3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.35508, size = 374, normalized size = 2.99

$$\frac{(a+4\,b)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{8\left(a^2bd+2\,ab^2d+b^3d\right)\sqrt{-ab-b^2}}+\frac{a^3e^{(6\,dx+6\,c)}+4\,a^2be^{(6\,dx+6\,c)}+3\,a^3e^{(4\,dx+4\,c)}+2\,a^2be^{(4\,dx+4\,c)}-8\,ab^2e^{(4\,dx+4\,c)}-16\cdots}{4\left(a^3bd+2\,a^2b^2d+ab^3d\right)\left(ae^{(4\,dx+4\,c)}+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(a + 4*b)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/((
a^2*b*d + 2*a*b^2*d + b^3*d)*sqrt(-a*b - b^2)) + 1/4*(a^3*e^(6*d*x + 6*c) +
 4*a^2*b*e^(6*d*x + 6*c) + 3*a^3*e^(4*d*x + 4*c) + 2*a^2*b*e^(4*d*x + 4*c)
- 8*a*b^2*e^(4*d*x + 4*c) - 16*b^3*e^(4*d*x + 4*c) + 3*a^3*e^(2*d*x + 2*c)
- 4*a^2*b*e^(2*d*x + 2*c) - 16*a*b^2*e^(2*d*x + 2*c) + a^3 - 2*a^2*b)/((a^3
*b*d + 2*a^2*b^2*d + a*b^3*d)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*
b*e^(2*d*x + 2*c) + a)^2)
```

3.99 $\int \frac{\text{sech}^5(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=106

$$\frac{3\sinh(c+dx)}{8d(a+b)^2\left(a\sinh^2(c+dx)+a+b\right)}+\frac{\sinh(c+dx)}{4d(a+b)\left(a\sinh^2(c+dx)+a+b\right)^2}+\frac{3\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8\sqrt{a}d(a+b)^{5/2}}$$

```
[Out] (3*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(8*Sqrt[a]*(a + b)^(5/2)*d)
 + Sinh[c + d*x]/(4*(a + b)*d*(a + b + a*Sinh[c + d*x]^2)^2) + (3*Sinh[c +
d*x])/(8*(a + b)^2*d*(a + b + a*Sinh[c + d*x]^2))
```

Rubi [A] time = 0.0921015, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4147, 199, 205}

$$\frac{3\sinh(c+dx)}{8d(a+b)^2\left(a\sinh^2(c+dx)+a+b\right)}+\frac{\sinh(c+dx)}{4d(a+b)\left(a\sinh^2(c+dx)+a+b\right)^2}+\frac{3\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8\sqrt{a}d(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^5/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (3*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(8*Sqrt[a]*(a + b)^(5/2)*d)
 + Sinh[c + d*x]/(4*(a + b)*d*(a + b + a*Sinh[c + d*x]^2)^2) + (3*Sinh[c +
d*x])/(8*(a + b)^2*d*(a + b + a*Sinh[c + d*x]^2))
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{\left(a+b+ax^2\right)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\sinh(c+dx)}{4(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} + \frac{3\operatorname{Subst}\left(\int \frac{1}{\left(a+b+ax^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{4(a+b)d}$$

$$= \frac{\sinh(c+dx)}{4(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} + \frac{3\sinh(c+dx)}{8(a+b)^2d\left(a+b+a\sinh^2(c+dx)\right)} + \text{3 Subst}$$

$$= \frac{3\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8\sqrt{a}(a+b)^{5/2}d} + \frac{\sinh(c+dx)}{4(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} + \frac{3\sinh(c+}{8(a+b)^2d\left(a+b+a\,\mathrm{s}\right.}$$

Mathematica [A] time = 0.281514, size = 125, normalized size = 1.18

$$\frac{\operatorname{sech}^6(c+dx)(a\cosh(2(c+dx))+a+2b)^3\left(\frac{5(a+b)\sinh(c+dx)+3a\sinh^3(c+dx)}{(a+b)^2\left(a\sinh^2(c+dx)+a+b\right)^2} + \frac{3\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}(a+b)^{5/2}}\right)}{64d\left(a+b\operatorname{sech}^2(c+dx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^5/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^3*Sech[c + d*x]^6*((3*ArcTan[(Sqrt[a]*Sinh
[c + d*x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(5/2)) + (5*(a + b)*Sinh[c + d*x]
 + 3*a*Sinh[c + d*x]^3)/((a + b)^2*(a + b + a*Sinh[c + d*x]^2)^2)))/(64*d*(
a + b*Sech[c + d*x]^2)^3)
```

Maple [B] time = 0.077, size = 592, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -5/4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*
c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^7+3/4/d/(
tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2
*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5*a+15/4/d/(tan
h(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*ta
nh(1/2*d*x+1/2*c)^2*b+a+b)^2*b/(a+b)^2*tanh(1/2*d*x+1/2*c)^5-3/4/d/(tanh(1/
2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1
/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3*a-15/4/d/(tanh(1/2*d
*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*
d*x+1/2*c)^2*b+a+b)^2*b/(a+b)^2*tanh(1/2*d*x+1/2*c)^3+5/4/d/(tanh(1/2*d*x+1
/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+
1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)+3/8/d/(a^2+2*a*b+b^2)/(a+b)^(1/
2)/a^(1/2)*arctan(1/2*(2*tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)+2*b^(1/2))/a^(1/2)
```

```
)+3/8/d/(a^2+2*a*b+b^2)/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*tanh(1/2*d*x+1/2*
c)*(a+b)^(1/2)-2*b^(1/2))/a^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(11\,ae^{(5\,c)}+20\,be^{(5\,c)}\right)e^{(5\,dx)}-}{4\left(a^4d+2\,a^3bd+a^2b^2d+\left(a^4de^{(8\,c)}+2\,a^3bde^{(8\,c)}+a^2b^2de^{(8\,c)}\right)e^{(8\,dx)}+4\left(a^4de^{(6\,c)}+4\,a^3bde^{(6\,c)}+5\,a^2b^2de^{(6\,c)}+2\,ab^3de^{(6\,c}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((11*a*e^(5*c) + 20*b*e^(5*c))*e^(5*d*x) - (11*a*e^(3*c) + 20*b*e^(3*c)
)*e^(3*d*x) + 3*a*e^(7*d*x + 7*c) - 3*a*e^(d*x + c))/(a^4*d + 2*a^3*b*d + a
^2*b^2*d + (a^4*d*e^(8*c) + 2*a^3*b*d*e^(8*c) + a^2*b^2*d*e^(8*c))*e^(8*d*x
) + 4*(a^4*d*e^(6*c) + 4*a^3*b*d*e^(6*c) + 5*a^2*b^2*d*e^(6*c) + 2*a*b^3*d*
e^(6*c))*e^(6*d*x) + 2*(3*a^4*d*e^(4*c) + 14*a^3*b*d*e^(4*c) + 27*a^2*b^2*d
*e^(4*c) + 24*a*b^3*d*e^(4*c) + 8*b^4*d*e^(4*c))*e^(4*d*x) + 4*(a^4*d*e^(2*
c) + 4*a^3*b*d*e^(2*c) + 5*a^2*b^2*d*e^(2*c) + 2*a*b^3*d*e^(2*c))*e^(2*d*x)
) + 32*integrate(3/128*(e^(3*d*x + 3*c) + e^(d*x + c))/(a^3 + 2*a^2*b + a*b
^2 + (a^3*e^(4*c) + 2*a^2*b*e^(4*c) + a*b^2*e^(4*c))*e^(4*d*x) + 2*(a^3*e^(
2*c) + 4*a^2*b*e^(2*c) + 5*a*b^2*e^(2*c) + 2*b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.7666, size = 12078, normalized size = 113.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(12*(a^3 + a^2*b)*cosh(d*x + c)^7 + 84*(a^3 + a^2*b)*cosh(d*x + c)*si
nh(d*x + c)^6 + 12*(a^3 + a^2*b)*sinh(d*x + c)^7 + 4*(11*a^3 + 31*a^2*b + 2
0*a*b^2)*cosh(d*x + c)^5 + 4*(11*a^3 + 31*a^2*b + 20*a*b^2 + 63*(a^3 + a^2*
b)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(21*(a^3 + a^2*b)*cosh(d*x + c)^3
+ (11*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*(11*a^3
 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^3 + 4*(105*(a^3 + a^2*b)*cosh(d*x + c
)^4 - 11*a^3 - 31*a^2*b - 20*a*b^2 + 10*(11*a^3 + 31*a^2*b + 20*a*b^2)*cosh
(d*x + c)^2)*sinh(d*x + c)^3 + 4*(63*(a^3 + a^2*b)*cosh(d*x + c)^5 + 10*(11
*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^3 - 3*(11*a^3 + 31*a^2*b + 20*a*b
^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 3*(a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*
x + c)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 + 4*(a^2 + 2*a*b)*cosh(d*x + c
)^6 + 4*(7*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^6 + 8*(7*a^2*co
sh(d*x + c)^3 + 3*(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^2 +
 8*a*b + 8*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*cosh(d*x + c)^4 + 30*(a^2 + 2*a
*b)*cosh(d*x + c)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(d*x + c)^4 + 8*(7*a^2*cos
h(d*x + c)^5 + 10*(a^2 + 2*a*b)*cosh(d*x + c)^3 + (3*a^2 + 8*a*b + 8*b^2)*c
osh(d*x + c))*sinh(d*x + c)^3 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 4*(7*a^2*
cosh(d*x + c)^6 + 15*(a^2 + 2*a*b)*cosh(d*x + c)^4 + 3*(3*a^2 + 8*a*b + 8*b
^2)*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*(a^2*cosh(d*x
+ c)^7 + 3*(a^2 + 2*a*b)*cosh(d*x + c)^5 + (3*a^2 + 8*a*b + 8*b^2)*cosh(d*x
 + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 - a*b)*log(
(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4
- 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(
```

```
d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c
) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3
+ (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) +
 a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c
)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(
d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c)
+ a)) - 12*(a^3 + a^2*b)*cosh(d*x + c) + 4*(21*(a^3 + a^2*b)*cosh(d*x + c)^
6 + 5*(11*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^4 - 3*a^3 - 3*a^2*b - 3*
(11*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^6 + 3*a^
5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^8 + 8*(a^6 + 3*a^5*b + 3*a^4*b^2
 + a^3*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^6 + 3*a^5*b + 3*a^4*b^2 +
a^3*b^3)*d*sinh(d*x + c)^8 + 4*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a
^2*b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*co
sh(d*x + c)^2 + (a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d)*sinh
(d*x + c)^6 + 2*(3*a^6 + 17*a^5*b + 41*a^4*b^2 + 51*a^3*b^3 + 32*a^2*b^4 +
8*a*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*c
osh(d*x + c)^3 + 3*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*co
sh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*
d*cosh(d*x + c)^4 + 30*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*
d*cosh(d*x + c)^2 + (3*a^6 + 17*a^5*b + 41*a^4*b^2 + 51*a^3*b^3 + 32*a^2*b^
4 + 8*a*b^5)*d)*sinh(d*x + c)^4 + 4*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3
+ 2*a^2*b^4)*d*cosh(d*x + c)^2 + 8*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)
*d*cosh(d*x + c)^5 + 10*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)
*d*cosh(d*x + c)^3 + (3*a^6 + 17*a^5*b + 41*a^4*b^2 + 51*a^3*b^3 + 32*a^2*b
^4 + 8*a*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^6 + 3*a^5*b + 3*a^
4*b^2 + a^3*b^3)*d*cosh(d*x + c)^6 + 15*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*
b^3 + 2*a^2*b^4)*d*cosh(d*x + c)^4 + 3*(3*a^6 + 17*a^5*b + 41*a^4*b^2 + 51*
a^3*b^3 + 32*a^2*b^4 + 8*a*b^5)*d*cosh(d*x + c)^2 + (a^6 + 5*a^5*b + 9*a^4*
b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d)*sinh(d*x + c)^2 + (a^6 + 3*a^5*b + 3*a^4*b^
2 + a^3*b^3)*d + 8*((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^7
 + 3*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*cosh(d*x + c)^5
+ (3*a^6 + 17*a^5*b + 41*a^4*b^2 + 51*a^3*b^3 + 32*a^2*b^4 + 8*a*b^5)*d*cos
h(d*x + c)^3 + (a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*cosh(d
*x + c))*sinh(d*x + c)), 1/8*(6*(a^3 + a^2*b)*cosh(d*x + c)^7 + 42*(a^3 + a
^2*b)*cosh(d*x + c)*sinh(d*x + c)^6 + 6*(a^3 + a^2*b)*sinh(d*x + c)^7 + 2*(
11*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^5 + 2*(11*a^3 + 31*a^2*b + 20*a
*b^2 + 63*(a^3 + a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 10*(21*(a^3 + a^
2*b)*cosh(d*x + c)^3 + (11*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c))*sinh(d
*x + c)^4 - 2*(11*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^3 + 2*(105*(a^3
+ a^2*b)*cosh(d*x + c)^4 - 11*a^3 - 31*a^2*b - 20*a*b^2 + 10*(11*a^3 + 31*a
^2*b + 20*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(63*(a^3 + a^2*b)*cos
h(d*x + c)^5 + 10*(11*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^3 - 3*(11*a^
3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 3*(a^2*cosh(d*x +
 c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 + 4*(a^2
+ 2*a*b)*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x
 + c)^6 + 8*(7*a^2*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*
x + c)^5 + 2*(3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*cosh(d*x +
 c)^4 + 30*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(d*x
+ c)^4 + 8*(7*a^2*cosh(d*x + c)^5 + 10*(a^2 + 2*a*b)*cosh(d*x + c)^3 + (3*a
^2 + 8*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^2 + 2*a*b)*cosh(d
*x + c)^2 + 4*(7*a^2*cosh(d*x + c)^6 + 15*(a^2 + 2*a*b)*cosh(d*x + c)^4 + 3
*(3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a
^2 + 8*(a^2*cosh(d*x + c)^7 + 3*(a^2 + 2*a*b)*cosh(d*x + c)^5 + (3*a^2 + 8*
a*b + 8*b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))*
sqrt(a^2 + a*b)*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x
+ c)^2 + a*sinh(d*x + c)^3 + (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)
^2 + 3*a + 4*b)*sinh(d*x + c))/sqrt(a^2 + a*b)) + 3*(a^2*cosh(d*x + c)^8 +
8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 + 4*(a^2 + 2*a*b)
*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^6
```

```
+ 8*(7*a^2*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^5
 + 2*(3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*cosh(d*x + c)^4 +
30*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(d*x + c)^4 +
 8*(7*a^2*cosh(d*x + c)^5 + 10*(a^2 + 2*a*b)*cosh(d*x + c)^3 + (3*a^2 + 8*a
*b + 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^
2 + 4*(7*a^2*cosh(d*x + c)^6 + 15*(a^2 + 2*a*b)*cosh(d*x + c)^4 + 3*(3*a^2
+ 8*a*b + 8*b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*(
a^2*cosh(d*x + c)^7 + 3*(a^2 + 2*a*b)*cosh(d*x + c)^5 + (3*a^2 + 8*a*b + 8*
b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2
 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/(a + b))
 - 6*(a^3 + a^2*b)*cosh(d*x + c) + 2*(21*(a^3 + a^2*b)*cosh(d*x + c)^6 + 5*
(11*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^4 - 3*a^3 - 3*a^2*b - 3*(11*a^
3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^6 + 3*a^5*b +
3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^8 + 8*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3
*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^
3)*d*sinh(d*x + c)^8 + 4*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4
)*d*cosh(d*x + c)^6 + 4*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x
 + c)^2 + (a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d)*sinh(d*x +
 c)^6 + 2*(3*a^6 + 17*a^5*b + 41*a^4*b^2 + 51*a^3*b^3 + 32*a^2*b^4 + 8*a*b^
5)*d*cosh(d*x + c)^4 + 8*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*
x + c)^3 + 3*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*cosh(d*x
 + c))*sinh(d*x + c)^5 + 2*(35*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh
(d*x + c)^4 + 30*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*cosh
(d*x + c)^2 + (3*a^6 + 17*a^5*b + 41*a^4*b^2 + 51*a^3*b^3 + 32*a^2*b^4 + 8*
a*b^5)*d)*sinh(d*x + c)^4 + 4*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^
2*b^4)*d*cosh(d*x + c)^2 + 8*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cos
h(d*x + c)^5 + 10*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*cos
h(d*x + c)^3 + (3*a^6 + 17*a^5*b + 41*a^4*b^2 + 51*a^3*b^3 + 32*a^2*b^4 + 8
*a*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^6 + 3*a^5*b + 3*a^4*b^2
+ a^3*b^3)*d*cosh(d*x + c)^6 + 15*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 +
2*a^2*b^4)*d*cosh(d*x + c)^4 + 3*(3*a^6 + 17*a^5*b + 41*a^4*b^2 + 51*a^3*b^
3 + 32*a^2*b^4 + 8*a*b^5)*d*cosh(d*x + c)^2 + (a^6 + 5*a^5*b + 9*a^4*b^2 +
7*a^3*b^3 + 2*a^2*b^4)*d)*sinh(d*x + c)^2 + (a^6 + 3*a^5*b + 3*a^4*b^2 + a^
3*b^3)*d + 8*((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^7 + 3*(
a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*cosh(d*x + c)^5 + (3*a
^6 + 17*a^5*b + 41*a^4*b^2 + 51*a^3*b^3 + 32*a^2*b^4 + 8*a*b^5)*d*cosh(d*x
+ c)^3 + (a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*cosh(d*x + c
))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**5/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.100 $\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=144

$$\frac{\left(3a^2+8ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8b^{5/2}d(a+b)^{5/2}}-\frac{3a(a+2b)\tanh(c+dx)}{8b^2d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)}-\frac{a\tanh(c+dx)\operatorname{sech}^2(c+dx)}{4bd(a+b)\left(a-b\tanh^2(c+dx)+b\right.}$$

```
[Out] ((3*a^2 + 8*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*b
^(5/2)*(a + b)^(5/2)*d) - (a*Sech[c + d*x]^2*Tanh[c + d*x])/(4*b*(a + b)*d*
(a + b - b*Tanh[c + d*x]^2)^2) - (3*a*(a + 2*b)*Tanh[c + d*x])/(8*b^2*(a +
b)^2*d*(a + b - b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.142384, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4146, 413, 385, 208}

$$\frac{\left(3a^2+8ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8b^{5/2}d(a+b)^{5/2}}-\frac{3a(a+2b)\tanh(c+dx)}{8b^2d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)}-\frac{a\tanh(c+dx)\operatorname{sech}^2(c+dx)}{4bd(a+b)\left(a-b\tanh^2(c+dx)+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^6/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((3*a^2 + 8*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*b
^(5/2)*(a + b)^(5/2)*d) - (a*Sech[c + d*x]^2*Tanh[c + d*x])/(4*b*(a + b)*d*
(a + b - b*Tanh[c + d*x]^2)^2) - (3*a*(a + 2*b)*Tanh[c + d*x])/(8*b^2*(a +
b)^2*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\text{sech}^6(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a+b-bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{a\,\text{sech}^2(c+dx)\tanh(c+dx)}{4b(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-a-4b+(3a+4b)x^2}{\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{4b(a+b)d}$$

$$= -\frac{a\,\text{sech}^2(c+dx)\tanh(c+dx)}{4b(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{3a(a+2b)\tanh(c+dx)}{8b^2(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{(3$$

$$= \frac{\left(3a^2+8ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8b^{5/2}(a+b)^{5/2}d} - \frac{a\,\text{sech}^2(c+dx)\tanh(c+dx)}{4b(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{}{8b}$$

Mathematica [A] time = 0.961206, size = 125, normalized size = 0.87

$$\frac{\frac{\left(3a^2+8ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{a\sqrt{b}\sinh(2(c+dx))\left(3a^2+3a(a+2b)\cosh(2(c+dx))+16ab+16b^2\right)}{(a+b)^2(a\cosh(2(c+dx))+a+2b)^2}}{8b^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^6/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (((3*a^2 + 8*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a
+ b)^(5/2) - (a*Sqrt[b]*(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cosh[2*(c
+ d*x)])*Sinh[2*(c + d*x)])/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2))/
(8*b^(5/2)*d)
```

Maple [B] time = 0.078, size = 1236, normalized size = 8.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -3/4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*
c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*a^2/(a+b)/b^2*tanh(1/2*d*x+1/2*c)^7
-2/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)
^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/b/(a+b)*tanh(1/2*d*x+1/2*c)^7*a-9/4/d
/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a
-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*a^3/(a+b)^2/b^2*tanh(1/2*d*x+1/2*c)^5-13/
4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^
2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2/b*tanh(1/2*d*x+1/2*c)^5*a^2+2/
```

```
d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*
a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5*a-9/4/d/(t
anh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*
tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*a^3/(a+b)^2/b^2*tanh(1/2*d*x+1/2*c)^3-13/4/d
/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a
-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2/b*tanh(1/2*d*x+1/2*c)^3*a^2+2/d/(
tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2
*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3*a-3/4/d/(tanh
(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tan
h(1/2*d*x+1/2*c)^2*b+a+b)^2*a^2/(a+b)/b^2*tanh(1/2*d*x+1/2*c)-2/d/(tanh(1/2
*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/
2*d*x+1/2*c)^2*b+a+b)^2/b/(a+b)*tanh(1/2*d*x+1/2*c)*a+3/16/d/b^(5/2)/(a^2+2
*a*b+b^2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1
/2*c)*b^(1/2)+(a+b)^(1/2))*a^2+1/2/d/b^(3/2)/(a^2+2*a*b+b^2)*a/(a+b)^(1/2)*
ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1
/2))+1/2/d/(a^2+2*a*b+b^2)/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+
1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-3/16/d/b^(5/2)/(a^2+2*a
*b+b^2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2
*c)*b^(1/2)+(a+b)^(1/2))*a^2-1/2/d/b^(3/2)/(a^2+2*a*b+b^2)/(a+b)^(1/2)*ln((
a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))
*a-1/2/d/(a^2+2*a*b+b^2)/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/
2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 2.86775, size = 13508, normalized size = 93.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [1/16*(4*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^6 + 24
*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^
5 + 4*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*sinh(d*x + c)^6 + 12*a^
4*b + 36*a^3*b^2 + 24*a^2*b^3 + 12*(3*a^4*b + 17*a^3*b^2 + 38*a^2*b^3 + 40*
a*b^4 + 16*b^5)*cosh(d*x + c)^4 + 12*(3*a^4*b + 17*a^3*b^2 + 38*a^2*b^3 + 4
0*a*b^4 + 16*b^5 + 5*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*cosh(d*x
 + c)^2)*sinh(d*x + c)^4 + 16*(5*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b
^4)*cosh(d*x + c)^3 + 3*(3*a^4*b + 17*a^3*b^2 + 38*a^2*b^3 + 40*a*b^4 + 16*
b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(9*a^4*b + 49*a^3*b^2 + 80*a^2*b^3
+ 40*a*b^4)*cosh(d*x + c)^2 + 4*(9*a^4*b + 49*a^3*b^2 + 80*a^2*b^3 + 40*a*b
^4 + 15*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^4 + 18*
(3*a^4*b + 17*a^3*b^2 + 38*a^2*b^3 + 40*a*b^4 + 16*b^5)*cosh(d*x + c)^2)*si
nh(d*x + c)^2 + ((3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^8 + 8*(3*a^4 +
 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^4 + 8*a^3*b + 8*
```

```
a^2*b^2)*sinh(d*x + c)^8 + 4*(3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3)*cos
h(d*x + c)^6 + 4*(3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3 + 7*(3*a^4 + 8*a
^3*b + 8*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^4 + 8*a^3*b
+ 8*a^2*b^2)*cosh(d*x + c)^3 + 3*(3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3)
*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(9*a^4 + 48*a^3*b + 112*a^2*b^2 + 128*a
*b^3 + 64*b^4)*cosh(d*x + c)^4 + 2*(35*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d
*x + c)^4 + 9*a^4 + 48*a^3*b + 112*a^2*b^2 + 128*a*b^3 + 64*b^4 + 30*(3*a^4
 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 3*a
^4 + 8*a^3*b + 8*a^2*b^2 + 8*(7*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)
^5 + 10*(3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^3 + (9*a^4
 + 48*a^3*b + 112*a^2*b^2 + 128*a*b^3 + 64*b^4)*cosh(d*x + c))*sinh(d*x + c
)^3 + 4*(3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^2 + 4*(7*(
3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^6 + 15*(3*a^4 + 14*a^3*b + 24*a^
2*b^2 + 16*a*b^3)*cosh(d*x + c)^4 + 3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^
3 + 3*(9*a^4 + 48*a^3*b + 112*a^2*b^2 + 128*a*b^3 + 64*b^4)*cosh(d*x + c)^2
)*sinh(d*x + c)^2 + 8*((3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^7 + 3*(3
*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^5 + (9*a^4 + 48*a^3*
b + 112*a^2*b^2 + 128*a*b^3 + 64*b^4)*cosh(d*x + c)^3 + (3*a^4 + 14*a^3*b +
 24*a^2*b^2 + 16*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b + b^2)*log((
a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x +
c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2
*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2
 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*
x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*cos
h(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a
 + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2
 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 8*
(3*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^5 + 6*(3*a^4
*b + 17*a^3*b^2 + 38*a^2*b^3 + 40*a*b^4 + 16*b^5)*cosh(d*x + c)^3 + (9*a^4*
b + 49*a^3*b^2 + 80*a^2*b^3 + 40*a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^5
*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^8 + 8*(a^5*b^3 + 3*
a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^5*b^3 +
 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*sinh(d*x + c)^8 + 4*(a^5*b^3 + 5*a^4*b^
4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d*cosh(d*x + c)^6 + 4*(7*(a^5*b^3 + 3*
a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^2 + (a^5*b^3 + 5*a^4*b^4 + 9
*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d)*sinh(d*x + c)^6 + 2*(3*a^5*b^3 + 17*a^4*
b^4 + 41*a^3*b^5 + 51*a^2*b^6 + 32*a*b^7 + 8*b^8)*d*cosh(d*x + c)^4 + 8*(7*
(a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^3 + 3*(a^5*b^3
+ 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d*cosh(d*x + c))*sinh(d*x +
c)^5 + 2*(35*(a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^4
+ 30*(a^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d*cosh(d*x + c
)^2 + (3*a^5*b^3 + 17*a^4*b^4 + 41*a^3*b^5 + 51*a^2*b^6 + 32*a*b^7 + 8*b^8)
*d)*sinh(d*x + c)^4 + 4*(a^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*
b^7)*d*cosh(d*x + c)^2 + 8*(7*(a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d
*cosh(d*x + c)^5 + 10*(a^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^
7)*d*cosh(d*x + c)^3 + (3*a^5*b^3 + 17*a^4*b^4 + 41*a^3*b^5 + 51*a^2*b^6 +
32*a*b^7 + 8*b^8)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^5*b^3 + 3*a^4*
b^4 + 3*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^6 + 15*(a^5*b^3 + 5*a^4*b^4 + 9*
a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d*cosh(d*x + c)^4 + 3*(3*a^5*b^3 + 17*a^4*b^
4 + 41*a^3*b^5 + 51*a^2*b^6 + 32*a*b^7 + 8*b^8)*d*cosh(d*x + c)^2 + (a^5*b^
3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d)*sinh(d*x + c)^2 + (a^5*
b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d + 8*((a^5*b^3 + 3*a^4*b^4 + 3*a^3*
b^5 + a^2*b^6)*d*cosh(d*x + c)^7 + 3*(a^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a
^2*b^6 + 2*a*b^7)*d*cosh(d*x + c)^5 + (3*a^5*b^3 + 17*a^4*b^4 + 41*a^3*b^5
+ 51*a^2*b^6 + 32*a*b^7 + 8*b^8)*d*cosh(d*x + c)^3 + (a^5*b^3 + 5*a^4*b^4 +
 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*(2*(
3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^6 + 12*(3*a^4*b
+ 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(3*a
^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*sinh(d*x + c)^6 + 6*a^4*b + 18*a^
```

3*b^2 + 12*a^2*b^3 + 6*(3*a^4*b + 17*a^3*b^2 + 38*a^2*b^3 + 40*a*b^4 + 16*b
^5)*cosh(d*x + c)^4 + 6*(3*a^4*b + 17*a^3*b^2 + 38*a^2*b^3 + 40*a*b^4 + 16*
b^5 + 5*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^2)*sinh
(d*x + c)^4 + 8*(5*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*cosh(d*x +
c)^3 + 3*(3*a^4*b + 17*a^3*b^2 + 38*a^2*b^3 + 40*a*b^4 + 16*b^5)*cosh(d*x
+ c))*sinh(d*x + c)^3 + 2*(9*a^4*b + 49*a^3*b^2 + 80*a^2*b^3 + 40*a*b^4)*co
sh(d*x + c)^2 + 2*(9*a^4*b + 49*a^3*b^2 + 80*a^2*b^3 + 40*a*b^4 + 15*(3*a^4
*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^4 + 18*(3*a^4*b + 17*
a^3*b^2 + 38*a^2*b^3 + 40*a*b^4 + 16*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2
+ ((3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^8 + 8*(3*a^4 + 8*a^3*b + 8*a
^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^4 + 8*a^3*b + 8*a^2*b^2)*sinh(
d*x + c)^8 + 4*(3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^6 +
4*(3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3 + 7*(3*a^4 + 8*a^3*b + 8*a^2*b
^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*c
osh(d*x + c)^3 + 3*(3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)
)*sinh(d*x + c)^5 + 2*(9*a^4 + 48*a^3*b + 112*a^2*b^2 + 128*a*b^3 + 64*b^4)
*cosh(d*x + c)^4 + 2*(35*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + 9*
a^4 + 48*a^3*b + 112*a^2*b^2 + 128*a*b^3 + 64*b^4 + 30*(3*a^4 + 14*a^3*b +
24*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 3*a^4 + 8*a^3*b +
8*a^2*b^2 + 8*(7*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^5 + 10*(3*a^4
+ 14*a^3*b + 24*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^3 + (9*a^4 + 48*a^3*b +
112*a^2*b^2 + 128*a*b^3 + 64*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a^4
+ 14*a^3*b + 24*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^2 + 4*(7*(3*a^4 + 8*a^3*
b + 8*a^2*b^2)*cosh(d*x + c)^6 + 15*(3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b
^3)*cosh(d*x + c)^4 + 3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3 + 3*(9*a^4 +
48*a^3*b + 112*a^2*b^2 + 128*a*b^3 + 64*b^4)*cosh(d*x + c)^2)*sinh(d*x + c
)^2 + 8*((3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^7 + 3*(3*a^4 + 14*a^3*
b + 24*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^5 + (9*a^4 + 48*a^3*b + 112*a^2*b^
2 + 128*a*b^3 + 64*b^4)*cosh(d*x + c)^3 + (3*a^4 + 14*a^3*b + 24*a^2*b^2 +
16*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b - b^2)*arctan(1/2*(a*cosh
(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b
)*sqrt(-a*b - b^2)/(a*b + b^2)) + 4*(3*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 +
8*a*b^4)*cosh(d*x + c)^5 + 6*(3*a^4*b + 17*a^3*b^2 + 38*a^2*b^3 + 40*a*b^4
+ 16*b^5)*cosh(d*x + c)^3 + (9*a^4*b + 49*a^3*b^2 + 80*a^2*b^3 + 40*a*b^4)
*cosh(d*x + c))*sinh(d*x + c))/((a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)
*d*cosh(d*x + c)^8 + 8*(a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*cosh(d
*x + c)*sinh(d*x + c)^7 + (a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*sin
h(d*x + c)^8 + 4*(a^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d*
cosh(d*x + c)^6 + 4*(7*(a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*cosh(d
*x + c)^2 + (a^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d)*sinh
(d*x + c)^6 + 2*(3*a^5*b^3 + 17*a^4*b^4 + 41*a^3*b^5 + 51*a^2*b^6 + 32*a*b^
7 + 8*b^8)*d*cosh(d*x + c)^4 + 8*(7*(a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*
b^6)*d*cosh(d*x + c)^3 + 3*(a^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2
*a*b^7)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^5*b^3 + 3*a^4*b^4 + 3*a
^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^4 + 30*(a^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 +
7*a^2*b^6 + 2*a*b^7)*d*cosh(d*x + c)^2 + (3*a^5*b^3 + 17*a^4*b^4 + 41*a^3*
b^5 + 51*a^2*b^6 + 32*a*b^7 + 8*b^8)*d)*sinh(d*x + c)^4 + 4*(a^5*b^3 + 5*a^
4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d*cosh(d*x + c)^2 + 8*(7*(a^5*b^3
+ 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^5 + 10*(a^5*b^3 + 5*a^4*
b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d*cosh(d*x + c)^3 + (3*a^5*b^3 + 17*
a^4*b^4 + 41*a^3*b^5 + 51*a^2*b^6 + 32*a*b^7 + 8*b^8)*d*cosh(d*x + c))*sinh
(d*x + c)^3 + 4*(7*(a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*cosh(d*x +
c)^6 + 15*(a^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d*cosh(d
x + c)^4 + 3(3*a^5*b^3 + 17*a^4*b^4 + 41*a^3*b^5 + 51*a^2*b^6 + 32*a*b^7
+ 8*b^8)*d*cosh(d*x + c)^2 + (a^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 +
2*a*b^7)*d)*sinh(d*x + c)^2 + (a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*
d + 8*((a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^7 + 3*(a
^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d*cosh(d*x + c)^5 + (
3*a^5*b^3 + 17*a^4*b^4 + 41*a^3*b^5 + 51*a^2*b^6 + 32*a*b^7 + 8*b^8)*d*cosh

```
(d*x + c)^3 + (a^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d*cos
h(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**6/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.34101, size = 413, normalized size = 2.87

$$\frac{\left(3\,a^2+8\,ab+8\,b^2\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{8\left(a^2b^2d+2\,ab^3d+b^4d\right)\sqrt{-ab-b^2}}+\frac{3\,a^3e^{(6\,dx+6\,c)}+8\,a^2be^{(6\,dx+6\,c)}+8\,ab^2e^{(6\,dx+6\,c)}+9\,a^3e^{(4\,dx+4\,c)}+42\,a^2be^{(4}}{4\left(a^2b^2d+2\,ab^3d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*a^2 + 8*a*b + 8*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-
a*b - b^2))/((a^2*b^2*d + 2*a*b^3*d + b^4*d)*sqrt(-a*b - b^2)) + 1/4*(3*a^3
*e^(6*d*x + 6*c) + 8*a^2*b*e^(6*d*x + 6*c) + 8*a*b^2*e^(6*d*x + 6*c) + 9*a^
3*e^(4*d*x + 4*c) + 42*a^2*b*e^(4*d*x + 4*c) + 72*a*b^2*e^(4*d*x + 4*c) + 4
8*b^3*e^(4*d*x + 4*c) + 9*a^3*e^(2*d*x + 2*c) + 40*a^2*b*e^(2*d*x + 2*c) +
40*a*b^2*e^(2*d*x + 2*c) + 3*a^3 + 6*a^2*b)/((a^2*b^2*d + 2*a*b^3*d + b^4*d
)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)^2)
```

3.101 $\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=153

$$-\frac{\sqrt{a}\left(8a^2+20ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8b^3d(a+b)^{5/2}}-\frac{a(4a+7b)\sinh(c+dx)}{8b^2d(a+b)^2\left(a\sinh^2(c+dx)+a+b\right)}-\frac{a\sinh(c+dx)}{4bd(a+b)\left(a\sinh^2(c+dx)+a\right.}$$

```
[Out] ArcTan[Sinh[c + d*x]]/(b^3*d) - (Sqrt[a]*(8*a^2 + 20*a*b + 15*b^2)*ArcTan[(
Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(8*b^3*(a + b)^(5/2)*d) - (a*Sinh[c +
d*x])/(4*b*(a + b)*d*(a + b + a*Sinh[c + d*x]^2)^2) - (a*(4*a + 7*b)*Sinh[c
 + d*x])/(8*b^2*(a + b)^2*d*(a + b + a*Sinh[c + d*x]^2))
```

Rubi [A] time = 0.204683, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4147, 414, 527, 522, 203, 205}

$$-\frac{\sqrt{a}\left(8a^2+20ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8b^3d(a+b)^{5/2}}-\frac{a(4a+7b)\sinh(c+dx)}{8b^2d(a+b)^2\left(a\sinh^2(c+dx)+a+b\right)}-\frac{a\sinh(c+dx)}{4bd(a+b)\left(a\sinh^2(c+dx)+a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^7/(a + b*Sech[c + d*x]^2)^3,x]
```


```
[Out] ArcTan[Sinh[c + d*x]]/(b^3*d) - (Sqrt[a]*(8*a^2 + 20*a*b + 15*b^2)*ArcTan[(
Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(8*b^3*(a + b)^(5/2)*d) - (a*Sinh[c +
d*x])/(4*b*(a + b)*d*(a + b + a*Sinh[c + d*x]^2)^2) - (a*(4*a + 7*b)*Sinh[c
 + d*x])/(8*b^2*(a + b)^2*d*(a + b + a*Sinh[c + d*x]^2))
```


Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
```

```
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+b+ax^2\right)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{a\sinh(c+dx)}{4b(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{a+4b-3ax^2}{(1+x^2)\left(a+b+ax^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{4b(a+b)d}$$

$$= -\frac{a\sinh(c+dx)}{4b(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} - \frac{a(4a+7b)\sinh(c+dx)}{8b^2(a+b)^2d\left(a+b+a\sinh^2(c+dx)\right)} + \text{Su}$$

$$= -\frac{a\sinh(c+dx)}{4b(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} - \frac{a(4a+7b)\sinh(c+dx)}{8b^2(a+b)^2d\left(a+b+a\sinh^2(c+dx)\right)} + \text{Su}$$

$$= \frac{\tan^{-1}(\sinh(c+dx))}{b^3d} - \frac{\sqrt{a}\left(8a^2+20ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8b^3(a+b)^{5/2}d} - \frac{a\sin}{4b(a+b)d\left(a+\right.}$$

Mathematica [A] time = 4.60144, size = 247, normalized size = 1.61

$$\operatorname{sech}^5(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\frac{\sqrt{a}\left(8a^2+20ab+15b^2\right)(\cosh(c)-\sinh(c))\operatorname{sech}(c+dx)(a\cosh(2(c+dx))+a+2b)^2\tan^{-1}\left(\frac{\sqrt{a+b}\sqrt{(\cosh(c)}}{}\right.}{(a+b)^{5/2}\sqrt{(\cosh(c)-\sinh(c))^2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^7/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^5*(16*ArcTan[Tanh[(c + d*x)/
2]]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x] + (Sqrt[a]*(8*a^2 + 20*
```

```
a*b + 15*b^2)*ArcTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]
*(Cosh[c] + Sinh[c]))/Sqrt[a]]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d
*x]*(Cosh[c] - Sinh[c]))/((a + b)^(5/2)*Sqrt[(Cosh[c] - Sinh[c])^2]) - (8*a
*b^2*Tanh[c + d*x])/(a + b) - (2*a*b*(4*a + 7*b)*(a + 2*b + a*Cosh[2*(c + d
*x)])*Tanh[c + d*x])/(a + b)^2))/(64*b^3*d*(a + b*Sech[c + d*x]^2)^3)
```

Maple [B] time = 0.084, size = 1202, normalized size = 7.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^
2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*a^2/(a+b)/b^2*tanh(1/2*d*x+1/2*c)^7+9/
4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^
2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/b/(a+b)*tanh(1/2*d*x+1/2*c)^7*a+1/d/(t
anh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*
tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*a^3/(a+b)^2/b^2*tanh(1/2*d*x+1/2*c)^5-11/4/d
/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a
-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2/b*tanh(1/2*d*x+1/2*c)^5*a^2-27/4/
d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*
a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5*a-1/d/(tan
h(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*ta
nh(1/2*d*x+1/2*c)^2*b+a+b)^2*a^3/(a+b)^2/b^2*tanh(1/2*d*x+1/2*c)^3+11/4/d/(
tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2
*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2/b*tanh(1/2*d*x+1/2*c)^3*a^2+27/4/d/
(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-
2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3*a-1/d/(tanh(
1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh
(1/2*d*x+1/2*c)^2*b+a+b)^2*a^2/(a+b)/b^2*tanh(1/2*d*x+1/2*c)-9/4/d/(tanh(1/
2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1
/2*d*x+1/2*c)^2*b+a+b)^2/b/(a+b)*tanh(1/2*d*x+1/2*c)*a-1/d*a^(5/2)/b^3/(a^2
+2*a*b+b^2)/(a+b)^(1/2)*arctan(1/2*(2*tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)+2*b^(
1/2))/a^(1/2))-5/2/d*a^(3/2)/b^2/(a^2+2*a*b+b^2)/(a+b)^(1/2)*arctan(1/2*(2*
tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)+2*b^(1/2))/a^(1/2))-15/8/d*a^(1/2)/b/(a^2+2
*a*b+b^2)/(a+b)^(1/2)*arctan(1/2*(2*tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)+2*b^(1/
2))/a^(1/2))-1/d*a^(5/2)/b^3/(a^2+2*a*b+b^2)/(a+b)^(1/2)*arctan(1/2*(2*tanh
(1/2*d*x+1/2*c)*(a+b)^(1/2)-2*b^(1/2))/a^(1/2))-5/2/d*a^(3/2)/b^2/(a^2+2*a*
b+b^2)/(a+b)^(1/2)*arctan(1/2*(2*tanh(1/2*d*x+1/2*c)*(a+b)^(1/2)-2*b^(1/2))
/a^(1/2))-15/8/d*a^(1/2)/b/(a^2+2*a*b+b^2)/(a+b)^(1/2)*arctan(1/2*(2*tanh(1
/2*d*x+1/2*c)*(a+b)^(1/2)-2*b^(1/2))/a^(1/2))+2/d/b^3*arctan(tanh(1/2*d*x+1
/2*c))
```

Maple [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 3.80044, size = 18826, normalized size = 123.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(4*a^3*b + 7*a^2*b^2)*cosh(d*x + c)^7 + 28*(4*a^3*b + 7*a^2*b^2)*
cosh(d*x + c)*sinh(d*x + c)^6 + 4*(4*a^3*b + 7*a^2*b^2)*sinh(d*x + c)^7 + 4
*(4*a^3*b + 31*a^2*b^2 + 36*a*b^3)*cosh(d*x + c)^5 + 4*(4*a^3*b + 31*a^2*b^
2 + 36*a*b^3 + 21*(4*a^3*b + 7*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^5 +
20*(7*(4*a^3*b + 7*a^2*b^2)*cosh(d*x + c)^3 + (4*a^3*b + 31*a^2*b^2 + 36*a*
b^3)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*(4*a^3*b + 31*a^2*b^2 + 36*a*b^3)*c
osh(d*x + c)^3 + 4*(35*(4*a^3*b + 7*a^2*b^2)*cosh(d*x + c)^4 - 4*a^3*b - 31
*a^2*b^2 - 36*a*b^3 + 10*(4*a^3*b + 31*a^2*b^2 + 36*a*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^3 + 4*(21*(4*a^3*b + 7*a^2*b^2)*cosh(d*x + c)^5 + 10*(4*a^3*
b + 31*a^2*b^2 + 36*a*b^3)*cosh(d*x + c)^3 - 3*(4*a^3*b + 31*a^2*b^2 + 36*a
*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - ((8*a^4 + 20*a^3*b + 15*a^2*b^2)*cos
h(d*x + c)^8 + 8*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c
)^7 + (8*a^4 + 20*a^3*b + 15*a^2*b^2)*sinh(d*x + c)^8 + 4*(8*a^4 + 36*a^3*b
 + 55*a^2*b^2 + 30*a*b^3)*cosh(d*x + c)^6 + 4*(8*a^4 + 36*a^3*b + 55*a^2*b^
2 + 30*a*b^3 + 7*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x
+ c)^6 + 8*(7*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^3 + 3*(8*a^4 +
36*a^3*b + 55*a^2*b^2 + 30*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(24*a^
4 + 124*a^3*b + 269*a^2*b^2 + 280*a*b^3 + 120*b^4)*cosh(d*x + c)^4 + 2*(35*
(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^4 + 24*a^4 + 124*a^3*b + 269*
a^2*b^2 + 280*a*b^3 + 120*b^4 + 30*(8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^
3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*a^4 + 20*a^3*b + 15*a^2*b^2 + 8*(7*
(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^5 + 10*(8*a^4 + 36*a^3*b + 55
*a^2*b^2 + 30*a*b^3)*cosh(d*x + c)^3 + (24*a^4 + 124*a^3*b + 269*a^2*b^2 +
280*a*b^3 + 120*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(8*a^4 + 36*a^3*b +
 55*a^2*b^2 + 30*a*b^3)*cosh(d*x + c)^2 + 4*(7*(8*a^4 + 20*a^3*b + 15*a^2*b
^2)*cosh(d*x + c)^6 + 15*(8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3)*cosh(d*
x + c)^4 + 8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3 + 3*(24*a^4 + 124*a^3*b
 + 269*a^2*b^2 + 280*a*b^3 + 120*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*
((8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^7 + 3*(8*a^4 + 36*a^3*b + 55
*a^2*b^2 + 30*a*b^3)*cosh(d*x + c)^5 + (24*a^4 + 124*a^3*b + 269*a^2*b^2 +
280*a*b^3 + 120*b^4)*cosh(d*x + c)^3 + (8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*
a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a/(a + b))*log((a*cosh(d*x + c)^
4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*c
osh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a
*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*co
sh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x
+ c)^3 - (a + b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d
*x + c))*sqrt(-a/(a + b)) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(
d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(
d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*co
sh(d*x + c))*sinh(d*x + c) + a)) - 32*((a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x +
 c)^8 + 8*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^4 +
2*a^3*b + a^2*b^2)*sinh(d*x + c)^8 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3
)*cosh(d*x + c)^6 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3 + 7*(a^4 + 2*a^3
*b + a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^4 + 2*a^3*b + a^2*
b^2)*cosh(d*x + c)^3 + 3*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(d*x + c
))*sinh(d*x + c)^5 + 2*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*c
osh(d*x + c)^4 + 2*(35*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^4 + 3*a^4 +
14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4 + 30*(a^4 + 4*a^3*b + 5*a^2*b^2 +
2*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + a^4 + 2*a^3*b + a^2*b^2 + 8*(7*
```

```
(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^5 + 10*(a^4 + 4*a^3*b + 5*a^2*b^2 +
 2*a*b^3)*cosh(d*x + c)^3 + (3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b
^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3
)*cosh(d*x + c)^2 + 4*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^6 + 15*(a^
4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^4 + a^4 + 4*a^3*b + 5*a^2*
b^2 + 2*a*b^3 + 3*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*cosh(d
*x + c)^2)*sinh(d*x + c)^2 + 8*((a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^7 +
 3*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^5 + (3*a^4 + 14*a^3*
b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*cosh(d*x + c)^3 + (a^4 + 4*a^3*b + 5*a^2
*b^2 + 2*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d
*x + c)) - 4*(4*a^3*b + 7*a^2*b^2)*cosh(d*x + c) + 4*(7*(4*a^3*b + 7*a^2*b^
2)*cosh(d*x + c)^6 + 5*(4*a^3*b + 31*a^2*b^2 + 36*a*b^3)*cosh(d*x + c)^4 -
4*a^3*b - 7*a^2*b^2 - 3*(4*a^3*b + 31*a^2*b^2 + 36*a*b^3)*cosh(d*x + c)^2)*
sinh(d*x + c))/((a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^8 + 8*(a^4*
b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^4*b^3 + 2*a
^3*b^4 + a^2*b^5)*d*sinh(d*x + c)^8 + 4*(a^4*b^3 + 4*a^3*b^4 + 5*a^2*b^5 +
2*a*b^6)*d*cosh(d*x + c)^6 + 4*(7*(a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*
x + c)^2 + (a^4*b^3 + 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d)*sinh(d*x + c)^6 +
 2*(3*a^4*b^3 + 14*a^3*b^4 + 27*a^2*b^5 + 24*a*b^6 + 8*b^7)*d*cosh(d*x + c)
^4 + 8*(7*(a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^3 + 3*(a^4*b^3 +
4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(
a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^4 + 30*(a^4*b^3 + 4*a^3*b^4
+ 5*a^2*b^5 + 2*a*b^6)*d*cosh(d*x + c)^2 + (3*a^4*b^3 + 14*a^3*b^4 + 27*a^2
*b^5 + 24*a*b^6 + 8*b^7)*d)*sinh(d*x + c)^4 + 4*(a^4*b^3 + 4*a^3*b^4 + 5*a^
2*b^5 + 2*a*b^6)*d*cosh(d*x + c)^2 + 8*(7*(a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d
*cosh(d*x + c)^5 + 10*(a^4*b^3 + 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d*cosh(d*
x + c)^3 + (3*a^4*b^3 + 14*a^3*b^4 + 27*a^2*b^5 + 24*a*b^6 + 8*b^7)*d*cosh(
d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x
 + c)^6 + 15*(a^4*b^3 + 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d*cosh(d*x + c)^4
+ 3*(3*a^4*b^3 + 14*a^3*b^4 + 27*a^2*b^5 + 24*a*b^6 + 8*b^7)*d*cosh(d*x + c
)^2 + (a^4*b^3 + 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d)*sinh(d*x + c)^2 + (a^4
*b^3 + 2*a^3*b^4 + a^2*b^5)*d + 8*((a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d
*x + c)^7 + 3*(a^4*b^3 + 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d*cosh(d*x + c)^5
 + (3*a^4*b^3 + 14*a^3*b^4 + 27*a^2*b^5 + 24*a*b^6 + 8*b^7)*d*cosh(d*x + c)
^3 + (a^4*b^3 + 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d*cosh(d*x + c))*sinh(d*x
+ c)), -1/8*(2*(4*a^3*b + 7*a^2*b^2)*cosh(d*x + c)^7 + 14*(4*a^3*b + 7*a^2*
b^2)*cosh(d*x + c)*sinh(d*x + c)^6 + 2*(4*a^3*b + 7*a^2*b^2)*sinh(d*x + c)^
7 + 2*(4*a^3*b + 31*a^2*b^2 + 36*a*b^3)*cosh(d*x + c)^5 + 2*(4*a^3*b + 31*a
^2*b^2 + 36*a*b^3 + 21*(4*a^3*b + 7*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)
^5 + 10*(7*(4*a^3*b + 7*a^2*b^2)*cosh(d*x + c)^3 + (4*a^3*b + 31*a^2*b^2 +
36*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 - 2*(4*a^3*b + 31*a^2*b^2 + 36*a*b
^3)*cosh(d*x + c)^3 + 2*(35*(4*a^3*b + 7*a^2*b^2)*cosh(d*x + c)^4 - 4*a^3*b
 - 31*a^2*b^2 - 36*a*b^3 + 10*(4*a^3*b + 31*a^2*b^2 + 36*a*b^3)*cosh(d*x +
c)^2)*sinh(d*x + c)^3 + 2*(21*(4*a^3*b + 7*a^2*b^2)*cosh(d*x + c)^5 + 10*(4
*a^3*b + 31*a^2*b^2 + 36*a*b^3)*cosh(d*x + c)^3 - 3*(4*a^3*b + 31*a^2*b^2 +
 36*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + ((8*a^4 + 20*a^3*b + 15*a^2*b^2
)*cosh(d*x + c)^8 + 8*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)*sinh(d*
x + c)^7 + (8*a^4 + 20*a^3*b + 15*a^2*b^2)*sinh(d*x + c)^8 + 4*(8*a^4 + 36*
a^3*b + 55*a^2*b^2 + 30*a*b^3)*cosh(d*x + c)^6 + 4*(8*a^4 + 36*a^3*b + 55*a
^2*b^2 + 30*a*b^3 + 7*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^2)*sinh
(d*x + c)^6 + 8*(7*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^3 + 3*(8*a
^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(
24*a^4 + 124*a^3*b + 269*a^2*b^2 + 280*a*b^3 + 120*b^4)*cosh(d*x + c)^4 + 2
*(35*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^4 + 24*a^4 + 124*a^3*b +
 269*a^2*b^2 + 280*a*b^3 + 120*b^4 + 30*(8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30
*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*a^4 + 20*a^3*b + 15*a^2*b^2 +
8*(7*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^5 + 10*(8*a^4 + 36*a^3*b
 + 55*a^2*b^2 + 30*a*b^3)*cosh(d*x + c)^3 + (24*a^4 + 124*a^3*b + 269*a^2*b
^2 + 280*a*b^3 + 120*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(8*a^4 + 36*a^
```

```
3*b + 55*a^2*b^2 + 30*a*b^3)*cosh(d*x + c)^2 + 4*(7*(8*a^4 + 20*a^3*b + 15*
a^2*b^2)*cosh(d*x + c)^6 + 15*(8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3)*co
sh(d*x + c)^4 + 8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3 + 3*(24*a^4 + 124*
a^3*b + 269*a^2*b^2 + 280*a*b^3 + 120*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2
 + 8*((8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^7 + 3*(8*a^4 + 36*a^3*b
 + 55*a^2*b^2 + 30*a*b^3)*cosh(d*x + c)^5 + (24*a^4 + 124*a^3*b + 269*a^2*b
^2 + 280*a*b^3 + 120*b^4)*cosh(d*x + c)^3 + (8*a^4 + 36*a^3*b + 55*a^2*b^2
+ 30*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a/(a + b))*arctan(1/2*sqrt(a
/(a + b))*(cosh(d*x + c) + sinh(d*x + c))) + ((8*a^4 + 20*a^3*b + 15*a^2*b^
2)*cosh(d*x + c)^8 + 8*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)*sinh(d
*x + c)^7 + (8*a^4 + 20*a^3*b + 15*a^2*b^2)*sinh(d*x + c)^8 + 4*(8*a^4 + 36
*a^3*b + 55*a^2*b^2 + 30*a*b^3)*cosh(d*x + c)^6 + 4*(8*a^4 + 36*a^3*b + 55*
a^2*b^2 + 30*a*b^3 + 7*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^2)*sin
h(d*x + c)^6 + 8*(7*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^3 + 3*(8*
a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*
(24*a^4 + 124*a^3*b + 269*a^2*b^2 + 280*a*b^3 + 120*b^4)*cosh(d*x + c)^4 +
2*(35*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^4 + 24*a^4 + 124*a^3*b
+ 269*a^2*b^2 + 280*a*b^3 + 120*b^4 + 30*(8*a^4 + 36*a^3*b + 55*a^2*b^2 + 3
0*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*a^4 + 20*a^3*b + 15*a^2*b^2 +
 8*(7*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^5 + 10*(8*a^4 + 36*a^3*
b + 55*a^2*b^2 + 30*a*b^3)*cosh(d*x + c)^3 + (24*a^4 + 124*a^3*b + 269*a^2*
b^2 + 280*a*b^3 + 120*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(8*a^4 + 36*a
^3*b + 55*a^2*b^2 + 30*a*b^3)*cosh(d*x + c)^2 + 4*(7*(8*a^4 + 20*a^3*b + 15
*a^2*b^2)*cosh(d*x + c)^6 + 15*(8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3)*c
osh(d*x + c)^4 + 8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3 + 3*(24*a^4 + 124
*a^3*b + 269*a^2*b^2 + 280*a*b^3 + 120*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^
2 + 8*((8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^7 + 3*(8*a^4 + 36*a^3*
b + 55*a^2*b^2 + 30*a*b^3)*cosh(d*x + c)^5 + (24*a^4 + 124*a^3*b + 269*a^2*
b^2 + 280*a*b^3 + 120*b^4)*cosh(d*x + c)^3 + (8*a^4 + 36*a^3*b + 55*a^2*b^2
 + 30*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a/(a + b))*arctan(1/2*(a*co
sh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (3*
a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*a + 4*b)*sinh(d*x + c))*s
qrt(a/(a + b))/a) - 16*((a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^8 + 8*(a^4
+ 2*a^3*b + a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^4 + 2*a^3*b + a^2*b
^2)*sinh(d*x + c)^8 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)
^6 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3 + 7*(a^4 + 2*a^3*b + a^2*b^2)*c
osh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x +
 c)^3 + 3*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(d*x + c))*sinh(d*x + c
)^5 + 2*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*cosh(d*x + c)^4
+ 2*(35*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^4 + 3*a^4 + 14*a^3*b + 27*a
^2*b^2 + 24*a*b^3 + 8*b^4 + 30*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(d
*x + c)^2)*sinh(d*x + c)^4 + a^4 + 2*a^3*b + a^2*b^2 + 8*(7*(a^4 + 2*a^3*b
+ a^2*b^2)*cosh(d*x + c)^5 + 10*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(
d*x + c)^3 + (3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*cosh(d*x +
c))*sinh(d*x + c)^3 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)
^2 + 4*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^6 + 15*(a^4 + 4*a^3*b + 5
*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^4 + a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3 +
 3*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*cosh(d*x + c)^2)*sinh
(d*x + c)^2 + 8*((a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^7 + 3*(a^4 + 4*a^3
*b + 5*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^5 + (3*a^4 + 14*a^3*b + 27*a^2*b^2
+ 24*a*b^3 + 8*b^4)*cosh(d*x + c)^3 + (a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)
*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 2*(4
*a^3*b + 7*a^2*b^2)*cosh(d*x + c) + 2*(7*(4*a^3*b + 7*a^2*b^2)*cosh(d*x + c
)^6 + 5*(4*a^3*b + 31*a^2*b^2 + 36*a*b^3)*cosh(d*x + c)^4 - 4*a^3*b - 7*a^2
*b^2 - 3*(4*a^3*b + 31*a^2*b^2 + 36*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/
((a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^8 + 8*(a^4*b^3 + 2*a^3*b^4
 + a^2*b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^4*b^3 + 2*a^3*b^4 + a^2*b^
5)*d*sinh(d*x + c)^8 + 4*(a^4*b^3 + 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d*cosh
(d*x + c)^6 + 4*(7*(a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^2 + (a^4
```

```
*b^3 + 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d)*sinh(d*x + c)^6 + 2*(3*a^4*b^3 +
 14*a^3*b^4 + 27*a^2*b^5 + 24*a*b^6 + 8*b^7)*d*cosh(d*x + c)^4 + 8*(7*(a^4*
b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^3 + 3*(a^4*b^3 + 4*a^3*b^4 + 5*a
^2*b^5 + 2*a*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^4*b^3 + 2*a^3
*b^4 + a^2*b^5)*d*cosh(d*x + c)^4 + 30*(a^4*b^3 + 4*a^3*b^4 + 5*a^2*b^5 + 2
*a*b^6)*d*cosh(d*x + c)^2 + (3*a^4*b^3 + 14*a^3*b^4 + 27*a^2*b^5 + 24*a*b^6
 + 8*b^7)*d)*sinh(d*x + c)^4 + 4*(a^4*b^3 + 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6
)*d*cosh(d*x + c)^2 + 8*(7*(a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^
5 + 10*(a^4*b^3 + 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d*cosh(d*x + c)^3 + (3*a
^4*b^3 + 14*a^3*b^4 + 27*a^2*b^5 + 24*a*b^6 + 8*b^7)*d*cosh(d*x + c))*sinh(
d*x + c)^3 + 4*(7*(a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^6 + 15*(a
^4*b^3 + 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d*cosh(d*x + c)^4 + 3*(3*a^4*b^3
+ 14*a^3*b^4 + 27*a^2*b^5 + 24*a*b^6 + 8*b^7)*d*cosh(d*x + c)^2 + (a^4*b^3
+ 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d)*sinh(d*x + c)^2 + (a^4*b^3 + 2*a^3*b^
4 + a^2*b^5)*d + 8*((a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^7 + 3*(
a^4*b^3 + 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d*cosh(d*x + c)^5 + (3*a^4*b^3 +
 14*a^3*b^4 + 27*a^2*b^5 + 24*a*b^6 + 8*b^7)*d*cosh(d*x + c)^3 + (a^4*b^3 +
 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**7/(a+b*sech(d*x+c)**2)**3,x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.102 $\int \left(a+b\text{sech}^2(c+dx)\right)\tanh^4(c+dx)\,dx$

Optimal. Leaf size=48

$$-\frac{a\tanh^3(c+dx)}{3d}-\frac{a\tanh(c+dx)}{d}+ax+\frac{b\tanh^5(c+dx)}{5d}$$

```
[Out] a*x - (a*Tanh[c + d*x])/d - (a*Tanh[c + d*x]^3)/(3*d) + (b*Tanh[c + d*x]^5)
/(5*d)
```

Rubi [A] time = 0.0638269, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4141, 1802, 206}

$$-\frac{a\tanh^3(c+dx)}{3d}-\frac{a\tanh(c+dx)}{d}+ax+\frac{b\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x]^4,x]
```

```
[Out] a*x - (a*Tanh[c + d*x])/d - (a*Tanh[c + d*x]^3)/(3*d) + (b*Tanh[c + d*x]^5)
/(5*d)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\text{sech}^2(c+dx)\right)\tanh^4(c+dx)\,dx &= \frac{\text{Subst}\left(\int \frac{x^4\left(a+b\left(1-x^2\right)\right)}{1-x^2}\,dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(-a-ax^2+bx^4+\frac{a}{1-x^2}\right)dx,x,\tanh(c+dx)\right)}{d}\\
&= -\frac{a\tanh(c+dx)}{d}-\frac{a\tanh^3(c+dx)}{3d}+\frac{b\tanh^5(c+dx)}{5d}+\frac{a\,\text{Subst}\left(\int \frac{1}{1-x^2}\right.}{}\\
&= ax-\frac{a\tanh(c+dx)}{d}-\frac{a\tanh^3(c+dx)}{3d}+\frac{b\tanh^5(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] time = 0.0243839, size = 57, normalized size = 1.19

$$-\frac{a\tanh^3(c+dx)}{3d}+\frac{a\tanh^{-1}(\tanh(c+dx))}{d}-\frac{a\tanh(c+dx)}{d}+\frac{b\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x]^4,x]
```

```
[Out] (a*ArcTanh[Tanh[c + d*x]])/d - (a*Tanh[c + d*x])/d - (a*Tanh[c + d*x]^3)/(3
*d) + (b*Tanh[c + d*x]^5)/(5*d)
```

Maple [B] time = 0.034, size = 98, normalized size = 2.

$$\frac{1}{d}\left(a\left(dx+c-\tanh(dx+c)-\frac{(\tanh(dx+c))^3}{3}\right)+b\left(-\frac{(\sinh(dx+c))^3}{2(\cosh(dx+c))^5}-\frac{3\sinh(dx+c)}{8(\cosh(dx+c))^5}+\frac{3\tanh(dx+c)}{8}\left(\frac{8}{15}+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)*tanh(d*x+c)^4,x)
```

```
[Out] 1/d*(a*(d*x+c-tanh(d*x+c)-1/3*tanh(d*x+c)^3)+b*(-1/2*sinh(d*x+c)^3/cosh(d*x
+c)^5-3/8*sinh(d*x+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d
*x+c)^2)*tanh(d*x+c)))
```

Maxima [B] time = 1.19444, size = 124, normalized size = 2.58

$$\frac{b\tanh(dx+c)^5}{5d}+\frac{1}{3}a\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+2\right)}{d\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/5*b*tanh(d*x + c)^5/d + 1/3*a*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^
(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x
 - 6*c) + 1)))
```

Fricas [B] time = 2.07739, size = 876, normalized size = 18.25

$$\frac{(15adx+20a-3b)\cosh(dx+c)^5+5(15adx+20a-3b)\cosh(dx+c)\sinh(dx+c)^4-(20a-3b)\sinh(dx+c)^5+5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/15*((15*a*d*x + 20*a - 3*b)*cosh(d*x + c)^5 + 5*(15*a*d*x + 20*a - 3*b)*c
osh(d*x + c)*sinh(d*x + c)^4 - (20*a - 3*b)*sinh(d*x + c)^5 + 5*(15*a*d*x +
 20*a - 3*b)*cosh(d*x + c)^3 - 5*(2*(20*a - 3*b)*cosh(d*x + c)^2 + 8*a + 3*
```

```
b)*sinh(d*x + c)^3 + 5*(2*(15*a*d*x + 20*a - 3*b)*cosh(d*x + c)^3 + 3*(15*a
*d*x + 20*a - 3*b)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(15*a*d*x + 20*a - 3
*b)*cosh(d*x + c) - 5*((20*a - 3*b)*cosh(d*x + c)^4 + 3*(8*a + 3*b)*cosh(d*
x + c)^2 + 4*a - 6*b)*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)
*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(
d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)\tanh^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)*tanh(d*x+c)**4,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*tanh(c + d*x)**4, x)
```

Giac [B] time = 1.317, size = 142, normalized size = 2.96

$$\frac{15\,adx+\frac{2\left(30\,ae^{(8\,dx+8\,c)}-15\,be^{(8\,dx+8\,c)}+90\,ae^{(6\,dx+6\,c)}+110\,ae^{(4\,dx+4\,c)}-30\,be^{(4\,dx+4\,c)}+70\,ae^{(2\,dx+2\,c)}+20\,a-3\,b\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/15*(15*a*d*x + 2*(30*a*e^(8*d*x + 8*c) - 15*b*e^(8*d*x + 8*c) + 90*a*e^(6
*d*x + 6*c) + 110*a*e^(4*d*x + 4*c) - 30*b*e^(4*d*x + 4*c) + 70*a*e^(2*d*x
+ 2*c) + 20*a - 3*b)/(e^(2*d*x + 2*c) + 1)^5)/d
```

3.103 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)\tanh^3(c+dx)\,dx$

Optimal. Leaf size=49

$$\frac{(a-b)\text{sech}^2(c+dx)}{2d}+\frac{a\log(\cosh(c+dx))}{d}+\frac{b\text{sech}^4(c+dx)}{4d}$$

```
[Out] (a*Log[Cosh[c + d*x]])/d + ((a - b)*Sech[c + d*x]^2)/(2*d) + (b*Sech[c + d*
x]^4)/(4*d)
```

Rubi [A] time = 0.0564093, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 446, 76}

$$\frac{(a-b)\text{sech}^2(c+dx)}{2d}+\frac{a\log(\cosh(c+dx))}{d}+\frac{b\text{sech}^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x]^3,x]
```

```
[Out] (a*Log[Cosh[c + d*x]])/d + ((a - b)*Sech[c + d*x]^2)/(2*d) + (b*Sech[c + d*
x]^4)/(4*d)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\int \left(a+b\,\text{sech}^2(c+dx)\right)\tanh^3(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)(b+ax^2)}{x^5}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{(1-x)(b+ax)}{x^3}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{b}{x^3}+\frac{a-b}{x^2}-\frac{a}{x}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{a\log(\cosh(c+dx))}{d}+\frac{(a-b)\text{sech}^2(c+dx)}{2d}+\frac{b\,\text{sech}^4(c+dx)}{4d}$$

Mathematica [A] time = 0.0225026, size = 45, normalized size = 0.92

$$-\frac{a\tanh^2(c+dx)}{2d}+\frac{a\log(\cosh(c+dx))}{d}+\frac{b\tanh^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x]^3,x]

[Out] (a*Log[Cosh[c + d*x]])/d - (a*Tanh[c + d*x]^2)/(2*d) + (b*Tanh[c + d*x]^4)/
(4*d)
```

Maple [A] time = 0.035, size = 72, normalized size = 1.5

$$\frac{a\ln(\cosh(dx+c))}{d}-\frac{(\tanh(dx+c))^2 a}{2d}-\frac{b(\sinh(dx+c))^2}{4d(\cosh(dx+c))^4}+\frac{b(\sinh(dx+c))^2}{4d(\cosh(dx+c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)*tanh(d*x+c)^3,x)

[Out] a*ln(cosh(d*x+c))/d-1/2/d*a*tanh(d*x+c)^2-1/4/d*b*sinh(d*x+c)^2/cosh(d*x+c)
^4+1/4/d*b*sinh(d*x+c)^2/cosh(d*x+c)^2
```

Maxima [A] time = 1.70759, size = 105, normalized size = 2.14

$$\frac{b\tanh(dx+c)^4}{4d}+a\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{2e^{(-2dx-2c)}}{d\left(2e^{(-2dx-2c)}+e^{(-4dx-4c)}+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^3,x, algorithm="maxima")

[Out] 1/4*b*tanh(d*x + c)^4/d + a*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-
2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1)))
```

Fricas [B] time = 2.39513, size = 2871, normalized size = 58.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -(a*d*x*cosh(d*x + c)^8 + 8*a*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + a*d*x*sin
h(d*x + c)^8 + 2*(2*a*d*x - a + b)*cosh(d*x + c)^6 + 2*(14*a*d*x*cosh(d*x +
 c)^2 + 2*a*d*x - a + b)*sinh(d*x + c)^6 + 4*(14*a*d*x*cosh(d*x + c)^3 + 3*
(2*a*d*x - a + b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a*d*x - 2*a)*cosh(d
*x + c)^4 + 2*(35*a*d*x*cosh(d*x + c)^4 + 3*a*d*x + 15*(2*a*d*x - a + b)*co
sh(d*x + c)^2 - 2*a)*sinh(d*x + c)^4 + 8*(7*a*d*x*cosh(d*x + c)^5 + 5*(2*a*
d*x - a + b)*cosh(d*x + c)^3 + (3*a*d*x - 2*a)*cosh(d*x + c))*sinh(d*x + c)
^3 + a*d*x + 2*(2*a*d*x - a + b)*cosh(d*x + c)^2 + 2*(14*a*d*x*cosh(d*x + c
)^6 + 15*(2*a*d*x - a + b)*cosh(d*x + c)^4 + 2*a*d*x + 6*(3*a*d*x - 2*a)*co
sh(d*x + c)^2 - a + b)*sinh(d*x + c)^2 - (a*cosh(d*x + c)^8 + 8*a*cosh(d*x
+ c)*sinh(d*x + c)^7 + a*sinh(d*x + c)^8 + 4*a*cosh(d*x + c)^6 + 4*(7*a*cos
h(d*x + c)^2 + a)*sinh(d*x + c)^6 + 8*(7*a*cosh(d*x + c)^3 + 3*a*cosh(d*x +
 c))*sinh(d*x + c)^5 + 6*a*cosh(d*x + c)^4 + 2*(35*a*cosh(d*x + c)^4 + 30*a
*cosh(d*x + c)^2 + 3*a)*sinh(d*x + c)^4 + 8*(7*a*cosh(d*x + c)^5 + 10*a*cos
h(d*x + c)^3 + 3*a*cosh(d*x + c))*sinh(d*x + c)^3 + 4*a*cosh(d*x + c)^2 + 4
*(7*a*cosh(d*x + c)^6 + 15*a*cosh(d*x + c)^4 + 9*a*cosh(d*x + c)^2 + a)*sin
h(d*x + c)^2 + 8*(a*cosh(d*x + c)^7 + 3*a*cosh(d*x + c)^5 + 3*a*cosh(d*x +
c)^3 + a*cosh(d*x + c))*sinh(d*x + c) + a)*log(2*cosh(d*x + c)/(cosh(d*x +
c) - sinh(d*x + c))) + 4*(2*a*d*x*cosh(d*x + c)^7 + 3*(2*a*d*x - a + b)*cos
h(d*x + c)^5 + 2*(3*a*d*x - 2*a)*cosh(d*x + c)^3 + (2*a*d*x - a + b)*cosh(d
*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c
)^7 + d*sinh(d*x + c)^8 + 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)
*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c
)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2
+ 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 3*
d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x +
c)^6 + 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 8*
(d*cosh(d*x + c)^7 + 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 + d*cosh(d*x
 + c))*sinh(d*x + c) + d)
```

Sympy [A] time = 2.73193, size = 80, normalized size = 1.63

$$\begin{cases} ax - \frac{a\log(\tanh(c+dx)+1)}{d} - \frac{a\tanh^2(c+dx)}{2d} - \frac{b\tanh^2(c+dx)\operatorname{sech}^2(c+dx)}{4d} - \frac{b\operatorname{sech}^2(c+dx)}{4d} & \text{for } d \neq 0 \\ x\left(a + b\operatorname{sech}^2(c)\right)\tanh^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)*tanh(d*x+c)**3,x)
```

```
[Out] Piecewise((a*x - a*log(tanh(c + d*x) + 1)/d - a*tanh(c + d*x)**2/(2*d) - b*
tanh(c + d*x)**2*sech(c + d*x)**2/(4*d) - b*sech(c + d*x)**2/(4*d), Ne(d, 0
)), (x*(a + b*sech(c)**2)*tanh(c)**3, True))
```

Giac [B] time = 1.25088, size = 157, normalized size = 3.2

$$-\frac{12\,adx - 12\,a\log\left(e^{(2\,dx+2\,c)}+1\right) + \frac{25\,ae^{(8\,dx+8\,c)}+76\,ae^{(6\,dx+6\,c)}+24\,be^{(6\,dx+6\,c)}+102\,ae^{(4\,dx+4\,c)}+76\,ae^{(2\,dx+2\,c)}+24\,be^{(2\,dx+2\,c)}+25\,a}{\left(e^{(2\,dx+2\,c)}+1\right)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/12*(12*a*d*x - 12*a*log(e^(2*d*x + 2*c) + 1) + (25*a*e^(8*d*x + 8*c) + 7
6*a*e^(6*d*x + 6*c) + 24*b*e^(6*d*x + 6*c) + 102*a*e^(4*d*x + 4*c) + 76*a*e
^(2*d*x + 2*c) + 24*b*e^(2*d*x + 2*c) + 25*a)/(e^(2*d*x + 2*c) + 1)^4)/d
```

3.104 $\int \left(a+b\,\mathrm{sech}^2(c+dx)\right)\tanh^2(c+dx)\,dx$

Optimal. Leaf size=32

$$-\frac{a\tanh(c+dx)}{d}+ax+\frac{b\tanh^3(c+dx)}{3d}$$

```
[Out] a*x - (a*Tanh[c + d*x])/d + (b*Tanh[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0570871, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4141, 1802, 206}

$$-\frac{a\tanh(c+dx)}{d}+ax+\frac{b\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x]^2,x]
```

```
[Out] a*x - (a*Tanh[c + d*x])/d + (b*Tanh[c + d*x]^3)/(3*d)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\,\mathrm{sech}^2(c+dx)\right)\tanh^2(c+dx)\,dx &= \frac{\mathrm{Subst}\left(\int \frac{x^2\left(a+b\left(1-x^2\right)\right)}{1-x^2}\,dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{\mathrm{Subst}\left(\int \left(-a+bx^2+\frac{a}{1-x^2}\right)\,dx,x,\tanh(c+dx)\right)}{d}\\
&= -\frac{a\tanh(c+dx)}{d}+\frac{b\tanh^3(c+dx)}{3d}+\frac{a\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\tanh(c+dx)\right)}{d}\\
&= ax-\frac{a\tanh(c+dx)}{d}+\frac{b\tanh^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.0161676, size = 41, normalized size = 1.28

$$\frac{a\tanh^{-1}(\tanh(c+dx))}{d}-\frac{a\tanh(c+dx)}{d}+\frac{b\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x]^2,x]
```

```
[Out] (a*ArcTanh[Tanh[c + d*x]])/d - (a*Tanh[c + d*x])/d + (b*Tanh[c + d*x]^3)/(3
*d)
```

Maple [A] time = 0.028, size = 60, normalized size = 1.9

$$\frac{1}{d}\left(a\left(dx+c-\tanh\left(dx+c\right)\right)+b\left(-\frac{\sinh\left(dx+c\right)}{2\left(\cosh\left(dx+c\right)\right)^{3}}+\frac{\tanh\left(dx+c\right)}{2}\left(\frac{2}{3}+\frac{\left(\operatorname{sech}\left(dx+c\right)\right)^{2}}{3}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)*tanh(d*x+c)^2,x)
```

```
[Out] 1/d*(a*(d*x+c-tanh(d*x+c))+b*(-1/2*sinh(d*x+c)/cosh(d*x+c)^3+1/2*(2/3+1/3*s
ech(d*x+c)^2)*tanh(d*x+c)))
```

Maxima [A] time = 1.17165, size = 57, normalized size = 1.78

$$\frac{b\tanh\left(dx+c\right)^{3}}{3\,d}+a\left(x+\frac{c}{d}-\frac{2}{d\left(e^{(-2\,dx-2\,c)}+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/3*b*tanh(d*x + c)^3/d + a*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1)))
```

Fricas [B] time = 2.23047, size = 396, normalized size = 12.38

$$\frac{(3\,adx+3\,a-b)\cosh\left(dx+c\right)^{3}+3\,(3\,adx+3\,a-b)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}-(3\,a-b)\sinh\left(dx+c\right)^{3}+3\,(3\,adx\ -}{3\left(d\cosh\left(dx+c\right)^{3}+3\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}+3\,d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/3*((3*a*d*x + 3*a - b)*cosh(d*x + c)^3 + 3*(3*a*d*x + 3*a - b)*cosh(d*x +
 c)*sinh(d*x + c)^2 - (3*a - b)*sinh(d*x + c)^3 + 3*(3*a*d*x + 3*a - b)*cos
h(d*x + c) - 3*((3*a - b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))/(d*cosh(d
*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\,\mathrm{sech}^2\left(c + dx\right)\right)\tanh^2\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)*tanh(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*tanh(c + d*x)**2, x)
```

Giac [B] time = 1.2492, size = 93, normalized size = 2.91

$$\frac{3\,adx + \frac{2\left(3\,ae^{(4\,dx+4\,c)} - 3\,be^{(4\,dx+4\,c)} + 6\,ae^{(2\,dx+2\,c)} + 3\,a - b\right)}{\left(e^{(2\,dx+2\,c)} + 1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*a*d*x + 2*(3*a*e^(4*d*x + 4*c) - 3*b*e^(4*d*x + 4*c) + 6*a*e^(2*d*x
+ 2*c) + 3*a - b)/(e^(2*d*x + 2*c) + 1)^3)/d
```

3.105 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)\tanh(c+dx)\,dx$

Optimal. Leaf size=29

$$\frac{a\log(\cosh(c+dx))}{d}-\frac{b\,\text{sech}^2(c+dx)}{2d}$$

```
[Out] (a*Log[Cosh[c + d*x]])/d - (b*Sech[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.0283861, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4138, 14}

$$\frac{a\log(\cosh(c+dx))}{d}-\frac{b\,\text{sech}^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x],x]
```

```
[Out] (a*Log[Cosh[c + d*x]])/d - (b*Sech[c + d*x]^2)/(2*d)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\,\text{sech}^2(c+dx)\right)\tanh(c+dx)\,dx &= \frac{\text{Subst}\left(\int \frac{b+ax^2}{x^3}\,dx, x, \cosh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{b}{x^3}+\frac{a}{x}\right)dx, x, \cosh(c+dx)\right)}{d}\\ &= \frac{a\log(\cosh(c+dx))}{d}-\frac{b\,\text{sech}^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0218577, size = 29, normalized size = 1.

$$\frac{a\log(\cosh(c+dx))}{d}-\frac{b\,\text{sech}^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x],x]
```

```
[Out] (a*Log[Cosh[c + d*x]])/d - (b*Sech[c + d*x]^2)/(2*d)
```

Maple [A] time = 0.013, size = 29, normalized size = 1.

$$-\frac{b\left(\mathrm{sech}\left(dx+c\right)\right)^{2}}{2d}-\frac{a\ln\left(\mathrm{sech}\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)*tanh(d*x+c),x)
```

```
[Out] -1/2*b*sech(d*x+c)^2/d-1/d*a*ln(sech(d*x+c))
```

Maxima [A] time = 1.1496, size = 36, normalized size = 1.24

$$\frac{b\tanh\left(dx+c\right)^{2}}{2d}+\frac{a\log\left(\cosh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*b*tanh(d*x + c)^2/d + a*log(cosh(d*x + c))/d
```

Fricas [B] time = 2.08514, size = 973, normalized size = 33.55

$$-\frac{adx\cosh\left(dx+c\right)^{4}+4\,adx\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+adx\sinh\left(dx+c\right)^{4}+adx+2\,(adx+b)\cosh\left(dx+c\right)^{2}+2\left(3\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(a*d*x*cosh(d*x + c)^4 + 4*a*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + a*d*x*sin
h(d*x + c)^4 + a*d*x + 2*(a*d*x + b)*cosh(d*x + c)^2 + 2*(3*a*d*x*cosh(d*x
+ c)^2 + a*d*x + b)*sinh(d*x + c)^2 - (a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c
)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*a*cosh(d*x + c)^2 + 2*(3*a*cosh(d
*x + c)^2 + a)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + a*cosh(d*x + c))*si
nh(d*x + c) + a)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(
a*d*x*cosh(d*x + c)^3 + (a*d*x + b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d
*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 + 2*d*cos
h(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x
+ c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [A] time = 0.726178, size = 42, normalized size = 1.45

$$\begin{cases} ax-\frac{a\log\left(\tanh\left(c+dx\right)+1\right)}{d}-\frac{b\,\mathrm{sech}^{2}\left(c+dx\right)}{2d} & \text{for } d\neq 0\\ x\left(a+b\,\mathrm{sech}^{2}\left(c\right)\right)\tanh\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)*tanh(d*x+c),x)
```

```
[Out] Piecewise((a*x - a*log(tanh(c + d*x) + 1)/d - b*sech(c + d*x)**2/(2*d), Ne(
d, 0)), (x*(a + b*sech(c)**2)*tanh(c), True))
```

Giac [B] time = 1.19476, size = 108, normalized size = 3.72

$$-\frac{2\,adx - 2\,a\log\left(e^{(2\,dx+2\,c)} + 1\right) + \frac{3\,ae^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+3\,a}{\left(e^{(2\,dx+2\,c)}+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c),x, algorithm="giac")
```

```
[Out] -1/2*(2*a*d*x - 2*a*log(e^(2*d*x + 2*c) + 1) + (3*a*e^(4*d*x + 4*c) + 6*a*e
^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + 3*a)/(e^(2*d*x + 2*c) + 1)^2)/d
```

3.106 $\int \left(a + b\,\mathbf{sech}^2(c + dx)\right) dx$

Optimal. Leaf size=15

$$ax + \frac{b\tanh(c + dx)}{d}$$

```
[Out] a*x + (b*Tanh[c + d*x])/d
```

Rubi [A] time = 0.0132529, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3767, 8}

$$ax + \frac{b\tanh(c + dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sech[c + d*x]^2,x]
```

```
[Out] a*x + (b*Tanh[c + d*x])/d
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a + b\text{sech}^2(c + dx)\right) dx &= ax + b\int \text{sech}^2(c + dx)\, dx\\
&= ax + \frac{(ib)\,\text{Subst}\left(\int 1\, dx, x, -i\tanh(c + dx)\right)}{d}\\
&= ax + \frac{b\tanh(c + dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.0027872, size = 15, normalized size = 1.

$$ax + \frac{b\tanh(c + dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sech[c + d*x]^2,x]
```

```
[Out] a*x + (b*Tanh[c + d*x])/d
```

Maple [A] time = 0.007, size = 16, normalized size = 1.1

$$ax+\frac{b\tanh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sech(d*x+c)^2,x)
```

```
[Out] a*x+b*tanh(d*x+c)/d
```

Maxima [A] time = 1.15253, size = 31, normalized size = 2.07

$$ax+\frac{2\,b}{d\left(e^{\left(-2\,dx-2\,c\right)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sech(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] a*x + 2*b/(d*(e^(-2*d*x - 2*c) + 1))
```

Fricas [B] time = 1.99825, size = 88, normalized size = 5.87

$$\frac{\left(adx-b\right)\cosh\left(dx+c\right)+b\sinh\left(dx+c\right)}{d\cosh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sech(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] ((a*d*x - b)*cosh(d*x + c) + b*sinh(d*x + c))/(d*cosh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\,\mathrm{sech}^{2}\left(c+dx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sech(d*x+c)**2,x)
```

```
[Out] Integral(a + b*sech(c + d*x)**2, x)
```

Giac [A] time = 1.14616, size = 31, normalized size = 2.07

$$ax-\frac{2\,b}{d\left(e^{\left(2\,dx+2\,c\right)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sech(d*x+c)^2,x, algorithm="giac")
```

```
[Out] a*x - 2*b/(d*(e^(2*d*x + 2*c) + 1))
```

3.107 $\int \coth(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=28

$$\frac{(a+b)\log(\sinh(c+dx))}{d}-\frac{b\log(\cosh(c+dx))}{d}$$

```
[Out] -((b*Log[Cosh[c + d*x]])/d) + ((a + b)*Log[Sinh[c + d*x]])/d
```

Rubi [A] time = 0.0514288, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4138, 446, 72}

$$\frac{(a+b)\log(\sinh(c+dx))}{d}-\frac{b\log(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((b*Log[Cosh[c + d*x]])/d) + ((a + b)*Log[Sinh[c + d*x]])/d
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \coth(c+dx)\left(a+b\,\text{sech}^2(c+dx)\right)\,dx &= -\frac{\text{Subst}\left(\int \frac{b+ax^2}{x\left(1-x^2\right)}\,dx,x,\cosh(c+dx)\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \frac{b+ax}{(1-x)x}\,dx,x,\cosh^2(c+dx)\right)}{2d}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{-a-b}{-1+x}+\frac{b}{x}\right)\,dx,x,\cosh^2(c+dx)\right)}{2d}\\
&= -\frac{b\log(\cosh(c+dx))}{d}+\frac{(a+b)\log(\sinh(c+dx))}{d}
\end{aligned}$$

Mathematica [A] time = 0.0379646, size = 44, normalized size = 1.57

$$\frac{a(\log(\tanh(c+dx))+\log(\cosh(c+dx)))}{d}-\frac{b(\log(\cosh(c+dx))-\log(\sinh(c+dx)))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((b*(Log[Cosh[c + d*x]] - Log[Sinh[c + d*x]]))/d) + (a*(Log[Cosh[c + d*x]]
 + Log[Tanh[c + d*x]]))/d
```

Maple [A] time = 0.033, size = 26, normalized size = 0.9

$$\frac{b\ln(\tanh(dx+c))}{d}+\frac{a\ln(\sinh(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)*(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/d*b*ln(tanh(d*x+c))+a*ln(sinh(d*x+c))/d
```

Maxima [B] time = 1.73326, size = 88, normalized size = 3.14

$$b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}\right)+\frac{a\log(\sinh(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] b*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x - 2*c)
 + 1)/d) + a*log(sinh(d*x + c))/d
```

Fricas [B] time = 2.09733, size = 178, normalized size = 6.36

$$-\frac{adx+b\log\left(\frac{2\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)-(a+b)\log\left(\frac{2\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -(a*d*x + b*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - (a + b)*
log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\operatorname{sech}^2(c+dx)\right)\coth(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*coth(c + d*x), x)
```

Giac [A] time = 1.17696, size = 76, normalized size = 2.71

$$-\frac{adx - \left(ae^{(2c)} + be^{(2c)}\right)e^{(-2c)} \log\left(\left|e^{(2dx+2c)} - 1\right|\right) + b\log\left(e^{(2dx+2c)} + 1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -(a*d*x - (a*e^(2*c) + b*e^(2*c))*e^(-2*c)*log(abs(e^(2*d*x + 2*c) - 1)) +
b*log(e^(2*d*x + 2*c) + 1))/d
```

3.108 $\int \coth^2(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)dx$

Optimal. Leaf size=18

$$ax-\frac{(a+b)\coth(c+dx)}{d}$$

```
[Out] a*x - ((a + b)*Coth[c + d*x])/d
```

Rubi [A] time = 0.0585628, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4141, 1802, 207}

$$ax-\frac{(a+b)\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] a*x - ((a + b)*Coth[c + d*x])/d
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \coth^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)dx &= \frac{\operatorname{Subst}\left(\int \frac{a+b\left(1-x^2\right)}{x^2\left(1-x^2\right)}\,dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{a+b}{x^2}-\frac{a}{-1+x^2}\right)dx,x,\tanh(c+dx)\right)}{d}\\
&= -\frac{(a+b)\coth(c+dx)}{d}-\frac{a\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\tanh(c+dx)\right)}{d}\\
&= ax-\frac{(a+b)\coth(c+dx)}{d}
\end{aligned}$$

Mathematica [C] time = 0.030999, size = 41, normalized size = 2.28

$$-\frac{a\coth(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2(c+dx)\right)}{d}-\frac{b\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^2*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((b*Coth[c + d*x])/d) - (a*Coth[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, T
anh[c + d*x]^2])/d
```

Maple [A] time = 0.032, size = 30, normalized size = 1.7

$$\frac{a\left(dx+c-\coth\left(dx+c\right)\right)-b\coth\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2*(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/d*(a*(d*x+c-coth(d*x+c))-b*coth(d*x+c))
```

Maxima [B] time = 1.15868, size = 63, normalized size = 3.5

$$a\left(x+\frac{c}{d}+\frac{2}{d\left(e^{(-2\,dx-2\,c)}-1\right)}\right)+\frac{2\,b}{d\left(e^{(-2\,dx-2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] a*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + 2*b/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [B] time = 2.02467, size = 103, normalized size = 5.72

$$-\frac{(a+b)\cosh\left(dx+c\right)-(adx+a+b)\sinh\left(dx+c\right)}{d\sinh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -((a + b)*cosh(d*x + c) - (a*d*x + a + b)*sinh(d*x + c))/(d*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19189, size = 36, normalized size = 2.

$$\frac{adx - \frac{2\,(a+b)}{e^{(2\,dx+2\,c)}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] (a*d*x - 2*(a + b)/(e^(2*d*x + 2*c) - 1))/d
```

3.109 $\int \coth^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=31

$$\frac{a\log(\sinh(c+dx))}{d}-\frac{(a+b)\operatorname{csch}^2(c+dx)}{2d}$$

```
[Out] -((a + b)*Csch[c + d*x]^2)/(2*d) + (a*Log[Sinh[c + d*x]])/d
```

Rubi [A] time = 0.0565121, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 444, 43}

$$\frac{a\log(\sinh(c+dx))}{d}-\frac{(a+b)\operatorname{csch}^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((a + b)*Csch[c + d*x]^2)/(2*d) + (a*Log[Sinh[c + d*x]])/d
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \coth^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)dx = \frac{\text{Subst}\left(\int \frac{x(b+ax^2)}{(1-x^2)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{b+ax}{(1-x)^2}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a+b}{(-1+x)^2}+\frac{a}{-1+x}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{(a+b)\text{csch}^2(c+dx)}{2d}+\frac{a\log(\sinh(c+dx))}{d}$$

Mathematica [A] time = 0.168375, size = 52, normalized size = 1.68

$$-\frac{a\left(\coth^2(c+dx)-2\log(\tanh(c+dx))-2\log(\cosh(c+dx))\right)}{2d}-\frac{b\text{csch}^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -(b*Csch[c + d*x]^2)/(2*d) - (a*(Coth[c + d*x]^2 - 2*Log[Cosh[c + d*x]] - 2
*Log[Tanh[c + d*x]]))/(2*d)
```

Maple [A] time = 0.04, size = 50, normalized size = 1.6

$$\frac{a\ln(\sinh(dx+c))}{d}-\frac{(\coth(dx+c))^2 a}{2d}-\frac{b(\cosh(dx+c))^2}{2d(\sinh(dx+c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3*(a+b*sech(d*x+c)^2),x)
```

```
[Out] a*ln(sinh(d*x+c))/d-1/2*a*coth(d*x+c)^2/d-1/2/d*b*cosh(d*x+c)^2/sinh(d*x+c)
^2
```

Maxima [B] time = 1.1838, size = 146, normalized size = 4.71

$$a\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2e^{(-2dx-2c)}}{d\left(2e^{(-2dx-2c)}-e^{(-4dx-4c)}-1\right)}\right)-\frac{2b}{d\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] a*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*
x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 2*b/(d*(e^(d*x
+ c) - e^(-d*x - c))^2)
```

Fricas [B] time = 2.1493, size = 990, normalized size = 31.94

$$-\frac{adx\cosh(dx+c)^4 + 4\,adx\cosh(dx+c)\sinh(dx+c)^3 + adx\sinh(dx+c)^4 + adx - 2\,(adx-a-b)\cosh(dx+c)^2 + \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -(a*d*x*cosh(d*x + c)^4 + 4*a*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + a*d*x*sin
h(d*x + c)^4 + a*d*x - 2*(a*d*x - a - b)*cosh(d*x + c)^2 + 2*(3*a*d*x*cosh(
d*x + c)^2 - a*d*x + a + b)*sinh(d*x + c)^2 - (a*cosh(d*x + c)^4 + 4*a*cosh
(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*a*cosh(d*x + c)^2 + 2*(3*
a*cosh(d*x + c)^2 - a)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - a*cosh(d*x
+ c))*sinh(d*x + c) + a)*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c)
)) + 4*(a*d*x*cosh(d*x + c)^3 - (a*d*x - a - b)*cosh(d*x + c))*sinh(d*x + c
))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)
^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 4*
(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2267, size = 109, normalized size = 3.52

$$-\frac{2\,adx - 2\,a\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right) + \frac{3\,ae^{(4\,dx+4\,c)} - 2\,ae^{(2\,dx+2\,c)} + 4\,be^{(2\,dx+2\,c)} + 3\,a}{\left(e^{(2\,dx+2\,c)}-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*(2*a*d*x - 2*a*log(abs(e^(2*d*x + 2*c) - 1)) + (3*a*e^(4*d*x + 4*c) -
2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + 3*a)/(e^(2*d*x + 2*c) - 1)^2)/d
```

3.110 $\int \coth^4(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=34

$$-\frac{(a+b)\coth^3(c+dx)}{3d}-\frac{a\coth(c+dx)}{d}+ax$$

```
[Out] a*x - (a*Coth[c + d*x])/d - ((a + b)*Coth[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0628673, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4141, 1802, 207}

$$-\frac{(a+b)\coth^3(c+dx)}{3d}-\frac{a\coth(c+dx)}{d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^4*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] a*x - (a*Coth[c + d*x])/d - ((a + b)*Coth[c + d*x]^3)/(3*d)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \coth^4(c+dx)\left(a+b\,\text{sech}^2(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int \frac{a+b\left(1-x^2\right)}{x^4\left(1-x^2\right)}\,dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{a+b}{x^4}+\frac{a}{x^2}-\frac{a}{-1+x^2}\right)dx,x,\tanh(c+dx)\right)}{d}\\
&= -\frac{a\coth(c+dx)}{d}-\frac{(a+b)\coth^3(c+dx)}{3d}-\frac{a\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\tanh(c+\right.}{d}\\
&= ax-\frac{a\coth(c+dx)}{d}-\frac{(a+b)\coth^3(c+dx)}{3d}
\end{aligned}
$$

Mathematica [C] time = 0.0226054, size = 49, normalized size = 1.44

$$-\frac{a\coth^3(c+dx)\,_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\tanh^2(c+dx)\right)}{3d}-\frac{b\coth^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^4*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -(b*Coth[c + d*x]^3)/(3*d) - (a*Coth[c + d*x]^3*Hypergeometric2F1[-3/2, 1,
-1/2, Tanh[c + d*x]^2])/(3*d)
```

Maple [B] time = 0.04, size = 70, normalized size = 2.1

$$\frac{1}{d}\left(a\left(dx+c-\coth(dx+c)-\frac{(\coth(dx+c))^3}{3}\right)+b\left(-\frac{\cosh(dx+c)}{2(\sinh(dx+c))^3}-\frac{\coth(dx+c)}{2}\left(\frac{2}{3}-\frac{(\operatorname{csch}(dx+c))^2}{3}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^4*(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/d*(a*(d*x+c-coth(d*x+c)-1/3*coth(d*x+c)^3)+b*(-1/2/sinh(d*x+c)^3*cosh(d*x
+c)-1/2*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)))
```

Maxima [B] time = 1.15464, size = 230, normalized size = 6.76

$$\frac{1}{3}a\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}-2\right)}{d\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1\right)}\right)+\frac{2}{3}b\left(\frac{3e^{(-4dx-4c)}}{d\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1\right)}+\frac{}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d*(3*
e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + 2/3*b*(3*
e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x -
6*c) - 1)) + 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*
c) - 1)))
```

Fricas [B] time = 2.04397, size = 370, normalized size = 10.88

$$-\frac{(4a+b)\cosh(dx+c)^3+3(4a+b)\cosh(dx+c)\sinh(dx+c)^2-(3adx+4a+b)\sinh(dx+c)^3+3b\cosh(dx+c}{3\left(d\sinh(dx+c)^3+3\left(d\cosh(dx+c)^2-d\right)\sinh(d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/3*((4*a + b)*cosh(d*x + c)^3 + 3*(4*a + b)*cosh(d*x + c)*sinh(d*x + c)^2
 - (3*a*d*x + 4*a + b)*sinh(d*x + c)^3 + 3*b*cosh(d*x + c) + 3*(3*a*d*x - (
```

```
3*a*d*x + 4*a + b)*cosh(d*x + c)^2 + 4*a + b)*sinh(d*x + c))/(d*sinh(d*x +
c)^3 + 3*(d*cosh(d*x + c)^2 - d)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**4*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24198, size = 90, normalized size = 2.65

$$\frac{3\,adx - \frac{2\left(6\,ae^{(4\,dx+4\,c)}+3\,be^{(4\,dx+4\,c)}-6\,ae^{(2\,dx+2\,c)}+4\,a+b\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*a*d*x - 2*(6*a*e^(4*d*x + 4*c) + 3*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x
+ 2*c) + 4*a + b)/(e^(2*d*x + 2*c) - 1)^3)/d
```

3.111 $\int \coth^5(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=51

$$-\frac{(a+b)\text{csch}^4(c+dx)}{4d}-\frac{(2a+b)\text{csch}^2(c+dx)}{2d}+\frac{a\log(\sinh(c+dx))}{d}$$

```
[Out] -((2*a + b)*Csch[c + d*x]^2)/(2*d) - ((a + b)*Csch[c + d*x]^4)/(4*d) + (a*L
og[Sinh[c + d*x]])/d
```

Rubi [A] time = 0.0781343, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 446, 77}

$$-\frac{(a+b)\text{csch}^4(c+dx)}{4d}-\frac{(2a+b)\text{csch}^2(c+dx)}{2d}+\frac{a\log(\sinh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^5*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((2*a + b)*Csch[c + d*x]^2)/(2*d) - ((a + b)*Csch[c + d*x]^4)/(4*d) + (a*L
og[Sinh[c + d*x]])/d
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \coth^5(c+dx)\left(a+b\text{sech}^2(c+dx)\right)dx = -\frac{\text{Subst}\left(\int \frac{x^3(b+ax^2)}{(1-x^2)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{x(b+ax)}{(1-x)^3}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{-a-b}{(-1+x)^3}+\frac{-2a-b}{(-1+x)^2}-\frac{a}{-1+x}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{(2a+b)\text{csch}^2(c+dx)}{2d}-\frac{(a+b)\text{csch}^4(c+dx)}{4d}+\frac{a\log(\sinh(c+dx))}{d}$$

Mathematica [A] time = 0.262425, size = 62, normalized size = 1.22

$$-\frac{a\left(\coth^4(c+dx)+2\coth^2(c+dx)-4\log(\tanh(c+dx))-4\log(\cosh(c+dx))\right)}{4d}-\frac{b\coth^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^5*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -(b*Coth[c + d*x]^4)/(4*d) - (a*(2*Coth[c + d*x]^2 + Coth[c + d*x]^4 - 4*Lo
g[Cosh[c + d*x]] - 4*Log[Tanh[c + d*x]]))/(4*d)
```

Maple [A] time = 0.041, size = 86, normalized size = 1.7

$$\frac{a\ln(\sinh(dx+c))}{d}-\frac{(\coth(dx+c))^2a}{2d}-\frac{a(\coth(dx+c))^4}{4d}-\frac{b(\cosh(dx+c))^2}{4d(\sinh(dx+c))^4}-\frac{b(\cosh(dx+c))^2}{4d(\sinh(dx+c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^5*(a+b*sech(d*x+c)^2),x)
```

```
[Out] a*ln(sinh(d*x+c))/d-1/2*a*coth(d*x+c)^2/d-1/4/d*a*coth(d*x+c)^4-1/4/d*b/sin
h(d*x+c)^4*cosh(d*x+c)^2-1/4/d*b*cosh(d*x+c)^2/sinh(d*x+c)^2
```

Maxima [B] time = 1.2081, size = 339, normalized size = 6.65

$$a\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{4\left(e^{(-2dx-2c)}-e^{(-4dx-4c)}+e^{(-6dx-6c)}\right)}{d\left(4e^{(-2dx-2c)}-6e^{(-4dx-4c)}+4e^{(-6dx-6c)}-e^{(-8dx-8c)}-1\right)}\right)+2b\left(\frac{}{d(4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] a*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-2*d
*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) - 6
*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + 2*b*(e^(
-2*d*x - 2*c)/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6
*c) - e^(-8*d*x - 8*c) - 1)) + e^(-6*d*x - 6*c)/(d*(4*e^(-2*d*x - 2*c) - 6*
```

```
e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1)))
```

Fricas [B] time = 2.24184, size = 2898, normalized size = 56.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -(a*d*x*cosh(d*x + c)^8 + 8*a*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + a*d*x*sin
h(d*x + c)^8 - 2*(2*a*d*x - 2*a - b)*cosh(d*x + c)^6 + 2*(14*a*d*x*cosh(d*x
 + c)^2 - 2*a*d*x + 2*a + b)*sinh(d*x + c)^6 + 4*(14*a*d*x*cosh(d*x + c)^3
- 3*(2*a*d*x - 2*a - b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a*d*x - 2*a)*
cosh(d*x + c)^4 + 2*(35*a*d*x*cosh(d*x + c)^4 + 3*a*d*x - 15*(2*a*d*x - 2*a
 - b)*cosh(d*x + c)^2 - 2*a)*sinh(d*x + c)^4 + 8*(7*a*d*x*cosh(d*x + c)^5 -
 5*(2*a*d*x - 2*a - b)*cosh(d*x + c)^3 + (3*a*d*x - 2*a)*cosh(d*x + c))*sin
h(d*x + c)^3 + a*d*x - 2*(2*a*d*x - 2*a - b)*cosh(d*x + c)^2 + 2*(14*a*d*x*
cosh(d*x + c)^6 - 15*(2*a*d*x - 2*a - b)*cosh(d*x + c)^4 - 2*a*d*x + 6*(3*a
*d*x - 2*a)*cosh(d*x + c)^2 + 2*a + b)*sinh(d*x + c)^2 - (a*cosh(d*x + c)^8
 + 8*a*cosh(d*x + c)*sinh(d*x + c)^7 + a*sinh(d*x + c)^8 - 4*a*cosh(d*x + c
)^6 + 4*(7*a*cosh(d*x + c)^2 - a)*sinh(d*x + c)^6 + 8*(7*a*cosh(d*x + c)^3
- 3*a*cosh(d*x + c))*sinh(d*x + c)^5 + 6*a*cosh(d*x + c)^4 + 2*(35*a*cosh(d
*x + c)^4 - 30*a*cosh(d*x + c)^2 + 3*a)*sinh(d*x + c)^4 + 8*(7*a*cosh(d*x +
 c)^5 - 10*a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c))*sinh(d*x + c)^3 - 4*a*cos
h(d*x + c)^2 + 4*(7*a*cosh(d*x + c)^6 - 15*a*cosh(d*x + c)^4 + 9*a*cosh(d*x
 + c)^2 - a)*sinh(d*x + c)^2 + 8*(a*cosh(d*x + c)^7 - 3*a*cosh(d*x + c)^5 +
 3*a*cosh(d*x + c)^3 - a*cosh(d*x + c))*sinh(d*x + c) + a)*log(2*sinh(d*x +
 c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(2*a*d*x*cosh(d*x + c)^7 - 3*(2*a*
d*x - 2*a - b)*cosh(d*x + c)^5 + 2*(3*a*d*x - 2*a)*cosh(d*x + c)^3 - (2*a*d
*x - 2*a - b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d
*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 4*d*cosh(d*x + c)^6 + 4*(7*d*
cosh(d*x + c)^2 - d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 - 3*d*cosh(d*
x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 - 3
0*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - 10*d*
cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 - 4*d*cosh(d*x + c)^2
+ 4*(7*d*cosh(d*x + c)^6 - 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 - d)*
sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 - 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x
 + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**5*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27999, size = 158, normalized size = 3.1

$$-\frac{12\,adx - 12\,a\log\left(\left|e^{(2\,dx+2\,c)} - 1\right|\right) + \frac{25\,ae^{(8\,dx+8\,c)} - 52\,ae^{(6\,dx+6\,c)} + 24\,be^{(6\,dx+6\,c)} + 102\,ae^{(4\,dx+4\,c)} - 52\,ae^{(2\,dx+2\,c)} + 24\,be^{(2\,dx+2\,c)} + 25\,a}{\left(e^{(2\,dx+2\,c)} - 1\right)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/12*(12*a*d*x - 12*a*log(abs(e^(2*d*x + 2*c) - 1)) + (25*a*e^(8*d*x + 8*c
) - 52*a*e^(6*d*x + 6*c) + 24*b*e^(6*d*x + 6*c) + 102*a*e^(4*d*x + 4*c) - 5
2*a*e^(2*d*x + 2*c) + 24*b*e^(2*d*x + 2*c) + 25*a)/(e^(2*d*x + 2*c) - 1)^4)
/d
```

3.112 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)^2 \tanh^4(c+dx)\,dx$

Optimal. Leaf size=77

$$-\frac{a^2\tanh^3(c+dx)}{3d}-\frac{a^2\tanh(c+dx)}{d}+a^2x+\frac{b(2a+b)\tanh^5(c+dx)}{5d}-\frac{b^2\tanh^7(c+dx)}{7d}$$

```
[Out] a^2*x - (a^2*Tanh[c + d*x])/d - (a^2*Tanh[c + d*x]^3)/(3*d) + (b*(2*a + b)*
Tanh[c + d*x]^5)/(5*d) - (b^2*Tanh[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.104635, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4141, 1802, 206}

$$-\frac{a^2\tanh^3(c+dx)}{3d}-\frac{a^2\tanh(c+dx)}{d}+a^2x+\frac{b(2a+b)\tanh^5(c+dx)}{5d}-\frac{b^2\tanh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x]^4,x]
```

```
[Out] a^2*x - (a^2*Tanh[c + d*x])/d - (a^2*Tanh[c + d*x]^3)/(3*d) + (b*(2*a + b)*
Tanh[c + d*x]^5)/(5*d) - (b^2*Tanh[c + d*x]^7)/(7*d)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2\tanh^4(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+b\left(1-x^2\right)\right)^2}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-a^2-a^2x^2+b(2a+b)x^4-b^2x^6+\frac{a^2}{1-x^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{a^2\tanh(c+dx)}{d}-\frac{a^2\tanh^3(c+dx)}{3d}+\frac{b(2a+b)\tanh^5(c+dx)}{5d}-\frac{b^2\tanh}{7}$$
$$= a^2x-\frac{a^2\tanh(c+dx)}{d}-\frac{a^2\tanh^3(c+dx)}{3d}+\frac{b(2a+b)\tanh^5(c+dx)}{5d}-\frac{b^2 t}{}$$

Mathematica [B] time = 1.16638, size = 395, normalized size = 5.13

$$\frac{\text{sech}(c)\text{sech}^7(c+dx)\left(4480a^2\sinh(2c+dx)-3780a^2\sinh(2c+3dx)+2100a^2\sinh(4c+3dx)-1540a^2\sinh(4c+5dx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x]^4,x]
```

```
[Out] (Sech[c]*Sech[c + d*x]^7*(3675*a^2*d*x*Cosh[d*x] + 3675*a^2*d*x*Cosh[2*c +
d*x] + 2205*a^2*d*x*Cosh[2*c + 3*d*x] + 2205*a^2*d*x*Cosh[4*c + 3*d*x] + 73
5*a^2*d*x*Cosh[4*c + 5*d*x] + 735*a^2*d*x*Cosh[6*c + 5*d*x] + 105*a^2*d*x*C
osh[6*c + 7*d*x] + 105*a^2*d*x*Cosh[8*c + 7*d*x] - 5320*a^2*Sinh[d*x] + 168
0*a*b*Sinh[d*x] + 840*b^2*Sinh[d*x] + 4480*a^2*Sinh[2*c + d*x] - 1260*a*b*S
inh[2*c + d*x] + 420*b^2*Sinh[2*c + d*x] - 3780*a^2*Sinh[2*c + 3*d*x] + 924
*a*b*Sinh[2*c + 3*d*x] - 168*b^2*Sinh[2*c + 3*d*x] + 2100*a^2*Sinh[4*c + 3*
d*x] - 840*a*b*Sinh[4*c + 3*d*x] - 420*b^2*Sinh[4*c + 3*d*x] - 1540*a^2*Sin
h[4*c + 5*d*x] + 168*a*b*Sinh[4*c + 5*d*x] + 84*b^2*Sinh[4*c + 5*d*x] + 420
*a^2*Sinh[6*c + 5*d*x] - 420*a*b*Sinh[6*c + 5*d*x] - 280*a^2*Sinh[6*c + 7*d
*x] + 84*a*b*Sinh[6*c + 7*d*x] + 12*b^2*Sinh[6*c + 7*d*x]))/(13440*d)
```

Maple [B] time = 0.046, size = 181, normalized size = 2.4

$$\frac{1}{d}\left(a^2\left(dx+c-\tanh(dx+c)-\frac{(\tanh(dx+c))^3}{3}\right)+2\,ab\left(-1/2\,\frac{(\sinh(dx+c))^3}{(\cosh(dx+c))^5}-3/8\,\frac{\sinh(dx+c)}{(\cosh(dx+c))^5}+3/8\left(\frac{8}{15}+1/5\,(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^4,x)
```

```
[Out] 1/d*(a^2*(d*x+c-tanh(d*x+c)-1/3*tanh(d*x+c)^3)+2*a*b*(-1/2*sinh(d*x+c)^3/co
sh(d*x+c)^5-3/8*sinh(d*x+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/15*
sech(d*x+c)^2)*tanh(d*x+c))+b^2*(-1/4*sinh(d*x+c)^3/cosh(d*x+c)^7-1/8*sinh(
d*x+c)/cosh(d*x+c)^7+1/8*(16/35+1/7*sech(d*x+c)^6+6/35*sech(d*x+c)^4+8/35*s
ech(d*x+c)^2)*tanh(d*x+c)))
```

Maxima [B] time = 1.22086, size = 876, normalized size = 11.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 2/5*a*b*tanh(d*x + c)^5/d + 1/3*a^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) +
3*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6
*d*x - 6*c) + 1))) + 4/35*b^2*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) +
21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10
*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 14*e^(-4*d
*x - 4*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c
) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^
(-14*d*x - 14*c) + 1)) + 70*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^
(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x
- 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 35*e^(-8*d*x -
8*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 3
5*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*
d*x - 14*c) + 1)) + 35*e^(-10*d*x - 10*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4
*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 1
0*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x
- 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) +
21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)))
```

Fricas [B] time = 2.08535, size = 1868, normalized size = 24.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/105*((105*a^2*d*x + 140*a^2 - 42*a*b - 6*b^2)*cosh(d*x + c)^7 + 7*(105*a^
2*d*x + 140*a^2 - 42*a*b - 6*b^2)*cosh(d*x + c)*sinh(d*x + c)^6 - 2*(70*a^2
 - 21*a*b - 3*b^2)*sinh(d*x + c)^7 + 7*(105*a^2*d*x + 140*a^2 - 42*a*b - 6*
b^2)*cosh(d*x + c)^5 - 14*(3*(70*a^2 - 21*a*b - 3*b^2)*cosh(d*x + c)^2 + 40
*a^2 + 9*a*b - 3*b^2)*sinh(d*x + c)^5 + 35*((105*a^2*d*x + 140*a^2 - 42*a*b
 - 6*b^2)*cosh(d*x + c)^3 + (105*a^2*d*x + 140*a^2 - 42*a*b - 6*b^2)*cosh(d
*x + c))*sinh(d*x + c)^4 + 21*(105*a^2*d*x + 140*a^2 - 42*a*b - 6*b^2)*cosh
(d*x + c)^3 - 14*(5*(70*a^2 - 21*a*b - 3*b^2)*cosh(d*x + c)^4 + 10*(40*a^2
+ 9*a*b - 3*b^2)*cosh(d*x + c)^2 + 60*a^2 - 3*a*b + 21*b^2)*sinh(d*x + c)^3
 + 7*(3*(105*a^2*d*x + 140*a^2 - 42*a*b - 6*b^2)*cosh(d*x + c)^5 + 10*(105*
a^2*d*x + 140*a^2 - 42*a*b - 6*b^2)*cosh(d*x + c)^3 + 9*(105*a^2*d*x + 140*
a^2 - 42*a*b - 6*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 35*(105*a^2*d*x + 14
0*a^2 - 42*a*b - 6*b^2)*cosh(d*x + c) - 14*((70*a^2 - 21*a*b - 3*b^2)*cosh(
d*x + c)^6 + 5*(40*a^2 + 9*a*b - 3*b^2)*cosh(d*x + c)^4 + 9*(20*a^2 - a*b +
 7*b^2)*cosh(d*x + c)^2 + 30*a^2 - 15*a*b - 45*b^2)*sinh(d*x + c))/(d*cosh(
d*x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + 7*d*cosh(d*x + c)^5 + 35*(
d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^4 + 21*d*cosh(d*x + c)^3
 + 7*(3*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 9*d*cosh(d*x + c))*sinh(
d*x + c)^2 + 35*d*cosh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\operatorname{sech}^2(c + dx)\right)^2 \tanh^4(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**2*tanh(d*x+c)**4,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*tanh(c + d*x)**4, x)
```

Giac [B] time = 1.32151, size = 371, normalized size = 4.82

$$105\,a^2dx + \frac{4\left(105\,a^2e^{(12\,dx+12\,c)} - 105\,abe^{(12\,dx+12\,c)} + 525\,a^2e^{(10\,dx+10\,c)} - 210\,abe^{(10\,dx+10\,c)} - 105\,b^2e^{(10\,dx+10\,c)} + 1120\,a^2e^{(8\,dx+8\,c)} - 315\,abe^{(8\,dx+8\,c)} + 105\,b^2e^{(8\,dx+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/105*(105*a^2*d*x + 4*(105*a^2*e^(12*d*x + 12*c) - 105*a*b*e^(12*d*x + 12*
c) + 525*a^2*e^(10*d*x + 10*c) - 210*a*b*e^(10*d*x + 10*c) - 105*b^2*e^(10*
d*x + 10*c) + 1120*a^2*e^(8*d*x + 8*c) - 315*a*b*e^(8*d*x + 8*c) + 105*b^2*
e^(8*d*x + 8*c) + 1330*a^2*e^(6*d*x + 6*c) - 420*a*b*e^(6*d*x + 6*c) - 210*
b^2*e^(6*d*x + 6*c) + 945*a^2*e^(4*d*x + 4*c) - 231*a*b*e^(4*d*x + 4*c) + 4
2*b^2*e^(4*d*x + 4*c) + 385*a^2*e^(2*d*x + 2*c) - 42*a*b*e^(2*d*x + 2*c) -
21*b^2*e^(2*d*x + 2*c) + 70*a^2 - 21*a*b - 3*b^2)/(e^(2*d*x + 2*c) + 1)^7)/
d
```

3.113 $\int \left(a+b\,\mathrm{sech}^2(c+dx)\right)^2 \tanh^3(c+dx)\,dx$

Optimal. Leaf size=77

$$\frac{a^2\log(\cosh(c+dx))}{d}+\frac{b(2a-b)\mathrm{sech}^4(c+dx)}{4d}+\frac{a(a-2b)\mathrm{sech}^2(c+dx)}{2d}+\frac{b^2\mathrm{sech}^6(c+dx)}{6d}$$

```
[Out] (a^2*Log[Cosh[c + d*x]])/d + (a*(a - 2*b)*Sech[c + d*x]^2)/(2*d) + ((2*a -
b)*b*Sech[c + d*x]^4)/(4*d) + (b^2*Sech[c + d*x]^6)/(6*d)
```

Rubi [A] time = 0.094726, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4138, 446, 76}

$$\frac{a^2\log(\cosh(c+dx))}{d}+\frac{b(2a-b)\mathrm{sech}^4(c+dx)}{4d}+\frac{a(a-2b)\mathrm{sech}^2(c+dx)}{2d}+\frac{b^2\mathrm{sech}^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x]^3,x]
```

```
[Out] (a^2*Log[Cosh[c + d*x]])/d + (a*(a - 2*b)*Sech[c + d*x]^2)/(2*d) + ((2*a -
b)*b*Sech[c + d*x]^4)/(4*d) + (b^2*Sech[c + d*x]^6)/(6*d)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2\tanh^3(c+dx)\,dx = -\frac{\text{Subst}\left(\int\frac{(1-x^2)(b+ax^2)^2}{x^7}dx,x,\cosh(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int\frac{(1-x)(b+ax)^2}{x^4}dx,x,\cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{\text{Subst}\left(\int\left(\frac{b^2}{x^4}+\frac{(2a-b)b}{x^3}+\frac{a(a-2b)}{x^2}-\frac{a^2}{x}\right)dx,x,\cosh^2(c+dx)\right)}{2d}$$
$$= \frac{a^2\log(\cosh(c+dx))}{d}+\frac{a(a-2b)\text{sech}^2(c+dx)}{2d}+\frac{(2a-b)b\text{sech}^4(c+dx)}{4d}+$$

Mathematica [A] time = 0.306637, size = 107, normalized size = 1.39

$$\frac{\cosh^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\left(12a^2\log(\cosh(c+dx))+3b(2a-b)\text{sech}^4(c+dx)+6a(a-2b)\text{sech}^2(c+dx)+2b^2\right.}{3d(a\cosh(2c+2dx)+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x]^3,x]
```

```
[Out] (Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2*(12*a^2*Log[Cosh[c + d*x]] + 6*a
*(a - 2*b)*Sech[c + d*x]^2 + 3*(2*a - b)*b*Sech[c + d*x]^4 + 2*b^2*Sech[c +
 d*x]^6))/(3*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^2)
```

Maple [B] time = 0.043, size = 150, normalized size = 2.

$$\frac{a^2\ln(\cosh(dx+c))}{d}-\frac{(\tanh(dx+c))^2a^2}{2d}-\frac{ab(\sinh(dx+c))^2}{2d(\cosh(dx+c))^4}+\frac{ab(\sinh(dx+c))^2}{2d(\cosh(dx+c))^2}-\frac{b^2(\sinh(dx+c))^2}{6d(\cosh(dx+c))^6}+\frac{b^2(\sin}{12d(\text{co}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^3,x)
```

```
[Out] a^2*ln(cosh(d*x+c))/d-1/2/d*tanh(d*x+c)^2*a^2-1/2/d*a*b*sinh(d*x+c)^2/cosh(
d*x+c)^4+1/2/d*a*b*sinh(d*x+c)^2/cosh(d*x+c)^2-1/6/d*b^2*sinh(d*x+c)^2/cosh
(d*x+c)^6+1/12/d*b^2*sinh(d*x+c)^2/cosh(d*x+c)^4+1/12/d*b^2*sinh(d*x+c)^2/c
osh(d*x+c)^2
```

Maxima [B] time = 2.35945, size = 450, normalized size = 5.84

$$\frac{ab\tanh(dx+c)^4}{2d}+a^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{2e^{(-2dx-2c)}}{d\left(2e^{(-2dx-2c)}+e^{(-4dx-4c)}+1\right)}\right)-\frac{4}{3}b^2\left(\frac{}{d\left(6e^{(-2dx-2c)}+15e^{(-4dx-}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*b*tanh(d*x + c)^4/d + a^2*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*
e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - 4/3*b^2
```

```
*(3*e^(-4*d*x - 4*c)/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-
6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12
*c) + 1)) - 2*e^(-6*d*x - 6*c)/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c)
 + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-1
2*d*x - 12*c) + 1)) + 3*e^(-8*d*x - 8*c)/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*
d*x - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*
c) + e^(-12*d*x - 12*c) + 1)))
```

Fricas [B] time = 2.43469, size = 6669, normalized size = 86.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^2*d*x*cosh(d*x + c)^12 + 36*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^1
1 + 3*a^2*d*x*sinh(d*x + c)^12 + 6*(3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x + c)^
10 + 6*(33*a^2*d*x*cosh(d*x + c)^2 + 3*a^2*d*x - a^2 + 2*a*b)*sinh(d*x + c)
^10 + 60*(11*a^2*d*x*cosh(d*x + c)^3 + (3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x +
 c))*sinh(d*x + c)^9 + 3*(15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)
^8 + 3*(495*a^2*d*x*cosh(d*x + c)^4 + 15*a^2*d*x + 90*(3*a^2*d*x - a^2 + 2*
a*b)*cosh(d*x + c)^2 - 8*a^2 + 8*a*b + 4*b^2)*sinh(d*x + c)^8 + 24*(99*a^2*
d*x*cosh(d*x + c)^5 + 30*(3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x + c)^3 + (15*a^
2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 4*(15*a^2*d
*x - 9*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^6 + 4*(693*a^2*d*x*cosh(d*x + c)^
6 + 315*(3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x + c)^4 + 15*a^2*d*x + 21*(15*a^2
*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^2 - 9*a^2 + 6*a*b - 2*b^2)*sinh
(d*x + c)^6 + 24*(99*a^2*d*x*cosh(d*x + c)^7 + 63*(3*a^2*d*x - a^2 + 2*a*b)
*cosh(d*x + c)^5 + 7*(15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^3 +
 (15*a^2*d*x - 9*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 3*(1
5*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^4 + 3*(495*a^2*d*x*cosh(d*
x + c)^8 + 420*(3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x + c)^6 + 70*(15*a^2*d*x -
 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^4 + 15*a^2*d*x + 20*(15*a^2*d*x - 9*a
^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^2 - 8*a^2 + 8*a*b + 4*b^2)*sinh(d*x + c)^
4 + 3*a^2*d*x + 4*(165*a^2*d*x*cosh(d*x + c)^9 + 180*(3*a^2*d*x - a^2 + 2*a
*b)*cosh(d*x + c)^7 + 42*(15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)
^5 + 20*(15*a^2*d*x - 9*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^3 + 3*(15*a^2*d*
x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(3*a^2*d*x -
a^2 + 2*a*b)*cosh(d*x + c)^2 + 6*(33*a^2*d*x*cosh(d*x + c)^10 + 45*(3*a^2*d
*x - a^2 + 2*a*b)*cosh(d*x + c)^8 + 14*(15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)
*cosh(d*x + c)^6 + 10*(15*a^2*d*x - 9*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^4
+ 3*a^2*d*x + 3*(15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^2 - a^2
+ 2*a*b)*sinh(d*x + c)^2 - 3*(a^2*cosh(d*x + c)^12 + 12*a^2*cosh(d*x + c)*s
inh(d*x + c)^11 + a^2*sinh(d*x + c)^12 + 6*a^2*cosh(d*x + c)^10 + 6*(11*a^2
*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^10 + 15*a^2*cosh(d*x + c)^8 + 20*(11*
a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^9 + 15*(33*a^2*cos
h(d*x + c)^4 + 18*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^8 + 20*a^2*cosh(
d*x + c)^6 + 24*(33*a^2*cosh(d*x + c)^5 + 30*a^2*cosh(d*x + c)^3 + 5*a^2*co
sh(d*x + c))*sinh(d*x + c)^7 + 4*(231*a^2*cosh(d*x + c)^6 + 315*a^2*cosh(d*
x + c)^4 + 105*a^2*cosh(d*x + c)^2 + 5*a^2)*sinh(d*x + c)^6 + 15*a^2*cosh(d
*x + c)^4 + 24*(33*a^2*cosh(d*x + c)^7 + 63*a^2*cosh(d*x + c)^5 + 35*a^2*co
sh(d*x + c)^3 + 5*a^2*cosh(d*x + c))*sinh(d*x + c)^5 + 15*(33*a^2*cosh(d*x
+ c)^8 + 84*a^2*cosh(d*x + c)^6 + 70*a^2*cosh(d*x + c)^4 + 20*a^2*cosh(d*x
+ c)^2 + a^2)*sinh(d*x + c)^4 + 6*a^2*cosh(d*x + c)^2 + 20*(11*a^2*cosh(d*x
 + c)^9 + 36*a^2*cosh(d*x + c)^7 + 42*a^2*cosh(d*x + c)^5 + 20*a^2*cosh(d*x
 + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(11*a^2*cosh(d*x + c)^10
```

```
 + 45*a^2*cosh(d*x + c)^8 + 70*a^2*cosh(d*x + c)^6 + 50*a^2*cosh(d*x + c)^4
 + 15*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^2 + a^2 + 12*(a^2*cosh(d*x +
 c)^11 + 5*a^2*cosh(d*x + c)^9 + 10*a^2*cosh(d*x + c)^7 + 10*a^2*cosh(d*x +
 c)^5 + 5*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c))*log(2*cos
h(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 12*(3*a^2*d*x*cosh(d*x + c)^1
1 + 5*(3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x + c)^9 + 2*(15*a^2*d*x - 8*a^2 + 8
*a*b + 4*b^2)*cosh(d*x + c)^7 + 2*(15*a^2*d*x - 9*a^2 + 6*a*b - 2*b^2)*cosh
(d*x + c)^5 + (15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^3 + (3*a^2
*d*x - a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^12 + 12*
d*cosh(d*x + c)*sinh(d*x + c)^11 + d*sinh(d*x + c)^12 + 6*d*cosh(d*x + c)^1
0 + 6*(11*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^10 + 20*(11*d*cosh(d*x + c)^
3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^9 + 15*d*cosh(d*x + c)^8 + 15*(33*d*co
sh(d*x + c)^4 + 18*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 24*(33*d*cosh(d
*x + c)^5 + 30*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^7 + 20*
d*cosh(d*x + c)^6 + 4*(231*d*cosh(d*x + c)^6 + 315*d*cosh(d*x + c)^4 + 105*
d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^6 + 24*(33*d*cosh(d*x + c)^7 + 63*d*
cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^5
 + 15*d*cosh(d*x + c)^4 + 15*(33*d*cosh(d*x + c)^8 + 84*d*cosh(d*x + c)^6 +
 70*d*cosh(d*x + c)^4 + 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 20*(11*
d*cosh(d*x + c)^9 + 36*d*cosh(d*x + c)^7 + 42*d*cosh(d*x + c)^5 + 20*d*cosh
(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 6*d*cosh(d*x + c)^2 + 6*
(11*d*cosh(d*x + c)^10 + 45*d*cosh(d*x + c)^8 + 70*d*cosh(d*x + c)^6 + 50*d
*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 12*(d*cosh(d
*x + c)^11 + 5*d*cosh(d*x + c)^9 + 10*d*cosh(d*x + c)^7 + 10*d*cosh(d*x + c
)^5 + 5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [A] time = 8.80859, size = 129, normalized size = 1.68

$$\begin{cases} a^2x - \frac{a^2\log(\tanh(c+dx)+1)}{d} - \frac{a^2\tanh^2(c+dx)}{2d} - \frac{ab\tanh^2(c+dx)\operatorname{sech}^2(c+dx)}{2d} - \frac{ab\operatorname{sech}^2(c+dx)}{2d} - \frac{b^2\tanh^2(c+dx)\operatorname{sech}^4(c+dx)}{6d} - \frac{b^2\operatorname{sech}^4(c+dx)}{12d} \\ x\left(a+b\operatorname{sech}^2(c)\right)^2\tanh^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**2*tanh(d*x+c)**3,x)
```

```
[Out] Piecewise((a**2*x - a**2*log(tanh(c + d*x) + 1)/d - a**2*tanh(c + d*x)**2/(
2*d) - a*b*tanh(c + d*x)**2*sech(c + d*x)**2/(2*d) - a*b*sech(c + d*x)**2/(
2*d) - b**2*tanh(c + d*x)**2*sech(c + d*x)**4/(6*d) - b**2*sech(c + d*x)**4
/(12*d), Ne(d, 0)), (x*(a + b*sech(c)**2)**2*tanh(c)**3, True))
```

Giac [B] time = 1.28324, size = 325, normalized size = 4.22

$$-\frac{60\,a^2dx - 60\,a^2\log\left(e^{(2\,dx+2\,c)}+1\right) + \frac{147\,a^2e^{(12\,dx+12\,c)}+762\,a^2e^{(10\,dx+10\,c)}+240\,abe^{(10\,dx+10\,c)}+1725\,a^2e^{(8\,dx+8\,c)}+480\,abe^{(8\,dx+8\,c)}+240\,b^2e^{(8\,dx+8\,c)}+\cdots}{\cdots}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/60*(60*a^2*d*x - 60*a^2*log(e^(2*d*x + 2*c) + 1) + (147*a^2*e^(12*d*x +
12*c) + 762*a^2*e^(10*d*x + 10*c) + 240*a*b*e^(10*d*x + 10*c) + 1725*a^2*e^
(8*d*x + 8*c) + 480*a*b*e^(8*d*x + 8*c) + 240*b^2*e^(8*d*x + 8*c) + 2220*a^
2*e^(6*d*x + 6*c) + 480*a*b*e^(6*d*x + 6*c) - 160*b^2*e^(6*d*x + 6*c) + 172
```

```
5*a^2*e^(4*d*x + 4*c) + 480*a*b*e^(4*d*x + 4*c) + 240*b^2*e^(4*d*x + 4*c) +
 762*a^2*e^(2*d*x + 2*c) + 240*a*b*e^(2*d*x + 2*c) + 147*a^2)/(e^(2*d*x + 2
*c) + 1)^6)/d
```

3.114 $\int \left(a+b\,\mathrm{sech}^2(c+dx)\right)^2 \tanh^2(c+dx)\,dx$

Optimal. Leaf size=59

$$-\frac{a^2\tanh(c+dx)}{d}+a^2x+\frac{b(2a+b)\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^5(c+dx)}{5d}$$

```
[Out] a^2*x - (a^2*Tanh[c + d*x])/d + (b*(2*a + b)*Tanh[c + d*x]^3)/(3*d) - (b^2*
Tanh[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0980225, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4141, 1802, 206}

$$-\frac{a^2\tanh(c+dx)}{d}+a^2x+\frac{b(2a+b)\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x]^2,x]
```

```
[Out] a^2*x - (a^2*Tanh[c + d*x])/d + (b*(2*a + b)*Tanh[c + d*x]^3)/(3*d) - (b^2*
Tanh[c + d*x]^5)/(5*d)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2\tanh^2(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+b\left(1-x^2\right)\right)^2}{1-x^2}dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-a^2+b(2a+b)x^2-b^2x^4+\frac{a^2}{1-x^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{a^2\tanh(c+dx)}{d}+\frac{b(2a+b)\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^5(c+dx)}{5d}+\frac{a^2\,\text{S}}{}$$
$$= a^2x-\frac{a^2\tanh(c+dx)}{d}+\frac{b(2a+b)\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^5(c+dx)}{5d}$$

Mathematica [B] time = 0.89966, size = 281, normalized size = 4.76

$$\text{sech}(c)\text{sech}^5(c+dx)\left(120a^2\sinh(2c+dx)-120a^2\sinh(2c+3dx)+30a^2\sinh(4c+3dx)-30a^2\sinh(4c+5dx)+15\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x]^2,x]
```

```
[Out] (Sech[c]*Sech[c + d*x]^5*(150*a^2*d*x*Cosh[d*x] + 150*a^2*d*x*Cosh[2*c + d*
x] + 75*a^2*d*x*Cosh[2*c + 3*d*x] + 75*a^2*d*x*Cosh[4*c + 3*d*x] + 15*a^2*d
*x*Cosh[4*c + 5*d*x] + 15*a^2*d*x*Cosh[6*c + 5*d*x] - 180*a^2*Sinh[d*x] + 8
0*a*b*Sinh[d*x] - 20*b^2*Sinh[d*x] + 120*a^2*Sinh[2*c + d*x] - 120*a*b*Sinh
[2*c + d*x] - 60*b^2*Sinh[2*c + d*x] - 120*a^2*Sinh[2*c + 3*d*x] + 40*a*b*S
inh[2*c + 3*d*x] + 20*b^2*Sinh[2*c + 3*d*x] + 30*a^2*Sinh[4*c + 3*d*x] - 60
*a*b*Sinh[4*c + 3*d*x] - 30*a^2*Sinh[4*c + 5*d*x] + 20*a*b*Sinh[4*c + 5*d*x
] + 4*b^2*Sinh[4*c + 5*d*x]))/(480*d)
```

Maple [B] time = 0.037, size = 115, normalized size = 2.

$$\frac{1}{d}\left(a^2(dx+c-\tanh(dx+c))+2ab\left(-1/2\frac{\sinh(dx+c)}{(\cosh(dx+c))^3}+1/2\left(2/3+1/3(\text{sech}(dx+c))^2\right)\tanh(dx+c)\right)+b^2\left(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^2,x)
```

```
[Out] 1/d*(a^2*(d*x+c-tanh(d*x+c))+2*a*b*(-1/2*sinh(d*x+c)/cosh(d*x+c)^3+1/2*(2/3
+1/3*sech(d*x+c)^2)*tanh(d*x+c))+b^2*(-1/4*sinh(d*x+c)/cosh(d*x+c)^5+1/4*(8
/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c)))
```

Maxima [B] time = 1.18822, size = 439, normalized size = 7.44

$$\frac{2ab\tanh(dx+c)^3}{3d}+a^2\left(x+\frac{c}{d}-\frac{2}{d\left(e^{(-2dx-2c)}+1\right)}\right)+\frac{4}{15}b^2\left(\frac{5e^{(-2dx-2c)}}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 2/3*a*b*tanh(d*x + c)^3/d + a^2*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) +
4/15*b^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) +
 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) - 5*e^
(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
- 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-6*d*x - 6*c
)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^
(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e
^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 1
0*c) + 1)))
```

Fricas [B] time = 2.07574, size = 1099, normalized size = 18.63

$$\frac{\left(15\,a^2dx + 15\,a^2 - 10\,ab - 2\,b^2\right)\cosh\left(dx + c\right)^5 + 5\left(15\,a^2dx + 15\,a^2 - 10\,ab - 2\,b^2\right)\cosh\left(dx + c\right)\sinh\left(dx + c\right)^4 - \left(15\,a^2 - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/15*((15*a^2*d*x + 15*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c)^5 + 5*(15*a^2*d*
x + 15*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 - (15*a^2 - 10*a
*b - 2*b^2)*sinh(d*x + c)^5 + 5*(15*a^2*d*x + 15*a^2 - 10*a*b - 2*b^2)*cosh
(d*x + c)^3 - 5*(2*(15*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c)^2 + 9*a^2 + 2*a*
b - 2*b^2)*sinh(d*x + c)^3 + 5*(2*(15*a^2*d*x + 15*a^2 - 10*a*b - 2*b^2)*co
sh(d*x + c)^3 + 3*(15*a^2*d*x + 15*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c))*sin
h(d*x + c)^2 + 10*(15*a^2*d*x + 15*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c) - 5*
((15*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c)^4 + 3*(9*a^2 + 2*a*b - 2*b^2)*cosh
(d*x + c)^2 + 6*a^2 + 4*a*b + 8*b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*
d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c
)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\operatorname{sech}^2\left(c + dx\right)\right)^2 \tanh^2\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**2*tanh(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*tanh(c + d*x)**2, x)
```

Giac [B] time = 1.23893, size = 261, normalized size = 4.42

$$\frac{15\,a^2dx + \frac{2\left(15\,a^2e^{(8\,dx+8\,c)} - 30\,abe^{(8\,dx+8\,c)} + 60\,a^2e^{(6\,dx+6\,c)} - 60\,abe^{(6\,dx+6\,c)} - 30\,b^2e^{(6\,dx+6\,c)} + 90\,a^2e^{(4\,dx+4\,c)} - 40\,abe^{(4\,dx+4\,c)} + 10\,b^2e^{(4\,dx+4\,c)} + 60\,a^2e^{(2\,dx+2\,c)} - 20\right.}{\left(e^{(2\,dx+2\,c)}+1\right)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/15*(15*a^2*d*x + 2*(15*a^2*e^(8*d*x + 8*c) - 30*a*b*e^(8*d*x + 8*c) + 60*
a^2*e^(6*d*x + 6*c) - 60*a*b*e^(6*d*x + 6*c) - 30*b^2*e^(6*d*x + 6*c) + 90*
a^2*e^(4*d*x + 4*c) - 40*a*b*e^(4*d*x + 4*c) + 10*b^2*e^(4*d*x + 4*c) + 60*
a^2*e^(2*d*x + 2*c) - 20*a*b*e^(2*d*x + 2*c) - 10*b^2*e^(2*d*x + 2*c) + 15*
a^2 - 10*a*b - 2*b^2)/(e^(2*d*x + 2*c) + 1)^5)/d
```

3.115 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)^2 \tanh(c+dx)\,dx$

Optimal. Leaf size=48

$$\frac{a^2\log(\cosh(c+dx))}{d}-\frac{ab\,\mathrm{sech}^2(c+dx)}{d}-\frac{b^2\mathrm{sech}^4(c+dx)}{4d}$$

```
[Out] (a^2*Log[Cosh[c + d*x]])/d - (a*b*Sech[c + d*x]^2)/d - (b^2*Sech[c + d*x]^4
)/(4*d)
```

Rubi [A] time = 0.0500898, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 266, 43}

$$\frac{a^2\log(\cosh(c+dx))}{d}-\frac{ab\,\mathrm{sech}^2(c+dx)}{d}-\frac{b^2\mathrm{sech}^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x],x]
```

```
[Out] (a^2*Log[Cosh[c + d*x]])/d - (a*b*Sech[c + d*x]^2)/d - (b^2*Sech[c + d*x]^4
)/(4*d)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2 \tanh(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(b+ax^2\right)^2}{x^5}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(b+ax)^2}{x^3}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{b^2}{x^3}+\frac{2ab}{x^2}+\frac{a^2}{x}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{a^2\log(\cosh(c+dx))}{d} - \frac{ab\text{sech}^2(c+dx)}{d} - \frac{b^2\text{sech}^4(c+dx)}{4d}$$

Mathematica [A] time = 0.131715, size = 81, normalized size = 1.69

$$\frac{\text{sech}^4(c+dx)\left(a\cosh^2(c+dx)+b\right)^2\left(4a^2\cosh^4(c+dx)\log(\cosh(c+dx))-4ab\cosh^2(c+dx)-b^2\right)}{d(a\cosh(2(c+dx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x],x]
```

```
[Out] ((b + a*Cosh[c + d*x]^2)^2*(-b^2 - 4*a*b*Cosh[c + d*x]^2 + 4*a^2*Cosh[c + d
*x]^4*Log[Cosh[c + d*x]])*Sech[c + d*x]^4)/(d*(a + 2*b + a*Cosh[2*(c + d*x)
])^2)
```

Maple [A] time = 0.011, size = 48, normalized size = 1.

$$-\frac{b^2(\text{sech}(dx+c))^4}{4d} - \frac{ab(\text{sech}(dx+c))^2}{d} - \frac{a^2\ln(\text{sech}(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^2*tanh(d*x+c),x)
```

```
[Out] -1/4*b^2*sech(d*x+c)^4/d-a*b*sech(d*x+c)^2/d-1/d*a^2*ln(sech(d*x+c))
```

Maxima [A] time = 1.11594, size = 74, normalized size = 1.54

$$\frac{ab\tanh(dx+c)^2}{d} + \frac{a^2\log(\cosh(dx+c))}{d} - \frac{4b^2}{d\left(e^{(dx+c)}+e^{(-dx-c)}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c),x, algorithm="maxima")
```

```
[Out] a*b*tanh(d*x + c)^2/d + a^2*log(cosh(d*x + c))/d - 4*b^2/(d*(e^(d*x + c) +
e^(-d*x - c))^4)
```

Fricas [B] time = 2.18418, size = 3012, normalized size = 62.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(a^2*d*x*cosh(d*x + c)^8 + 8*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*d
*x*sinh(d*x + c)^8 + 4*(a^2*d*x + a*b)*cosh(d*x + c)^6 + 4*(7*a^2*d*x*cosh(
d*x + c)^2 + a^2*d*x + a*b)*sinh(d*x + c)^6 + 8*(7*a^2*d*x*cosh(d*x + c)^3
+ 3*(a^2*d*x + a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^2*d*x + 4*a*b +
 2*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*d*x*cosh(d*x + c)^4 + 3*a^2*d*x + 30*(a
^2*d*x + a*b)*cosh(d*x + c)^2 + 4*a*b + 2*b^2)*sinh(d*x + c)^4 + a^2*d*x +
8*(7*a^2*d*x*cosh(d*x + c)^5 + 10*(a^2*d*x + a*b)*cosh(d*x + c)^3 + (3*a^2*
d*x + 4*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^2*d*x + a*b)*cos
h(d*x + c)^2 + 4*(7*a^2*d*x*cosh(d*x + c)^6 + 15*(a^2*d*x + a*b)*cosh(d*x +
 c)^4 + a^2*d*x + 3*(3*a^2*d*x + 4*a*b + 2*b^2)*cosh(d*x + c)^2 + a*b)*sinh
(d*x + c)^2 - (a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 +
a^2*sinh(d*x + c)^8 + 4*a^2*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 + a^
2)*sinh(d*x + c)^6 + 6*a^2*cosh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^3 + 3*a
^2*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^2*cosh(d*x + c)^4 + 30*a^2*cosh
(d*x + c)^2 + 3*a^2)*sinh(d*x + c)^4 + 4*a^2*cosh(d*x + c)^2 + 8*(7*a^2*cos
h(d*x + c)^5 + 10*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^
3 + 4*(7*a^2*cosh(d*x + c)^6 + 15*a^2*cosh(d*x + c)^4 + 9*a^2*cosh(d*x + c)
^2 + a^2)*sinh(d*x + c)^2 + a^2 + 8*(a^2*cosh(d*x + c)^7 + 3*a^2*cosh(d*x +
 c)^5 + 3*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c))*log(2*cos
h(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(a^2*d*x*cosh(d*x + c)^7 +
3*(a^2*d*x + a*b)*cosh(d*x + c)^5 + (3*a^2*d*x + 4*a*b + 2*b^2)*cosh(d*x +
c)^3 + (a^2*d*x + a*b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8
*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 + 4*d*cosh(d*x + c)^6
+ 4*(7*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 + 3*
d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x +
 c)^4 + 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^
5 + 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*d*cosh(d*
x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c
)^2 + d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 + 3*d*cosh(d*x + c)^5 + 3*d
*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [A] time = 2.91853, size = 63, normalized size = 1.31

$$\begin{cases} a^2x - \frac{a^2\log(\tanh(c+dx)+1)}{d} - \frac{ab\,\text{sech}^2(c+dx)}{d} - \frac{b^2\,\text{sech}^4(c+dx)}{4d} & \text{for } d \neq 0 \\ x\left(a + b\,\text{sech}^2(c)\right)^2 \tanh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**2*tanh(d*x+c),x)
```

```
[Out] Piecewise((a**2*x - a**2*log(tanh(c + d*x) + 1)/d - a*b*sech(c + d*x)**2/d
- b**2*sech(c + d*x)**4/(4*d), Ne(d, 0)), (x*(a + b*sech(c)**2)**2*tanh(c),
 True))
```

Giac [B] time = 1.16215, size = 215, normalized size = 4.48

$$-\frac{12\,a^2dx-12\,a^2\log\left(e^{(2\,dx+2\,c)}+1\right)+\frac{25\,a^2e^{(8\,dx+8\,c)}+100\,a^2e^{(6\,dx+6\,c)}+48\,abe^{(6\,dx+6\,c)}+150\,a^2e^{(4\,dx+4\,c)}+96\,abe^{(4\,dx+4\,c)}+48\,b^2e^{(4\,dx+4\,c)}+100\,a^2e^{(}}{\left(e^{(2\,dx+2\,c)}+1\right)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c),x, algorithm="giac")
```

```
[Out] -1/12*(12*a^2*d*x - 12*a^2*log(e^(2*d*x + 2*c) + 1) + (25*a^2*e^(8*d*x + 8*
c) + 100*a^2*e^(6*d*x + 6*c) + 48*a*b*e^(6*d*x + 6*c) + 150*a^2*e^(4*d*x +
4*c) + 96*a*b*e^(4*d*x + 4*c) + 48*b^2*e^(4*d*x + 4*c) + 100*a^2*e^(2*d*x +
 2*c) + 48*a*b*e^(2*d*x + 2*c) + 25*a^2)/(e^(2*d*x + 2*c) + 1)^4)/d
```

3.116 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=40

$$a^2x+\frac{b(2a+b)\tanh(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}$$

```
[Out] a^2*x + (b*(2*a + b)*Tanh[c + d*x])/d - (b^2*Tanh[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0341698, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4128, 390, 206}

$$a^2x+\frac{b(2a+b)\tanh(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] a^2*x + (b*(2*a + b)*Tanh[c + d*x])/d - (b^2*Tanh[c + d*x]^3)/(3*d)
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\,\mathrm{sech}^2(c+dx)\right)^2\,dx &= \frac{\mathrm{Subst}\left(\int \frac{\left(a+b-bx^2\right)^2}{1-x^2}\,dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{\mathrm{Subst}\left(\int \left(b(2a+b)-b^2x^2+\frac{a^2}{1-x^2}\right)\,dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{b(2a+b)\tanh(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}+\frac{a^2\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\tanh(c+dx)\right)}{d}\\
&= a^2x+\frac{b(2a+b)\tanh(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [B] time = 0.420214, size = 106, normalized size = 2.65

$$\frac{4\operatorname{sech}^3(c+dx)\left(a\cosh^2(c+dx)+b\right)^2\left(3a^2dx\cosh^3(c+dx)+2b(3a+b)\operatorname{sech}(c)\sinh(dx)\cosh^2(c+dx)+b^2\tanh(c)\right.}{3d(a\cosh(2(c+dx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (4*(b + a*Cosh[c + d*x]^2)^2*Sech[c + d*x]^3*(3*a^2*d*x*Cosh[c + d*x]^3 + b
^2*Sech[c]*Sinh[d*x] + 2*b*(3*a + b)*Cosh[c + d*x]^2*Sech[c]*Sinh[d*x] + b^
2*Cosh[c + d*x]*Tanh[c]))/(3*d*(a + 2*b + a*Cosh[2*(c + d*x)])^2)
```

Maple [A] time = 0.022, size = 47, normalized size = 1.2

$$\frac{1}{d}\left(a^2(dx+c)+2ab\tanh(dx+c)+b^2\left(\frac{2}{3}+\frac{(\operatorname{sech}(dx+c))^2}{3}\right)\tanh(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(d*x+c)+2*a*b*tanh(d*x+c)+b^2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c))
```

Maxima [B] time = 1.11098, size = 162, normalized size = 4.05

$$a^2x+\frac{4}{3}b^2\left(\frac{3e^{(-2dx-2c)}}{d\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1\right)}+\frac{1}{d\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1\right)}\right)+\frac{4a}{d\left(e^{(-2dx-\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 4/3*b^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x -
4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*
c) + e^(-6*d*x - 6*c) + 1))) + 4*a*b/(d*(e^(-2*d*x - 2*c) + 1))
```

Fricas [B] time = 2.00682, size = 447, normalized size = 11.18

$$\frac{\left(3a^2dx-6ab-2b^2\right)\cosh(dx+c)^3+3\left(3a^2dx-6ab-2b^2\right)\cosh(dx+c)\sinh(dx+c)^2+2\left(3ab+b^2\right)\sinh(dx+}{3\left(d\cosh(dx+c)^3+3d\cosh(dx+c)\sinh(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*((3*a^2*d*x - 6*a*b - 2*b^2)*cosh(d*x + c)^3 + 3*(3*a^2*d*x - 6*a*b - 2
*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(3*a*b + b^2)*sinh(d*x + c)^3 + 3*(
3*a^2*d*x - 6*a*b - 2*b^2)*cosh(d*x + c) + 6*((3*a*b + b^2)*cosh(d*x + c)^2
 + a*b + b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*
```

```
x + c)^2 + 3*d*cosh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\operatorname{sech}^2(c + dx)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2, x)
```

Giac [B] time = 1.1386, size = 107, normalized size = 2.68

$$\frac{(dx + c)a^2}{d} - \frac{4\left(3\,abe^{(4\,dx+4\,c)} + 6\,abe^{(2\,dx+2\,c)} + 3\,b^2e^{(2\,dx+2\,c)} + 3\,ab + b^2\right)}{3\,d\left(e^{(2\,dx+2\,c)} + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] (d*x + c)*a^2/d - 4/3*(3*a*b*e^(4*d*x + 4*c) + 6*a*b*e^(2*d*x + 2*c) + 3*b^
2*e^(2*d*x + 2*c) + 3*a*b + b^2)/(d*(e^(2*d*x + 2*c) + 1)^3)
```

3.117 $\int \coth(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=53

$$\frac{(a+b)^2\log(\sinh(c+dx))}{d}-\frac{b(2a+b)\log(\cosh(c+dx))}{d}+\frac{b^2\mathrm{sech}^2(c+dx)}{2d}$$

```
[Out] -((b*(2*a + b)*Log[Cosh[c + d*x]])/d) + ((a + b)^2*Log[Sinh[c + d*x]])/d +
(b^2*Sech[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.0801266, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 446, 88}

$$\frac{(a+b)^2\log(\sinh(c+dx))}{d}-\frac{b(2a+b)\log(\cosh(c+dx))}{d}+\frac{b^2\mathrm{sech}^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -((b*(2*a + b)*Log[Cosh[c + d*x]])/d) + ((a + b)^2*Log[Sinh[c + d*x]])/d +
(b^2*Sech[c + d*x]^2)/(2*d)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \coth(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx = -\frac{\text{Subst}\left(\int \frac{\left(b+ax^2\right)^2}{x^3(1-x^2)}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{(b+ax)^2}{(1-x)x^2}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{\text{Subst}\left(\int \left(-\frac{(a+b)^2}{-1+x}+\frac{b^2}{x^2}+\frac{b(2a+b)}{x}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{b(2a+b)\log(\cosh(c+dx))}{d}+\frac{(a+b)^2\log(\sinh(c+dx))}{d}+\frac{b^2\text{sech}^2(c+dx)}{2d}$$

Mathematica [A] time = 0.242977, size = 84, normalized size = 1.58

$$\frac{2(a\cosh(c+dx)+b\text{sech}(c+dx))^2\left(2\cosh^2(c+dx)\left((a+b)^2\log(\sinh(c+dx))-b(2a+b)\log(\cosh(c+dx))\right)+b^2\right)}{d(a\cosh(2(c+dx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (2*(b^2 + 2*Cosh[c + d*x]^2*(-(b*(2*a + b)*Log[Cosh[c + d*x]]) + (a + b)^2*
Log[Sinh[c + d*x]]))*(a*Cosh[c + d*x] + b*Sech[c + d*x])^2)/(d*(a + 2*b + a
*Cosh[2*(c + d*x)])^2)
```

Maple [A] time = 0.041, size = 60, normalized size = 1.1

$$\frac{a^2\ln(\sinh(dx+c))}{d}+2\,\frac{ab\ln(\tanh(dx+c))}{d}+\frac{b^2}{2\,d\,(\cosh(dx+c))^2}+\frac{b^2\ln(\tanh(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d*a^2*ln(sinh(d*x+c))+2/d*a*b*ln(tanh(d*x+c))+1/2/d*b^2/cosh(d*x+c)^2+1/d
*b^2*ln(tanh(d*x+c))
```

Maxima [B] time = 1.86205, size = 217, normalized size = 4.09

$$b^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1\right)}\right)+2\,ab\left(\frac{\log\left(e^{(-dx-c)}+1\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] b^2*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x - 2*
c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) +
1))) + 2*a*b*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2
*d*x - 2*c) + 1)/d) + a^2*log(sinh(d*x + c))/d
```

Fricas [B] time = 2.21251, size = 1709, normalized size = 32.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -(a^2*d*x*cosh(d*x + c)^4 + 4*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*d
*x*sinh(d*x + c)^4 + a^2*d*x + 2*(a^2*d*x - b^2)*cosh(d*x + c)^2 + 2*(3*a^2
*d*x*cosh(d*x + c)^2 + a^2*d*x - b^2)*sinh(d*x + c)^2 + ((2*a*b + b^2)*cosh
(d*x + c)^4 + 4*(2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a*b + b^2)
*sinh(d*x + c)^4 + 2*(2*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(2*a*b + b^2)*cos
h(d*x + c)^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + 2*a*b + b^2 + 4*((2*a*b + b^2
)*cosh(d*x + c)^3 + (2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(
d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - ((a^2 + 2*a*b + b^2)*cosh(d*x +
 c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b
+ b^2)*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 +
 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b + b^2)
*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*
x + c))) + 4*(a^2*d*x*cosh(d*x + c)^3 + (a^2*d*x - b^2)*cosh(d*x + c))*sinh
(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(
d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x + c
)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18386, size = 231, normalized size = 4.36

$$-\frac{2\,a^2dx + 2\left(2\,abe^{(2\,c)} + b^2e^{(2\,c)}\right)e^{(-2\,c)}\log\left(e^{(2\,dx+2\,c)} + 1\right) - 2\left(a^2e^{(2\,c)} + 2\,abe^{(2\,c)} + b^2e^{(2\,c)}\right)e^{(-2\,c)}\log\left(\left|e^{(2\,dx+2\,c)} - 1\right|\right) - \frac{6}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*a^2*d*x + 2*(2*a*b*e^(2*c) + b^2*e^(2*c))*e^(-2*c)*log(e^(2*d*x + 2
*c) + 1) - 2*(a^2*e^(2*c) + 2*a*b*e^(2*c) + b^2*e^(2*c))*e^(-2*c)*log(abs(e
^(2*d*x + 2*c) - 1)) - (6*a*b*e^(4*d*x + 4*c) + 3*b^2*e^(4*d*x + 4*c) + 12*
a*b*e^(2*d*x + 2*c) + 10*b^2*e^(2*d*x + 2*c) + 6*a*b + 3*b^2)/(e^(2*d*x + 2
*c) + 1)^2)/d
```

3.118 $\int \coth^2(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=36

$$a^2x-\frac{(a+b)^2\coth(c+dx)}{d}-\frac{b^2\tanh(c+dx)}{d}$$

```
[Out] a^2*x - ((a + b)^2*Coth[c + d*x])/d - (b^2*Tanh[c + d*x])/d
```

Rubi [A] time = 0.0851578, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4141, 1802, 207}

$$a^2x-\frac{(a+b)^2\coth(c+dx)}{d}-\frac{b^2\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] a^2*x - ((a + b)^2*Coth[c + d*x])/d - (b^2*Tanh[c + d*x])/d
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \coth^2(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2\,dx &= \frac{\mathrm{Subst}\left(\int \frac{\left(a+b\left(1-x^2\right)\right)^2}{x^2\left(1-x^2\right)}\,dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{\mathrm{Subst}\left(\int \left(-b^2+\frac{(a+b)^2}{x^2}-\frac{a^2}{-1+x^2}\right)dx,x,\tanh(c+dx)\right)}{d}\\
&= -\frac{(a+b)^2\coth(c+dx)}{d}-\frac{b^2\tanh(c+dx)}{d}-\frac{a^2\,\mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\tanh(
}{d}\\
&= a^2x-\frac{(a+b)^2\coth(c+dx)}{d}-\frac{b^2\tanh(c+dx)}{d}
\end{aligned}$$

Mathematica [B] time = 0.700138, size = 82, normalized size = 2.28

$$\frac{4\text{sech}(c+dx)\left(a\cosh^2(c+dx)+b\right)^2\left(a^2dx\cosh(c+dx)+\sinh(dx)\left((a+b)^2\text{csch}(c)\coth(c+dx)-b^2\text{sech}(c)\right)\right)}{d(a\cosh(2(c+dx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (4*(b + a*Cosh[c + d*x]^2)^2*Sech[c + d*x]*(a^2*d*x*Cosh[c + d*x] + ((a + b
)^2*Coth[c + d*x]*Csch[c] - b^2*Sech[c])*Sinh[d*x]))/(d*(a + 2*b + a*Cosh[2
*(c + d*x)])^2)
```

Maple [A] time = 0.033, size = 64, normalized size = 1.8

$$\frac{1}{d}\left(a^2\left(dx+c-\coth\left(dx+c\right)\right)-2\,ab\coth\left(dx+c\right)+b^2\left(-\frac{1}{\cosh\left(dx+c\right)\sinh\left(dx+c\right)}-2\,\tanh\left(dx+c\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(d*x+c-coth(d*x+c))-2*a*b*coth(d*x+c)+b^2*(-1/sinh(d*x+c)/cosh(d*x
+c)-2*tanh(d*x+c)))
```

Maxima [A] time = 1.20152, size = 96, normalized size = 2.67

$$a^2\left(x+\frac{c}{d}+\frac{2}{d\left(e^{(-2\,dx-2\,c)}-1\right)}\right)+\frac{4\,ab}{d\left(e^{(-2\,dx-2\,c)}-1\right)}+\frac{4\,b^2}{d\left(e^{(-4\,dx-4\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + 4*a*b/(d*(e^(-2*d*x - 2*c) -
 1)) + 4*b^2/(d*(e^(-4*d*x - 4*c) - 1))
```

Fricas [B] time = 2.11021, size = 265, normalized size = 7.36

$$-\frac{\left(a^2+2\,ab+2\,b^2\right)\cosh\left(dx+c\right)^2-2\left(a^2dx+a^2+2\,ab+2\,b^2\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\left(a^2+2\,ab+2\,b^2\right)\sinh\left(dx+c\right)^2+a^2+2\,ab}{2\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((a^2 + 2*a*b + 2*b^2)*cosh(d*x + c)^2 - 2*(a^2*d*x + a^2 + 2*a*b + 2*
b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + 2*a*b + 2*b^2)*sinh(d*x + c)^2 +
a^2 + 2*a*b)/(d*cosh(d*x + c)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26052, size = 88, normalized size = 2.44

$$\frac{a^2dx - \frac{2\left(a^2e^{(2\,dx+2\,c)}+2\,abe^{(2\,dx+2\,c)}+a^2+2\,ab+2\,b^2\right)}{e^{(4\,dx+4\,c)}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] (a^2*d*x - 2*(a^2*e^(2*d*x + 2*c) + 2*a*b*e^(2*d*x + 2*c) + a^2 + 2*a*b + 2
*b^2)/(e^(4*d*x + 4*c) - 1))/d
```

3.119 $\int \coth^3(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=55

$$\frac{(a^2-b^2)\log(\sinh(c+dx))}{d}-\frac{(a+b)^2\mathrm{csch}^2(c+dx)}{2d}+\frac{b^2\log(\cosh(c+dx))}{d}$$

```
[Out] -((a + b)^2*Csch[c + d*x]^2)/(2*d) + (b^2*Log[Cosh[c + d*x]])/d + ((a^2 - b
^2)*Log[Sinh[c + d*x]])/d
```

Rubi [A] time = 0.089335, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4138, 446, 88}

$$\frac{(a^2-b^2)\log(\sinh(c+dx))}{d}-\frac{(a+b)^2\mathrm{csch}^2(c+dx)}{2d}+\frac{b^2\log(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]
```


```
[Out] -((a + b)^2*Csch[c + d*x]^2)/(2*d) + (b^2*Log[Cosh[c + d*x]])/d + ((a^2 - b
^2)*Log[Sinh[c + d*x]])/d
```


Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \coth^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(b+ax^2\right)^2}{x\left(1-x^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(b+ax)^2}{(1-x)^2x}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{(a+b)^2}{(-1+x)^2}+\frac{a^2-b^2}{-1+x}+\frac{b^2}{x}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{(a+b)^2\text{csch}^2(c+dx)}{2d}+\frac{b^2\log(\cosh(c+dx))}{d}+\frac{\left(a^2-b^2\right)\log(\sinh(c+dx))}{d}$$

Mathematica [A] time = 0.208623, size = 82, normalized size = 1.49

$$-\frac{2\left(a\cosh^2(c+dx)+b\right)^2\left((a+b)^2\text{csch}^2(c+dx)-2\left(\left(a^2-b^2\right)\log(\sinh(c+dx))+b^2\log(\cosh(c+dx))\right)\right)}{d(a\cosh(2(c+dx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (-2*(b + a*Cosh[c + d*x]^2)^2*((a + b)^2*Csch[c + d*x]^2 - 2*(b^2*Log[Cosh[
c + d*x]] + (a^2 - b^2)*Log[Sinh[c + d*x]])))/(d*(a + 2*b + a*Cosh[2*(c + d
*x)])^2)
```

Maple [A] time = 0.044, size = 86, normalized size = 1.6

$$\frac{a^2\ln(\sinh(dx+c))}{d}-\frac{a^2(\coth(dx+c))^2}{2d}-\frac{ab(\cosh(dx+c))^2}{d(\sinh(dx+c))^2}-\frac{b^2}{2d(\sinh(dx+c))^2}-\frac{b^2\ln(\tanh(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d*a^2*ln(sinh(d*x+c))-1/2*a^2*coth(d*x+c)^2/d-1/d*a*b*cosh(d*x+c)^2/sinh(
d*x+c)^2-1/2/d*b^2/sinh(d*x+c)^2-1/d*b^2*ln(tanh(d*x+c))
```

Maxima [B] time = 1.79006, size = 278, normalized size = 5.05

$$a^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1\right)}\right)-b^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*
d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - b^2*(log(e^(-
d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x - 2*c) + 1)/d - 2
*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 4*a*b/
```

```
(d*(e^(d*x + c) - e^(-d*x - c))^2)
```

Fricas [B] time = 2.27925, size = 1553, normalized size = 28.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -(a^2*d*x*cosh(d*x + c)^4 + 4*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*d
*x*sinh(d*x + c)^4 + a^2*d*x - 2*(a^2*d*x - a^2 - 2*a*b - b^2)*cosh(d*x + c
)^2 + 2*(3*a^2*d*x*cosh(d*x + c)^2 - a^2*d*x + a^2 + 2*a*b + b^2)*sinh(d*x
+ c)^2 - (b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*s
inh(d*x + c)^4 - 2*b^2*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 - b^2)*si
nh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 - b^2*cosh(d*x + c))*sinh(d*x
+ c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - ((a^2 - b^2)*c
osh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 - b^2)*
sinh(d*x + c)^4 - 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 - b^2)*cosh(d*x
 + c)^2 - a^2 + b^2)*sinh(d*x + c)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(d*x
+ c)^3 - (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cos
h(d*x + c) - sinh(d*x + c))) + 4*(a^2*d*x*cosh(d*x + c)^3 - (a^2*d*x - a^2
- 2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(
d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d
*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x +
 c))*sinh(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31819, size = 217, normalized size = 3.95

$$-\frac{2\,a^2dx - 2\,b^2\log\left(e^{(2\,dx+2\,c)}+1\right) - 2\left(a^2e^{(2\,c)} - b^2e^{(2\,c)}\right)e^{(-2\,c)}\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right) + \frac{3\,a^2e^{(4\,dx+4\,c)} - 3\,b^2e^{(4\,dx+4\,c)} - 2\,a^2e^{(2\,dx+2\,c)} + 8\,a}{\left(e^{(2\,dx+2\,c)} - 1\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*a^2*d*x - 2*b^2*log(e^(2*d*x + 2*c) + 1) - 2*(a^2*e^(2*c) - b^2*e^(
2*c))*e^(-2*c)*log(abs(e^(2*d*x + 2*c) - 1)) + (3*a^2*e^(4*d*x + 4*c) - 3*b
^2*e^(4*d*x + 4*c) - 2*a^2*e^(2*d*x + 2*c) + 8*a*b*e^(2*d*x + 2*c) + 10*b^2
*e^(2*d*x + 2*c) + 3*a^2 - 3*b^2)/(e^(2*d*x + 2*c) - 1)^2)/d
```

3.120 $\int \coth^4(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=46

$$-\frac{\left(a^2-b^2\right)\coth(c+dx)}{d}+a^2x-\frac{(a+b)^2\coth^3(c+dx)}{3d}$$

```
[Out] a^2*x - ((a^2 - b^2)*Coth[c + d*x])/d - ((a + b)^2*Coth[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0917147, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4141, 1802, 207}

$$-\frac{\left(a^2-b^2\right)\coth(c+dx)}{d}+a^2x-\frac{(a+b)^2\coth^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] a^2*x - ((a^2 - b^2)*Coth[c + d*x])/d - ((a + b)^2*Coth[c + d*x]^3)/(3*d)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \coth^4(c+dx)\left(a+b\,\text{sech}^2(c+dx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\left(1-x^2\right)\right)^2}{x^4\left(1-x^2\right)}\,dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{(a+b)^2}{x^4}+\frac{a^2-b^2}{x^2}-\frac{a^2}{-1+x^2}\right)dx,x,\tanh(c+dx)\right)}{d}\\
&= -\frac{\left(a^2-b^2\right)\coth(c+dx)}{d}-\frac{(a+b)^2\coth^3(c+dx)}{3d}-\frac{a^2\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x\right.}{d}\\
&= a^2x-\frac{\left(a^2-b^2\right)\coth(c+dx)}{d}-\frac{(a+b)^2\coth^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [B] time = 0.818269, size = 160, normalized size = 3.48

$$\frac{\operatorname{csch}(c)\operatorname{csch}^3(c+dx)\left(-12a^2\sinh(2c+dx)+8a^2\sinh(2c+3dx)-9a^2dx\cosh(2c+dx)-3a^2dx\cosh(2c+3dx)+3a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (Csch[c]*Csch[c + d*x]^3*(9*a^2*d*x*Cosh[d*x] - 9*a^2*d*x*Cosh[2*c + d*x] -
 3*a^2*d*x*Cosh[2*c + 3*d*x] + 3*a^2*d*x*Cosh[4*c + 3*d*x] - 12*a^2*Sinh[d*
x] + 12*b^2*Sinh[d*x] - 12*a^2*Sinh[2*c + d*x] - 12*a*b*Sinh[2*c + d*x] + 8
*a^2*Sinh[2*c + 3*d*x] + 4*a*b*Sinh[2*c + 3*d*x] - 4*b^2*Sinh[2*c + 3*d*x])
)/(24*d)
```

Maple [B] time = 0.04, size = 96, normalized size = 2.1

$$\frac{1}{d}\left(a^2\left(dx+c-\coth(dx+c)-\frac{(\coth(dx+c))^3}{3}\right)+2\,ab\left(-1/2\,\frac{\cosh(dx+c)}{(\sinh(dx+c))^3}-1/2\,\left(2/3-1/3\,(\operatorname{csch}(dx+c))^2\right)\operatorname{coth}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(d*x+c-coth(d*x+c)-1/3*coth(d*x+c)^3)+2*a*b*(-1/2/sinh(d*x+c)^3*co
sh(d*x+c)-1/2*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c))+b^2*(2/3-1/3*csch(d*x+c)
^2)*coth(d*x+c))
```

Maxima [B] time = 1.16095, size = 362, normalized size = 7.87

$$\frac{1}{3}\,a^2\left(3\,x+\frac{3\,c}{d}-\frac{4\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}-2\right)}{d\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1\right)}\right)+\frac{4}{3}\,b^2\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1\right)}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d*(
3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + 4/3*b^2
*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*
x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x
- 6*c) - 1))) + 4/3*a*b*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-
4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) + 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*
d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

Fricas [B] time = 2.06739, size = 478, normalized size = 10.39

$$-\frac{2\left(2\,a^2+ab-b^2\right)\cosh(dx+c)^3+6\left(2\,a^2+ab-b^2\right)\cosh(dx+c)\sinh(dx+c)^2-\left(3\,a^2dx+4\,a^2+2\,ab-2\,b^2\right)\sinh}{3\left(d\sinh(dx+c)^3+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*(2*a^2 + a*b - b^2)*cosh(d*x + c)^3 + 6*(2*a^2 + a*b - b^2)*cosh(d*
x + c)*sinh(d*x + c)^2 - (3*a^2*d*x + 4*a^2 + 2*a*b - 2*b^2)*sinh(d*x + c)^
3 + 6*(a*b + b^2)*cosh(d*x + c) + 3*(3*a^2*d*x - (3*a^2*d*x + 4*a^2 + 2*a*b
 - 2*b^2)*cosh(d*x + c)^2 + 4*a^2 + 2*a*b - 2*b^2)*sinh(d*x + c))/(d*sinh(d
*x + c)^3 + 3*(d*cosh(d*x + c)^2 - d)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**4*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32449, size = 131, normalized size = 2.85

$$\frac{3\,a^2dx - \frac{4\left(3\,a^2e^{(4\,dx+4\,c)}+3\,abe^{(4\,dx+4\,c)}-3\,a^2e^{(2\,dx+2\,c)}+3\,b^2e^{(2\,dx+2\,c)}+2\,a^2+ab-b^2\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*d*x - 4*(3*a^2*e^(4*d*x + 4*c) + 3*a*b*e^(4*d*x + 4*c) - 3*a^2*e
^(2*d*x + 2*c) + 3*b^2*e^(2*d*x + 2*c) + 2*a^2 + a*b - b^2)/(e^(2*d*x + 2*c
) - 1)^3)/d
```

3.121 $\int \coth^5(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=52

$$\frac{a^2\log(\sinh(c+dx))}{d}-\frac{(a+b)^2\operatorname{csch}^4(c+dx)}{4d}-\frac{a(a+b)\operatorname{csch}^2(c+dx)}{d}$$

```
[Out] -((a*(a + b)*Csch[c + d*x]^2)/d) - ((a + b)^2*Csch[c + d*x]^4)/(4*d) + (a^2
*Log[Sinh[c + d*x]])/d
```

Rubi [A] time = 0.0939137, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4138, 444, 43}

$$\frac{a^2\log(\sinh(c+dx))}{d}-\frac{(a+b)^2\operatorname{csch}^4(c+dx)}{4d}-\frac{a(a+b)\operatorname{csch}^2(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^5*(a + b*Sech[c + d*x]^2)^2,x]
```


```
[Out] -((a*(a + b)*Csch[c + d*x]^2)/d) - ((a + b)^2*Csch[c + d*x]^4)/(4*d) + (a^2
*Log[Sinh[c + d*x]])/d
```


Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \coth^5(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2 dx = -\frac{\text{Subst}\left(\int \frac{x(b+ax^2)^2}{(1-x^2)^3} dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{(b+ax)^2}{(1-x)^3} dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{\text{Subst}\left(\int \left(-\frac{(a+b)^2}{(-1+x)^3} - \frac{2a(a+b)}{(-1+x)^2} - \frac{a^2}{-1+x}\right) dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{a(a+b)\text{csch}^2(c+dx)}{d} - \frac{(a+b)^2\text{csch}^4(c+dx)}{4d} + \frac{a^2\log(\sinh(c+dx))}{d}$$

Mathematica [A] time = 0.232371, size = 77, normalized size = 1.48

$$-\frac{\left(a\cosh^2(c+dx)+b\right)^2\left(-4a^2\log(\sinh(c+dx))+(a+b)^2\text{csch}^4(c+dx)+4a(a+b)\text{csch}^2(c+dx)\right)}{d(a\cosh(2(c+dx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^5*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -(((b + a*Cosh[c + d*x]^2)^2*(4*a*(a + b)*Csch[c + d*x]^2 + (a + b)^2*Csch[
c + d*x]^4 - 4*a^2*Log[Sinh[c + d*x]]))/(d*(a + 2*b + a*Cosh[2*(c + d*x)])^
2))
```

Maple [B] time = 0.046, size = 142, normalized size = 2.7

$$\frac{a^2\ln(\sinh(dx+c))}{d} - \frac{a^2(\coth(dx+c))^2}{2d} - \frac{a^2(\coth(dx+c))^4}{4d} - \frac{ab(\cosh(dx+c))^2}{2d(\sinh(dx+c))^4} - \frac{ab(\cosh(dx+c))^2}{2d(\sinh(dx+c))^2} - \frac{b^2(\cosh(}{4d(\sinh(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d*a^2*ln(sinh(d*x+c))-1/2*a^2*coth(d*x+c)^2/d-1/4*a^2*coth(d*x+c)^4/d-1/2
/d*a*b/sinh(d*x+c)^4*cosh(d*x+c)^2-1/2/d*a*b*cosh(d*x+c)^2/sinh(d*x+c)^2-1/
4/d*b^2/sinh(d*x+c)^4*cosh(d*x+c)^2+1/4/d*b^2*cosh(d*x+c)^2/sinh(d*x+c)^2
```

Maxima [B] time = 1.15157, size = 381, normalized size = 7.33

$$a^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{4\left(e^{(-2dx-2c)}-e^{(-4dx-4c)}+e^{(-6dx-6c)}\right)}{d\left(4e^{(-2dx-2c)}-6e^{(-4dx-4c)}+4e^{(-6dx-6c)}-e^{(-8dx-8c)}-1\right)}\right)+4ab\left(-\frac{}{d(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-2
*d*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) -
 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + 4*a*b*
```

```
(e^(-2*d*x - 2*c)/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x
 - 6*c) - e^(-8*d*x - 8*c) - 1)) + e^(-6*d*x - 6*c)/(d*(4*e^(-2*d*x - 2*c)
- 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) - 4*b^2
/(d*(e^(d*x + c) - e^(-d*x - c))^4)
```

Fricas [B] time = 2.23174, size = 3093, normalized size = 59.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -(a^2*d*x*cosh(d*x + c)^8 + 8*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*d
*x*sinh(d*x + c)^8 - 4*(a^2*d*x - a^2 - a*b)*cosh(d*x + c)^6 + 4*(7*a^2*d*x
*cosh(d*x + c)^2 - a^2*d*x + a^2 + a*b)*sinh(d*x + c)^6 + 8*(7*a^2*d*x*cosh
(d*x + c)^3 - 3*(a^2*d*x - a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3
*a^2*d*x - 2*a^2 + 2*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*d*x*cosh(d*x + c)^4 +
 3*a^2*d*x - 30*(a^2*d*x - a^2 - a*b)*cosh(d*x + c)^2 - 2*a^2 + 2*b^2)*sinh
(d*x + c)^4 + a^2*d*x + 8*(7*a^2*d*x*cosh(d*x + c)^5 - 10*(a^2*d*x - a^2 -
a*b)*cosh(d*x + c)^3 + (3*a^2*d*x - 2*a^2 + 2*b^2)*cosh(d*x + c))*sinh(d*x
+ c)^3 - 4*(a^2*d*x - a^2 - a*b)*cosh(d*x + c)^2 + 4*(7*a^2*d*x*cosh(d*x +
c)^6 - 15*(a^2*d*x - a^2 - a*b)*cosh(d*x + c)^4 - a^2*d*x + 3*(3*a^2*d*x -
2*a^2 + 2*b^2)*cosh(d*x + c)^2 + a^2 + a*b)*sinh(d*x + c)^2 - (a^2*cosh(d*x
 + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 - 4*a^2
*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 - a^2)*sinh(d*x + c)^6 + 6*a^2*
cosh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^3 - 3*a^2*cosh(d*x + c))*sinh(d*x
+ c)^5 + 2*(35*a^2*cosh(d*x + c)^4 - 30*a^2*cosh(d*x + c)^2 + 3*a^2)*sinh(d
*x + c)^4 - 4*a^2*cosh(d*x + c)^2 + 8*(7*a^2*cosh(d*x + c)^5 - 10*a^2*cosh(
d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a^2*cosh(d*x + c)^
6 - 15*a^2*cosh(d*x + c)^4 + 9*a^2*cosh(d*x + c)^2 - a^2)*sinh(d*x + c)^2 +
 a^2 + 8*(a^2*cosh(d*x + c)^7 - 3*a^2*cosh(d*x + c)^5 + 3*a^2*cosh(d*x + c)
^3 - a^2*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) -
 sinh(d*x + c))) + 8*(a^2*d*x*cosh(d*x + c)^7 - 3*(a^2*d*x - a^2 - a*b)*cos
h(d*x + c)^5 + (3*a^2*d*x - 2*a^2 + 2*b^2)*cosh(d*x + c)^3 - (a^2*d*x - a^2
 - a*b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c
)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d
*x + c)^2 - d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c)
)*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 - 30*d*co
sh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - 10*d*cosh(d
*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 - 4*d*cosh(d*x + c)^2 + 4*(7
*d*cosh(d*x + c)^6 - 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 - d)*sinh(d
*x + c)^2 + 8*(d*cosh(d*x + c)^7 - 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^
3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**5*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.43632, size = 198, normalized size = 3.81

$$-\frac{12\,a^2dx-12\,a^2\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+\frac{25\,a^2e^{(8\,dx+8\,c)}-52\,a^2e^{(6\,dx+6\,c)}+48\,abe^{(6\,dx+6\,c)}+102\,a^2e^{(4\,dx+4\,c)}+48\,b^2e^{(4\,dx+4\,c)}-52\,a^2e^{(2\,dx+2\,c)}+48\,abe^{(2\,dx+2}}{\left(e^{(2\,dx+2\,c)}-1\right)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/12*(12*a^2*d*x - 12*a^2*log(abs(e^(2*d*x + 2*c) - 1)) + (25*a^2*e^(8*d*x
 + 8*c) - 52*a^2*e^(6*d*x + 6*c) + 48*a*b*e^(6*d*x + 6*c) + 102*a^2*e^(4*d*
x + 4*c) + 48*b^2*e^(4*d*x + 4*c) - 52*a^2*e^(2*d*x + 2*c) + 48*a*b*e^(2*d*
x + 2*c) + 25*a^2)/(e^(2*d*x + 2*c) - 1)^4)/d
```

3.122 $\int \coth^6(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=64

$$-\frac{(a^2-b^2)\coth^3(c+dx)}{3d}-\frac{a^2\coth(c+dx)}{d}+a^2x-\frac{(a+b)^2\coth^5(c+dx)}{5d}$$

```
[Out] a^2*x - (a^2*Coth[c + d*x])/d - ((a^2 - b^2)*Coth[c + d*x]^3)/(3*d) - ((a +
 b)^2*Coth[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0970039, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4141, 1802, 207}

$$-\frac{(a^2-b^2)\coth^3(c+dx)}{3d}-\frac{a^2\coth(c+dx)}{d}+a^2x-\frac{(a+b)^2\coth^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^6*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] a^2*x - (a^2*Coth[c + d*x])/d - ((a^2 - b^2)*Coth[c + d*x]^3)/(3*d) - ((a +
 b)^2*Coth[c + d*x]^5)/(5*d)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \coth^6(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\left(1-x^2\right)\right)^2}{x^6\left(1-x^2\right)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{(a+b)^2}{x^6}+\frac{a^2-b^2}{x^4}+\frac{a^2}{x^2}-\frac{a^2}{-1+x^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{a^2\coth(c+dx)}{d}-\frac{\left(a^2-b^2\right)\coth^3(c+dx)}{3d}-\frac{(a+b)^2\coth^5(c+dx)}{5d}-\frac{a^2 \text{S}}{}$$
$$= a^2x-\frac{a^2\coth(c+dx)}{d}-\frac{\left(a^2-b^2\right)\coth^3(c+dx)}{3d}-\frac{(a+b)^2\coth^5(c+dx)}{5d}$$

Mathematica [B] time = 1.1011, size = 256, normalized size = 4.

$$\frac{\text{csch}(c)\text{csch}^5(c+dx)\left(180a^2\sinh(2c+dx)-140a^2\sinh(2c+3dx)-90a^2\sinh(4c+3dx)+46a^2\sinh(4c+5dx)+150a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^6*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (Csch[c]*Csch[c + d*x]^5*(-150*a^2*d*x*Cosh[d*x] + 150*a^2*d*x*Cosh[2*c + d
*x] + 75*a^2*d*x*Cosh[2*c + 3*d*x] - 75*a^2*d*x*Cosh[4*c + 3*d*x] - 15*a^2*
d*x*Cosh[4*c + 5*d*x] + 15*a^2*d*x*Cosh[6*c + 5*d*x] + 280*a^2*Sinh[d*x] +
120*a*b*Sinh[d*x] + 20*b^2*Sinh[d*x] + 180*a^2*Sinh[2*c + d*x] - 60*b^2*Sin
h[2*c + d*x] - 140*a^2*Sinh[2*c + 3*d*x] + 20*b^2*Sinh[2*c + 3*d*x] - 90*a^
2*Sinh[4*c + 3*d*x] - 60*a*b*Sinh[4*c + 3*d*x] + 46*a^2*Sinh[4*c + 5*d*x] +
 12*a*b*Sinh[4*c + 5*d*x] - 4*b^2*Sinh[4*c + 5*d*x]))/(480*d)
```

Maple [B] time = 0.049, size = 163, normalized size = 2.6

$$\frac{1}{d}\left(a^2\left(dx+c-\coth(dx+c)-\frac{(\coth(dx+c))^3}{3}-\frac{(\coth(dx+c))^5}{5}\right)+2\,ab\left(-1/2\,\frac{(\cosh(dx+c))^3}{(\sinh(dx+c))^5}+3/8\,\frac{\cosh(dx+c)}{(\sinh(dx+c))^5}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(d*x+c-coth(d*x+c)-1/3*coth(d*x+c)^3-1/5*coth(d*x+c)^5)+2*a*b*(-1/
2/sinh(d*x+c)^5*cosh(d*x+c)^3+3/8/sinh(d*x+c)^5*cosh(d*x+c)+3/8*(-8/15-1/5*
csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c))+b^2*(-1/4/sinh(d*x+c)^5*cosh
(d*x+c)-1/4*(-8/15-1/5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c)))
```

Maxima [B] time = 1.19317, size = 828, normalized size = 12.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*a^2*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) - 140*e^(-4*d*x - 4*c) + 9
0*e^(-6*d*x - 6*c) - 45*e^(-8*d*x - 8*c) - 23)/(d*(5*e^(-2*d*x - 2*c) - 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) - 1))) + 4/15*b^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(
-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*
c) - 1)) + 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c)
+ 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) + 15*
e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*
x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 1/(d*(5*e^(-2*d*
x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) +
 e^(-10*d*x - 10*c) - 1))) + 4/5*a*b*(10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x -
 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^
(-10*d*x - 10*c) - 1)) + 5*e^(-8*d*x - 8*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(
-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*
c) - 1)) + 1/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)))
```

Fricas [B] time = 2.16501, size = 1050, normalized size = 16.41

$$-\frac{\left(23\,a^2+6\,ab-2\,b^2\right)\cosh\left(dx+c\right)^5+5\left(23\,a^2+6\,ab-2\,b^2\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4-\left(15\,a^2dx+23\,a^2+6\,ab-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/15*((23*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^5 + 5*(23*a^2 + 6*a*b - 2*b^2
)*cosh(d*x + c)*sinh(d*x + c)^4 - (15*a^2*d*x + 23*a^2 + 6*a*b - 2*b^2)*sin
h(d*x + c)^5 - 5*(5*a^2 - 6*a*b - 2*b^2)*cosh(d*x + c)^3 + 5*(15*a^2*d*x -
2*(15*a^2*d*x + 23*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^2 + 23*a^2 + 6*a*b -
2*b^2)*sinh(d*x + c)^3 + 5*(2*(23*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^3 - 3*
(5*a^2 - 6*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(5*a^2 + 6*a*b
+ 4*b^2)*cosh(d*x + c) - 5*((15*a^2*d*x + 23*a^2 + 6*a*b - 2*b^2)*cosh(d*x
+ c)^4 + 30*a^2*d*x - 3*(15*a^2*d*x + 23*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)
^2 + 46*a^2 + 12*a*b - 4*b^2)*sinh(d*x + c))/(d*sinh(d*x + c)^5 + 5*(2*d*co
sh(d*x + c)^2 - d)*sinh(d*x + c)^3 + 5*(d*cosh(d*x + c)^4 - 3*d*cosh(d*x +
c)^2 + 2*d)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**6*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.45845, size = 225, normalized size = 3.52

$$\frac{15\,a^2dx-\frac{2\left(45\,a^2e^{(8\,dx+8\,c)}+30\,abe^{(8\,dx+8\,c)}-90\,a^2e^{(6\,dx+6\,c)}+30\,b^2e^{(6\,dx+6\,c)}+140\,a^2e^{(4\,dx+4\,c)}+60\,abe^{(4\,dx+4\,c)}+10\,b^2e^{(4\,dx+4\,c)}-70\,a^2e^{(2\,dx+2\,c)}+10\,b^2e^{(2\,dx+2}}{\left(e^{(2\,dx+2\,c)}-1\right)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/15*(15*a^2*d*x - 2*(45*a^2*e^(8*d*x + 8*c) + 30*a*b*e^(8*d*x + 8*c) - 90*
a^2*e^(6*d*x + 6*c) + 30*b^2*e^(6*d*x + 6*c) + 140*a^2*e^(4*d*x + 4*c) + 60
*a*b*e^(4*d*x + 4*c) + 10*b^2*e^(4*d*x + 4*c) - 70*a^2*e^(2*d*x + 2*c) + 10
*b^2*e^(2*d*x + 2*c) + 23*a^2 + 6*a*b - 2*b^2)/(e^(2*d*x + 2*c) - 1)^5)/d
```

3.123 $\int \coth^7(c+dx)\left(a+b\mathbf{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=86

$$\frac{a^2\log(\sinh(c+dx))}{d}-\frac{(a+b)^2\text{csch}^4(c+dx)}{4d}-\frac{a(a+b)\text{csch}^2(c+dx)}{d}-\frac{\text{csch}^6(c+dx)\left(a\cosh^2(c+dx)+b\right)^3}{6d(a+b)}$$

```
[Out] -((a*(a + b)*Csch[c + d*x]^2)/d) - ((a + b)^2*Csch[c + d*x]^4)/(4*d) - ((b
+ a*Cosh[c + d*x]^2)^3*Csch[c + d*x]^6)/(6*(a + b)*d) + (a^2*Log[Sinh[c + d
*x]])/d
```

Rubi [A] time = 0.124845, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4138, 446, 78, 43}

$$\frac{a^2\log(\sinh(c+dx))}{d}-\frac{(a+b)^2\text{csch}^4(c+dx)}{4d}-\frac{a(a+b)\text{csch}^2(c+dx)}{d}-\frac{\text{csch}^6(c+dx)\left(a\cosh^2(c+dx)+b\right)^3}{6d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^7*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -((a*(a + b)*Csch[c + d*x]^2)/d) - ((a + b)^2*Csch[c + d*x]^4)/(4*d) - ((b
+ a*Cosh[c + d*x]^2)^3*Csch[c + d*x]^6)/(6*(a + b)*d) + (a^2*Log[Sinh[c + d
*x]])/d
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \coth^7(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{x^3\left(b+ax^2\right)^2}{\left(1-x^2\right)^4}dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x(b+ax)^2}{(1-x)^4}dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{\left(b+a\cosh^2(c+dx)\right)^3\text{csch}^6(c+dx)}{6(a+b)d} - \frac{\text{Subst}\left(\int \frac{(b+ax)^2}{(1-x)^3}dx, x, \cosh^2(c+\right.}{2d}$$
$$= -\frac{\left(b+a\cosh^2(c+dx)\right)^3\text{csch}^6(c+dx)}{6(a+b)d} - \frac{\text{Subst}\left(\int\left(-\frac{(a+b)^2}{(-1+x)^3} - \frac{2a(a+b)}{(-1+x)^2} - \frac{a}{-1}\right.\right.}{2d}$$
$$= -\frac{a(a+b)\text{csch}^2(c+dx)}{d} - \frac{(a+b)^2\text{csch}^4(c+dx)}{4d} - \frac{\left(b+a\cosh^2(c+dx)\right)^3 \text{c}}{6(a+b)d}$$

Mathematica [A] time = 0.50803, size = 107, normalized size = 1.24

$$-\frac{\left(a\cosh^2(c+dx)+b\right)^2\left(3\left(3a^2+4ab+b^2\right)\text{csch}^4(c+dx)-12a^2\log(\sinh(c+dx))+2(a+b)^2\text{csch}^6(c+dx)+6a(3a+2\right.}{3d(a\cosh(2(c+dx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^7*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -((b + a*Cosh[c + d*x]^2)^2*(6*a*(3*a + 2*b)*Csch[c + d*x]^2 + 3*(3*a^2 + 4
*a*b + b^2)*Csch[c + d*x]^4 + 2*(a + b)^2*Csch[c + d*x]^6 - 12*a^2*Log[Sinh
[c + d*x]]))/(3*d*(a + 2*b + a*Cosh[2*(c + d*x)])^2)
```

Maple [B] time = 0.053, size = 228, normalized size = 2.7

$$\frac{a^2\ln(\sinh(dx+c))}{d} - \frac{a^2(\coth(dx+c))^2}{2d} - \frac{a^2(\coth(dx+c))^4}{4d} - \frac{a^2(\coth(dx+c))^6}{6d} - \frac{ab(\cosh(dx+c))^4}{d(\sinh(dx+c))^6} + \frac{2ab(\cosh}{3d(\sinh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d*a^2*ln(sinh(d*x+c))-1/2*a^2*coth(d*x+c)^2/d-1/4*a^2*coth(d*x+c)^4/d-1/6
/d*a^2*coth(d*x+c)^6-1/d*a*b/sinh(d*x+c)^6*cosh(d*x+c)^4+2/3/d*a*b/sinh(d*x
+c)^6*cosh(d*x+c)^2+1/3/d*a*b/sinh(d*x+c)^4*cosh(d*x+c)^2-1/3/d*a*b*cosh(d*
x+c)^2/sinh(d*x+c)^2-1/6/d*b^2/sinh(d*x+c)^6*cosh(d*x+c)^2-1/12/d*b^2/sinh(
d*x+c)^4*cosh(d*x+c)^2+1/12/d*b^2*cosh(d*x+c)^2/sinh(d*x+c)^2
```

Maxima [B] time = 1.21822, size = 940, normalized size = 10.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*(3*x + 3*c/d + 3*log(e^(-d*x - c) + 1)/d + 3*log(e^(-d*x - c) - 1)/
d + 2*(9*e^(-2*d*x - 2*c) - 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) - 18*
e^(-8*d*x - 8*c) + 9*e^(-10*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*
d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*
c) - e^(-12*d*x - 12*c) - 1))) + 4/3*a*b*(3*e^(-2*d*x - 2*c)/(d*(6*e^(-2*d*
x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c)
+ 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1)) + 10*e^(-6*d*x - 6*c)/(d*
(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*
d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1)) + 3*e^(-10*d*x
 - 10*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c)
 - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1))) +
 4/3*b^2*(3*e^(-4*d*x - 4*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) +
 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*
d*x - 12*c) - 1)) + 2*e^(-6*d*x - 6*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*
x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c)
 - e^(-12*d*x - 12*c) - 1)) + 3*e^(-8*d*x - 8*c)/(d*(6*e^(-2*d*x - 2*c) - 1
5*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d
*x - 10*c) - e^(-12*d*x - 12*c) - 1)))
```

Fricas [B] time = 2.46665, size = 6593, normalized size = 76.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^2*d*x*cosh(d*x + c)^12 + 36*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^1
1 + 3*a^2*d*x*sinh(d*x + c)^12 - 6*(3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c
)^10 + 6*(33*a^2*d*x*cosh(d*x + c)^2 - 3*a^2*d*x + 3*a^2 + 2*a*b)*sinh(d*x
+ c)^10 + 60*(11*a^2*d*x*cosh(d*x + c)^3 - (3*a^2*d*x - 3*a^2 - 2*a*b)*cosh
(d*x + c))*sinh(d*x + c)^9 + 3*(15*a^2*d*x - 12*a^2 + 4*b^2)*cosh(d*x + c)^
8 + 3*(495*a^2*d*x*cosh(d*x + c)^4 + 15*a^2*d*x - 90*(3*a^2*d*x - 3*a^2 - 2
*a*b)*cosh(d*x + c)^2 - 12*a^2 + 4*b^2)*sinh(d*x + c)^8 + 24*(99*a^2*d*x*co
sh(d*x + c)^5 - 30*(3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c)^3 + (15*a^2*d*
x - 12*a^2 + 4*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(15*a^2*d*x - 17*a^2
 - 10*a*b - 2*b^2)*cosh(d*x + c)^6 + 4*(693*a^2*d*x*cosh(d*x + c)^6 - 315*(
3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c)^4 - 15*a^2*d*x + 21*(15*a^2*d*x -
12*a^2 + 4*b^2)*cosh(d*x + c)^2 + 17*a^2 + 10*a*b + 2*b^2)*sinh(d*x + c)^6
+ 24*(99*a^2*d*x*cosh(d*x + c)^7 - 63*(3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x
+ c)^5 + 7*(15*a^2*d*x - 12*a^2 + 4*b^2)*cosh(d*x + c)^3 - (15*a^2*d*x - 17
*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 3*(15*a^2*d*x - 12*
a^2 + 4*b^2)*cosh(d*x + c)^4 + 3*(495*a^2*d*x*cosh(d*x + c)^8 - 420*(3*a^2*
d*x - 3*a^2 - 2*a*b)*cosh(d*x + c)^6 + 70*(15*a^2*d*x - 12*a^2 + 4*b^2)*cos
h(d*x + c)^4 + 15*a^2*d*x - 20*(15*a^2*d*x - 17*a^2 - 10*a*b - 2*b^2)*cosh(
d*x + c)^2 - 12*a^2 + 4*b^2)*sinh(d*x + c)^4 + 3*a^2*d*x + 4*(165*a^2*d*x*c
osh(d*x + c)^9 - 180*(3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c)^7 + 42*(15*a
^2*d*x - 12*a^2 + 4*b^2)*cosh(d*x + c)^5 - 20*(15*a^2*d*x - 17*a^2 - 10*a*b
 - 2*b^2)*cosh(d*x + c)^3 + 3*(15*a^2*d*x - 12*a^2 + 4*b^2)*cosh(d*x + c))*
sinh(d*x + c)^3 - 6*(3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c)^2 + 6*(33*a^2
*d*x*cosh(d*x + c)^10 - 45*(3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c)^8 + 14
*(15*a^2*d*x - 12*a^2 + 4*b^2)*cosh(d*x + c)^6 - 10*(15*a^2*d*x - 17*a^2 -
10*a*b - 2*b^2)*cosh(d*x + c)^4 - 3*a^2*d*x + 3*(15*a^2*d*x - 12*a^2 + 4*b^
```

```
2)*cosh(d*x + c)^2 + 3*a^2 + 2*a*b)*sinh(d*x + c)^2 - 3*(a^2*cosh(d*x + c)^
12 + 12*a^2*cosh(d*x + c)*sinh(d*x + c)^11 + a^2*sinh(d*x + c)^12 - 6*a^2*c
osh(d*x + c)^10 + 6*(11*a^2*cosh(d*x + c)^2 - a^2)*sinh(d*x + c)^10 + 15*a^
2*cosh(d*x + c)^8 + 20*(11*a^2*cosh(d*x + c)^3 - 3*a^2*cosh(d*x + c))*sinh(
d*x + c)^9 + 15*(33*a^2*cosh(d*x + c)^4 - 18*a^2*cosh(d*x + c)^2 + a^2)*sin
h(d*x + c)^8 - 20*a^2*cosh(d*x + c)^6 + 24*(33*a^2*cosh(d*x + c)^5 - 30*a^2
*cosh(d*x + c)^3 + 5*a^2*cosh(d*x + c))*sinh(d*x + c)^7 + 4*(231*a^2*cosh(d
*x + c)^6 - 315*a^2*cosh(d*x + c)^4 + 105*a^2*cosh(d*x + c)^2 - 5*a^2)*sinh
(d*x + c)^6 + 15*a^2*cosh(d*x + c)^4 + 24*(33*a^2*cosh(d*x + c)^7 - 63*a^2*
cosh(d*x + c)^5 + 35*a^2*cosh(d*x + c)^3 - 5*a^2*cosh(d*x + c))*sinh(d*x +
c)^5 + 15*(33*a^2*cosh(d*x + c)^8 - 84*a^2*cosh(d*x + c)^6 + 70*a^2*cosh(d*
x + c)^4 - 20*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 - 6*a^2*cosh(d*x +
 c)^2 + 20*(11*a^2*cosh(d*x + c)^9 - 36*a^2*cosh(d*x + c)^7 + 42*a^2*cosh(d
*x + c)^5 - 20*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^3 +
 6*(11*a^2*cosh(d*x + c)^10 - 45*a^2*cosh(d*x + c)^8 + 70*a^2*cosh(d*x + c)
^6 - 50*a^2*cosh(d*x + c)^4 + 15*a^2*cosh(d*x + c)^2 - a^2)*sinh(d*x + c)^2
 + a^2 + 12*(a^2*cosh(d*x + c)^11 - 5*a^2*cosh(d*x + c)^9 + 10*a^2*cosh(d*x
 + c)^7 - 10*a^2*cosh(d*x + c)^5 + 5*a^2*cosh(d*x + c)^3 - a^2*cosh(d*x + c
))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 12
*(3*a^2*d*x*cosh(d*x + c)^11 - 5*(3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c)^
9 + 2*(15*a^2*d*x - 12*a^2 + 4*b^2)*cosh(d*x + c)^7 - 2*(15*a^2*d*x - 17*a^
2 - 10*a*b - 2*b^2)*cosh(d*x + c)^5 + (15*a^2*d*x - 12*a^2 + 4*b^2)*cosh(d*
x + c)^3 - (3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c))*sinh(d*x + c))/(d*cos
h(d*x + c)^12 + 12*d*cosh(d*x + c)*sinh(d*x + c)^11 + d*sinh(d*x + c)^12 -
6*d*cosh(d*x + c)^10 + 6*(11*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^10 + 20*(
11*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^9 + 15*d*cosh(d*x +
 c)^8 + 15*(33*d*cosh(d*x + c)^4 - 18*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^
8 + 24*(33*d*cosh(d*x + c)^5 - 30*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*si
nh(d*x + c)^7 - 20*d*cosh(d*x + c)^6 + 4*(231*d*cosh(d*x + c)^6 - 315*d*cos
h(d*x + c)^4 + 105*d*cosh(d*x + c)^2 - 5*d)*sinh(d*x + c)^6 + 24*(33*d*cosh
(d*x + c)^7 - 63*d*cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 - 5*d*cosh(d*x +
c))*sinh(d*x + c)^5 + 15*d*cosh(d*x + c)^4 + 15*(33*d*cosh(d*x + c)^8 - 84*
d*cosh(d*x + c)^6 + 70*d*cosh(d*x + c)^4 - 20*d*cosh(d*x + c)^2 + d)*sinh(d
*x + c)^4 + 20*(11*d*cosh(d*x + c)^9 - 36*d*cosh(d*x + c)^7 + 42*d*cosh(d*x
 + c)^5 - 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 - 6*d*c
osh(d*x + c)^2 + 6*(11*d*cosh(d*x + c)^10 - 45*d*cosh(d*x + c)^8 + 70*d*cos
h(d*x + c)^6 - 50*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 - d)*sinh(d*x +
c)^2 + 12*(d*cosh(d*x + c)^11 - 5*d*cosh(d*x + c)^9 + 10*d*cosh(d*x + c)^7
- 10*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x +
c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**7*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.65332, size = 292, normalized size = 3.4

$$-\frac{60\,a^2dx - 60\,a^2\log\left(\left|e^{(2\,dx+2\,c)} - 1\right|\right) + \frac{147\,a^2e^{(12\,dx+12\,c)} - 522\,a^2e^{(10\,dx+10\,c)} + 240\,abe^{(10\,dx+10\,c)} + 1485\,a^2e^{(8\,dx+8\,c)} + 240\,b^2e^{(8\,dx+8\,c)} - 1580\,a^2e^{(6\,dx+6\,c}}{\left(e^{(2\,dx}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/60*(60*a^2*d*x - 60*a^2*log(abs(e^(2*d*x + 2*c) - 1)) + (147*a^2*e^(12*d
*x + 12*c) - 522*a^2*e^(10*d*x + 10*c) + 240*a*b*e^(10*d*x + 10*c) + 1485*a
^2*e^(8*d*x + 8*c) + 240*b^2*e^(8*d*x + 8*c) - 1580*a^2*e^(6*d*x + 6*c) + 8
00*a*b*e^(6*d*x + 6*c) + 160*b^2*e^(6*d*x + 6*c) + 1485*a^2*e^(4*d*x + 4*c)
 + 240*b^2*e^(4*d*x + 4*c) - 522*a^2*e^(2*d*x + 2*c) + 240*a*b*e^(2*d*x + 2
*c) + 147*a^2)/(e^(2*d*x + 2*c) - 1)^6)/d
```

3.124 $\int \left(a+b\,\mathrm{sech}^2(c+dx)\right)^3 \tanh^4(c+dx)\,dx$

Optimal. Leaf size=110

$$\frac{b\left(3a^2+3ab+b^2\right)\tanh^5(c+dx)}{5d}-\frac{a^3\tanh^3(c+dx)}{3d}-\frac{a^3\tanh(c+dx)}{d}+a^3x-\frac{b^2(3a+2b)\tanh^7(c+dx)}{7d}+\frac{b^3\tanh^9(c+dx)}{9d}$$

```
[Out] a^3*x - (a^3*Tanh[c + d*x])/d - (a^3*Tanh[c + d*x]^3)/(3*d) + (b*(3*a^2 + 3
*a*b + b^2)*Tanh[c + d*x]^5)/(5*d) - (b^2*(3*a + 2*b)*Tanh[c + d*x]^7)/(7*d
) + (b^3*Tanh[c + d*x]^9)/(9*d)
```

Rubi [A] time = 0.118163, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4141, 1802, 206}

$$\frac{b\left(3a^2+3ab+b^2\right)\tanh^5(c+dx)}{5d}-\frac{a^3\tanh^3(c+dx)}{3d}-\frac{a^3\tanh(c+dx)}{d}+a^3x-\frac{b^2(3a+2b)\tanh^7(c+dx)}{7d}+\frac{b^3\tanh^9(c+dx)}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x]^4,x]
```

```
[Out] a^3*x - (a^3*Tanh[c + d*x])/d - (a^3*Tanh[c + d*x]^3)/(3*d) + (b*(3*a^2 + 3
*a*b + b^2)*Tanh[c + d*x]^5)/(5*d) - (b^2*(3*a + 2*b)*Tanh[c + d*x]^7)/(7*d
) + (b^3*Tanh[c + d*x]^9)/(9*d)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^3\tanh^4(c+dx)\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^4\left(a+b\left(1-x^2\right)\right)^3}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(-a^3-a^3x^2+b\left(3a^2+3ab+b^2\right)x^4-b^2(3a+2b)x^6+b^3x^8+\right.\right.}{d}$$

$$= -\frac{a^3\tanh(c+dx)}{d}-\frac{a^3\tanh^3(c+dx)}{3d}+\frac{b\left(3a^2+3ab+b^2\right)\tanh^5(c+d}{5d}$$

$$= a^3x-\frac{a^3\tanh(c+dx)}{d}-\frac{a^3\tanh^3(c+dx)}{3d}+\frac{b\left(3a^2+3ab+b^2\right)\tanh^5(c}{5d}$$

Mathematica [B] time = 6.14437, size = 301, normalized size = 2.74

$$8\operatorname{sech}^9(c+dx)\left(a\cosh^2(c+dx)+b\right)^3\left(\left(-378a^2b+105a^3+27ab^2+4b^3\right)\tanh(c)\cosh^7(c+dx)+3b\left(63a^2-72ab+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x]^4,x]
```

```
[Out] (8*(b + a*Cosh[c + d*x]^2)^3*Sech[c + d*x]^9*(315*a^3*d*x*Cosh[c + d*x]^9 +
 35*b^3*Sech[c]*Sinh[d*x] + 5*(27*a - 10*b)*b^2*Cosh[c + d*x]^2*Sech[c]*Sin
h[d*x] + 3*b*(63*a^2 - 72*a*b + b^2)*Cosh[c + d*x]^4*Sech[c]*Sinh[d*x] + (1
05*a^3 - 378*a^2*b + 27*a*b^2 + 4*b^3)*Cosh[c + d*x]^6*Sech[c]*Sinh[d*x] -
(420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*Cosh[c + d*x]^8*Sech[c]*Sinh[d*x]
+ 35*b^3*Cosh[c + d*x]*Tanh[c] + 5*(27*a - 10*b)*b^2*Cosh[c + d*x]^3*Tanh[c
] + 3*b*(63*a^2 - 72*a*b + b^2)*Cosh[c + d*x]^5*Tanh[c] + (105*a^3 - 378*a^
2*b + 27*a*b^2 + 4*b^3)*Cosh[c + d*x]^7*Tanh[c]))/(315*d*(a + 2*b + a*Cosh[
2*(c + d*x)])^3)
```

Maple [B] time = 0.049, size = 274, normalized size = 2.5

$$\frac{1}{d}\left(a^3\left(dx+c-\tanh(dx+c)-\frac{(\tanh(dx+c))^3}{3}\right)+3\,a^2b\left(-1/2\,\frac{(\sinh(dx+c))^3}{(\cosh(dx+c))^5}-3/8\,\frac{\sinh(dx+c)}{(\cosh(dx+c))^5}+3/8\left(\frac{8}{15}+1\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^4,x)
```

```
[Out] 1/d*(a^3*(d*x+c-tanh(d*x+c)-1/3*tanh(d*x+c)^3)+3*a^2*b*(-1/2*sinh(d*x+c)^3/
cosh(d*x+c)^5-3/8*sinh(d*x+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/1
5*sech(d*x+c)^2)*tanh(d*x+c))+3*a*b^2*(-1/4*sinh(d*x+c)^3/cosh(d*x+c)^7-1/8
*sinh(d*x+c)/cosh(d*x+c)^7+1/8*(16/35+1/7*sech(d*x+c)^6+6/35*sech(d*x+c)^4+
8/35*sech(d*x+c)^2)*tanh(d*x+c))+b^3*(-1/6*sinh(d*x+c)^3/cosh(d*x+c)^9-1/16
*sinh(d*x+c)/cosh(d*x+c)^9+1/16*(128/315+1/9*sech(d*x+c)^8+8/63*sech(d*x+c)
^6+16/105*sech(d*x+c)^4+64/315*sech(d*x+c)^2)*tanh(d*x+c)))
```

Maxima [B] time = 1.18216, size = 1962, normalized size = 17.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 3/5*a^2*b*tanh(d*x + c)^5/d + 1/3*a^3*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c)
+ 3*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) + 1))) + 16/315*b^3*(9*e^(-2*d*x - 2*c)/(d*(9*e^(-2*d*x - 2*c
) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*
e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-
16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) + 36*e^(-4*d*x - 4*c)/(d*(9*e^(-2
*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8
*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c
) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) - 126*e^(-6*d*x - 6*c)/
(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^
(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14
*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) + 441*e^(-8*
d*x - 8*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*
c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c)
+ 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) -
 315*e^(-10*d*x - 10*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e
^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12
*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x -
18*c) + 1)) + 210*e^(-12*d*x - 12*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x
- 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c
) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) +
e^(-18*d*x - 18*c) + 1)) + 1/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) +
 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e
^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d
*x - 18*c) + 1))) + 12/35*a*b^2*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c)
+ 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-
10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 14*e^(-4
*d*x - 4*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6
*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) +
e^(-14*d*x - 14*c) + 1)) + 70*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*
e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*
x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 35*e^(-8*d*x
- 8*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) +
 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-1
4*d*x - 14*c) + 1)) + 35*e^(-10*d*x - 10*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(
-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x -
 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*
x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c)
+ 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)))
```

Fricas [B] time = 2.21913, size = 3414, normalized size = 31.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/315*((315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)
^9 + 9*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)
*sinh(d*x + c)^8 - (420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*sinh(d*x + c)^9
 + 9*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^7
 - 9*(280*a^3 + 21*a^2*b - 54*a*b^2 - 8*b^3 + 4*(420*a^3 - 189*a^2*b - 54*a
```

```
*b^2 - 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 21*(4*(315*a^3*d*x + 420*a
^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^3 + 3*(315*a^3*d*x + 420*a
^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 36*(315
*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^5 - 9*(14*
(420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^4 + 700*a^3 + 84*a^2
*b + 204*a*b^2 - 32*b^3 + 21*(280*a^3 + 21*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d
*x + c)^2)*sinh(d*x + c)^5 + 9*(14*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*
a*b^2 - 8*b^3)*cosh(d*x + c)^5 + 35*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54
*a*b^2 - 8*b^3)*cosh(d*x + c)^3 + 20*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 5
4*a*b^2 - 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 84*(315*a^3*d*x + 420*a^3
 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^3 - 3*(28*(420*a^3 - 189*a^2
*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^6 + 105*(280*a^3 + 21*a^2*b - 54*a*b^2
 - 8*b^3)*cosh(d*x + c)^4 + 2660*a^3 - 252*a^2*b - 252*a*b^2 + 896*b^3 + 12
0*(175*a^3 + 21*a^2*b + 51*a*b^2 - 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3
+ 9*(4*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)
^7 + 21*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c
)^5 + 40*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x +
c)^3 + 28*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x +
 c))*sinh(d*x + c)^2 + 126*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 -
8*b^3)*cosh(d*x + c) - 9*((420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x
 + c)^8 + 7*(280*a^3 + 21*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^6 + 20*(1
75*a^3 + 21*a^2*b + 51*a*b^2 - 8*b^3)*cosh(d*x + c)^4 + 420*a^3 - 126*a^2*b
 - 336*a*b^2 - 672*b^3 + 28*(95*a^3 - 9*a^2*b - 9*a*b^2 + 32*b^3)*cosh(d*x
+ c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^9 + 9*d*cosh(d*x + c)*sinh(d*x + c)
^8 + 9*d*cosh(d*x + c)^7 + 21*(4*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sin
h(d*x + c)^6 + 36*d*cosh(d*x + c)^5 + 9*(14*d*cosh(d*x + c)^5 + 35*d*cosh(d
*x + c)^3 + 20*d*cosh(d*x + c))*sinh(d*x + c)^4 + 84*d*cosh(d*x + c)^3 + 9*
(4*d*cosh(d*x + c)^7 + 21*d*cosh(d*x + c)^5 + 40*d*cosh(d*x + c)^3 + 28*d*c
osh(d*x + c))*sinh(d*x + c)^2 + 126*d*cosh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\operatorname{sech}^2(c + dx)\right)^3 \tanh^4(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**3*tanh(d*x+c)**4,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*tanh(c + d*x)**4, x)
```

Giac [B] time = 1.58587, size = 637, normalized size = 5.79

$$315\,a^3dx + \frac{2\left(630\,a^3e^{(16\,dx+16\,c)} - 945\,a^2be^{(16\,dx+16\,c)} + 4410\,a^3e^{(14\,dx+14\,c)} - 3780\,a^2be^{(14\,dx+14\,c)} - 1890\,ab^2e^{(14\,dx+14\,c)} + 13650\,a^3e^{(12\,dx+12\,c)} - 7560\,a^2be^{(12\,dx}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/315*(315*a^3*d*x + 2*(630*a^3*e^(16*d*x + 16*c) - 945*a^2*b*e^(16*d*x + 1
6*c) + 4410*a^3*e^(14*d*x + 14*c) - 3780*a^2*b*e^(14*d*x + 14*c) - 1890*a*b
^2*e^(14*d*x + 14*c) + 13650*a^3*e^(12*d*x + 12*c) - 7560*a^2*b*e^(12*d*x +
 12*c) - 1890*a*b^2*e^(12*d*x + 12*c) - 1680*b^3*e^(12*d*x + 12*c) + 24570*
```

```
a^3*e^(10*d*x + 10*c) - 11340*a^2*b*e^(10*d*x + 10*c) - 1890*a*b^2*e^(10*d*
x + 10*c) + 2520*b^3*e^(10*d*x + 10*c) + 28350*a^3*e^(8*d*x + 8*c) - 12474*
a^2*b*e^(8*d*x + 8*c) - 4914*a*b^2*e^(8*d*x + 8*c) - 3528*b^3*e^(8*d*x + 8*
c) + 21630*a^3*e^(6*d*x + 6*c) - 8316*a^2*b*e^(6*d*x + 6*c) - 2646*a*b^2*e^
(6*d*x + 6*c) + 1008*b^3*e^(6*d*x + 6*c) + 10710*a^3*e^(4*d*x + 4*c) - 3024
*a^2*b*e^(4*d*x + 4*c) - 54*a*b^2*e^(4*d*x + 4*c) - 288*b^3*e^(4*d*x + 4*c)
 + 3150*a^3*e^(2*d*x + 2*c) - 756*a^2*b*e^(2*d*x + 2*c) - 486*a*b^2*e^(2*d*
x + 2*c) - 72*b^3*e^(2*d*x + 2*c) + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)
/(e^(2*d*x + 2*c) + 1)^9)/d
```

3.125 $\int \left(a+b\text{sech}^2(c+dx)\right)^3 \tanh^3(c+dx)\,dx$

Optimal. Leaf size=103

$$-\frac{3a^2b\text{sech}^2(c+dx)}{2d}+\frac{a^3\log(\cosh(c+dx))}{d}-\frac{3ab^2\text{sech}^4(c+dx)}{4d}+\frac{\text{sech}^8(c+dx)\left(a\cosh^2(c+dx)+b\right)^4}{8bd}-\frac{b^3\text{sech}^6(}{6c}$$

```
[Out] (a^3*Log[Cosh[c + d*x]])/d - (3*a^2*b*Sech[c + d*x]^2)/(2*d) - (3*a*b^2*Sec
h[c + d*x]^4)/(4*d) - (b^3*Sech[c + d*x]^6)/(6*d) + ((b + a*Cosh[c + d*x]^2
)^4*Sech[c + d*x]^8)/(8*b*d)
```

Rubi [A] time = 0.0976315, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4138, 446, 78, 43}

$$-\frac{3a^2b\text{sech}^2(c+dx)}{2d}+\frac{a^3\log(\cosh(c+dx))}{d}-\frac{3ab^2\text{sech}^4(c+dx)}{4d}+\frac{\text{sech}^8(c+dx)\left(a\cosh^2(c+dx)+b\right)^4}{8bd}-\frac{b^3\text{sech}^6(}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x]^3,x]
```

```
[Out] (a^3*Log[Cosh[c + d*x]])/d - (3*a^2*b*Sech[c + d*x]^2)/(2*d) - (3*a*b^2*Sec
h[c + d*x]^4)/(4*d) - (b^3*Sech[c + d*x]^6)/(6*d) + ((b + a*Cosh[c + d*x]^2
)^4*Sech[c + d*x]^8)/(8*b*d)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3\tanh^3(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)(b+ax^2)^3}{x^9}\,dx,x,\cosh(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{(1-x)(b+ax)^3}{x^5}\,dx,x,\cosh^2(c+dx)\right)}{2d}$$
$$= \frac{\left(b+a\cosh^2(c+dx)\right)^4\text{sech}^8(c+dx)}{8bd} + \frac{\text{Subst}\left(\int \frac{(b+ax)^3}{x^4}\,dx,x,\cosh^2(c+d$$
$$= \frac{\left(b+a\cosh^2(c+dx)\right)^4\text{sech}^8(c+dx)}{8bd} + \frac{\text{Subst}\left(\int\left(\frac{b^3}{x^4}+\frac{3ab^2}{x^3}+\frac{3a^2b}{x^2}+\frac{a^3}{x}\right)d$$
$$= \frac{a^3\log(\cosh(c+dx))}{d} - \frac{3a^2b\text{sech}^2(c+dx)}{2d} - \frac{3ab^2\text{sech}^4(c+dx)}{4d} - \frac{b^3\text{sech}^6}{6$$

Mathematica [A] time = 0.885562, size = 128, normalized size = 1.24

$$\frac{\cosh^6(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\left(12a^2(a-3b)\text{sech}^2(c+dx)+24a^3\log(\cosh(c+dx))+4b^2(3a-b)\text{sech}^6(c+dx)+\right.}{3d(a\cosh(2c+2dx)+a+2b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x]^3,x]
```

```
[Out] (Cosh[c + d*x]^6*(a + b*Sech[c + d*x]^2)^3*(24*a^3*Log[Cosh[c + d*x]] + 12*
a^2*(a - 3*b)*Sech[c + d*x]^2 + 18*a*(a - b)*b*Sech[c + d*x]^4 + 4*(3*a - b
)*b^2*Sech[c + d*x]^6 + 3*b^3*Sech[c + d*x]^8))/(3*d*(a + 2*b + a*Cosh[2*c
+ 2*d*x])^3)
```

Maple [B] time = 0.051, size = 253, normalized size = 2.5

$$\frac{a^3\ln(\cosh(dx+c))}{d} - \frac{(\tanh(dx+c))^2a^3}{2d} - \frac{3a^2b(\sinh(dx+c))^2}{4d(\cosh(dx+c))^4} + \frac{3a^2b(\sinh(dx+c))^2}{4d(\cosh(dx+c))^2} - \frac{ab^2(\sinh(dx+c))^2}{2d(\cosh(dx+c))^6} + \frac{ab^2}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^3,x)
```

```
[Out] a^3*ln(cosh(d*x+c))/d-1/2/d*tanh(d*x+c)^2*a^3-3/4/d*a^2*b*sinh(d*x+c)^2/cos
h(d*x+c)^4+3/4/d*a^2*b*sinh(d*x+c)^2/cosh(d*x+c)^2-1/2/d*a*b^2*sinh(d*x+c)^
2/cosh(d*x+c)^6+1/4/d*a*b^2*sinh(d*x+c)^2/cosh(d*x+c)^4+1/4/d*a*b^2*sinh(d*
x+c)^2/cosh(d*x+c)^2-1/8/d*b^3*sinh(d*x+c)^2/cosh(d*x+c)^8+1/24/d*b^3*sinh(
d*x+c)^2/cosh(d*x+c)^6+1/24/d*b^3*sinh(d*x+c)^2/cosh(d*x+c)^4+1/24/d*b^3*si
nh(d*x+c)^2/cosh(d*x+c)^2
```

Maxima [B] time = 1.56819, size = 880, normalized size = 8.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 3/4*a^2*b*tanh(d*x + c)^4/d + a^3*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d +
2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - 4*a*b
^2*(3*e^(-4*d*x - 4*c)/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^
(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x -
12*c) + 1)) - 2*e^(-6*d*x - 6*c)/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*
c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(
-12*d*x - 12*c) + 1)) + 3*e^(-8*d*x - 8*c)/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-
4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 1
0*c) + e^(-12*d*x - 12*c) + 1))) - 32/3*b^3*(e^(-6*d*x - 6*c)/(d*(8*e^(-2*d
*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c)
 + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e
^(-16*d*x - 16*c) + 1)) - e^(-8*d*x - 8*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-
4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x -
10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) +
 1)) + e^(-10*d*x - 10*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56
*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12
*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1)))
```

Fricas [B] time = 2.69868, size = 12142, normalized size = 117.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^3*d*x*cosh(d*x + c)^16 + 48*a^3*d*x*cosh(d*x + c)*sinh(d*x + c)^1
5 + 3*a^3*d*x*sinh(d*x + c)^16 + 6*(4*a^3*d*x - a^3 + 3*a^2*b)*cosh(d*x + c
)^14 + 6*(60*a^3*d*x*cosh(d*x + c)^2 + 4*a^3*d*x - a^3 + 3*a^2*b)*sinh(d*x
+ c)^14 + 84*(20*a^3*d*x*cosh(d*x + c)^3 + (4*a^3*d*x - a^3 + 3*a^2*b)*cosh
(d*x + c))*sinh(d*x + c)^13 + 12*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*co
sh(d*x + c)^12 + 6*(910*a^3*d*x*cosh(d*x + c)^4 + 14*a^3*d*x - 6*a^3 + 12*a
^2*b + 6*a*b^2 + 91*(4*a^3*d*x - a^3 + 3*a^2*b)*cosh(d*x + c)^2)*sinh(d*x +
 c)^12 + 24*(546*a^3*d*x*cosh(d*x + c)^5 + 91*(4*a^3*d*x - a^3 + 3*a^2*b)*c
osh(d*x + c)^3 + 6*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d*x + c))*s
inh(d*x + c)^11 + 2*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*co
sh(d*x + c)^10 + 2*(12012*a^3*d*x*cosh(d*x + c)^6 + 84*a^3*d*x + 3003*(4*a^
3*d*x - a^3 + 3*a^2*b)*cosh(d*x + c)^4 - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*
b^3 + 396*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d*x + c)^2)*sinh(d*x
 + c)^10 + 4*(8580*a^3*d*x*cosh(d*x + c)^7 + 3003*(4*a^3*d*x - a^3 + 3*a^2*
b)*cosh(d*x + c)^5 + 660*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d*x +
 c)^3 + 5*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*cosh(d*x + c
))*sinh(d*x + c)^9 + 2*(105*a^3*d*x - 60*a^3 + 72*a^2*b + 12*a*b^2 - 16*b^3
)*cosh(d*x + c)^8 + 2*(19305*a^3*d*x*cosh(d*x + c)^8 + 9009*(4*a^3*d*x - a^
3 + 3*a^2*b)*cosh(d*x + c)^6 + 105*a^3*d*x + 2970*(7*a^3*d*x - 3*a^3 + 6*a^
2*b + 3*a*b^2)*cosh(d*x + c)^4 - 60*a^3 + 72*a^2*b + 12*a*b^2 - 16*b^3 + 45
*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*cosh(d*x + c)^2)*sinh
(d*x + c)^8 + 16*(2145*a^3*d*x*cosh(d*x + c)^9 + 1287*(4*a^3*d*x - a^3 + 3*
a^2*b)*cosh(d*x + c)^7 + 594*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d
*x + c)^5 + 15*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*cosh(d*
x + c)^3 + (105*a^3*d*x - 60*a^3 + 72*a^2*b + 12*a*b^2 - 16*b^3)*cosh(d*x +
 c))*sinh(d*x + c)^7 + 2*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^
3)*cosh(d*x + c)^6 + 2*(12012*a^3*d*x*cosh(d*x + c)^10 + 9009*(4*a^3*d*x -
```

a^3 + 3*a^2*b)*cosh(d*x + c)^8 + 5544*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^
2)*cosh(d*x + c)^6 + 84*a^3*d*x + 210*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*
a*b^2 + 16*b^3)*cosh(d*x + c)^4 - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3 + 2
8*(105*a^3*d*x - 60*a^3 + 72*a^2*b + 12*a*b^2 - 16*b^3)*cosh(d*x + c)^2)*si
nh(d*x + c)^6 + 4*(3276*a^3*d*x*cosh(d*x + c)^11 + 3003*(4*a^3*d*x - a^3 +
3*a^2*b)*cosh(d*x + c)^9 + 2376*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cos
h(d*x + c)^7 + 126*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*cos
h(d*x + c)^5 + 28*(105*a^3*d*x - 60*a^3 + 72*a^2*b + 12*a*b^2 - 16*b^3)*cos
h(d*x + c)^3 + 3*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*cosh(
d*x + c))*sinh(d*x + c)^5 + 3*a^3*d*x + 12*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3
*a*b^2)*cosh(d*x + c)^4 + 2*(2730*a^3*d*x*cosh(d*x + c)^12 + 3003*(4*a^3*d*
x - a^3 + 3*a^2*b)*cosh(d*x + c)^10 + 2970*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3
*a*b^2)*cosh(d*x + c)^8 + 210*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 +
16*b^3)*cosh(d*x + c)^6 + 42*a^3*d*x + 70*(105*a^3*d*x - 60*a^3 + 72*a^2*b
+ 12*a*b^2 - 16*b^3)*cosh(d*x + c)^4 - 18*a^3 + 36*a^2*b + 18*a*b^2 + 15*(8
4*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*cosh(d*x + c)^2)*sinh(d*
x + c)^4 + 8*(210*a^3*d*x*cosh(d*x + c)^13 + 273*(4*a^3*d*x - a^3 + 3*a^2*b
)*cosh(d*x + c)^11 + 330*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d*x +
c)^9 + 30*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*cosh(d*x +
c)^7 + 14*(105*a^3*d*x - 60*a^3 + 72*a^2*b + 12*a*b^2 - 16*b^3)*cosh(d*x +
c)^5 + 5*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*cosh(d*x + c)
^3 + 6*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d*x + c))*sinh(d*x + c)
^3 + 6*(4*a^3*d*x - a^3 + 3*a^2*b)*cosh(d*x + c)^2 + 2*(180*a^3*d*x*cosh(d*
x + c)^14 + 273*(4*a^3*d*x - a^3 + 3*a^2*b)*cosh(d*x + c)^12 + 396*(7*a^3*d
*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d*x + c)^10 + 45*(84*a^3*d*x - 45*a^3
+ 63*a^2*b + 24*a*b^2 + 16*b^3)*cosh(d*x + c)^8 + 28*(105*a^3*d*x - 60*a^3
+ 72*a^2*b + 12*a*b^2 - 16*b^3)*cosh(d*x + c)^6 + 12*a^3*d*x + 15*(84*a^3*d
*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*cosh(d*x + c)^4 - 3*a^3 + 9*a^2
b + 36(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d*x + c)^2)*sinh(d*x +
c)^2 - 3*(a^3*cosh(d*x + c)^16 + 16*a^3*cosh(d*x + c)*sinh(d*x + c)^15 + a
^3*sinh(d*x + c)^16 + 8*a^3*cosh(d*x + c)^14 + 28*a^3*cosh(d*x + c)^12 + 8*
(15*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^14 + 112*(5*a^3*cosh(d*x + c)^
3 + a^3*cosh(d*x + c))*sinh(d*x + c)^13 + 56*a^3*cosh(d*x + c)^10 + 28*(65*
a^3*cosh(d*x + c)^4 + 26*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^12 + 112*
(39*a^3*cosh(d*x + c)^5 + 26*a^3*cosh(d*x + c)^3 + 3*a^3*cosh(d*x + c))*sin
h(d*x + c)^11 + 70*a^3*cosh(d*x + c)^8 + 56*(143*a^3*cosh(d*x + c)^6 + 143*
a^3*cosh(d*x + c)^4 + 33*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^10 + 16*(
715*a^3*cosh(d*x + c)^7 + 1001*a^3*cosh(d*x + c)^5 + 385*a^3*cosh(d*x + c)^
3 + 35*a^3*cosh(d*x + c))*sinh(d*x + c)^9 + 56*a^3*cosh(d*x + c)^6 + 2*(643
5*a^3*cosh(d*x + c)^8 + 12012*a^3*cosh(d*x + c)^6 + 6930*a^3*cosh(d*x + c)^
4 + 1260*a^3*cosh(d*x + c)^2 + 35*a^3)*sinh(d*x + c)^8 + 16*(715*a^3*cosh(d
*x + c)^9 + 1716*a^3*cosh(d*x + c)^7 + 1386*a^3*cosh(d*x + c)^5 + 420*a^3*c
osh(d*x + c)^3 + 35*a^3*cosh(d*x + c))*sinh(d*x + c)^7 + 28*a^3*cosh(d*x +
c)^4 + 56*(143*a^3*cosh(d*x + c)^10 + 429*a^3*cosh(d*x + c)^8 + 462*a^3*cos
h(d*x + c)^6 + 210*a^3*cosh(d*x + c)^4 + 35*a^3*cosh(d*x + c)^2 + a^3)*sinh
(d*x + c)^6 + 112*(39*a^3*cosh(d*x + c)^11 + 143*a^3*cosh(d*x + c)^9 + 198*
a^3*cosh(d*x + c)^7 + 126*a^3*cosh(d*x + c)^5 + 35*a^3*cosh(d*x + c)^3 + 3*
a^3*cosh(d*x + c))*sinh(d*x + c)^5 + 8*a^3*cosh(d*x + c)^2 + 28*(65*a^3*cos
h(d*x + c)^12 + 286*a^3*cosh(d*x + c)^10 + 495*a^3*cosh(d*x + c)^8 + 420*a^
3*cosh(d*x + c)^6 + 175*a^3*cosh(d*x + c)^4 + 30*a^3*cosh(d*x + c)^2 + a^3)
*sinh(d*x + c)^4 + 112*(5*a^3*cosh(d*x + c)^13 + 26*a^3*cosh(d*x + c)^11 +
55*a^3*cosh(d*x + c)^9 + 60*a^3*cosh(d*x + c)^7 + 35*a^3*cosh(d*x + c)^5 +
10*a^3*cosh(d*x + c)^3 + a^3*cosh(d*x + c))*sinh(d*x + c)^3 + a^3 + 8*(15*a
^3*cosh(d*x + c)^14 + 91*a^3*cosh(d*x + c)^12 + 231*a^3*cosh(d*x + c)^10 +
315*a^3*cosh(d*x + c)^8 + 245*a^3*cosh(d*x + c)^6 + 105*a^3*cosh(d*x + c)^4
+ 21*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^2 + 16*(a^3*cosh(d*x + c)^15
+ 7*a^3*cosh(d*x + c)^13 + 21*a^3*cosh(d*x + c)^11 + 35*a^3*cosh(d*x + c)^
9 + 35*a^3*cosh(d*x + c)^7 + 21*a^3*cosh(d*x + c)^5 + 7*a^3*cosh(d*x + c)^3
+ a^3*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - s

```
inh(d*x + c))) + 4*(12*a^3*d*x*cosh(d*x + c)^15 + 21*(4*a^3*d*x - a^3 + 3*a
^2*b)*cosh(d*x + c)^13 + 36*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d*
x + c)^11 + 5*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*cosh(d*x
 + c)^9 + 4*(105*a^3*d*x - 60*a^3 + 72*a^2*b + 12*a*b^2 - 16*b^3)*cosh(d*x
+ c)^7 + 3*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*cosh(d*x +
c)^5 + 12*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d*x + c)^3 + 3*(4*a^
3*d*x - a^3 + 3*a^2*b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^16 +
16*d*cosh(d*x + c)*sinh(d*x + c)^15 + d*sinh(d*x + c)^16 + 8*d*cosh(d*x + c
)^14 + 8*(15*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^14 + 112*(5*d*cosh(d*x +
c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^13 + 28*d*cosh(d*x + c)^12 + 28*(65*d
*cosh(d*x + c)^4 + 26*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^12 + 112*(39*d*c
osh(d*x + c)^5 + 26*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^11
 + 56*d*cosh(d*x + c)^10 + 56*(143*d*cosh(d*x + c)^6 + 143*d*cosh(d*x + c)^
4 + 33*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^10 + 16*(715*d*cosh(d*x + c)^7
+ 1001*d*cosh(d*x + c)^5 + 385*d*cosh(d*x + c)^3 + 35*d*cosh(d*x + c))*sinh
(d*x + c)^9 + 70*d*cosh(d*x + c)^8 + 2*(6435*d*cosh(d*x + c)^8 + 12012*d*co
sh(d*x + c)^6 + 6930*d*cosh(d*x + c)^4 + 1260*d*cosh(d*x + c)^2 + 35*d)*sin
h(d*x + c)^8 + 16*(715*d*cosh(d*x + c)^9 + 1716*d*cosh(d*x + c)^7 + 1386*d*
cosh(d*x + c)^5 + 420*d*cosh(d*x + c)^3 + 35*d*cosh(d*x + c))*sinh(d*x + c)
^7 + 56*d*cosh(d*x + c)^6 + 56*(143*d*cosh(d*x + c)^10 + 429*d*cosh(d*x + c
)^8 + 462*d*cosh(d*x + c)^6 + 210*d*cosh(d*x + c)^4 + 35*d*cosh(d*x + c)^2
+ d)*sinh(d*x + c)^6 + 112*(39*d*cosh(d*x + c)^11 + 143*d*cosh(d*x + c)^9 +
 198*d*cosh(d*x + c)^7 + 126*d*cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 + 3*d
*cosh(d*x + c))*sinh(d*x + c)^5 + 28*d*cosh(d*x + c)^4 + 28*(65*d*cosh(d*x
+ c)^12 + 286*d*cosh(d*x + c)^10 + 495*d*cosh(d*x + c)^8 + 420*d*cosh(d*x +
 c)^6 + 175*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 +
 112*(5*d*cosh(d*x + c)^13 + 26*d*cosh(d*x + c)^11 + 55*d*cosh(d*x + c)^9 +
 60*d*cosh(d*x + c)^7 + 35*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + d*cos
h(d*x + c))*sinh(d*x + c)^3 + 8*d*cosh(d*x + c)^2 + 8*(15*d*cosh(d*x + c)^1
4 + 91*d*cosh(d*x + c)^12 + 231*d*cosh(d*x + c)^10 + 315*d*cosh(d*x + c)^8
+ 245*d*cosh(d*x + c)^6 + 105*d*cosh(d*x + c)^4 + 21*d*cosh(d*x + c)^2 + d)
*sinh(d*x + c)^2 + 16*(d*cosh(d*x + c)^15 + 7*d*cosh(d*x + c)^13 + 21*d*cos
h(d*x + c)^11 + 35*d*cosh(d*x + c)^9 + 35*d*cosh(d*x + c)^7 + 21*d*cosh(d*x
 + c)^5 + 7*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [A] time = 22.6259, size = 178, normalized size = 1.73

$$\begin{cases} a^3x - \frac{a^3\log(\tanh(c+dx)+1)}{d} - \frac{a^3\tanh^2(c+dx)}{2d} - \frac{3a^2b\tanh^2(c+dx)\operatorname{sech}^2(c+dx)}{4d} - \frac{3a^2b\operatorname{sech}^2(c+dx)}{4d} - \frac{ab^2\tanh^2(c+dx)\operatorname{sech}^4(c+dx)}{2d} - \frac{ab^2 \mathrm{s}\cdots}{} \\ x\left(a + b\operatorname{sech}^2(c)\right)^3\tanh^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**3*tanh(d*x+c)**3,x)
```

```
[Out] Piecewise((a**3*x - a**3*log(tanh(c + d*x) + 1)/d - a**3*tanh(c + d*x)**2/(
2*d) - 3*a**2*b*tanh(c + d*x)**2*sech(c + d*x)**2/(4*d) - 3*a**2*b*sech(c +
 d*x)**2/(4*d) - a*b**2*tanh(c + d*x)**2*sech(c + d*x)**4/(2*d) - a*b**2*se
ch(c + d*x)**4/(4*d) - b**3*tanh(c + d*x)**2*sech(c + d*x)**6/(8*d) - b**3*
sech(c + d*x)**6/(24*d), Ne(d, 0)), (x*(a + b*sech(c)**2)**3*tanh(c)**3, Tr
ue))
```

Giac [B] time = 1.35544, size = 518, normalized size = 5.03

$$-\frac{840\,a^3dx - 840\,a^3\log\left(e^{(2\,dx+2\,c)}+1\right) + \frac{2283\,a^3e^{(16\,dx+16\,c)}+16584\,a^3e^{(14\,dx+14\,c)}+5040\,a^2be^{(14\,dx+14\,c)}+53844\,a^3e^{(12\,dx+12\,c)}+20160\,a^2be^{(12\,dx+\cdots}}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/840*(840*a^3*d*x - 840*a^3*log(e^(2*d*x + 2*c) + 1) + (2283*a^3*e^(16*d*
x + 16*c) + 16584*a^3*e^(14*d*x + 14*c) + 5040*a^2*b*e^(14*d*x + 14*c) + 53
844*a^3*e^(12*d*x + 12*c) + 20160*a^2*b*e^(12*d*x + 12*c) + 10080*a*b^2*e^(
12*d*x + 12*c) + 102648*a^3*e^(10*d*x + 10*c) + 35280*a^2*b*e^(10*d*x + 10*
c) + 13440*a*b^2*e^(10*d*x + 10*c) + 8960*b^3*e^(10*d*x + 10*c) + 126210*a^
3*e^(8*d*x + 8*c) + 40320*a^2*b*e^(8*d*x + 8*c) + 6720*a*b^2*e^(8*d*x + 8*c
) - 8960*b^3*e^(8*d*x + 8*c) + 102648*a^3*e^(6*d*x + 6*c) + 35280*a^2*b*e^(
6*d*x + 6*c) + 13440*a*b^2*e^(6*d*x + 6*c) + 8960*b^3*e^(6*d*x + 6*c) + 538
44*a^3*e^(4*d*x + 4*c) + 20160*a^2*b*e^(4*d*x + 4*c) + 10080*a*b^2*e^(4*d*x
 + 4*c) + 16584*a^3*e^(2*d*x + 2*c) + 5040*a^2*b*e^(2*d*x + 2*c) + 2283*a^3
)/(e^(2*d*x + 2*c) + 1)^8)/d
```

3.126 $\int \left(a+b\,\mathrm{sech}^2(c+dx)\right)^3 \tanh^2(c+dx)\,dx$

Optimal. Leaf size=92

$$\frac{b\left(3a^2+3ab+b^2\right)\tanh^3(c+dx)}{3d}-\frac{a^3\tanh(c+dx)}{d}+a^3x-\frac{b^2(3a+2b)\tanh^5(c+dx)}{5d}+\frac{b^3\tanh^7(c+dx)}{7d}$$

```
[Out] a^3*x - (a^3*Tanh[c + d*x])/d + (b*(3*a^2 + 3*a*b + b^2)*Tanh[c + d*x]^3)/(
3*d) - (b^2*(3*a + 2*b)*Tanh[c + d*x]^5)/(5*d) + (b^3*Tanh[c + d*x]^7)/(7*d
)
```

Rubi [A] time = 0.110217, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4141, 1802, 206}

$$\frac{b\left(3a^2+3ab+b^2\right)\tanh^3(c+dx)}{3d}-\frac{a^3\tanh(c+dx)}{d}+a^3x-\frac{b^2(3a+2b)\tanh^5(c+dx)}{5d}+\frac{b^3\tanh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x]^2,x]
```

```
[Out] a^3*x - (a^3*Tanh[c + d*x])/d + (b*(3*a^2 + 3*a*b + b^2)*Tanh[c + d*x]^3)/(
3*d) - (b^2*(3*a + 2*b)*Tanh[c + d*x]^5)/(5*d) + (b^3*Tanh[c + d*x]^7)/(7*d
)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3 \tanh^2(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+b\left(1-x^2\right)\right)^3}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-a^3+b\left(3a^2+3ab+b^2\right)x^2-b^2(3a+2b)x^4+b^3x^6+\frac{a^3}{1-x^2}\right)dx, x\right.}{d}$$

$$= -\frac{a^3\tanh(c+dx)}{d}+\frac{b\left(3a^2+3ab+b^2\right)\tanh^3(c+dx)}{3d}-\frac{b^2(3a+2b)\tanh^5(}{5d}$$

$$= a^3x-\frac{a^3\tanh(c+dx)}{d}+\frac{b\left(3a^2+3ab+b^2\right)\tanh^3(c+dx)}{3d}-\frac{b^2(3a+2b)\,\text{ta}}{5d}$$

Mathematica [B] time = 1.80319, size = 479, normalized size = 5.21

$$\text{sech}(c)\text{sech}^7(c+dx)\left(-3990a^2b\sinh(2c+dx)+1890a^2b\sinh(2c+3dx)-2520a^2b\sinh(4c+3dx)+840a^2b\sinh(4c+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x]^2,x]
```

```
[Out] (Sech[c]*Sech[c + d*x]^7*(3675*a^3*d*x*Cosh[d*x] + 3675*a^3*d*x*Cosh[2*c +
d*x] + 2205*a^3*d*x*Cosh[2*c + 3*d*x] + 2205*a^3*d*x*Cosh[4*c + 3*d*x] + 73
5*a^3*d*x*Cosh[4*c + 5*d*x] + 735*a^3*d*x*Cosh[6*c + 5*d*x] + 105*a^3*d*x*C
osh[6*c + 7*d*x] + 105*a^3*d*x*Cosh[8*c + 7*d*x] - 4200*a^3*Sinh[d*x] + 336
0*a^2*b*Sinh[d*x] + 840*a*b^2*Sinh[d*x] - 560*b^3*Sinh[d*x] + 3150*a^3*Sinh
[2*c + d*x] - 3990*a^2*b*Sinh[2*c + d*x] - 2100*a*b^2*Sinh[2*c + d*x] - 112
0*b^3*Sinh[2*c + d*x] - 3150*a^3*Sinh[2*c + 3*d*x] + 1890*a^2*b*Sinh[2*c +
3*d*x] + 504*a*b^2*Sinh[2*c + 3*d*x] + 336*b^3*Sinh[2*c + 3*d*x] + 1260*a^3
*Sinh[4*c + 3*d*x] - 2520*a^2*b*Sinh[4*c + 3*d*x] - 1260*a*b^2*Sinh[4*c + 3
*d*x] - 1260*a^3*Sinh[4*c + 5*d*x] + 840*a^2*b*Sinh[4*c + 5*d*x] + 588*a*b^
2*Sinh[4*c + 5*d*x] + 112*b^3*Sinh[4*c + 5*d*x] + 210*a^3*Sinh[6*c + 5*d*x]
 - 630*a^2*b*Sinh[6*c + 5*d*x] - 210*a^3*Sinh[6*c + 7*d*x] + 210*a^2*b*Sinh
[6*c + 7*d*x] + 84*a*b^2*Sinh[6*c + 7*d*x] + 16*b^3*Sinh[6*c + 7*d*x]))/(13
440*d)
```

Maple [B] time = 0.042, size = 180, normalized size = 2.

$$\frac{1}{d}\left(a^3\left(dx+c-\tanh\left(dx+c\right)\right)+3\,a^2b\left(-1/2\,\frac{\sinh\left(dx+c\right)}{\left(\cosh\left(dx+c\right)\right)^3}+1/2\,\left(2/3+1/3\,\left(\text{sech}\left(dx+c\right)\right)^2\right)\tanh\left(dx+c\right)\right)+3\,ab^2\left(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^2,x)
```

```
[Out] 1/d*(a^3*(d*x+c-tanh(d*x+c))+3*a^2*b*(-1/2*sinh(d*x+c)/cosh(d*x+c)^3+1/2*(2
/3+1/3*sech(d*x+c)^2)*tanh(d*x+c))+3*a*b^2*(-1/4*sinh(d*x+c)/cosh(d*x+c)^5+
1/4*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c))+b^3*(-1/6*sinh
(d*x+c)/cosh(d*x+c)^7+1/6*(16/35+1/7*sech(d*x+c)^6+6/35*sech(d*x+c)^4+8/35*
sech(d*x+c)^2)*tanh(d*x+c)))
```

Maxima [B] time = 1.24013, size = 1064, normalized size = 11.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] a^2*b*tanh(d*x + c)^3/d + a^3*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + 16
/105*b^3*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) +
 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-
12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 21*e^(-4*d*x - 4*c)/(d*(7*e^(-2
*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*
c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)
) - 35*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e
^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*
x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 70*e^(-8*d*x - 8*c)/(d*(7*e^(-2*d*x
- 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) +
21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1
/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^
(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x
- 14*c) + 1))) + 4/5*a*b^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) + 1)) - 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*
c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) +
15*e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6
*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2
*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c
) + e^(-10*d*x - 10*c) + 1)))
```

Fricas [B] time = 2.18119, size = 2249, normalized size = 24.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/105*((105*a^3*d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)
^7 + 7*(105*a^3*d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)
*sinh(d*x + c)^6 - (105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*sinh(d*x + c)^7
 + 7*(105*a^3*d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)^5
 - 7*(75*a^3 - 15*a^2*b - 42*a*b^2 - 8*b^3 + 3*(105*a^3 - 105*a^2*b - 42*a*
b^2 - 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 35*((105*a^3*d*x + 105*a^3
- 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)^3 + (105*a^3*d*x + 105*a^3 -
105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 21*(105*a^3*
d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)^3 - 7*(5*(105*a
^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)^4 + 135*a^3 + 45*a^2*b + 5
4*a*b^2 - 24*b^3 + 10*(75*a^3 - 15*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^3 + 7*(3*(105*a^3*d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 - 8
*b^3)*cosh(d*x + c)^5 + 10*(105*a^3*d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 -
8*b^3)*cosh(d*x + c)^3 + 9*(105*a^3*d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 -
8*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 35*(105*a^3*d*x + 105*a^3 - 105*a^2
*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c) - 7*((105*a^3 - 105*a^2*b - 42*a*b^2 -
 8*b^3)*cosh(d*x + c)^6 + 5*(75*a^3 - 15*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x
 + c)^4 + 75*a^3 + 45*a^2*b + 90*a*b^2 + 120*b^3 + 9*(45*a^3 + 15*a^2*b + 1
8*a*b^2 - 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*c
```

```
osh(d*x + c)*sinh(d*x + c)^6 + 7*d*cosh(d*x + c)^5 + 35*(d*cosh(d*x + c)^3
+ d*cosh(d*x + c))*sinh(d*x + c)^4 + 21*d*cosh(d*x + c)^3 + 7*(3*d*cosh(d*x
 + c)^5 + 10*d*cosh(d*x + c)^3 + 9*d*cosh(d*x + c))*sinh(d*x + c)^2 + 35*d*
cosh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\,\mathrm{sech}^2\left(c + dx\right)\right)^3 \tanh^2\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**3*tanh(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*tanh(c + d*x)**2, x)
```

Giac [B] time = 1.30873, size = 481, normalized size = 5.23

$$105\,a^3dx + \frac{2\left(105\,a^3e^{(12\,dx+12\,c)} - 315\,a^2be^{(12\,dx+12\,c)} + 630\,a^3e^{(10\,dx+10\,c)} - 1260\,a^2be^{(10\,dx+10\,c)} - 630\,ab^2e^{(10\,dx+10\,c)} + 1575\,a^3e^{(8\,dx+8\,c)} - 1995\,a^2be^{(8\,dx+8\,c)} - 1050\,\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/105*(105*a^3*d*x + 2*(105*a^3*e^(12*d*x + 12*c) - 315*a^2*b*e^(12*d*x + 1
2*c) + 630*a^3*e^(10*d*x + 10*c) - 1260*a^2*b*e^(10*d*x + 10*c) - 630*a*b^2
*e^(10*d*x + 10*c) + 1575*a^3*e^(8*d*x + 8*c) - 1995*a^2*b*e^(8*d*x + 8*c)
- 1050*a*b^2*e^(8*d*x + 8*c) - 560*b^3*e^(8*d*x + 8*c) + 2100*a^3*e^(6*d*x
+ 6*c) - 1680*a^2*b*e^(6*d*x + 6*c) - 420*a*b^2*e^(6*d*x + 6*c) + 280*b^3*e
^(6*d*x + 6*c) + 1575*a^3*e^(4*d*x + 4*c) - 945*a^2*b*e^(4*d*x + 4*c) - 252
*a*b^2*e^(4*d*x + 4*c) - 168*b^3*e^(4*d*x + 4*c) + 630*a^3*e^(2*d*x + 2*c)
- 420*a^2*b*e^(2*d*x + 2*c) - 294*a*b^2*e^(2*d*x + 2*c) - 56*b^3*e^(2*d*x +
 2*c) + 105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)/(e^(2*d*x + 2*c) + 1)^7)/d
```

3.127 $\int \left(a+b\,\mathrm{sech}^2(c+dx)\right)^3 \tanh(c+dx)\,dx$

Optimal. Leaf size=71

$$-\frac{3a^2b\,\mathrm{sech}^2(c+dx)}{2d}+\frac{a^3\log(\cosh(c+dx))}{d}-\frac{3ab^2\,\mathrm{sech}^4(c+dx)}{4d}-\frac{b^3\,\mathrm{sech}^6(c+dx)}{6d}$$

```
[Out] (a^3*Log[Cosh[c + d*x]])/d - (3*a^2*b*Sech[c + d*x]^2)/(2*d) - (3*a*b^2*Sec
h[c + d*x]^4)/(4*d) - (b^3*Sech[c + d*x]^6)/(6*d)
```

Rubi [A] time = 0.0581538, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 266, 43}

$$-\frac{3a^2b\,\mathrm{sech}^2(c+dx)}{2d}+\frac{a^3\log(\cosh(c+dx))}{d}-\frac{3ab^2\,\mathrm{sech}^4(c+dx)}{4d}-\frac{b^3\,\mathrm{sech}^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x],x]
```

```
[Out] (a^3*Log[Cosh[c + d*x]])/d - (3*a^2*b*Sech[c + d*x]^2)/(2*d) - (3*a*b^2*Sec
h[c + d*x]^4)/(4*d) - (b^3*Sech[c + d*x]^6)/(6*d)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \left(a+b\,\mathrm{sech}^2(c+dx)\right)^3\tanh(c+dx)\,dx = \frac{\mathrm{Subst}\left(\int \frac{\left(b+ax^2\right)^3}{x^7}\,dx,x,\cosh(c+dx)\right)}{d}$$
$$= \frac{\mathrm{Subst}\left(\int \frac{(b+ax)^3}{x^4}\,dx,x,\cosh^2(c+dx)\right)}{2d}$$
$$= \frac{\mathrm{Subst}\left(\int \left(\frac{b^3}{x^4}+\frac{3ab^2}{x^3}+\frac{3a^2b}{x^2}+\frac{a^3}{x}\right)dx,x,\cosh^2(c+dx)\right)}{2d}$$
$$= \frac{a^3\log(\cosh(c+dx))}{d}-\frac{3a^2b\,\mathrm{sech}^2(c+dx)}{2d}-\frac{3ab^2\,\mathrm{sech}^4(c+dx)}{4d}-\frac{b^3\mathrm{sech}^6(c+dx)}{6d}$$

Mathematica [A] time = 0.291873, size = 100, normalized size = 1.41

$$\frac{2\,\mathrm{sech}^6(c+dx)\left(a\cosh^2(c+dx)+b\right)^3\left(-18a^2b\cosh^4(c+dx)+12a^3\cosh^6(c+dx)\log(\cosh(c+dx))-9ab^2\cosh^2(c+dx)\ldots\right)}{3d(a\cosh(2(c+dx))+a+2b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x],x]
```

```
[Out] (2*(b + a*Cosh[c + d*x]^2)^3*(-2*b^3 - 9*a*b^2*Cosh[c + d*x]^2 - 18*a^2*b*C
osh[c + d*x]^4 + 12*a^3*Cosh[c + d*x]^6*Log[Cosh[c + d*x]])*Sech[c + d*x]^6
)/(3*d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)
```

Maple [A] time = 0.013, size = 67, normalized size = 0.9

$$-\frac{b^3\left(\mathrm{sech}\left(dx+c\right)\right)^6}{6\,d}-\frac{3\,ab^2\left(\mathrm{sech}\left(dx+c\right)\right)^4}{4\,d}-\frac{3\,a^2b\left(\mathrm{sech}\left(dx+c\right)\right)^2}{2\,d}-\frac{a^3\ln\left(\mathrm{sech}\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^3*tanh(d*x+c),x)
```

```
[Out] -1/6*b^3*sech(d*x+c)^6/d-3/4*a*b^2*sech(d*x+c)^4/d-3/2*a^2*b*sech(d*x+c)^2/
d-1/d*a^3*ln(sech(d*x+c))
```

Maxima [A] time = 1.12722, size = 115, normalized size = 1.62

$$\frac{3\,a^2b\tanh\left(dx+c\right)^2}{2\,d}+\frac{a^3\log\left(\cosh\left(dx+c\right)\right)}{d}-\frac{12\,ab^2}{d\left(e^{(dx+c)}+e^{(-dx-c)}\right)^4}-\frac{32\,b^3}{3\,d\left(e^{(dx+c)}+e^{(-dx-c)}\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c),x, algorithm="maxima")
```

```
[Out] 3/2*a^2*b*tanh(d*x + c)^2/d + a^3*log(cosh(d*x + c))/d - 12*a*b^2/(d*(e^(d*
x + c) + e^(-d*x - c))^4) - 32/3*b^3/(d*(e^(d*x + c) + e^(-d*x - c))^6)
```

Fricas [B] time = 2.75446, size = 6511, normalized size = 91.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^3*d*x*cosh(d*x + c)^12 + 36*a^3*d*x*cosh(d*x + c)*sinh(d*x + c)^1
1 + 3*a^3*d*x*sinh(d*x + c)^12 + 18*(a^3*d*x + a^2*b)*cosh(d*x + c)^10 + 18
*(11*a^3*d*x*cosh(d*x + c)^2 + a^3*d*x + a^2*b)*sinh(d*x + c)^10 + 60*(11*a
^3*d*x*cosh(d*x + c)^3 + 3*(a^3*d*x + a^2*b)*cosh(d*x + c))*sinh(d*x + c)^9
 + 9*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c)^8 + 9*(165*a^3*d*x*cosh(
d*x + c)^4 + 5*a^3*d*x + 8*a^2*b + 4*a*b^2 + 90*(a^3*d*x + a^2*b)*cosh(d*x
+ c)^2)*sinh(d*x + c)^8 + 72*(33*a^3*d*x*cosh(d*x + c)^5 + 30*(a^3*d*x + a^
2*b)*cosh(d*x + c)^3 + (5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c))*sinh(
d*x + c)^7 + 4*(15*a^3*d*x + 27*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c)^6 +
 4*(693*a^3*d*x*cosh(d*x + c)^6 + 15*a^3*d*x + 945*(a^3*d*x + a^2*b)*cosh(d
*x + c)^4 + 27*a^2*b + 18*a*b^2 + 8*b^3 + 63*(5*a^3*d*x + 8*a^2*b + 4*a*b^2
)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 24*(99*a^3*d*x*cosh(d*x + c)^7 + 189*(
a^3*d*x + a^2*b)*cosh(d*x + c)^5 + 21*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(
d*x + c)^3 + (15*a^3*d*x + 27*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c))*sinh
(d*x + c)^5 + 3*a^3*d*x + 9*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c)^4
 + 3*(495*a^3*d*x*cosh(d*x + c)^8 + 1260*(a^3*d*x + a^2*b)*cosh(d*x + c)^6
+ 15*a^3*d*x + 210*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c)^4 + 24*a^2
*b + 12*a*b^2 + 20*(15*a^3*d*x + 27*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c)
^2)*sinh(d*x + c)^4 + 4*(165*a^3*d*x*cosh(d*x + c)^9 + 540*(a^3*d*x + a^2*b
)*cosh(d*x + c)^7 + 126*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c)^5 + 2
0*(15*a^3*d*x + 27*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c)^3 + 9*(5*a^3*d*x
 + 8*a^2*b + 4*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 18*(a^3*d*x + a^2*b)
*cosh(d*x + c)^2 + 6*(33*a^3*d*x*cosh(d*x + c)^10 + 135*(a^3*d*x + a^2*b)*c
osh(d*x + c)^8 + 42*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c)^6 + 3*a^3
*d*x + 10*(15*a^3*d*x + 27*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c)^4 + 3*a^
2*b + 9*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 -
3*(a^3*cosh(d*x + c)^12 + 12*a^3*cosh(d*x + c)*sinh(d*x + c)^11 + a^3*sinh(
d*x + c)^12 + 6*a^3*cosh(d*x + c)^10 + 15*a^3*cosh(d*x + c)^8 + 6*(11*a^3*c
osh(d*x + c)^2 + a^3)*sinh(d*x + c)^10 + 20*(11*a^3*cosh(d*x + c)^3 + 3*a^3
*cosh(d*x + c))*sinh(d*x + c)^9 + 20*a^3*cosh(d*x + c)^6 + 15*(33*a^3*cosh(
d*x + c)^4 + 18*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^8 + 24*(33*a^3*cos
h(d*x + c)^5 + 30*a^3*cosh(d*x + c)^3 + 5*a^3*cosh(d*x + c))*sinh(d*x + c)^
7 + 15*a^3*cosh(d*x + c)^4 + 4*(231*a^3*cosh(d*x + c)^6 + 315*a^3*cosh(d*x
+ c)^4 + 105*a^3*cosh(d*x + c)^2 + 5*a^3)*sinh(d*x + c)^6 + 24*(33*a^3*cosh
(d*x + c)^7 + 63*a^3*cosh(d*x + c)^5 + 35*a^3*cosh(d*x + c)^3 + 5*a^3*cosh(
d*x + c))*sinh(d*x + c)^5 + 6*a^3*cosh(d*x + c)^2 + 15*(33*a^3*cosh(d*x + c
)^8 + 84*a^3*cosh(d*x + c)^6 + 70*a^3*cosh(d*x + c)^4 + 20*a^3*cosh(d*x + c
)^2 + a^3)*sinh(d*x + c)^4 + 20*(11*a^3*cosh(d*x + c)^9 + 36*a^3*cosh(d*x +
 c)^7 + 42*a^3*cosh(d*x + c)^5 + 20*a^3*cosh(d*x + c)^3 + 3*a^3*cosh(d*x +
c))*sinh(d*x + c)^3 + a^3 + 6*(11*a^3*cosh(d*x + c)^10 + 45*a^3*cosh(d*x +
c)^8 + 70*a^3*cosh(d*x + c)^6 + 50*a^3*cosh(d*x + c)^4 + 15*a^3*cosh(d*x +
c)^2 + a^3)*sinh(d*x + c)^2 + 12*(a^3*cosh(d*x + c)^11 + 5*a^3*cosh(d*x + c
)^9 + 10*a^3*cosh(d*x + c)^7 + 10*a^3*cosh(d*x + c)^5 + 5*a^3*cosh(d*x + c)
^3 + a^3*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) -
 sinh(d*x + c))) + 12*(3*a^3*d*x*cosh(d*x + c)^11 + 15*(a^3*d*x + a^2*b)*co
sh(d*x + c)^9 + 6*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c)^7 + 2*(15*a
^3*d*x + 27*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c)^5 + 3*(5*a^3*d*x + 8*a^
2*b + 4*a*b^2)*cosh(d*x + c)^3 + 3*(a^3*d*x + a^2*b)*cosh(d*x + c))*sinh(d*
x + c))/(d*cosh(d*x + c)^12 + 12*d*cosh(d*x + c)*sinh(d*x + c)^11 + d*sinh(
d*x + c)^12 + 6*d*cosh(d*x + c)^10 + 6*(11*d*cosh(d*x + c)^2 + d)*sinh(d*x
+ c)^10 + 20*(11*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^9 + 1
```

```
5*d*cosh(d*x + c)^8 + 15*(33*d*cosh(d*x + c)^4 + 18*d*cosh(d*x + c)^2 + d)*
sinh(d*x + c)^8 + 24*(33*d*cosh(d*x + c)^5 + 30*d*cosh(d*x + c)^3 + 5*d*cos
h(d*x + c))*sinh(d*x + c)^7 + 20*d*cosh(d*x + c)^6 + 4*(231*d*cosh(d*x + c)
^6 + 315*d*cosh(d*x + c)^4 + 105*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^6 +
 24*(33*d*cosh(d*x + c)^7 + 63*d*cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 + 5
*d*cosh(d*x + c))*sinh(d*x + c)^5 + 15*d*cosh(d*x + c)^4 + 15*(33*d*cosh(d*
x + c)^8 + 84*d*cosh(d*x + c)^6 + 70*d*cosh(d*x + c)^4 + 20*d*cosh(d*x + c)
^2 + d)*sinh(d*x + c)^4 + 20*(11*d*cosh(d*x + c)^9 + 36*d*cosh(d*x + c)^7 +
 42*d*cosh(d*x + c)^5 + 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x
+ c)^3 + 6*d*cosh(d*x + c)^2 + 6*(11*d*cosh(d*x + c)^10 + 45*d*cosh(d*x + c
)^8 + 70*d*cosh(d*x + c)^6 + 50*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 +
d)*sinh(d*x + c)^2 + 12*(d*cosh(d*x + c)^11 + 5*d*cosh(d*x + c)^9 + 10*d*co
sh(d*x + c)^7 + 10*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^3 + d*cosh(d*x + c
))*sinh(d*x + c) + d)
```

Sympy [A] time = 8.92085, size = 87, normalized size = 1.23

$$\begin{cases} a^3x - \frac{a^3\log(\tanh(c+dx)+1)}{d} - \frac{3a^2b\operatorname{sech}^2(c+dx)}{2d} - \frac{3ab^2\operatorname{sech}^4(c+dx)}{4d} - \frac{b^3\operatorname{sech}^6(c+dx)}{6d} & \text{for } d \neq 0 \\ x\left(a+b\operatorname{sech}^2(c)\right)^3\tanh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**3*tanh(d*x+c),x)
```

```
[Out] Piecewise((a**3*x - a**3*log(tanh(c + d*x) + 1)/d - 3*a**2*b*sech(c + d*x)*
*2/(2*d) - 3*a*b**2*sech(c + d*x)**4/(4*d) - b**3*sech(c + d*x)**6/(6*d), N
e(d, 0)), (x*(a + b*sech(c)**2)**3*tanh(c), True))
```

Giac [B] time = 1.23772, size = 362, normalized size = 5.1

$$-\frac{60\,a^3dx - 60\,a^3\log\left(e^{(2\,dx+2\,c)}+1\right) + \frac{147\,a^3e^{(12\,dx+12\,c)} + 882\,a^3e^{(10\,dx+10\,c)} + 360\,a^2be^{(10\,dx+10\,c)} + 2205\,a^3e^{(8\,dx+8\,c)} + 1440\,a^2be^{(8\,dx+8\,c)} + 720\,ab^2e^{(8\,dx+ \cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c),x, algorithm="giac")
```

```
[Out] -1/60*(60*a^3*d*x - 60*a^3*log(e^(2*d*x + 2*c) + 1) + (147*a^3*e^(12*d*x +
12*c) + 882*a^3*e^(10*d*x + 10*c) + 360*a^2*b*e^(10*d*x + 10*c) + 2205*a^3*
e^(8*d*x + 8*c) + 1440*a^2*b*e^(8*d*x + 8*c) + 720*a*b^2*e^(8*d*x + 8*c) +
2940*a^3*e^(6*d*x + 6*c) + 2160*a^2*b*e^(6*d*x + 6*c) + 1440*a*b^2*e^(6*d*x
 + 6*c) + 640*b^3*e^(6*d*x + 6*c) + 2205*a^3*e^(4*d*x + 4*c) + 1440*a^2*b*e
^(4*d*x + 4*c) + 720*a*b^2*e^(4*d*x + 4*c) + 882*a^3*e^(2*d*x + 2*c) + 360*
a^2*b*e^(2*d*x + 2*c) + 147*a^3)/(e^(2*d*x + 2*c) + 1)^6)/d
```

3.128 $\int \left(a+b\,\mathrm{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=73

$$\frac{b\left(3a^2+3ab+b^2\right)\tanh(c+dx)}{d}+a^3x-\frac{b^2(3a+2b)\tanh^3(c+dx)}{3d}+\frac{b^3\tanh^5(c+dx)}{5d}$$

```
[Out] a^3*x + (b*(3*a^2 + 3*a*b + b^2)*Tanh[c + d*x])/d - (b^2*(3*a + 2*b)*Tanh[c
 + d*x]^3)/(3*d) + (b^3*Tanh[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0469913, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4128, 390, 206}

$$\frac{b\left(3a^2+3ab+b^2\right)\tanh(c+dx)}{d}+a^3x-\frac{b^2(3a+2b)\tanh^3(c+dx)}{3d}+\frac{b^3\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] a^3*x + (b*(3*a^2 + 3*a*b + b^2)*Tanh[c + d*x])/d - (b^2*(3*a + 2*b)*Tanh[c
 + d*x]^3)/(3*d) + (b^3*Tanh[c + d*x]^5)/(5*d)
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int \frac{(a+b-bx^2)^3}{1-x^2}dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(b\left(3a^2+3ab+b^2\right)-b^2(3a+2b)x^2+b^3x^4+\frac{a^3}{1-x^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{b\left(3a^2+3ab+b^2\right)\tanh(c+dx)}{d} - \frac{b^2(3a+2b)\tanh^3(c+dx)}{3d} + \frac{b^3\tanh^5(c+dx)}{5d} + \frac{a^3\,\text{Su}}{}$$
$$= a^3x + \frac{b\left(3a^2+3ab+b^2\right)\tanh(c+dx)}{d} - \frac{b^2(3a+2b)\tanh^3(c+dx)}{3d} + \frac{b^3\tanh^5(c+dx)}{5d}$$

Mathematica [B] time = 0.91406, size = 268, normalized size = 3.67

$$\operatorname{sech}(c)\operatorname{sech}^5(c+dx)\left(-360a^2b\sinh(2c+dx)+360a^2b\sinh(2c+3dx)-90a^2b\sinh(4c+3dx)+90a^2b\sinh(4c+5dx)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (Sech[c]*Sech[c + d*x]^5*(150*a^3*d*x*Cosh[d*x] + 150*a^3*d*x*Cosh[2*c + d*
x] + 75*a^3*d*x*Cosh[2*c + 3*d*x] + 75*a^3*d*x*Cosh[4*c + 3*d*x] + 15*a^3*d
*x*Cosh[4*c + 5*d*x] + 15*a^3*d*x*Cosh[6*c + 5*d*x] + 540*a^2*b*Sinh[d*x] +
 420*a*b^2*Sinh[d*x] + 160*b^3*Sinh[d*x] - 360*a^2*b*Sinh[2*c + d*x] - 180*
a*b^2*Sinh[2*c + d*x] + 360*a^2*b*Sinh[2*c + 3*d*x] + 300*a*b^2*Sinh[2*c +
3*d*x] + 80*b^3*Sinh[2*c + 3*d*x] - 90*a^2*b*Sinh[4*c + 3*d*x] + 90*a^2*b*S
inh[4*c + 5*d*x] + 60*a*b^2*Sinh[4*c + 5*d*x] + 16*b^3*Sinh[4*c + 5*d*x]))/
(480*d)
```

Maple [A] time = 0.026, size = 83, normalized size = 1.1

$$\frac{1}{d}\left(a^3(dx+c)+3a^2b\tanh(dx+c)+3ab^2\left(2/3+1/3(\operatorname{sech}(dx+c))^2\right)\tanh(dx+c)+b^3\left(\frac{8}{15}+\frac{(\operatorname{sech}(dx+c))^4}{5}+\frac{4\,(\text{s}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(d*x+c)+3*a^2*b*tanh(d*x+c)+3*a*b^2*(2/3+1/3*sech(d*x+c)^2)*tanh(d
*x+c)+b^3*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c))
```

Maxima [B] time = 1.03359, size = 448, normalized size = 6.14

$$a^3x + \frac{16}{15}b^3\left(\frac{5e^{(-2dx-2c)}}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)} + \frac{}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 16/15*b^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x
 - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1
)) + 10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*
e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*
e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x
- 8*c) + e^(-10*d*x - 10*c) + 1))) + 4*a*b^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-
2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*
d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 6*a^2*b/(d*(e^(
-2*d*x - 2*c) + 1))
```

Fricas [B] time = 2.49725, size = 1173, normalized size = 16.07

$$\frac{\left(15\,a^3dx - 45\,a^2b - 30\,ab^2 - 8\,b^3\right)\cosh\left(dx+c\right)^5 + 5\left(15\,a^3dx - 45\,a^2b - 30\,ab^2 - 8\,b^3\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4 + \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*((15*a^3*d*x - 45*a^2*b - 30*a*b^2 - 8*b^3)*cosh(d*x + c)^5 + 5*(15*a^
3*d*x - 45*a^2*b - 30*a*b^2 - 8*b^3)*cosh(d*x + c)*sinh(d*x + c)^4 + (45*a^
2*b + 30*a*b^2 + 8*b^3)*sinh(d*x + c)^5 + 5*(15*a^3*d*x - 45*a^2*b - 30*a*b
^2 - 8*b^3)*cosh(d*x + c)^3 + 5*(27*a^2*b + 30*a*b^2 + 8*b^3 + 2*(45*a^2*b
+ 30*a*b^2 + 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 5*(2*(15*a^3*d*x - 4
5*a^2*b - 30*a*b^2 - 8*b^3)*cosh(d*x + c)^3 + 3*(15*a^3*d*x - 45*a^2*b - 30
*a*b^2 - 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(15*a^3*d*x - 45*a^2*b
- 30*a*b^2 - 8*b^3)*cosh(d*x + c) + 5*((45*a^2*b + 30*a*b^2 + 8*b^3)*cosh(d
*x + c)^4 + 18*a^2*b + 24*a*b^2 + 16*b^3 + 3*(27*a^2*b + 30*a*b^2 + 8*b^3)*
cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh
(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x +
 c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\,\mathrm{sech}^2\left(c+dx\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3, x)
```

Giac [B] time = 1.16311, size = 246, normalized size = 3.37

$$\frac{(dx+c)a^3}{d} - \frac{2\left(45\,a^2be^{(8\,dx+8\,c)} + 180\,a^2be^{(6\,dx+6\,c)} + 90\,ab^2e^{(6\,dx+6\,c)} + 270\,a^2be^{(4\,dx+4\,c)} + 210\,ab^2e^{(4\,dx+4\,c)} + 80\,b^3e^{(4\,dx+\cdots} \cdots\right)}{15\,d\left(e^{(2\,dx+2\,c)}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] (d*x + c)*a^3/d - 2/15*(45*a^2*b*e^(8*d*x + 8*c) + 180*a^2*b*e^(6*d*x + 6*c
) + 90*a*b^2*e^(6*d*x + 6*c) + 270*a^2*b*e^(4*d*x + 4*c) + 210*a*b^2*e^(4*d
*x + 4*c) + 80*b^3*e^(4*d*x + 4*c) + 180*a^2*b*e^(2*d*x + 2*c) + 150*a*b^2*
e^(2*d*x + 2*c) + 40*b^3*e^(2*d*x + 2*c) + 45*a^2*b + 30*a*b^2 + 8*b^3)/(d*
(e^(2*d*x + 2*c) + 1)^5)
```

3.129 $\int \coth(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=84

$$-\frac{b\left(3a^2+3ab+b^2\right)\log(\cosh(c+dx))}{d}+\frac{b^2(3a+b)\text{sech}^2(c+dx)}{2d}+\frac{(a+b)^3\log(\sinh(c+dx))}{d}+\frac{b^3\text{sech}^4(c+dx)}{4d}$$

```
[Out] -((b*(3*a^2 + 3*a*b + b^2)*Log[Cosh[c + d*x]])/d) + ((a + b)^3*Log[Sinh[c +
 d*x]])/d + (b^2*(3*a + b)*Sech[c + d*x]^2)/(2*d) + (b^3*Sech[c + d*x]^4)/(
4*d)
```

Rubi [A] time = 0.106783, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 446, 88}

$$-\frac{b\left(3a^2+3ab+b^2\right)\log(\cosh(c+dx))}{d}+\frac{b^2(3a+b)\text{sech}^2(c+dx)}{2d}+\frac{(a+b)^3\log(\sinh(c+dx))}{d}+\frac{b^3\text{sech}^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]
```


```
[Out] -((b*(3*a^2 + 3*a*b + b^2)*Log[Cosh[c + d*x]])/d) + ((a + b)^3*Log[Sinh[c +
 d*x]])/d + (b^2*(3*a + b)*Sech[c + d*x]^2)/(2*d) + (b^3*Sech[c + d*x]^4)/(
4*d)
```


Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \coth(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^3\,dx = -\frac{\mathrm{Subst}\left(\int \frac{(b+ax^2)^3}{x^5(1-x^2)}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\mathrm{Subst}\left(\int \frac{(b+ax)^3}{(1-x)x^3}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{\mathrm{Subst}\left(\int \left(-\frac{(a+b)^3}{-1+x}+\frac{b^3}{x^3}+\frac{b^2(3a+b)}{x^2}+\frac{b(3a^2+3ab+b^2)}{x}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{b\left(3a^2+3ab+b^2\right)\log(\cosh(c+dx))}{d}+\frac{(a+b)^3\log(\sinh(c+dx))}{d}+\frac{b^2(3a$$

Mathematica [A] time = 0.619703, size = 114, normalized size = 1.36

$$-\frac{2\cosh^6(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^3\left(4b\left(3a^2+3ab+b^2\right)\log(\cosh(c+dx))-2b^2(3a+b)\mathrm{sech}^2(c+dx)-4(a+b)^3\log\right.}{d(a\cosh(2c+2dx)+a+2b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (-2*Cosh[c + d*x]^6*(a + b*Sech[c + d*x]^2)^3*(4*b*(3*a^2 + 3*a*b + b^2)*Lo
g[Cosh[c + d*x]] - 4*(a + b)^3*Log[Sinh[c + d*x]] - 2*b^2*(3*a + b)*Sech[c
+ d*x]^2 - b^3*Sech[c + d*x]^4))/(d*(a + 2*b + a*Cosh[2*c + 2*d*x])^3)
```

Maple [A] time = 0.048, size = 111, normalized size = 1.3

$$\frac{a^3\ln(\sinh(dx+c))}{d}+3\,\frac{a^2b\ln(\tanh(dx+c))}{d}+\frac{3\,ab^2}{2\,d(\cosh(dx+c))^2}+3\,\frac{ab^2\ln(\tanh(dx+c))}{d}+\frac{b^3}{4\,d(\cosh(dx+c))^4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d*a^3*ln(sinh(d*x+c))+3/d*a^2*b*ln(tanh(d*x+c))+3/2/d*a*b^2/cosh(d*x+c)^2
+3/d*a*b^2*ln(tanh(d*x+c))+1/4/d*b^3/cosh(d*x+c)^4+1/2/d*b^3/cosh(d*x+c)^2+
1/d*b^3*ln(tanh(d*x+c))
```

Maxima [B] time = 1.62742, size = 405, normalized size = 4.82

$$b^3\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\left(e^{(-2\,dx-2\,c)}+4\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}{d\left(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] b^3*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x - 2*
c) + 1)/d + 2*(e^(-2*d*x - 2*c) + 4*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d
*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x
```

```
- 8*c) + 1))) + 3*a*b^2*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d
- log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) +
 e^(-4*d*x - 4*c) + 1))) + 3*a^2*b*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x -
 c) - 1)/d - log(e^(-2*d*x - 2*c) + 1)/d) + a^3*log(sinh(d*x + c))/d
```

Fricas [B] time = 2.72209, size = 5819, normalized size = 69.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -(a^3*d*x*cosh(d*x + c)^8 + 8*a^3*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + a^3*d
*x*sinh(d*x + c)^8 + 2*(2*a^3*d*x - 3*a*b^2 - b^3)*cosh(d*x + c)^6 + 2*(14*
a^3*d*x*cosh(d*x + c)^2 + 2*a^3*d*x - 3*a*b^2 - b^3)*sinh(d*x + c)^6 + 4*(1
4*a^3*d*x*cosh(d*x + c)^3 + 3*(2*a^3*d*x - 3*a*b^2 - b^3)*cosh(d*x + c))*si
nh(d*x + c)^5 + a^3*d*x + 2*(3*a^3*d*x - 6*a*b^2 - 4*b^3)*cosh(d*x + c)^4 +
 2*(35*a^3*d*x*cosh(d*x + c)^4 + 3*a^3*d*x - 6*a*b^2 - 4*b^3 + 15*(2*a^3*d*
x - 3*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*a^3*d*x*cosh(d*x
 + c)^5 + 5*(2*a^3*d*x - 3*a*b^2 - b^3)*cosh(d*x + c)^3 + (3*a^3*d*x - 6*a*
b^2 - 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(2*a^3*d*x - 3*a*b^2 - b^3)
*cosh(d*x + c)^2 + 2*(14*a^3*d*x*cosh(d*x + c)^6 + 2*a^3*d*x + 15*(2*a^3*d*
x - 3*a*b^2 - b^3)*cosh(d*x + c)^4 - 3*a*b^2 - b^3 + 6*(3*a^3*d*x - 6*a*b^2
 - 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((3*a^2*b + 3*a*b^2 + b^3)*cos
h(d*x + c)^8 + 8*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^7 +
(3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^8 + 4*(3*a^2*b + 3*a*b^2 + b^3)*cos
h(d*x + c)^6 + 4*(3*a^2*b + 3*a*b^2 + b^3 + 7*(3*a^2*b + 3*a*b^2 + b^3)*cos
h(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c
)^3 + 3*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(3*a^2
*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 2*(35*(3*a^2*b + 3*a*b^2 + b^3)*cosh(
d*x + c)^4 + 9*a^2*b + 9*a*b^2 + 3*b^3 + 30*(3*a^2*b + 3*a*b^2 + b^3)*cosh(
d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^
5 + 10*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + 3*(3*a^2*b + 3*a*b^2 + b
^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*a^2*b + 3*a*b^2 + b^3 + 4*(3*a^2*b +
 3*a*b^2 + b^3)*cosh(d*x + c)^2 + 4*(7*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x +
 c)^6 + 15*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 3*a^2*b + 3*a*b^2 +
b^3 + 9*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*
a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 3*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d
*x + c)^5 + 3*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (3*a^2*b + 3*a*b^
2 + b^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) -
 sinh(d*x + c))) - ((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 8*(a^
3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*sinh(d*x + c)^8 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(
d*x + c)^6 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 7*(a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b
^3)*cosh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh
(d*x + c)^5 + 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 2*(35*(a^
3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 3*a^3 + 9*a^2*b + 9*a*b^2 +
3*b^3 + 30*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4
 + 8*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 10*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(
d*x + c))*sinh(d*x + c)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 4*(a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*cosh(d*x + c)^2 + 4*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c
osh(d*x + c)^6 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + a^3 +
 3*a^2*b + 3*a*b^2 + b^3 + 9*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^2 + 8*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 3
```

```
*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 3*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*cosh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*
sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(2*
a^3*d*x*cosh(d*x + c)^7 + 3*(2*a^3*d*x - 3*a*b^2 - b^3)*cosh(d*x + c)^5 + 2
*(3*a^3*d*x - 6*a*b^2 - 4*b^3)*cosh(d*x + c)^3 + (2*a^3*d*x - 3*a*b^2 - b^3
)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh
(d*x + c)^7 + d*sinh(d*x + c)^8 + 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c
)^2 + d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh
(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 + 30*d*cosh(d*x
 + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c
)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*d*cosh(d*x + c)^2 + 4*(7*d*cos
h(d*x + c)^6 + 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c
)^2 + 8*(d*cosh(d*x + c)^7 + 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 + d*
cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20856, size = 439, normalized size = 5.23

$$-\frac{12\,a^3dx+12\left(3\,a^2be^{(2\,c)}+3\,ab^2e^{(2\,c)}+b^3e^{(2\,c)}\right)e^{(-2\,c)}\log\left(e^{(2\,dx+2\,c)}+1\right)-12\left(a^3e^{(2\,c)}+3\,a^2be^{(2\,c)}+3\,ab^2e^{(2\,c)}+b^3e^{(2\,c)}\right)e^{(-}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/12*(12*a^3*d*x + 12*(3*a^2*b*e^(2*c) + 3*a*b^2*e^(2*c) + b^3*e^(2*c))*e^
(-2*c)*log(e^(2*d*x + 2*c) + 1) - 12*(a^3*e^(2*c) + 3*a^2*b*e^(2*c) + 3*a*b
^2*e^(2*c) + b^3*e^(2*c))*e^(-2*c)*log(abs(e^(2*d*x + 2*c) - 1)) - (75*a^2*
b*e^(8*d*x + 8*c) + 75*a*b^2*e^(8*d*x + 8*c) + 25*b^3*e^(8*d*x + 8*c) + 300
*a^2*b*e^(6*d*x + 6*c) + 372*a*b^2*e^(6*d*x + 6*c) + 124*b^3*e^(6*d*x + 6*c
) + 450*a^2*b*e^(4*d*x + 4*c) + 594*a*b^2*e^(4*d*x + 4*c) + 246*b^3*e^(4*d*
x + 4*c) + 300*a^2*b*e^(2*d*x + 2*c) + 372*a*b^2*e^(2*d*x + 2*c) + 124*b^3*
e^(2*d*x + 2*c) + 75*a^2*b + 75*a*b^2 + 25*b^3)/(e^(2*d*x + 2*c) + 1)^4)/d
```

3.130 $\int \coth^2(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=61

$$a^3x-\frac{b^2(3a+2b)\tanh(c+dx)}{d}-\frac{(a+b)^3\coth(c+dx)}{d}+\frac{b^3\tanh^3(c+dx)}{3d}$$

```
[Out] a^3*x - ((a + b)^3*Coth[c + d*x])/d - (b^2*(3*a + 2*b)*Tanh[c + d*x])/d + (
b^3*Tanh[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.102519, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4141, 1802, 207}

$$a^3x-\frac{b^2(3a+2b)\tanh(c+dx)}{d}-\frac{(a+b)^3\coth(c+dx)}{d}+\frac{b^3\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] a^3*x - ((a + b)^3*Coth[c + d*x])/d - (b^2*(3*a + 2*b)*Tanh[c + d*x])/d + (
b^3*Tanh[c + d*x]^3)/(3*d)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \coth^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b(1-x^2)\right)^3}{x^2(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-b^2(3a+2b)+\frac{(a+b)^3}{x^2}+b^3x^2-\frac{a^3}{-1+x^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{(a+b)^3\coth(c+dx)}{d}-\frac{b^2(3a+2b)\tanh(c+dx)}{d}+\frac{b^3\tanh^3(c+dx)}{3d}-$$

$$= a^3x-\frac{(a+b)^3\coth(c+dx)}{d}-\frac{b^2(3a+2b)\tanh(c+dx)}{d}+\frac{b^3\tanh^3(c+dx)}{3d}$$

Mathematica [B] time = 1.76005, size = 126, normalized size = 2.07

$$\frac{8(a\cosh(c+dx)+b\text{sech}(c+dx))^3\left(3a^3dx\cosh^3(c+dx)+\sinh(dx)\cosh^2(c+dx)\left(3(a+b)^3\text{csch}(c)\coth(c+dx)-b^2(9\right.\right.}{3d(a\cosh(2(c+dx))+a+2b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (8*(a*Cosh[c + d*x] + b*Sech[c + d*x])^3*(3*a^3*d*x*Cosh[c + d*x]^3 - b^3*S
ech[c]*Sinh[d*x] + Cosh[c + d*x]^2*(3*(a + b)^3*Coth[c + d*x]*Csch[c] - b^2
*(9*a + 5*b)*Sech[c])*Sinh[d*x] - b^3*Cosh[c + d*x]*Tanh[c]))/(3*d*(a + 2*b
 + a*Cosh[2*(c + d*x)])^3)
```

Maple [A] time = 0.043, size = 111, normalized size = 1.8

$$\frac{1}{d}\left(a^3\left(dx+c-\coth\left(dx+c\right)\right)-3a^2b\coth\left(dx+c\right)+3ab^2\left(-\frac{1}{\cosh\left(dx+c\right)\sinh\left(dx+c\right)}-2\tanh\left(dx+c\right)\right)+b^3\left(-\frac{}{\sinh}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(d*x+c-coth(d*x+c))-3*a^2*b*coth(d*x+c)+3*a*b^2*(-1/sinh(d*x+c)/co
sh(d*x+c)-2*tanh(d*x+c))+b^3*(-1/sinh(d*x+c)/cosh(d*x+c)^3-4*(2/3+1/3*sech(
d*x+c)^2)*tanh(d*x+c)))
```

Maxima [B] time = 1.17231, size = 232, normalized size = 3.8

$$a^3\left(x+\frac{c}{d}+\frac{2}{d\left(e^{(-2\,dx-2\,c)}-1\right)}\right)-\frac{16}{3}b^3\left(\frac{2\,e^{(-2\,dx-2\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)}-2\,e^{(-6\,dx-6\,c)}-e^{(-8\,dx-8\,c)}+1\right)}+\frac{1}{d\left(2\,e^{(-2\,dx-2\,c)}-2\,e^{(-6\,dx-6\,c)}-e^{(-8\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a^3*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) - 16/3*b^3*(2*e^(-2*d*x - 2*c)
/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) + 1)) + 1/(
```

```
d*(2*e^(-2*d*x - 2*c) - 2*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) + 1))) + 6*a^
2*b/(d*(e^(-2*d*x - 2*c) - 1)) + 12*a*b^2/(d*(e^(-4*d*x - 4*c) - 1))
```

Fricas [B] time = 2.4562, size = 863, normalized size = 14.15

$$-\frac{\left(3\,a^{3}+9\,a^{2}b+18\,ab^{2}+8\,b^{3}\right)\cosh\left(dx+c\right)^{4}-4\left(3\,a^{3}dx+3\,a^{3}+9\,a^{2}b+18\,ab^{2}+8\,b^{3}\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/12*((3*a^3 + 9*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c)^4 - 4*(3*a^3*d*x
+ 3*a^3 + 9*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^
3 + 9*a^2*b + 18*a*b^2 + 8*b^3)*sinh(d*x + c)^4 + 9*a^3 + 27*a^2*b + 18*a*b
^2 + 4*(3*a^3 + 9*a^2*b + 9*a*b^2 + 4*b^3)*cosh(d*x + c)^2 + 2*(6*a^3 + 18*
a^2*b + 18*a*b^2 + 8*b^3 + 3*(3*a^3 + 9*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x
+ c)^2)*sinh(d*x + c)^2 - 4*((3*a^3*d*x + 3*a^3 + 9*a^2*b + 18*a*b^2 + 8*b^
3)*cosh(d*x + c)^3 + (3*a^3*d*x + 3*a^3 + 9*a^2*b + 18*a*b^2 + 8*b^3)*cosh(
d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)*sinh(d*x + c)^3 + (d*cosh(d*x + c
)^3 + d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26706, size = 178, normalized size = 2.92

$$\frac{3\,a^{3}dx-\frac{6\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)}{e^{\left(2\,dx+2\,c\right)}-1}+\frac{2\left(9\,ab^{2}e^{\left(4\,dx+4\,c\right)}+3\,b^{3}e^{\left(4\,dx+4\,c\right)}+18\,ab^{2}e^{\left(2\,dx+2\,c\right)}+12\,b^{3}e^{\left(2\,dx+2\,c\right)}+9\,ab^{2}+5\,b^{3}\right)}{\left(e^{\left(2\,dx+2\,c\right)}+1\right)^{3}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^3*d*x - 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)/(e^(2*d*x + 2*c) - 1) +
2*(9*a*b^2*e^(4*d*x + 4*c) + 3*b^3*e^(4*d*x + 4*c) + 18*a*b^2*e^(2*d*x + 2*
c) + 12*b^3*e^(2*d*x + 2*c) + 9*a*b^2 + 5*b^3)/(e^(2*d*x + 2*c) + 1)^3)/d
```

3.131 $\int \coth^3(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=81

$$\frac{b^2(3a+2b)\log(\cosh(c+dx))}{d}-\frac{(a+b)^3\mathrm{csch}^2(c+dx)}{2d}+\frac{(a-2b)(a+b)^2\log(\sinh(c+dx))}{d}-\frac{b^3\mathrm{sech}^2(c+dx)}{2d}$$

```
[Out] -((a + b)^3*Csch[c + d*x]^2)/(2*d) + (b^2*(3*a + 2*b)*Log[Cosh[c + d*x]])/d
 + ((a - 2*b)*(a + b)^2*Log[Sinh[c + d*x]])/d - (b^3*Sech[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.116445, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4138, 446, 88}

$$\frac{b^2(3a+2b)\log(\cosh(c+dx))}{d}-\frac{(a+b)^3\mathrm{csch}^2(c+dx)}{2d}+\frac{(a-2b)(a+b)^2\log(\sinh(c+dx))}{d}-\frac{b^3\mathrm{sech}^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]
```


```
[Out] -((a + b)^3*Csch[c + d*x]^2)/(2*d) + (b^2*(3*a + 2*b)*Log[Cosh[c + d*x]])/d
 + ((a - 2*b)*(a + b)^2*Log[Sinh[c + d*x]])/d - (b^3*Sech[c + d*x]^2)/(2*d)
```


Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \coth^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{(b+ax^2)^3}{x^3(1-x^2)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(b+ax)^3}{(1-x)^2x^2}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{(a+b)^3}{(-1+x)^2} + \frac{(a-2b)(a+b)^2}{-1+x} + \frac{b^3}{x^2} + \frac{b^2(3a+2b)}{x}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{(a+b)^3\text{csch}^2(c+dx)}{2d} + \frac{b^2(3a+2b)\log(\cosh(c+dx))}{d} + \frac{(a-2b)(a+$$

Mathematica [A] time = 1.22554, size = 110, normalized size = 1.36

$$-\frac{4\cosh^6(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\left(-2b^2(3a+2b)\log(\cosh(c+dx)) + (a+b)^3\text{csch}^2(c+dx) - 2(a-2b)(a+b)^2}{d(a\cosh(2c+2dx)+a+2b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (-4*Cosh[c + d*x]^6*(a + b*Sech[c + d*x]^2)^3*((a + b)^3*Csch[c + d*x]^2 -
2*b^2*(3*a + 2*b)*Log[Cosh[c + d*x]] - 2*(a - 2*b)*(a + b)^2*Log[Sinh[c + d
*x]] + b^3*Sech[c + d*x]^2))/(d*(a + 2*b + a*Cosh[2*c + 2*d*x])^3)
```

Maple [A] time = 0.053, size = 145, normalized size = 1.8

$$\frac{a^3\ln(\sinh(dx+c))}{d} - \frac{a^3(\coth(dx+c))^2}{2d} - \frac{3a^2b(\cosh(dx+c))^2}{2d(\sinh(dx+c))^2} - \frac{3ab^2}{2d(\sinh(dx+c))^2} - 3\frac{ab^2\ln(\tanh(dx+c))}{d} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d*a^3*ln(sinh(d*x+c))-1/2*a^3*coth(d*x+c)^2/d-3/2/d*a^2*b*cosh(d*x+c)^2/s
inh(d*x+c)^2-3/2/d*a*b^2/sinh(d*x+c)^2-3/d*a*b^2*ln(tanh(d*x+c))-1/2/d*b^3/
sinh(d*x+c)^2/cosh(d*x+c)^2-1/d*b^3/cosh(d*x+c)^2-2/d*b^3*ln(tanh(d*x+c))
```

Maxima [B] time = 1.78659, size = 424, normalized size = 5.23

$$a^3\left(x + \frac{c}{d} + \frac{\log\left(e^{(-dx-c)}+1\right)}{d} + \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2e^{(-2dx-2c)}}{d\left(2e^{(-2dx-2c)} - e^{(-4dx-4c)} - 1\right)}\right) - 2b^3\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} + \frac{\log\left(e^{(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a^3*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*
d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 2*b^3*(log(e^
(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x - 2*c) + 1)/d -
```

```
 2*(e^(-2*d*x - 2*c) + e^(-6*d*x - 6*c))/(d*(2*e^(-4*d*x - 4*c) - e^(-8*d*x
 - 8*c) - 1))) - 3*a*b^2*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d
 - log(e^(-2*d*x - 2*c) + 1)/d - 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c)
- e^(-4*d*x - 4*c) - 1))) - 6*a^2*b/(d*(e^(d*x + c) - e^(-d*x - c))^2)
```

Fricas [B] time = 2.72197, size = 4127, normalized size = 50.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -(a^3*d*x*cosh(d*x + c)^8 + 8*a^3*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + a^3*d
*x*sinh(d*x + c)^8 + 2*(a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)^6 +
2*(14*a^3*d*x*cosh(d*x + c)^2 + a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*sinh(d*x +
 c)^6 + 4*(14*a^3*d*x*cosh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)
*cosh(d*x + c))*sinh(d*x + c)^5 + a^3*d*x - 2*(a^3*d*x - 2*a^3 - 6*a^2*b -
6*a*b^2)*cosh(d*x + c)^4 + 2*(35*a^3*d*x*cosh(d*x + c)^4 - a^3*d*x + 2*a^3
+ 6*a^2*b + 6*a*b^2 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^4 + 8*(7*a^3*d*x*cosh(d*x + c)^5 + 5*(a^3 + 3*a^2*b + 3*a*b^
2 + 2*b^3)*cosh(d*x + c)^3 - (a^3*d*x - 2*a^3 - 6*a^2*b - 6*a*b^2)*cosh(d*x
 + c))*sinh(d*x + c)^3 + 2*(a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)^
2 + 2*(14*a^3*d*x*cosh(d*x + c)^6 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*co
sh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3 - 6*(a^3*d*x - 2*a^3 - 6*a^
2*b - 6*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((3*a*b^2 + 2*b^3)*cosh(d
*x + c)^8 + 56*(3*a*b^2 + 2*b^3)*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*(3*a*
b^2 + 2*b^3)*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*(3*a*b^2 + 2*b^3)*cosh(d*x
 + c)*sinh(d*x + c)^7 + (3*a*b^2 + 2*b^3)*sinh(d*x + c)^8 - 2*(3*a*b^2 + 2*
b^3)*cosh(d*x + c)^4 + 2*(35*(3*a*b^2 + 2*b^3)*cosh(d*x + c)^4 - 3*a*b^2 -
2*b^3)*sinh(d*x + c)^4 + 8*(7*(3*a*b^2 + 2*b^3)*cosh(d*x + c)^5 - (3*a*b^2
+ 2*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*a*b^2 + 2*b^3 + 4*(7*(3*a*b^2 +
 2*b^3)*cosh(d*x + c)^6 - 3*(3*a*b^2 + 2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c
)^2 + 8*((3*a*b^2 + 2*b^3)*cosh(d*x + c)^7 - (3*a*b^2 + 2*b^3)*cosh(d*x + c
)^3)*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) -
((a^3 - 3*a*b^2 - 2*b^3)*cosh(d*x + c)^8 + 56*(a^3 - 3*a*b^2 - 2*b^3)*cosh(
d*x + c)^3*sinh(d*x + c)^5 + 28*(a^3 - 3*a*b^2 - 2*b^3)*cosh(d*x + c)^2*sin
h(d*x + c)^6 + 8*(a^3 - 3*a*b^2 - 2*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (a
^3 - 3*a*b^2 - 2*b^3)*sinh(d*x + c)^8 - 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(d*x
+ c)^4 + 2*(35*(a^3 - 3*a*b^2 - 2*b^3)*cosh(d*x + c)^4 - a^3 + 3*a*b^2 + 2*
b^3)*sinh(d*x + c)^4 + 8*(7*(a^3 - 3*a*b^2 - 2*b^3)*cosh(d*x + c)^5 - (a^3
- 3*a*b^2 - 2*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + a^3 - 3*a*b^2 - 2*b^3 +
 4*(7*(a^3 - 3*a*b^2 - 2*b^3)*cosh(d*x + c)^6 - 3*(a^3 - 3*a*b^2 - 2*b^3)*c
osh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^3 - 3*a*b^2 - 2*b^3)*cosh(d*x + c)^
7 - (a^3 - 3*a*b^2 - 2*b^3)*cosh(d*x + c)^3)*sinh(d*x + c))*log(2*sinh(d*x
+ c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(2*a^3*d*x*cosh(d*x + c)^7 + 3*(a
^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)^5 - 2*(a^3*d*x - 2*a^3 - 6*a^
2*b - 6*a*b^2)*cosh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x
 + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 56*d*cosh(d*x + c)^3*sinh(d*x +
c)^5 + 28*d*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*d*cosh(d*x + c)*sinh(d*x +
c)^7 + d*sinh(d*x + c)^8 - 2*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 -
d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - d*cosh(d*x + c))*sinh(d*x + c
)^3 + 4*(7*d*cosh(d*x + c)^6 - 3*d*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(d*
cosh(d*x + c)^7 - d*cosh(d*x + c)^3)*sinh(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.40154, size = 394, normalized size = 4.86

$$-\frac{4\,a^3dx - 4\left(3\,ab^2e^{(2\,c)} + 2\,b^3e^{(2\,c)}\right)e^{(-2\,c)}\log\left(e^{(2\,dx+2\,c)} + 1\right) - 4\left(a^3e^{(2\,c)} - 3\,ab^2e^{(2\,c)} - 2\,b^3e^{(2\,c)}\right)e^{(-2\,c)}\log\left(\left|e^{(2\,dx+2\,c)} - 1\right|\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/4*(4*a^3*d*x - 4*(3*a*b^2*e^(2*c) + 2*b^3*e^(2*c))*e^(-2*c)*log(e^(2*d*x
 + 2*c) + 1) - 4*(a^3*e^(2*c) - 3*a*b^2*e^(2*c) - 2*b^3*e^(2*c))*e^(-2*c)*l
og(abs(e^(2*d*x + 2*c) - 1)) + (3*a^3*e^(8*d*x + 8*c) + 8*a^3*e^(6*d*x + 6*
c) + 24*a^2*b*e^(6*d*x + 6*c) + 24*a*b^2*e^(6*d*x + 6*c) + 16*b^3*e^(6*d*x
+ 6*c) + 10*a^3*e^(4*d*x + 4*c) + 48*a^2*b*e^(4*d*x + 4*c) + 48*a*b^2*e^(4*
d*x + 4*c) + 8*a^3*e^(2*d*x + 2*c) + 24*a^2*b*e^(2*d*x + 2*c) + 24*a*b^2*e^
(2*d*x + 2*c) + 16*b^3*e^(2*d*x + 2*c) + 3*a^3)/(e^(4*d*x + 4*c) - 1)^2)/d
```

3.132 $\int \coth^4(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=60

$$a^3x-\frac{(a+b)^3\coth^3(c+dx)}{3d}-\frac{(a-2b)(a+b)^2\coth(c+dx)}{d}+\frac{b^3\tanh(c+dx)}{d}$$

```
[Out] a^3*x - ((a - 2*b)*(a + b)^2*Coth[c + d*x])/d - ((a + b)^3*Coth[c + d*x]^3)
/(3*d) + (b^3*Tanh[c + d*x])/d
```

Rubi [A] time = 0.100946, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4141, 1802, 207}

$$a^3x-\frac{(a+b)^3\coth^3(c+dx)}{3d}-\frac{(a-2b)(a+b)^2\coth(c+dx)}{d}+\frac{b^3\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] a^3*x - ((a - 2*b)*(a + b)^2*Coth[c + d*x])/d - ((a + b)^3*Coth[c + d*x]^3)
/(3*d) + (b^3*Tanh[c + d*x])/d
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \coth^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx=\frac{\text{Subst}\left(\int\frac{\left(a+b(1-x^2)\right)^3}{x^4(1-x^2)}\,dx,x,\tanh(c+dx)\right)}{d}$$
$$=\frac{\text{Subst}\left(\int\left(b^3+\frac{(a+b)^3}{x^4}+\frac{(a-2b)(a+b)^2}{x^2}-\frac{a^3}{-1+x^2}\right)dx,x,\tanh(c+dx)\right)}{d}$$
$$=-\frac{(a-2b)(a+b)^2\coth(c+dx)}{d}-\frac{(a+b)^3\coth^3(c+dx)}{3d}+\frac{b^3\tanh(c+\ldots}{d}$$
$$=a^3x-\frac{(a-2b)(a+b)^2\coth(c+dx)}{d}-\frac{(a+b)^3\coth^3(c+dx)}{3d}+\frac{b^3\tanh\ldots}{d}$$

Mathematica [B] time = 1.70692, size = 343, normalized size = 5.72

$$\frac{\text{csch}(c)\text{sech}(c)\text{csch}^3(c+dx)\text{sech}(c+dx)\left(-12a^2b\sinh(2(c+dx))+6a^2b\sinh(4(c+dx))+6a^2b\sinh(2(c+2dx))-\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (Csch[c]*Csch[c + d*x]^3*Sech[c]*Sech[c + d*x]*(6*a^3*d*x*Cosh[2*d*x] - 3*a
^3*d*x*Cosh[2*(c + 2*d*x)] - 6*a^3*d*x*Cosh[4*c + 2*d*x] + 3*a^3*d*x*Cosh[6
*c + 4*d*x] - 18*a^2*b*Sinh[2*c] - 36*a*b^2*Sinh[2*c] - 4*a^3*Sinh[2*d*x] +
 6*a^2*b*Sinh[2*d*x] + 24*a*b^2*Sinh[2*d*x] + 32*b^3*Sinh[2*d*x] - 16*a^3*S
inh[2*(c + d*x)] - 12*a^2*b*Sinh[2*(c + d*x)] + 24*a*b^2*Sinh[2*(c + d*x)]
+ 8*b^3*Sinh[2*(c + d*x)] + 8*a^3*Sinh[4*(c + d*x)] + 6*a^2*b*Sinh[4*(c + d
*x)] - 12*a*b^2*Sinh[4*(c + d*x)] - 4*b^3*Sinh[4*(c + d*x)] + 8*a^3*Sinh[2*
(c + 2*d*x)] + 6*a^2*b*Sinh[2*(c + 2*d*x)] - 12*a*b^2*Sinh[2*(c + 2*d*x)] -
 16*b^3*Sinh[2*(c + 2*d*x)] - 12*a^3*Sinh[4*c + 2*d*x] - 18*a^2*b*Sinh[4*c
+ 2*d*x]))/(96*d)
```

Maple [B] time = 0.043, size = 149, normalized size = 2.5

$$\frac{1}{d}\left(a^3\left(dx+c-\coth(dx+c)-\frac{(\coth(dx+c))^3}{3}\right)+3a^2b\left(-1/2\frac{\cosh(dx+c)}{(\sinh(dx+c))^3}-1/2\left(2/3-1/3(\text{csch}(dx+c))^2\right)\text{cot}\ldots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(d*x+c-coth(d*x+c)-1/3*coth(d*x+c)^3)+3*a^2*b*(-1/2/sinh(d*x+c)^3*
cosh(d*x+c)-1/2*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c))+3*a*b^2*(2/3-1/3*csch(
d*x+c)^2)*coth(d*x+c)+b^3*(-1/3/sinh(d*x+c)^3/cosh(d*x+c)+4/3/sinh(d*x+c)/c
osh(d*x+c)+8/3*tanh(d*x+c)))
```

Maxima [B] time = 1.2279, size = 494, normalized size = 8.23

$$\frac{1}{3}a^3\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}-2\right)}{d\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1\right)}\right)+4ab^2\left(\frac{3e^{(-2dx-2c)}}{d\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1\right)}-\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d*(
3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + 4*a*b^2
*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*
x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x
- 6*c) - 1))) + 16/3*b^3*(2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - 2*e^(
-6*d*x - 6*c) + e^(-8*d*x - 8*c) - 1)) - 1/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-6
*d*x - 6*c) + e^(-8*d*x - 8*c) - 1))) + 2*a^2*b*(3*e^(-4*d*x - 4*c)/(d*(3*e
^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) + 1/(d*(3*e^(
-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

Fricas [B] time = 2.37433, size = 837, normalized size = 13.95

$$-\frac{\left(4\,a^3+3\,a^2b-6\,ab^2-8\,b^3\right)\cosh\left(dx+c\right)^4-4\left(3\,a^3dx+4\,a^3+3\,a^2b-6\,ab^2-8\,b^3\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+\left(4\,a^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/12*((4*a^3 + 3*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c)^4 - 4*(3*a^3*d*x +
 4*a^3 + 3*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (4*a^3
+ 3*a^2*b - 6*a*b^2 - 8*b^3)*sinh(d*x + c)^4 + 9*a^2*b + 18*a*b^2 + 4*(a^3
+ 3*a^2*b + 3*a*b^2 + 4*b^3)*cosh(d*x + c)^2 + 2*(2*a^3 + 6*a^2*b + 6*a*b^2
 + 8*b^3 + 3*(4*a^3 + 3*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c)^2)*sinh(d*x
+ c)^2 - 4*((3*a^3*d*x + 4*a^3 + 3*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c)^3
 - (3*a^3*d*x + 4*a^3 + 3*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c))*sinh(d*x
+ c))/(d*cosh(d*x + c)*sinh(d*x + c)^3 + (d*cosh(d*x + c)^3 - d*cosh(d*x +
c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**4*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.42265, size = 209, normalized size = 3.48

$$\frac{3\,a^3dx-\frac{6\,b^3}{e^{(2\,dx+2\,c)}+1}-\frac{2\left(6\,a^3e^{(4\,dx+4\,c)}+9\,a^2be^{(4\,dx+4\,c)}-3\,b^3e^{(4\,dx+4\,c)}-6\,a^3e^{(2\,dx+2\,c)}+18\,ab^2e^{(2\,dx+2\,c)}+12\,b^3e^{(2\,dx+2\,c)}+4\,a^3+3\,a^2b-6\,ab^2-5\,b^3\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^3*d*x - 6*b^3/(e^(2*d*x + 2*c) + 1) - 2*(6*a^3*e^(4*d*x + 4*c) + 9
*a^2*b*e^(4*d*x + 4*c) - 3*b^3*e^(4*d*x + 4*c) - 6*a^3*e^(2*d*x + 2*c) + 18
*a*b^2*e^(2*d*x + 2*c) + 12*b^3*e^(2*d*x + 2*c) + 4*a^3 + 3*a^2*b - 6*a*b^2
 - 5*b^3)/(e^(2*d*x + 2*c) - 1)^3)/d
```

3.133 $\int \coth^5(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=81

$$\frac{\left(a^3+b^3\right)\log(\sinh(c+dx))}{d}-\frac{(a+b)^3\text{csch}^4(c+dx)}{4d}-\frac{(2a-b)(a+b)^2\text{csch}^2(c+dx)}{2d}-\frac{b^3\log(\cosh(c+dx))}{d}$$

```
[Out] -((2*a - b)*(a + b)^2*Csch[c + d*x]^2)/(2*d) - ((a + b)^3*Csch[c + d*x]^4)/
(4*d) - (b^3*Log[Cosh[c + d*x]])/d + ((a^3 + b^3)*Log[Sinh[c + d*x]])/d
```

Rubi [A] time = 0.120995, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4138, 446, 88}

$$\frac{\left(a^3+b^3\right)\log(\sinh(c+dx))}{d}-\frac{(a+b)^3\text{csch}^4(c+dx)}{4d}-\frac{(2a-b)(a+b)^2\text{csch}^2(c+dx)}{2d}-\frac{b^3\log(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^5*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -((2*a - b)*(a + b)^2*Csch[c + d*x]^2)/(2*d) - ((a + b)^3*Csch[c + d*x]^4)/
(4*d) - (b^3*Log[Cosh[c + d*x]])/d + ((a^3 + b^3)*Log[Sinh[c + d*x]])/d
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \coth^5(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx = -\frac{\operatorname{Subst}\left(\int \frac{\left(b+ax^2\right)^3}{x\left(1-x^2\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\operatorname{Subst}\left(\int \frac{(b+ax)^3}{(1-x)^3 x}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{\operatorname{Subst}\left(\int \left(-\frac{(a+b)^3}{(-1+x)^3} - \frac{(2a-b)(a+b)^2}{(-1+x)^2} + \frac{-a^3-b^3}{-1+x} + \frac{b^3}{x}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{(2a-b)(a+b)^2\operatorname{csch}^2(c+dx)}{2d} - \frac{(a+b)^3\operatorname{csch}^4(c+dx)}{4d} - \frac{b^3\log(\cosh(}{d}$$

Mathematica [A] time = 0.858402, size = 101, normalized size = 1.25

$$-\frac{2\left(a\cosh^2(c+dx)+b\right)^3\left(-4\left(a^3+b^3\right)\log(\sinh(c+dx)) + (a+b)^3\operatorname{csch}^4(c+dx) + 2(2a-b)(a+b)^2\operatorname{csch}^2(c+dx) +\right.}{d(a\cosh(2(c+dx))+a+2b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^5*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (-2*(b + a*Cosh[c + d*x]^2)^3*(2*(2*a - b)*(a + b)^2*Csch[c + d*x]^2 + (a +
 b)^3*Csch[c + d*x]^4 + 4*b^3*Log[Cosh[c + d*x]] - 4*(a^3 + b^3)*Log[Sinh[c
 + d*x]]))/(d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)
```

Maple [B] time = 0.047, size = 194, normalized size = 2.4

$$\frac{a^3\ln(\sinh(dx+c))}{d} - \frac{a^3(\coth(dx+c))^2}{2d} - \frac{a^3(\coth(dx+c))^4}{4d} - \frac{3a^2b(\cosh(dx+c))^2}{4d(\sinh(dx+c))^4} - \frac{3a^2b(\cosh(dx+c))^2}{4d(\sinh(dx+c))^2} - \frac{3}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d*a^3*ln(sinh(d*x+c))-1/2*a^3*coth(d*x+c)^2/d-1/4*a^3*coth(d*x+c)^4/d-3/4
/d*a^2*b/sinh(d*x+c)^4*cosh(d*x+c)^2-3/4/d*a^2*b*cosh(d*x+c)^2/sinh(d*x+c)^
2-3/4/d*a*b^2/sinh(d*x+c)^4*cosh(d*x+c)^2+3/4/d*a*b^2*cosh(d*x+c)^2/sinh(d*
x+c)^2-1/4/d*b^3/sinh(d*x+c)^4+1/2/d*b^3/sinh(d*x+c)^2+1/d*b^3*ln(tanh(d*x+
c))
```

Maxima [B] time = 1.70119, size = 570, normalized size = 7.04

$$a^3\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{4\left(e^{(-2dx-2c)}-e^{(-4dx-4c)}+e^{(-6dx-6c)}\right)}{d\left(4e^{(-2dx-2c)}-6e^{(-4dx-4c)}+4e^{(-6dx-6c)}-e^{(-8dx-8c)}-1\right)}\right)+b^3\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a^3*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-2
*d*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) -
 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + b^3*(l
og(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x - 2*c) + 1
)/d - 2*(e^(-2*d*x - 2*c) - 4*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^
(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c)
 - 1))) + 6*a^2*b*(e^(-2*d*x - 2*c)/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x -
4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1)) + e^(-6*d*x - 6*c)/(d*(4
*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8
*c) - 1))) - 12*a*b^2/(d*(e^(d*x + c) - e^(-d*x - c))^4)
```

Fricas [B] time = 2.74878, size = 4574, normalized size = 56.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -(a^3*d*x*cosh(d*x + c)^8 + 8*a^3*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + a^3*d
*x*sinh(d*x + c)^8 - 2*(2*a^3*d*x - 2*a^3 - 3*a^2*b + b^3)*cosh(d*x + c)^6
+ 2*(14*a^3*d*x*cosh(d*x + c)^2 - 2*a^3*d*x + 2*a^3 + 3*a^2*b - b^3)*sinh(d
*x + c)^6 + 4*(14*a^3*d*x*cosh(d*x + c)^3 - 3*(2*a^3*d*x - 2*a^3 - 3*a^2*b
+ b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + a^3*d*x + 2*(3*a^3*d*x - 2*a^3 + 6*
a*b^2 + 4*b^3)*cosh(d*x + c)^4 + 2*(35*a^3*d*x*cosh(d*x + c)^4 + 3*a^3*d*x
- 2*a^3 + 6*a*b^2 + 4*b^3 - 15*(2*a^3*d*x - 2*a^3 - 3*a^2*b + b^3)*cosh(d*x
 + c)^2)*sinh(d*x + c)^4 + 8*(7*a^3*d*x*cosh(d*x + c)^5 - 5*(2*a^3*d*x - 2*
a^3 - 3*a^2*b + b^3)*cosh(d*x + c)^3 + (3*a^3*d*x - 2*a^3 + 6*a*b^2 + 4*b^3
)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(2*a^3*d*x - 2*a^3 - 3*a^2*b + b^3)*co
sh(d*x + c)^2 + 2*(14*a^3*d*x*cosh(d*x + c)^6 - 2*a^3*d*x - 15*(2*a^3*d*x -
 2*a^3 - 3*a^2*b + b^3)*cosh(d*x + c)^4 + 2*a^3 + 3*a^2*b - b^3 + 6*(3*a^3*
d*x - 2*a^3 + 6*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + (b^3*cosh
(d*x + c)^8 + 8*b^3*cosh(d*x + c)*sinh(d*x + c)^7 + b^3*sinh(d*x + c)^8 - 4
*b^3*cosh(d*x + c)^6 + 6*b^3*cosh(d*x + c)^4 + 4*(7*b^3*cosh(d*x + c)^2 - b
^3)*sinh(d*x + c)^6 + 8*(7*b^3*cosh(d*x + c)^3 - 3*b^3*cosh(d*x + c))*sinh(
d*x + c)^5 - 4*b^3*cosh(d*x + c)^2 + 2*(35*b^3*cosh(d*x + c)^4 - 30*b^3*cos
h(d*x + c)^2 + 3*b^3)*sinh(d*x + c)^4 + 8*(7*b^3*cosh(d*x + c)^5 - 10*b^3*c
osh(d*x + c)^3 + 3*b^3*cosh(d*x + c))*sinh(d*x + c)^3 + b^3 + 4*(7*b^3*cosh
(d*x + c)^6 - 15*b^3*cosh(d*x + c)^4 + 9*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*
x + c)^2 + 8*(b^3*cosh(d*x + c)^7 - 3*b^3*cosh(d*x + c)^5 + 3*b^3*cosh(d*x
+ c)^3 - b^3*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x +
c) - sinh(d*x + c))) - ((a^3 + b^3)*cosh(d*x + c)^8 + 8*(a^3 + b^3)*cosh(d*
x + c)*sinh(d*x + c)^7 + (a^3 + b^3)*sinh(d*x + c)^8 - 4*(a^3 + b^3)*cosh(d
*x + c)^6 - 4*(a^3 + b^3 - 7*(a^3 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 +
 8*(7*(a^3 + b^3)*cosh(d*x + c)^3 - 3*(a^3 + b^3)*cosh(d*x + c))*sinh(d*x +
 c)^5 + 6*(a^3 + b^3)*cosh(d*x + c)^4 + 2*(35*(a^3 + b^3)*cosh(d*x + c)^4 +
 3*a^3 + 3*b^3 - 30*(a^3 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^
3 + b^3)*cosh(d*x + c)^5 - 10*(a^3 + b^3)*cosh(d*x + c)^3 + 3*(a^3 + b^3)*c
osh(d*x + c))*sinh(d*x + c)^3 + a^3 + b^3 - 4*(a^3 + b^3)*cosh(d*x + c)^2 +
 4*(7*(a^3 + b^3)*cosh(d*x + c)^6 - 15*(a^3 + b^3)*cosh(d*x + c)^4 - a^3 -
b^3 + 9*(a^3 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^3 + b^3)*cosh(
d*x + c)^7 - 3*(a^3 + b^3)*cosh(d*x + c)^5 + 3*(a^3 + b^3)*cosh(d*x + c)^3
- (a^3 + b^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x +
 c) - sinh(d*x + c))) + 4*(2*a^3*d*x*cosh(d*x + c)^7 - 3*(2*a^3*d*x - 2*a^3
 - 3*a^2*b + b^3)*cosh(d*x + c)^5 + 2*(3*a^3*d*x - 2*a^3 + 6*a*b^2 + 4*b^3)
*cosh(d*x + c)^3 - (2*a^3*d*x - 2*a^3 - 3*a^2*b + b^3)*cosh(d*x + c))*sinh(
d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d
```

```
*x + c)^8 - 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)
^6 + 8*(7*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh
(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 - 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*
x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c
))*sinh(d*x + c)^3 - 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 - 15*d*co
sh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x +
c)^7 - 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*
x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**5*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.58104, size = 335, normalized size = 4.14

$$-\frac{12\,a^3dx + 12\,b^3\log\left(e^{(2\,dx+2\,c)} + 1\right) - 12\left(a^3e^{(2\,c)} + b^3e^{(2\,c)}\right)e^{(-2\,c)}\log\left(\left|e^{(2\,dx+2\,c)} - 1\right|\right) + \frac{25\,a^3e^{(8\,dx+8\,c)} + 25\,b^3e^{(8\,dx+8\,c)} - 52\,a^3e^{(6\,dx}}{}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/12*(12*a^3*d*x + 12*b^3*log(e^(2*d*x + 2*c) + 1) - 12*(a^3*e^(2*c) + b^3
*e^(2*c))*e^(-2*c)*log(abs(e^(2*d*x + 2*c) - 1)) + (25*a^3*e^(8*d*x + 8*c)
+ 25*b^3*e^(8*d*x + 8*c) - 52*a^3*e^(6*d*x + 6*c) + 72*a^2*b*e^(6*d*x + 6*c
) - 124*b^3*e^(6*d*x + 6*c) + 102*a^3*e^(4*d*x + 4*c) + 144*a*b^2*e^(4*d*x
+ 4*c) + 246*b^3*e^(4*d*x + 4*c) - 52*a^3*e^(2*d*x + 2*c) + 72*a^2*b*e^(2*d
*x + 2*c) - 124*b^3*e^(2*d*x + 2*c) + 25*a^3 + 25*b^3)/(e^(2*d*x + 2*c) - 1
)^4)/d
```

3.134 $\int \coth^6(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=69

$$-\frac{\left(a^3+b^3\right)\coth(c+dx)}{d}+a^3x-\frac{(a+b)^3\coth^5(c+dx)}{5d}-\frac{(a-2b)(a+b)^2\coth^3(c+dx)}{3d}$$

```
[Out] a^3*x - ((a^3 + b^3)*Coth[c + d*x])/d - ((a - 2*b)*(a + b)^2*Coth[c + d*x]^
3)/(3*d) - ((a + b)^3*Coth[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.109348, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4141, 1802, 207}

$$-\frac{\left(a^3+b^3\right)\coth(c+dx)}{d}+a^3x-\frac{(a+b)^3\coth^5(c+dx)}{5d}-\frac{(a-2b)(a+b)^2\coth^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^6*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] a^3*x - ((a^3 + b^3)*Coth[c + d*x])/d - ((a - 2*b)*(a + b)^2*Coth[c + d*x]^
3)/(3*d) - ((a + b)^3*Coth[c + d*x]^5)/(5*d)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \coth^6(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b(1-x^2)\right)^3}{x^6(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{(a+b)^3}{x^6}+\frac{(a-2b)(a+b)^2}{x^4}+\frac{a^3+b^3}{x^2}-\frac{a^3}{-1+x^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{\left(a^3+b^3\right)\coth(c+dx)}{d}-\frac{(a-2b)(a+b)^2\coth^3(c+dx)}{3d}-\frac{(a+b)^3\cot}{5d}$$
$$= a^3x-\frac{\left(a^3+b^3\right)\coth(c+dx)}{d}-\frac{(a-2b)(a+b)^2\coth^3(c+dx)}{3d}-\frac{(a+b)}{}$$

Mathematica [B] time = 1.0977, size = 303, normalized size = 4.39

$$\frac{\text{csch}(c)\text{csch}^5(c+dx)\left(-90a^2b\sinh(4c+3dx)+18a^2b\sinh(4c+5dx)+180a^2b\sinh(dx)+180a^3\sinh(2c+dx)-140\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^6*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (Csch[c]*Csch[c + d*x]^5*(-150*a^3*d*x*Cosh[d*x] + 150*a^3*d*x*Cosh[2*c + d
*x] + 75*a^3*d*x*Cosh[2*c + 3*d*x] - 75*a^3*d*x*Cosh[4*c + 3*d*x] - 15*a^3*
d*x*Cosh[4*c + 5*d*x] + 15*a^3*d*x*Cosh[6*c + 5*d*x] + 280*a^3*Sinh[d*x] +
180*a^2*b*Sinh[d*x] + 60*a*b^2*Sinh[d*x] + 160*b^3*Sinh[d*x] + 180*a^3*Sinh
[2*c + d*x] - 180*a*b^2*Sinh[2*c + d*x] - 140*a^3*Sinh[2*c + 3*d*x] + 60*a*
b^2*Sinh[2*c + 3*d*x] - 80*b^3*Sinh[2*c + 3*d*x] - 90*a^3*Sinh[4*c + 3*d*x]
 - 90*a^2*b*Sinh[4*c + 3*d*x] + 46*a^3*Sinh[4*c + 5*d*x] + 18*a^2*b*Sinh[4*
c + 5*d*x] - 12*a*b^2*Sinh[4*c + 5*d*x] + 16*b^3*Sinh[4*c + 5*d*x]))/(480*d
)
```

Maple [B] time = 0.046, size = 199, normalized size = 2.9

$$\frac{1}{d}\left(a^3\left(dx+c-\coth(dx+c)-\frac{(\coth(dx+c))^3}{3}-\frac{(\coth(dx+c))^5}{5}\right)+3a^2b\left(-1/2\frac{(\cosh(dx+c))^3}{(\sinh(dx+c))^5}+3/8\frac{\cosh(dx+}{(\sinh(dx+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(d*x+c-coth(d*x+c)-1/3*coth(d*x+c)^3-1/5*coth(d*x+c)^5)+3*a^2*b*(-
1/2/sinh(d*x+c)^5*cosh(d*x+c)^3+3/8/sinh(d*x+c)^5*cosh(d*x+c)+3/8*(-8/15-1/
5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c))+3*a*b^2*(-1/4/sinh(d*x+c)^
5*cosh(d*x+c)-1/4*(-8/15-1/5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c))
+b^3*(-8/15-1/5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c))
```

Maxima [B] time = 1.25417, size = 1115, normalized size = 16.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/15*a^3*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) - 140*e^(-4*d*x - 4*c) + 9
0*e^(-6*d*x - 6*c) - 45*e^(-8*d*x - 8*c) - 23)/(d*(5*e^(-2*d*x - 2*c) - 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) - 1))) + 4/5*a*b^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^
(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10
*c) - 1)) + 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c)
 + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) + 15
*e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d
*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 1/(d*(5*e^(-2*d
*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c)
+ e^(-10*d*x - 10*c) - 1))) - 16/15*b^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x
 - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) +
e^(-10*d*x - 10*c) - 1)) - 10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) - 1)) - 1/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
 - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1))) + 6/5*a^2*b*(10*e^
(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
- 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) + 5*e^(-8*d*x - 8*c)
/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(
-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) + 1/(d*(5*e^(-2*d*x - 2*c) - 10*e^
(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10
*c) - 1)))
```

Fricas [B] time = 2.37112, size = 1249, normalized size = 18.1

$$-\frac{\left(23\,a^3+9\,a^2b-6\,ab^2+8\,b^3\right)\cosh\left(dx+c\right)^5+5\left(23\,a^3+9\,a^2b-6\,ab^2+8\,b^3\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4-\left(15\,a^3dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/15*((23*a^3 + 9*a^2*b - 6*a*b^2 + 8*b^3)*cosh(d*x + c)^5 + 5*(23*a^3 + 9
*a^2*b - 6*a*b^2 + 8*b^3)*cosh(d*x + c)*sinh(d*x + c)^4 - (15*a^3*d*x + 23*
a^3 + 9*a^2*b - 6*a*b^2 + 8*b^3)*sinh(d*x + c)^5 - 5*(5*a^3 - 9*a^2*b - 6*a
*b^2 + 8*b^3)*cosh(d*x + c)^3 + 5*(15*a^3*d*x + 23*a^3 + 9*a^2*b - 6*a*b^2
+ 8*b^3 - 2*(15*a^3*d*x + 23*a^3 + 9*a^2*b - 6*a*b^2 + 8*b^3)*cosh(d*x + c)
^2)*sinh(d*x + c)^3 + 5*(2*(23*a^3 + 9*a^2*b - 6*a*b^2 + 8*b^3)*cosh(d*x +
c)^3 - 3*(5*a^3 - 9*a^2*b - 6*a*b^2 + 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^2
 + 10*(5*a^3 + 9*a^2*b + 12*a*b^2 + 8*b^3)*cosh(d*x + c) - 5*(30*a^3*d*x +
(15*a^3*d*x + 23*a^3 + 9*a^2*b - 6*a*b^2 + 8*b^3)*cosh(d*x + c)^4 + 46*a^3
+ 18*a^2*b - 12*a*b^2 + 16*b^3 - 3*(15*a^3*d*x + 23*a^3 + 9*a^2*b - 6*a*b^2
 + 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*sinh(d*x + c)^5 + 5*(2*d*cosh(
d*x + c)^2 - d)*sinh(d*x + c)^3 + 5*(d*cosh(d*x + c)^4 - 3*d*cosh(d*x + c)^
2 + 2*d)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**6*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.60703, size = 284, normalized size = 4.12

$$\frac{15\,a^3dx - \frac{2\left(45\,a^3e^{(8\,dx+8\,c)}+45\,a^2be^{(8\,dx+8\,c)}-90\,a^3e^{(6\,dx+6\,c)}+90\,ab^2e^{(6\,dx+6\,c)}+140\,a^3e^{(4\,dx+4\,c)}+90\,a^2be^{(4\,dx+4\,c)}+30\,ab^2e^{(4\,dx+4\,c)}+80\,b^3e^{(4\,dx+4\,c)}-70\,a^3e^{(2}}{\left(e^{(2\,dx+2\,c)}-1\right)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/15*(15*a^3*d*x - 2*(45*a^3*e^(8*d*x + 8*c) + 45*a^2*b*e^(8*d*x + 8*c) - 9
0*a^3*e^(6*d*x + 6*c) + 90*a*b^2*e^(6*d*x + 6*c) + 140*a^3*e^(4*d*x + 4*c)
+ 90*a^2*b*e^(4*d*x + 4*c) + 30*a*b^2*e^(4*d*x + 4*c) + 80*b^3*e^(4*d*x + 4
*c) - 70*a^3*e^(2*d*x + 2*c) + 30*a*b^2*e^(2*d*x + 2*c) - 40*b^3*e^(2*d*x +
 2*c) + 23*a^3 + 9*a^2*b - 6*a*b^2 + 8*b^3)/(e^(2*d*x + 2*c) - 1)^5)/d
```

3.135 $\int \coth^7(c+dx)\left(a+b\,\text{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=77

$$-\frac{3a^2(a+b)\text{csch}^2(c+dx)}{2d}+\frac{a^3\log(\sinh(c+dx))}{d}-\frac{(a+b)^3\text{csch}^6(c+dx)}{6d}-\frac{3a(a+b)^2\text{csch}^4(c+dx)}{4d}$$

```
[Out] (-3*a^2*(a + b)*Csch[c + d*x]^2)/(2*d) - (3*a*(a + b)^2*Csch[c + d*x]^4)/(4
*d) - ((a + b)^3*Csch[c + d*x]^6)/(6*d) + (a^3*Log[Sinh[c + d*x]])/d
```

Rubi [A] time = 0.121003, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4138, 444, 43}

$$-\frac{3a^2(a+b)\text{csch}^2(c+dx)}{2d}+\frac{a^3\log(\sinh(c+dx))}{d}-\frac{(a+b)^3\text{csch}^6(c+dx)}{6d}-\frac{3a(a+b)^2\text{csch}^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^7*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (-3*a^2*(a + b)*Csch[c + d*x]^2)/(2*d) - (3*a*(a + b)^2*Csch[c + d*x]^4)/(4
*d) - ((a + b)^3*Csch[c + d*x]^6)/(6*d) + (a^3*Log[Sinh[c + d*x]])/d
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \coth^7(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int \frac{x\left(b+ax^2\right)^3}{\left(1-x^2\right)^4} dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(b+ax)^3}{(1-x)^4} dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{(a+b)^3}{(-1+x)^4} + \frac{3a(a+b)^2}{(-1+x)^3} + \frac{3a^2(a+b)}{(-1+x)^2} + \frac{a^3}{-1+x}\right) dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{3a^2(a+b)\text{csch}^2(c+dx)}{2d} - \frac{3a(a+b)^2\text{csch}^4(c+dx)}{4d} - \frac{(a+b)^3\text{csch}^6(c}{6d}$$

Mathematica [A] time = 0.773075, size = 98, normalized size = 1.27

$$-\frac{2\left(a\cosh^2(c+dx)+b\right)^3\left(18a^2(a+b)\text{csch}^2(c+dx)-12a^3\log(\sinh(c+dx))+2(a+b)^3\text{csch}^6(c+dx)+9a(a+b)^2\text{c}\right.}{3d(a\cosh(2(c+dx))+a+2b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^7*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (-2*(b + a*Cosh[c + d*x]^2)^3*(18*a^2*(a + b)*Csch[c + d*x]^2 + 9*a*(a + b)
^2*Csch[c + d*x]^4 + 2*(a + b)^3*Csch[c + d*x]^6 - 12*a^3*Log[Sinh[c + d*x]
]))/(3*d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)
```

Maple [B] time = 0.05, size = 310, normalized size = 4.

$$\frac{a^3\ln(\sinh(dx+c))}{d} - \frac{a^3(\coth(dx+c))^2}{2d} - \frac{a^3(\coth(dx+c))^4}{4d} - \frac{a^3(\coth(dx+c))^6}{6d} - \frac{3a^2b(\cosh(dx+c))^4}{2d(\sinh(dx+c))^6} + \frac{a^2b(}{d(s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d*a^3*ln(sinh(d*x+c))-1/2*a^3*coth(d*x+c)^2/d-1/4*a^3*coth(d*x+c)^4/d-1/6
*a^3*coth(d*x+c)^6/d-3/2/d*a^2*b/sinh(d*x+c)^6*cosh(d*x+c)^4+1/d*a^2*b/sinh
(d*x+c)^6*cosh(d*x+c)^2+1/2/d*a^2*b/sinh(d*x+c)^4*cosh(d*x+c)^2-1/2/d*a^2*b
*cosh(d*x+c)^2/sinh(d*x+c)^2-1/2/d*a*b^2/sinh(d*x+c)^6*cosh(d*x+c)^2-1/4/d*
a*b^2/sinh(d*x+c)^4*cosh(d*x+c)^2+1/4/d*a*b^2*cosh(d*x+c)^2/sinh(d*x+c)^2-1
/6/d*b^3/sinh(d*x+c)^6*cosh(d*x+c)^2+1/6/d*b^3/sinh(d*x+c)^4*cosh(d*x+c)^2-
1/6/d*b^3*cosh(d*x+c)^2/sinh(d*x+c)^2
```

Maxima [B] time = 1.26945, size = 981, normalized size = 12.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*(3*x + 3*c/d + 3*log(e^(-d*x - c) + 1)/d + 3*log(e^(-d*x - c) - 1)/
d + 2*(9*e^(-2*d*x - 2*c) - 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) - 18*
e^(-8*d*x - 8*c) + 9*e^(-10*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*
d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*
c) - e^(-12*d*x - 12*c) - 1))) + 2*a^2*b*(3*e^(-2*d*x - 2*c)/(d*(6*e^(-2*d*
x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c)
+ 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1)) + 10*e^(-6*d*x - 6*c)/(d*
(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*
d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1)) + 3*e^(-10*d*x
 - 10*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c)
 - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1))) +
 4*a*b^2*(3*e^(-4*d*x - 4*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) +
 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*
d*x - 12*c) - 1)) + 2*e^(-6*d*x - 6*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*
x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c)
 - e^(-12*d*x - 12*c) - 1)) + 3*e^(-8*d*x - 8*c)/(d*(6*e^(-2*d*x - 2*c) - 1
5*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d
*x - 10*c) - e^(-12*d*x - 12*c) - 1))) - 32/3*b^3/(d*(e^(d*x + c) - e^(-d*x
 - c))^6)
```

Fricas [B] time = 2.47787, size = 6665, normalized size = 86.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^3*d*x*cosh(d*x + c)^12 + 36*a^3*d*x*cosh(d*x + c)*sinh(d*x + c)^1
1 + 3*a^3*d*x*sinh(d*x + c)^12 - 18*(a^3*d*x - a^3 - a^2*b)*cosh(d*x + c)^1
0 + 18*(11*a^3*d*x*cosh(d*x + c)^2 - a^3*d*x + a^3 + a^2*b)*sinh(d*x + c)^1
0 + 60*(11*a^3*d*x*cosh(d*x + c)^3 - 3*(a^3*d*x - a^3 - a^2*b)*cosh(d*x + c
))*sinh(d*x + c)^9 + 9*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^8 + 9*(1
65*a^3*d*x*cosh(d*x + c)^4 + 5*a^3*d*x - 4*a^3 + 4*a*b^2 - 90*(a^3*d*x - a^
3 - a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 72*(33*a^3*d*x*cosh(d*x + c)^
5 - 30*(a^3*d*x - a^3 - a^2*b)*cosh(d*x + c)^3 + (5*a^3*d*x - 4*a^3 + 4*a*b
^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(15*a^3*d*x - 17*a^3 - 15*a^2*b - 6*
a*b^2 - 8*b^3)*cosh(d*x + c)^6 + 4*(693*a^3*d*x*cosh(d*x + c)^6 - 15*a^3*d*
x - 945*(a^3*d*x - a^3 - a^2*b)*cosh(d*x + c)^4 + 17*a^3 + 15*a^2*b + 6*a*b
^2 + 8*b^3 + 63*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c
)^6 + 24*(99*a^3*d*x*cosh(d*x + c)^7 - 189*(a^3*d*x - a^3 - a^2*b)*cosh(d*x
 + c)^5 + 21*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^3 - (15*a^3*d*x -
17*a^3 - 15*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 3*a^3
*d*x + 9*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^4 + 3*(495*a^3*d*x*cos
h(d*x + c)^8 - 1260*(a^3*d*x - a^3 - a^2*b)*cosh(d*x + c)^6 + 15*a^3*d*x +
210*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^4 - 12*a^3 + 12*a*b^2 - 20*
(15*a^3*d*x - 17*a^3 - 15*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c)^2)*sinh(d*
x + c)^4 + 4*(165*a^3*d*x*cosh(d*x + c)^9 - 540*(a^3*d*x - a^3 - a^2*b)*cos
h(d*x + c)^7 + 126*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^5 - 20*(15*a
^3*d*x - 17*a^3 - 15*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c)^3 + 9*(5*a^3*d*
x - 4*a^3 + 4*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 18*(a^3*d*x - a^3 - a
^2*b)*cosh(d*x + c)^2 + 6*(33*a^3*d*x*cosh(d*x + c)^10 - 135*(a^3*d*x - a^3
 - a^2*b)*cosh(d*x + c)^8 + 42*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^
6 - 3*a^3*d*x - 10*(15*a^3*d*x - 17*a^3 - 15*a^2*b - 6*a*b^2 - 8*b^3)*cosh(
d*x + c)^4 + 3*a^3 + 3*a^2*b + 9*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c
)^2)*sinh(d*x + c)^2 - 3*(a^3*cosh(d*x + c)^12 + 12*a^3*cosh(d*x + c)*sinh(
d*x + c)^11 + a^3*sinh(d*x + c)^12 - 6*a^3*cosh(d*x + c)^10 + 15*a^3*cosh(d
```

```
*x + c)^8 + 6*(11*a^3*cosh(d*x + c)^2 - a^3)*sinh(d*x + c)^10 + 20*(11*a^3*
cosh(d*x + c)^3 - 3*a^3*cosh(d*x + c))*sinh(d*x + c)^9 - 20*a^3*cosh(d*x +
c)^6 + 15*(33*a^3*cosh(d*x + c)^4 - 18*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x
+ c)^8 + 24*(33*a^3*cosh(d*x + c)^5 - 30*a^3*cosh(d*x + c)^3 + 5*a^3*cosh(d
*x + c))*sinh(d*x + c)^7 + 15*a^3*cosh(d*x + c)^4 + 4*(231*a^3*cosh(d*x + c
)^6 - 315*a^3*cosh(d*x + c)^4 + 105*a^3*cosh(d*x + c)^2 - 5*a^3)*sinh(d*x +
 c)^6 + 24*(33*a^3*cosh(d*x + c)^7 - 63*a^3*cosh(d*x + c)^5 + 35*a^3*cosh(d
*x + c)^3 - 5*a^3*cosh(d*x + c))*sinh(d*x + c)^5 - 6*a^3*cosh(d*x + c)^2 +
15*(33*a^3*cosh(d*x + c)^8 - 84*a^3*cosh(d*x + c)^6 + 70*a^3*cosh(d*x + c)^
4 - 20*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^4 + 20*(11*a^3*cosh(d*x + c
)^9 - 36*a^3*cosh(d*x + c)^7 + 42*a^3*cosh(d*x + c)^5 - 20*a^3*cosh(d*x + c
)^3 + 3*a^3*cosh(d*x + c))*sinh(d*x + c)^3 + a^3 + 6*(11*a^3*cosh(d*x + c)^
10 - 45*a^3*cosh(d*x + c)^8 + 70*a^3*cosh(d*x + c)^6 - 50*a^3*cosh(d*x + c)
^4 + 15*a^3*cosh(d*x + c)^2 - a^3)*sinh(d*x + c)^2 + 12*(a^3*cosh(d*x + c)^
11 - 5*a^3*cosh(d*x + c)^9 + 10*a^3*cosh(d*x + c)^7 - 10*a^3*cosh(d*x + c)^
5 + 5*a^3*cosh(d*x + c)^3 - a^3*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*
x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 12*(3*a^3*d*x*cosh(d*x + c)^11 -
15*(a^3*d*x - a^3 - a^2*b)*cosh(d*x + c)^9 + 6*(5*a^3*d*x - 4*a^3 + 4*a*b^2
)*cosh(d*x + c)^7 - 2*(15*a^3*d*x - 17*a^3 - 15*a^2*b - 6*a*b^2 - 8*b^3)*co
sh(d*x + c)^5 + 3*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^3 - 3*(a^3*d*
x - a^3 - a^2*b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^12 + 12*d*c
osh(d*x + c)*sinh(d*x + c)^11 + d*sinh(d*x + c)^12 - 6*d*cosh(d*x + c)^10 +
 6*(11*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^10 + 20*(11*d*cosh(d*x + c)^3 -
 3*d*cosh(d*x + c))*sinh(d*x + c)^9 + 15*d*cosh(d*x + c)^8 + 15*(33*d*cosh(
d*x + c)^4 - 18*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 24*(33*d*cosh(d*x
+ c)^5 - 30*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^7 - 20*d*c
osh(d*x + c)^6 + 4*(231*d*cosh(d*x + c)^6 - 315*d*cosh(d*x + c)^4 + 105*d*c
osh(d*x + c)^2 - 5*d)*sinh(d*x + c)^6 + 24*(33*d*cosh(d*x + c)^7 - 63*d*cos
h(d*x + c)^5 + 35*d*cosh(d*x + c)^3 - 5*d*cosh(d*x + c))*sinh(d*x + c)^5 +
15*d*cosh(d*x + c)^4 + 15*(33*d*cosh(d*x + c)^8 - 84*d*cosh(d*x + c)^6 + 70
*d*cosh(d*x + c)^4 - 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 20*(11*d*c
osh(d*x + c)^9 - 36*d*cosh(d*x + c)^7 + 42*d*cosh(d*x + c)^5 - 20*d*cosh(d*
x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 - 6*d*cosh(d*x + c)^2 + 6*(11
*d*cosh(d*x + c)^10 - 45*d*cosh(d*x + c)^8 + 70*d*cosh(d*x + c)^6 - 50*d*co
sh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 12*(d*cosh(d*x
+ c)^11 - 5*d*cosh(d*x + c)^9 + 10*d*cosh(d*x + c)^7 - 10*d*cosh(d*x + c)^5
 + 5*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**7*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.7339, size = 323, normalized size = 4.19

$$-\frac{60\,a^3dx - 60\,a^3\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right) + \frac{147\,a^3e^{(12\,dx+12\,c)} - 522\,a^3e^{(10\,dx+10\,c)} + 360\,a^2be^{(10\,dx+10\,c)} + 1485\,a^3e^{(8\,dx+8\,c)} + 720\,ab^2e^{(8\,dx+8\,c)} - 1580\,a^3e^{(\cdots}}{\cdots}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/60*(60*a^3*d*x - 60*a^3*log(abs(e^(2*d*x + 2*c) - 1)) + (147*a^3*e^(12*d
*x + 12*c) - 522*a^3*e^(10*d*x + 10*c) + 360*a^2*b*e^(10*d*x + 10*c) + 1485
*a^3*e^(8*d*x + 8*c) + 720*a*b^2*e^(8*d*x + 8*c) - 1580*a^3*e^(6*d*x + 6*c)
 + 1200*a^2*b*e^(6*d*x + 6*c) + 480*a*b^2*e^(6*d*x + 6*c) + 640*b^3*e^(6*d*
x + 6*c) + 1485*a^3*e^(4*d*x + 4*c) + 720*a*b^2*e^(4*d*x + 4*c) - 522*a^3*e
^(2*d*x + 2*c) + 360*a^2*b*e^(2*d*x + 2*c) + 147*a^3)/(e^(2*d*x + 2*c) - 1)
^6)/d
```

3.136 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)^4\,dx$

Optimal. Leaf size=111

$$-\frac{b^2\left(6a^2+8ab+3b^2\right)\tanh^3(c+dx)}{3d}+\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tanh(c+dx)}{d}+a^4x+\frac{b^3(4a+3b)\tanh^5(c+dx)}{5d}-$$

```
[Out] a^4*x + (b*(2*a + b)*(2*a^2 + 2*a*b + b^2)*Tanh[c + d*x])/d - (b^2*(6*a^2 +
 8*a*b + 3*b^2)*Tanh[c + d*x]^3)/(3*d) + (b^3*(4*a + 3*b)*Tanh[c + d*x]^5)/
(5*d) - (b^4*Tanh[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.0700309, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4128, 390, 206}

$$-\frac{b^2\left(6a^2+8ab+3b^2\right)\tanh^3(c+dx)}{3d}+\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tanh(c+dx)}{d}+a^4x+\frac{b^3(4a+3b)\tanh^5(c+dx)}{5d}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^4,x]
```

```
[Out] a^4*x + (b*(2*a + b)*(2*a^2 + 2*a*b + b^2)*Tanh[c + d*x])/d - (b^2*(6*a^2 +
 8*a*b + 3*b^2)*Tanh[c + d*x]^3)/(3*d) + (b^3*(4*a + 3*b)*Tanh[c + d*x]^5)/
(5*d) - (b^4*Tanh[c + d*x]^7)/(7*d)
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)^4 dx = \frac{\text{Subst}\left(\int \frac{\left(a+b-bx^2\right)^4}{1-x^2}dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(b(2a+b)\left(2a^2+2ab+b^2\right)-b^2\left(6a^2+8ab+3b^2\right)x^2+b^3(4a+3b)x^4-b^4x^6+\right.\right.}{d}$$
$$= \frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tanh(c+dx)}{d} - \frac{b^2\left(6a^2+8ab+3b^2\right)\tanh^3(c+dx)}{3d} + \frac{b^3(4a+}{}$$
$$= a^4x + \frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tanh(c+dx)}{d} - \frac{b^2\left(6a^2+8ab+3b^2\right)\tanh^3(c+dx)}{3d} + \frac{b^3}{}$$

Mathematica [B] time = 1.59919, size = 455, normalized size = 4.1

$$\text{sech}(c)\text{sech}^7(c+dx)\left(-10920a^2b^2\sinh(2c+dx)+15120a^2b^2\sinh(2c+3dx)-2520a^2b^2\sinh(4c+3dx)+5880a^2b^2\sin\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^4,x]
```

```
[Out] (Sech[c]*Sech[c + d*x]^7*(3675*a^4*d*x*Cosh[d*x] + 3675*a^4*d*x*Cosh[2*c +
d*x] + 2205*a^4*d*x*Cosh[2*c + 3*d*x] + 2205*a^4*d*x*Cosh[4*c + 3*d*x] + 73
5*a^4*d*x*Cosh[4*c + 5*d*x] + 735*a^4*d*x*Cosh[6*c + 5*d*x] + 105*a^4*d*x*C
osh[6*c + 7*d*x] + 105*a^4*d*x*Cosh[8*c + 7*d*x] + 16800*a^3*b*Sinh[d*x] +
18480*a^2*b^2*Sinh[d*x] + 11200*a*b^3*Sinh[d*x] + 3360*b^4*Sinh[d*x] - 1260
0*a^3*b*Sinh[2*c + d*x] - 10920*a^2*b^2*Sinh[2*c + d*x] - 4480*a*b^3*Sinh[2
*c + d*x] + 12600*a^3*b*Sinh[2*c + 3*d*x] + 15120*a^2*b^2*Sinh[2*c + 3*d*x]
 + 9408*a*b^3*Sinh[2*c + 3*d*x] + 2016*b^4*Sinh[2*c + 3*d*x] - 5040*a^3*b*S
inh[4*c + 3*d*x] - 2520*a^2*b^2*Sinh[4*c + 3*d*x] + 5040*a^3*b*Sinh[4*c + 5
*d*x] + 5880*a^2*b^2*Sinh[4*c + 5*d*x] + 3136*a*b^3*Sinh[4*c + 5*d*x] + 672
*b^4*Sinh[4*c + 5*d*x] - 840*a^3*b*Sinh[6*c + 5*d*x] + 840*a^3*b*Sinh[6*c +
 7*d*x] + 840*a^2*b^2*Sinh[6*c + 7*d*x] + 448*a*b^3*Sinh[6*c + 7*d*x] + 96*
b^4*Sinh[6*c + 7*d*x]))/(13440*d)
```

Maple [A] time = 0.032, size = 129, normalized size = 1.2

$$\frac{1}{d}\left(a^4(dx+c)+4a^3b\tanh(dx+c)+6a^2b^2\left(2/3+1/3(\text{sech}(dx+c))^2\right)\tanh(dx+c)+4ab^3\left(\frac{8}{15}+1/5(\text{sech}(dx+c))^4\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^4,x)
```

```
[Out] 1/d*(a^4*(d*x+c)+4*a^3*b*tanh(d*x+c)+6*a^2*b^2*(2/3+1/3*sech(d*x+c)^2)*tanh
(d*x+c)+4*a*b^3*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c)+b^4
*(16/35+1/7*sech(d*x+c)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c
))
```

Maxima [B] time = 1.46072, size = 949, normalized size = 8.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 32/35*b^4*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x
 - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c)
 + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 21*e^(-4*d*x - 4*c)/(d
*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8
*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 1
4*c) + 1)) + 35*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*
c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*
e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) +
21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10
*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 64/15*a*b
^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^
(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 10*e^(-4*d
*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c
) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c
) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10
*d*x - 10*c) + 1))) + 8*a^2*b^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c)
+ 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) +
3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 8*a^3*b/(d*(e^(-2*d*x - 2*c)
 + 1))
```

Fricas [B] time = 2.17041, size = 2410, normalized size = 21.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] 1/105*((105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224*a*b^3 - 48*b^4)*cosh(d*
x + c)^7 + 7*(105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224*a*b^3 - 48*b^4)*c
osh(d*x + c)*sinh(d*x + c)^6 + 4*(105*a^3*b + 105*a^2*b^2 + 56*a*b^3 + 12*b
^4)*sinh(d*x + c)^7 + 7*(105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224*a*b^3
- 48*b^4)*cosh(d*x + c)^5 + 28*(75*a^3*b + 105*a^2*b^2 + 56*a*b^3 + 12*b^4
+ 3*(105*a^3*b + 105*a^2*b^2 + 56*a*b^3 + 12*b^4)*cosh(d*x + c)^2)*sinh(d*x
 + c)^5 + 35*((105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224*a*b^3 - 48*b^4)*
cosh(d*x + c)^3 + (105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224*a*b^3 - 48*b
^4)*cosh(d*x + c))*sinh(d*x + c)^4 + 21*(105*a^4*d*x - 420*a^3*b - 420*a^2*
b^2 - 224*a*b^3 - 48*b^4)*cosh(d*x + c)^3 + 28*(5*(105*a^3*b + 105*a^2*b^2
+ 56*a*b^3 + 12*b^4)*cosh(d*x + c)^4 + 135*a^3*b + 225*a^2*b^2 + 168*a*b^3
+ 36*b^4 + 10*(75*a^3*b + 105*a^2*b^2 + 56*a*b^3 + 12*b^4)*cosh(d*x + c)^2)
*sinh(d*x + c)^3 + 7*(3*(105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224*a*b^3
- 48*b^4)*cosh(d*x + c)^5 + 10*(105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224
*a*b^3 - 48*b^4)*cosh(d*x + c)^3 + 9*(105*a^4*d*x - 420*a^3*b - 420*a^2*b^2
 - 224*a*b^3 - 48*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + 35*(105*a^4*d*x - 4
20*a^3*b - 420*a^2*b^2 - 224*a*b^3 - 48*b^4)*cosh(d*x + c) + 28*((105*a^3*b
 + 105*a^2*b^2 + 56*a*b^3 + 12*b^4)*cosh(d*x + c)^6 + 5*(75*a^3*b + 105*a^2
*b^2 + 56*a*b^3 + 12*b^4)*cosh(d*x + c)^4 + 75*a^3*b + 135*a^2*b^2 + 120*a*
b^3 + 60*b^4 + 9*(45*a^3*b + 75*a^2*b^2 + 56*a*b^3 + 12*b^4)*cosh(d*x + c)^
2)*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 +
7*d*cosh(d*x + c)^5 + 35*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c
)^4 + 21*d*cosh(d*x + c)^3 + 7*(3*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3
+ 9*d*cosh(d*x + c))*sinh(d*x + c)^2 + 35*d*cosh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\,\text{sech}^2\,(c + dx)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**4,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**4, x)
```

Giac [B] time = 1.15895, size = 451, normalized size = 4.06

$$\frac{(dx+c)a^4}{d} - \frac{8\left(105\,a^3be^{(12\,dx+12\,c)} + 630\,a^3be^{(10\,dx+10\,c)} + 315\,a^2b^2e^{(10\,dx+10\,c)} + 1575\,a^3be^{(8\,dx+8\,c)} + 1365\,a^2b^2e^{(8\,dx+8\,c)} + 5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] (d*x + c)*a^4/d - 8/105*(105*a^3*b*e^(12*d*x + 12*c) + 630*a^3*b*e^(10*d*x
+ 10*c) + 315*a^2*b^2*e^(10*d*x + 10*c) + 1575*a^3*b*e^(8*d*x + 8*c) + 1365
*a^2*b^2*e^(8*d*x + 8*c) + 560*a*b^3*e^(8*d*x + 8*c) + 2100*a^3*b*e^(6*d*x
+ 6*c) + 2310*a^2*b^2*e^(6*d*x + 6*c) + 1400*a*b^3*e^(6*d*x + 6*c) + 420*b^
4*e^(6*d*x + 6*c) + 1575*a^3*b*e^(4*d*x + 4*c) + 1890*a^2*b^2*e^(4*d*x + 4*
c) + 1176*a*b^3*e^(4*d*x + 4*c) + 252*b^4*e^(4*d*x + 4*c) + 630*a^3*b*e^(2*
d*x + 2*c) + 735*a^2*b^2*e^(2*d*x + 2*c) + 392*a*b^3*e^(2*d*x + 2*c) + 84*b
^4*e^(2*d*x + 2*c) + 105*a^3*b + 105*a^2*b^2 + 56*a*b^3 + 12*b^4)/(d*(e^(2*
d*x + 2*c) + 1)^7)
```

3.137 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)^5\,dx$

Optimal. Leaf size=163

$$\frac{b^3\left(10a^2+15ab+6b^2\right)\tanh^5(c+dx)}{5d}-\frac{b^2\left(20a^2b+10a^3+15ab^2+4b^3\right)\tanh^3(c+dx)}{3d}+\frac{b\left(10a^2b^2+10a^3b+5a^4+\right.}{d}$$

```
[Out] a^5*x + (b*(5*a^4 + 10*a^3*b + 10*a^2*b^2 + 5*a*b^3 + b^4)*Tanh[c + d*x])/d
 - (b^2*(10*a^3 + 20*a^2*b + 15*a*b^2 + 4*b^3)*Tanh[c + d*x]^3)/(3*d) + (b^
3*(10*a^2 + 15*a*b + 6*b^2)*Tanh[c + d*x]^5)/(5*d) - (b^4*(5*a + 4*b)*Tanh[
c + d*x]^7)/(7*d) + (b^5*Tanh[c + d*x]^9)/(9*d)
```

Rubi [A] time = 0.0952181, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4128, 390, 206}

$$\frac{b^3\left(10a^2+15ab+6b^2\right)\tanh^5(c+dx)}{5d}-\frac{b^2\left(20a^2b+10a^3+15ab^2+4b^3\right)\tanh^3(c+dx)}{3d}+\frac{b\left(10a^2b^2+10a^3b+5a^4+\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^5,x]
```

```
[Out] a^5*x + (b*(5*a^4 + 10*a^3*b + 10*a^2*b^2 + 5*a*b^3 + b^4)*Tanh[c + d*x])/d
 - (b^2*(10*a^3 + 20*a^2*b + 15*a*b^2 + 4*b^3)*Tanh[c + d*x]^3)/(3*d) + (b^
3*(10*a^2 + 15*a*b + 6*b^2)*Tanh[c + d*x]^5)/(5*d) - (b^4*(5*a + 4*b)*Tanh[
c + d*x]^7)/(7*d) + (b^5*Tanh[c + d*x]^9)/(9*d)
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)^5 dx = \frac{\text{Subst}\left(\int \frac{(a+b-bx^2)^5}{1-x^2} dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(b\left(5a^4+10a^3b+10a^2b^2+5ab^3+b^4\right)-b^2\left(10a^3+20a^2b+15ab^2+4b^3\right)x^2+\right.\right.}{d}$$
$$= \frac{b\left(5a^4+10a^3b+10a^2b^2+5ab^3+b^4\right)\tanh(c+dx)}{d} - \frac{b^2\left(10a^3+20a^2b+15ab^2+4b^3\right)\tanh}{3d}$$
$$= a^5x + \frac{b\left(5a^4+10a^3b+10a^2b^2+5ab^3+b^4\right)\tanh(c+dx)}{d} - \frac{b^2\left(10a^3+20a^2b+15ab^2+4\right.}{3d}$$

Mathematica [B] time = 6.54179, size = 724, normalized size = 4.44

$$\frac{32\text{sech}(c)\cosh^9(c+dx)\left(1680a^2b^3\sinh(dx)+2100a^3b^2\sinh(dx)+1575a^4b\sinh(dx)+720ab^4\sinh(dx)+128b^5\sinh(d\right.}{315d(a\cosh(2c+2dx)+a+2b)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^5,x]
```

```
[Out] (32*a^5*x*Cosh[c + d*x]^10*(a + b*Sech[c + d*x]^2)^5)/(a + 2*b + a*Cosh[2*c
 + 2*d*x])^5 + (32*Cosh[c + d*x]^4*Sech[c]*(a + b*Sech[c + d*x]^2)^5*(45*a*
b^4*Sinh[c] + 8*b^5*Sinh[c]))/(63*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^5) + (6
4*Cosh[c + d*x]^6*Sech[c]*(a + b*Sech[c + d*x]^2)^5*(105*a^2*b^3*Sinh[c] +
45*a*b^4*Sinh[c] + 8*b^5*Sinh[c]))/(105*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^5
) + (64*Cosh[c + d*x]^8*Sech[c]*(a + b*Sech[c + d*x]^2)^5*(525*a^3*b^2*Sinh
[c] + 420*a^2*b^3*Sinh[c] + 180*a*b^4*Sinh[c] + 32*b^5*Sinh[c]))/(315*d*(a
+ 2*b + a*Cosh[2*c + 2*d*x])^5) + (32*b^5*Cosh[c + d*x]*Sech[c]*(a + b*Sech
[c + d*x]^2)^5*Sinh[d*x])/(9*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^5) + (32*Cos
h[c + d*x]^3*Sech[c]*(a + b*Sech[c + d*x]^2)^5*(45*a*b^4*Sinh[d*x] + 8*b^5*
Sinh[d*x]))/(63*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^5) + (64*Cosh[c + d*x]^5*
Sech[c]*(a + b*Sech[c + d*x]^2)^5*(105*a^2*b^3*Sinh[d*x] + 45*a*b^4*Sinh[d*
x] + 8*b^5*Sinh[d*x]))/(105*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^5) + (64*Cosh
[c + d*x]^7*Sech[c]*(a + b*Sech[c + d*x]^2)^5*(525*a^3*b^2*Sinh[d*x] + 420*
a^2*b^3*Sinh[d*x] + 180*a*b^4*Sinh[d*x] + 32*b^5*Sinh[d*x]))/(315*d*(a + 2*
b + a*Cosh[2*c + 2*d*x])^5) + (32*Cosh[c + d*x]^9*Sech[c]*(a + b*Sech[c + d
*x]^2)^5*(1575*a^4*b*Sinh[d*x] + 2100*a^3*b^2*Sinh[d*x] + 1680*a^2*b^3*Sinh
[d*x] + 720*a*b^4*Sinh[d*x] + 128*b^5*Sinh[d*x]))/(315*d*(a + 2*b + a*Cosh[
2*c + 2*d*x])^5) + (32*b^5*Cosh[c + d*x]^2*(a + b*Sech[c + d*x]^2)^5*Tanh[c
])/(9*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^5)
```

Maple [A] time = 0.039, size = 185, normalized size = 1.1

$$\frac{1}{d}\left(a^5(dx+c)+5a^4b\tanh(dx+c)+10a^3b^2\left(2/3+1/3(\text{sech}(dx+c))^2\right)\tanh(dx+c)+10a^2b^3\left(\frac{8}{15}+1/5(\text{sech}(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^5,x)
```

```
[Out] 1/d*(a^5*(d*x+c)+5*a^4*b*tanh(d*x+c)+10*a^3*b^2*(2/3+1/3*sech(d*x+c)^2)*tan
h(d*x+c)+10*a^2*b^3*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c)
```

```
+5*a*b^4*(16/35+1/7*sech(d*x+c)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*ta
nh(d*x+c)+b^5*(128/315+1/9*sech(d*x+c)^8+8/63*sech(d*x+c)^6+16/105*sech(d*x
+c)^4+64/315*sech(d*x+c)^2)*tanh(d*x+c))
```

Maxima [B] time = 1.21151, size = 1724, normalized size = 10.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^5,x, algorithm="maxima")
```

```
[Out] a^5*x + 256/315*b^5*(9*e^(-2*d*x - 2*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d
*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 1
0*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c)
 + e^(-18*d*x - 18*c) + 1)) + 36*e^(-4*d*x - 4*c)/(d*(9*e^(-2*d*x - 2*c) +
36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-
10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d
*x - 16*c) + e^(-18*d*x - 18*c) + 1)) + 84*e^(-6*d*x - 6*c)/(d*(9*e^(-2*d*x
 - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c)
+ 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) +
9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) + 126*e^(-8*d*x - 8*c)/(d*(
9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*
d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x
 - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) + 1/(d*(9*e^(-2*
d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*
c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c)
 + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1))) + 32/7*a*b^4*(7*e^(-2*d
*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c
) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^
(-14*d*x - 14*c) + 1)) + 21*e^(-4*d*x - 4*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^
(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x
- 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 35*e^(-6*d*x -
6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 3
5*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*
d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(
-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x
- 12*c) + e^(-14*d*x - 14*c) + 1))) + 32/3*a^2*b^3*(5*e^(-2*d*x - 2*c)/(d*(
5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*
x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x -
 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^
(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 1
0*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 40/3*
a^3*b^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e
^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) + 1))) + 10*a^4*b/(d*(e^(-2*d*x - 2*c) + 1))
```

Fricas [B] time = 2.15477, size = 4396, normalized size = 26.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^5,x, algorithm="fricas")
```

```
[Out] 1/315*((315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4
 - 128*b^5)*cosh(d*x + c)^9 + 9*(315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1
680*a^2*b^3 - 720*a*b^4 - 128*b^5)*cosh(d*x + c)*sinh(d*x + c)^8 + (1575*a^
4*b + 2100*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 + 128*b^5)*sinh(d*x + c)^9 +
9*(315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128
*b^5)*cosh(d*x + c)^7 + 9*(1225*a^4*b + 2100*a^3*b^2 + 1680*a^2*b^3 + 720*a
*b^4 + 128*b^5 + 4*(1575*a^4*b + 2100*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 +
128*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 21*(4*(315*a^5*d*x - 1575*a^4*b
 - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*cosh(d*x + c)^3 + 3*(
315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^
5)*cosh(d*x + c))*sinh(d*x + c)^6 + 36*(315*a^5*d*x - 1575*a^4*b - 2100*a^3
*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*cosh(d*x + c)^5 + 9*(3500*a^4*b
+ 7000*a^3*b^2 + 6720*a^2*b^3 + 2880*a*b^4 + 512*b^5 + 14*(1575*a^4*b + 210
0*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 + 128*b^5)*cosh(d*x + c)^4 + 21*(1225*
a^4*b + 2100*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 + 128*b^5)*cosh(d*x + c)^2)
*sinh(d*x + c)^5 + 9*(14*(315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^
2*b^3 - 720*a*b^4 - 128*b^5)*cosh(d*x + c)^5 + 35*(315*a^5*d*x - 1575*a^4*b
 - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*cosh(d*x + c)^3 + 20*
(315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b
^5)*cosh(d*x + c))*sinh(d*x + c)^4 + 84*(315*a^5*d*x - 1575*a^4*b - 2100*a^
3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*cosh(d*x + c)^3 + 3*(28*(1575*a
^4*b + 2100*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 + 128*b^5)*cosh(d*x + c)^6 +
 14700*a^4*b + 32200*a^3*b^2 + 35840*a^2*b^3 + 20160*a*b^4 + 3584*b^5 + 105
*(1225*a^4*b + 2100*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 + 128*b^5)*cosh(d*x
+ c)^4 + 120*(875*a^4*b + 1750*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 + 128*b^5
)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 9*(4*(315*a^5*d*x - 1575*a^4*b - 2100*
a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*cosh(d*x + c)^7 + 21*(315*a^5
*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*cosh
(d*x + c)^5 + 40*(315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3 -
720*a*b^4 - 128*b^5)*cosh(d*x + c)^3 + 28*(315*a^5*d*x - 1575*a^4*b - 2100*
a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*cosh(d*x + c))*sinh(d*x + c)^
2 + 126*(315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4
 - 128*b^5)*cosh(d*x + c) + 9*((1575*a^4*b + 2100*a^3*b^2 + 1680*a^2*b^3 +
720*a*b^4 + 128*b^5)*cosh(d*x + c)^8 + 7*(1225*a^4*b + 2100*a^3*b^2 + 1680*
a^2*b^3 + 720*a*b^4 + 128*b^5)*cosh(d*x + c)^6 + 2450*a^4*b + 5600*a^3*b^2
+ 6720*a^2*b^3 + 4480*a*b^4 + 1792*b^5 + 20*(875*a^4*b + 1750*a^3*b^2 + 168
0*a^2*b^3 + 720*a*b^4 + 128*b^5)*cosh(d*x + c)^4 + 28*(525*a^4*b + 1150*a^3
*b^2 + 1280*a^2*b^3 + 720*a*b^4 + 128*b^5)*cosh(d*x + c)^2)*sinh(d*x + c))/
(d*cosh(d*x + c)^9 + 9*d*cosh(d*x + c)*sinh(d*x + c)^8 + 9*d*cosh(d*x + c)^
7 + 21*(4*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^6 + 36*d*cos
h(d*x + c)^5 + 9*(14*d*cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 + 20*d*cosh(d
*x + c))*sinh(d*x + c)^4 + 84*d*cosh(d*x + c)^3 + 9*(4*d*cosh(d*x + c)^7 +
21*d*cosh(d*x + c)^5 + 40*d*cosh(d*x + c)^3 + 28*d*cosh(d*x + c))*sinh(d*x
+ c)^2 + 126*d*cosh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\operatorname{sech}^2(c + dx)\right)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**5,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**5, x)
```

Giac [B] time = 1.18922, size = 725, normalized size = 4.45

$$\frac{(dx+c)a^5}{d} - \frac{2\left(1575\,a^4be^{(16\,dx+16\,c)} + 12600\,a^4be^{(14\,dx+14\,c)} + 6300\,a^3b^2e^{(14\,dx+14\,c)} + 44100\,a^4be^{(12\,dx+12\,c)} + 39900\,a^3b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^5,x, algorithm="giac")
```

```
[Out] (d*x + c)*a^5/d - 2/315*(1575*a^4*b*e^(16*d*x + 16*c) + 12600*a^4*b*e^(14*d
*x + 14*c) + 6300*a^3*b^2*e^(14*d*x + 14*c) + 44100*a^4*b*e^(12*d*x + 12*c)
 + 39900*a^3*b^2*e^(12*d*x + 12*c) + 16800*a^2*b^3*e^(12*d*x + 12*c) + 8820
0*a^4*b*e^(10*d*x + 10*c) + 107100*a^3*b^2*e^(10*d*x + 10*c) + 75600*a^2*b^
3*e^(10*d*x + 10*c) + 25200*a*b^4*e^(10*d*x + 10*c) + 110250*a^4*b*e^(8*d*x
 + 8*c) + 157500*a^3*b^2*e^(8*d*x + 8*c) + 136080*a^2*b^3*e^(8*d*x + 8*c) +
 65520*a*b^4*e^(8*d*x + 8*c) + 16128*b^5*e^(8*d*x + 8*c) + 88200*a^4*b*e^(6
*d*x + 6*c) + 136500*a^3*b^2*e^(6*d*x + 6*c) + 124320*a^2*b^3*e^(6*d*x + 6*
c) + 60480*a*b^4*e^(6*d*x + 6*c) + 10752*b^5*e^(6*d*x + 6*c) + 44100*a^4*b*
e^(4*d*x + 4*c) + 69300*a^3*b^2*e^(4*d*x + 4*c) + 60480*a^2*b^3*e^(4*d*x +
4*c) + 25920*a*b^4*e^(4*d*x + 4*c) + 4608*b^5*e^(4*d*x + 4*c) + 12600*a^4*b
*e^(2*d*x + 2*c) + 18900*a^3*b^2*e^(2*d*x + 2*c) + 15120*a^2*b^3*e^(2*d*x +
 2*c) + 6480*a*b^4*e^(2*d*x + 2*c) + 1152*b^5*e^(2*d*x + 2*c) + 1575*a^4*b
+ 2100*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 + 128*b^5)/(d*(e^(2*d*x + 2*c) +
1)^9)
```

3.138 $\int \frac{\tanh^5(c+dx)}{a+b\,\mathbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=70

$$\frac{(a+b)^2\log\left(a\cosh^2(c+dx)+b\right)}{2ab^2d}-\frac{(a+2b)\log(\cosh(c+dx))}{b^2d}-\frac{\mathrm{sech}^2(c+dx)}{2bd}$$

```
[Out] -(((a + 2*b)*Log[Cosh[c + d*x]])/(b^2*d)) + ((a + b)^2*Log[b + a*Cosh[c + d
*x]^2])/(2*a*b^2*d) - Sech[c + d*x]^2/(2*b*d)
```

Rubi [A] time = 0.111549, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4138, 446, 88}

$$\frac{(a+b)^2\log\left(a\cosh^2(c+dx)+b\right)}{2ab^2d}-\frac{(a+2b)\log(\cosh(c+dx))}{b^2d}-\frac{\mathrm{sech}^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^5/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -(((a + 2*b)*Log[Cosh[c + d*x]])/(b^2*d)) + ((a + b)^2*Log[b + a*Cosh[c + d
*x]^2])/(2*a*b^2*d) - Sech[c + d*x]^2/(2*b*d)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\tanh^5(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^2}{x^3(b+ax^2)}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \frac{(1-x)^2}{x^2(b+ax)}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{bx^2}+\frac{-a-2b}{b^2x}+\frac{(a+b)^2}{b^2(b+ax)}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{(a+2b)\log(\cosh(c+dx))}{b^2d} + \frac{(a+b)^2\log\left(b+a\cosh^2(c+dx)\right)}{2ab^2d} - \frac{\operatorname{sech}^2(c+dx)}{2bd}$$

Mathematica [A] time = 0.289927, size = 98, normalized size = 1.4

$$-\frac{\operatorname{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b)\left(ab\operatorname{sech}^2(c+dx)+(a+b)^2\left(-\log\left(a\sinh^2(c+dx)+a+b\right)\right)+2a(a+2b\right.}{4ab^2d\left(a+b\operatorname{sech}^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^5/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(2*a*(a + 2*b)*Log[Cosh[c
 + d*x]] - (a + b)^2*Log[a + b + a*Sinh[c + d*x]^2] + a*b*Sech[c + d*x]^2))
/(4*a*b^2*d*(a + b*Sech[c + d*x]^2))
```

Maple [B] time = 0.062, size = 331, normalized size = 4.7

$$-\frac{1}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{a}{2\,db^2}\ln\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4a+b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4+2\left(\tanh\left(1/2\,dx+c/2\right)\right)^2a-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/d/a*ln(tanh(1/2*d*x+1/2*c)+1)+1/2/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)^4*a+b*t
anh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+
b)+1/d/b*ln(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+
1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+1/2/d/a*ln(tanh(1/2*d*x+1/2*c)^4*
a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2
*b+a+b)-1/d/a*ln(tanh(1/2*d*x+1/2*c)-1)-2/d/b/(tanh(1/2*d*x+1/2*c)^2+1)^2-1
/d/b^2*ln(tanh(1/2*d*x+1/2*c)^2+1)*a-2/d/b*ln(tanh(1/2*d*x+1/2*c)^2+1)+2/d/
b/(tanh(1/2*d*x+1/2*c)^2+1)
```

Maxima [A] time = 1.76718, size = 177, normalized size = 2.53

$$\frac{dx+c}{ad}-\frac{2\,e^{(-2\,dx-2\,c)}}{\left(2\,be^{(-2\,dx-2\,c)}+be^{(-4\,dx-4\,c)}+b\right)d}-\frac{(a+2\,b)\log\left(e^{(-2\,dx-2\,c)}+1\right)}{b^2d}+\frac{\left(a^2+2\,ab+b^2\right)\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}\right.}{2\,ab^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] (d*x + c)/(a*d) - 2*e^(-2*d*x - 2*c)/((2*b*e^(-2*d*x - 2*c) + b*e^(-4*d*x -
 4*c) + b)*d) - (a + 2*b)*log(e^(-2*d*x - 2*c) + 1)/(b^2*d) + 1/2*(a^2 + 2*
a*b + b^2)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a*b^
2*d)
```

Fricas [B] time = 2.66465, size = 1895, normalized size = 27.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*d*x*cosh(d*x + c)^4 + 8*b^2*d*x*cosh(d*x + c)*sinh(d*x + c)^3 +
 2*b^2*d*x*sinh(d*x + c)^4 + 2*b^2*d*x + 4*(b^2*d*x + a*b)*cosh(d*x + c)^2
+ 4*(3*b^2*d*x*cosh(d*x + c)^2 + b^2*d*x + a*b)*sinh(d*x + c)^2 - ((a^2 + 2
*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x
+ c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b + b^2)*cosh(d
*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*s
inh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3
 + (a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*(a*cosh(d*x + c)
^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d
*x + c) + sinh(d*x + c)^2)) + 2*((a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 + 2
*a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b)*sinh(d*x + c)^4 + 2*(a^
2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b)*cosh(d*x + c)^2 + a^2 + 2*a
*b)*sinh(d*x + c)^2 + a^2 + 2*a*b + 4*((a^2 + 2*a*b)*cosh(d*x + c)^3 + (a^2
 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c)
- sinh(d*x + c))) + 8*(b^2*d*x*cosh(d*x + c)^3 + (b^2*d*x + a*b)*cosh(d*x +
 c))*sinh(d*x + c))/(a*b^2*d*cosh(d*x + c)^4 + 4*a*b^2*d*cosh(d*x + c)*sinh
(d*x + c)^3 + a*b^2*d*sinh(d*x + c)^4 + 2*a*b^2*d*cosh(d*x + c)^2 + a*b^2*d
 + 2*(3*a*b^2*d*cosh(d*x + c)^2 + a*b^2*d)*sinh(d*x + c)^2 + 4*(a*b^2*d*cos
h(d*x + c)^3 + a*b^2*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^5(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**5/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(tanh(c + d*x)**5/(a + b*sech(c + d*x)**2), x)
```

Giac [B] time = 2.24284, size = 240, normalized size = 3.43

$$-\frac{\frac{2\,dx}{a} + \frac{2\left(ae^{(2\,c)}+2\,be^{(2\,c)}\right)e^{(-2\,c)}\log\left(e^{(2\,dx+2\,c)}+1\right)}{b^2} - \frac{\left(a^2+2\,ab+b^2\right)\log\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}{ab^2} - \frac{3\,ae^{(4\,dx+4\,c)}+6\,be^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}+}{b^2\left(e^{(2\,dx+2\,c)}+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*(2*d*x/a + 2*(a*e^(2*c) + 2*b*e^(2*c))*e^(-2*c)*log(e^(2*d*x + 2*c) +
1)/b^2 - (a^2 + 2*a*b + b^2)*log(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) +
4*b*e^(2*d*x + 2*c) + a)/(a*b^2) - (3*a*e^(4*d*x + 4*c) + 6*b*e^(4*d*x + 4*
c) + 6*a*e^(2*d*x + 2*c) + 8*b*e^(2*d*x + 2*c) + 3*a + 6*b)/(b^2*(e^(2*d*x
+ 2*c) + 1)^2))/d
```

3.139 $\int \frac{\tanh^4(c+dx)}{a+b\,\mathbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=59

$$-\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{ab^{3/2}d}+\frac{x}{a}+\frac{\tanh(c+dx)}{bd}$$

```
[Out] x/a - ((a + b)^(3/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*b^(3/
2)*d) + Tanh[c + d*x]/(b*d)
```

Rubi [A] time = 0.178189, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4141, 1975, 479, 522, 206, 208}

$$-\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{ab^{3/2}d}+\frac{x}{a}+\frac{\tanh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```


```
[Out] x/a - ((a + b)^(3/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*b^(3/
2)*d) + Tanh[c + d*x]/(b*d)
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)\left(a+b(1-x^2)\right)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\tanh(c+dx)}{bd} - \frac{\operatorname{Subst}\left(\int \frac{a+b+(-a-2b)x^2}{(1-x^2)(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{bd}\\
&= \frac{\tanh(c+dx)}{bd} + \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{ad} - \frac{(a+b)^2\operatorname{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \tan\right.}{abd}\\
&= \frac{x}{a} - \frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{ab^{3/2}d} + \frac{\tanh(c+dx)}{bd}
\end{aligned}$$

Mathematica [B] time = 1.10741, size = 196, normalized size = 3.32

$$\frac{\operatorname{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}(a\operatorname{sech}(c)\sinh(dx)\operatorname{sech}(c+dx)+bdx)+(a\right.}{2abd\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}\left(a+b\operatorname{se}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*((a + b)^2*ArcTanh[(Sech[d
*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*S
qrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(-Cosh[2*c] + Sinh[2*c]) + Sqrt[
a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]*(b*d*x + a*Sech[c]*Sech[c + d*x]*Sinh[
d*x])))/(2*a*b*Sqrt[a + b]*d*(a + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh
[c])^4])
```

Maple [B] time = 0.067, size = 386, normalized size = 6.5

$$\frac{1}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right) - \frac{a}{2d}\ln\left(\sqrt{a+b}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 + 2\tanh(1/2\,dx+c/2)\sqrt{b}+\sqrt{a+b}\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{a+b}} - \frac{1}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/d/a*ln(tanh(1/2*d*x+1/2*c)+1)-1/2/d*a/b^(3/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*
tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-1/d/b^(1/2
)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^
(1/2)+(a+b)^(1/2))-1/2/d*b^(1/2)/a/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+
1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/2/d*a/b^(3/2)/(a+b)^(
1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+
b)^(1/2))+1/d/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*ta
nh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/2/d*b^(1/2)/a/(a+b)^(1/2)*ln((a+b)
^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-1/d
/a*ln(tanh(1/2*d*x+1/2*c)-1)+2/d/b*tanh(1/2*d*x+1/2*c)/(tanh(1/2*d*x+1/2*c)
^2+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.3162, size = 1854, normalized size = 31.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*b*d*x*cosh(d*x + c)^2 + 4*b*d*x*cosh(d*x + c)*sinh(d*x + c) + 2*b*d
*x*sinh(d*x + c)^2 + 2*b*d*x + ((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*
x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a + b)*sqrt((a + b)/b)*log
((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x
+ c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 +
 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a
^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*cosh(d*x + c)^2 + 2*a*b*c
osh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + a*b + 2*b^2)*sqrt((a + b
)/b))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x +
 c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sin
h(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c
) + a)) - 4*a)/(a*b*d*cosh(d*x + c)^2 + 2*a*b*d*cosh(d*x + c)*sinh(d*x + c)
 + a*b*d*sinh(d*x + c)^2 + a*b*d), (b*d*x*cosh(d*x + c)^2 + 2*b*d*x*cosh(d*
x + c)*sinh(d*x + c) + b*d*x*sinh(d*x + c)^2 + b*d*x - ((a + b)*cosh(d*x +
c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a
+ b)*sqrt(-(a + b)/b)*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sin
h(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-(a + b)/b)/(a + b)) - 2*a)/
(a*b*d*cosh(d*x + c)^2 + 2*a*b*d*cosh(d*x + c)*sinh(d*x + c) + a*b*d*sinh(d
*x + c)^2 + a*b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**4/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(tanh(c + d*x)**4/(a + b*sech(c + d*x)**2), x)
```

Giac [B] time = 1.91454, size = 147, normalized size = 2.49

$$\frac{\frac{dx}{a}-\frac{\left(a^2e^{(2c)}+2\,abe^{(2c)}+b^2e^{(2c)}\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\sqrt{-ab-b^2}}\right)e^{(-2\,c)}}{\sqrt{-ab-b^2}ab}-\frac{2}{b\left(e^{(2\,dx+2\,c)}+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] (d*x/a - (a^2*e^(2*c) + 2*a*b*e^(2*c) + b^2*e^(2*c))*arctan(1/2*(a*e^(2*d*x
 + 2*c) + a + 2*b)/sqrt(-a*b - b^2))*e^(-2*c)/(sqrt(-a*b - b^2)*a*b) - 2/(b
*(e^(2*d*x + 2*c) + 1)))/d
```

3.140 $\int \frac{\tanh^3(c+dx)}{a+b\,\mathbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=45

$$\frac{(a+b)\log\left(a\cosh^2(c+dx)+b\right)}{2abd}-\frac{\log(\cosh(c+dx))}{bd}$$

```
[Out] -(Log[Cosh[c + d*x]]/(b*d)) + ((a + b)*Log[b + a*Cosh[c + d*x]^2])/(2*a*b*d
)
```

Rubi [A] time = 0.0883408, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4138, 446, 72}

$$\frac{(a+b)\log\left(a\cosh^2(c+dx)+b\right)}{2abd}-\frac{\log(\cosh(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```


```
[Out] -(Log[Cosh[c + d*x]]/(b*d)) + ((a + b)*Log[b + a*Cosh[c + d*x]^2])/(2*a*b*d
)
```


Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\tanh^3(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{x(b+ax^2)}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{1-x}{x(b+ax)}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{bx}+\frac{-a-b}{b(b+ax)}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{\log(\cosh(c+dx))}{bd}+\frac{(a+b)\log\left(b+a\cosh^2(c+dx)\right)}{2abd}$$

Mathematica [A] time = 0.106356, size = 41, normalized size = 0.91

$$\frac{(a+b)\log\left(a\cosh^2(c+dx)+b\right)-2a\log(\cosh(c+dx))}{2abd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (-2*a*Log[Cosh[c + d*x]] + (a + b)*Log[b + a*Cosh[c + d*x]^2])/(2*a*b*d)
```

Maple [B] time = 0.054, size = 196, normalized size = 4.4

$$-\frac{1}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{1}{2\,bd}\ln\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 a+b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4+2\left(\tanh(1/2\,dx+c/2)\right)^2 a-2\,($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/d/a*ln(tanh(1/2*d*x+1/2*c)+1)+1/2/d/b*ln(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(
1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+1
/2/d/a*ln(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/
2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)-1/d/a*ln(tanh(1/2*d*x+1/2*c)-1)-1/d
/b*ln(tanh(1/2*d*x+1/2*c)^2+1)
```

Maxima [A] time = 1.75167, size = 104, normalized size = 2.31

$$\frac{dx+c}{ad}+\frac{(a+b)\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\,abd}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] (d*x + c)/(a*d) + 1/2*(a + b)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*
x - 4*c) + a)/(a*b*d) - log(e^(-2*d*x - 2*c) + 1)/(b*d)
```

Fricas [B] time = 2.32985, size = 294, normalized size = 6.53

$$-\frac{2\,bdx-(a+b)\log\left(\frac{2\left(a\cosh(dx+c)^2+a\sinh(dx+c)^2+a+2\,b\right)}{\cosh(dx+c)^2-2\,\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2}\right)+2\,a\log\left(\frac{2\,\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{2\,abd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*x - (a + b)*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a +
2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) +
 2*a*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/(a*b*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^3(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**3/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(tanh(c + d*x)**3/(a + b*sech(c + d*x)**2), x)
```

Giac [A] time = 1.79071, size = 107, normalized size = 2.38

$$-\frac{\frac{2\,dx}{a}-\frac{(a+b)\log\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}{ab}+\frac{2\,\log\left(e^{(2\,dx+2\,c)}+1\right)}{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*(2*d*x/a - (a + b)*log(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e
^(2*d*x + 2*c) + a)/(a*b) + 2*log(e^(2*d*x + 2*c) + 1)/b)/d
```

3.141 $\int \frac{\tanh^2(c+dx)}{a+b\,\mathbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=46

$$\frac{x}{a}-\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{b}d}$$

```
[Out] x/a - (Sqrt[a + b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*Sqrt[b]
*d)
```

Rubi [A] time = 0.140398, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4141, 1975, 481, 206, 208}

$$\frac{x}{a}-\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] x/a - (Sqrt[a + b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*Sqrt[b]
*d)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tanh^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)(a+b(1-x^2))}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{ad} - \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \tanh(c+dx)\right)}{ad}$$
$$= \frac{x}{a} - \frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{b}d}$$

Mathematica [B] time = 0.29852, size = 174, normalized size = 3.78

$$\frac{\operatorname{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b)\left(dx\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}+(a+b)(\sinh(2c)-\cosh(2c))\tanh^{-1}\left(\frac{(\cos}{}\right.\right.}{2ad\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}\left(a+b\operatorname{sech}^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(Sqrt[a + b]*d*x*Sqrt[b*(C
osh[c] - Sinh[c])^4] + (a + b)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*(
(a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] -
Sinh[c])^4])]*(-Cosh[2*c] + Sinh[2*c])))/(2*a*Sqrt[a + b]*d*(a + b*Sech[c +
 d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])
```

Maple [B] time = 0.059, size = 257, normalized size = 5.6

$$\frac{1}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right) - \frac{1}{2d}\ln\left(\sqrt{a+b}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 + 2\tanh(1/2\,dx+c/2)\sqrt{b}+\sqrt{a+b}\right)\frac{1}{\sqrt{b}}\frac{1}{\sqrt{a+b}} + \frac{1}{2d}\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/d/a*ln(tanh(1/2*d*x+1/2*c)+1)-1/2/d/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*ta
nh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/2/d/b^(1/2
)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b
^(1/2)-(a+b)^(1/2))-1/2/d*b^(1/2)/a/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x
+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/2/d/a*b^(1/2)/(a+b)^
(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)-(
a+b)^(1/2))-1/d/a*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.18677, size = 1087, normalized size = 23.63

$$\left[\frac{2\,dx + \sqrt{\frac{a+b}{b}}\log\left(\frac{a^2\cosh(dx+c)^4+4\,a^2\cosh(dx+c)\sinh(dx+c)^3+a^2\sinh(dx+c)^4+2\left(a^2+2\,ab\right)\cosh(dx+c)^2+2\left(3\,a^2\cosh(dx+c)^2+a^2+2\,ab\right)\sinh(dx+c}{a\cosh(dx+c)^4+4\,a\cosh(dx+c)\sinh(dx+c)^3+a\sinh(dx+c)^4+2\,(a+2\,b)\cosh(d}\right.}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*d*x + sqrt((a + b)/b)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c
)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 +
 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*
b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) +
 4*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x
+ c)^2 + a*b + 2*b^2)*sqrt((a + b)/b))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x +
c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3
*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a +
 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)))/(a*d), (d*x - sqrt(-(a + b)/b)*ar
ctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x
+ c)^2 + a + 2*b)*sqrt(-(a + b)/b)/(a + b)))/(a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(tanh(c + d*x)**2/(a + b*sech(c + d*x)**2), x)
```

Giac [B] time = 1.48974, size = 104, normalized size = 2.26

$$-\frac{\frac{\left(ae^{(2c)}+be^{(2c)}\right)\arctan\left(\frac{ae^{(2dx+2c)}+a+2b}{2\sqrt{-ab-b^2}}\right)e^{(-2c)}}{\sqrt{-ab-b^2}\,a} - \frac{dx}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -((a*e^(2*c) + b*e^(2*c))*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*
b - b^2))*e^(-2*c)/(sqrt(-a*b - b^2)*a) - d*x/a)/d
```

3.142 $\int \frac{\tanh(c+dx)}{a+b\,\mathbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=23

$$\frac{\log\left(a\cosh^2(c+dx)+b\right)}{2ad}$$

```
[Out] Log[b + a*Cosh[c + d*x]^2]/(2*a*d)
```

Rubi [A] time = 0.0347621, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4138, 260}

$$\frac{\log\left(a\cosh^2(c+dx)+b\right)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] Log[b + a*Cosh[c + d*x]^2]/(2*a*d)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x}{b+ax^2}\,dx, x, \cosh(c+dx)\right)}{d}\\ &= \frac{\log\left(b+a\cosh^2(c+dx)\right)}{2ad}\end{aligned}$$

Mathematica [A] time = 0.175158, size = 26, normalized size = 1.13

$$\frac{\log(a\cosh(2(c+dx))+a+2b)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] Log[a + 2*b + a*Cosh[2*(c + d*x)]]/(2*a*d)
```

Maple [A] time = 0.017, size = 38, normalized size = 1.7

$$\frac{\ln\left(a+b\left(\operatorname{sech}\left(dx+c\right)\right)^{2}\right)}{2\,da}-\frac{\ln\left(\operatorname{sech}\left(dx+c\right)\right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)/(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/2/d/a*ln(a+b*sech(d*x+c)^2)-1/d/a*ln(sech(d*x+c))
```

Maxima [B] time = 1.13346, size = 69, normalized size = 3.

$$\frac{dx+c}{ad}+\frac{\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] (d*x + c)/(a*d) + 1/2*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c)
 + a)/(a*d)
```

Fricas [B] time = 2.18883, size = 198, normalized size = 8.61

$$-\frac{2\,dx-\log\left(\frac{2\,\left(a\cosh(dx+c)^{2}+a\sinh(dx+c)^{2}+a+2\,b\right)}{\cosh(dx+c)^{2}-2\,\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^{2}}\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*d*x - log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh
(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)))/(a*d)
```

Sympy [A] time = 9.33816, size = 124, normalized size = 5.39

$$\begin{cases}\frac{\tilde{\infty}x\tanh(c)}{\operatorname{sech}^{2}(c)} & \text{for}\; a=0\wedge b=0\wedge d=0\\ \frac{1}{2bd\operatorname{sech}^{2}(c+dx)} & \text{for}\; a=0\\ \frac{x\tanh(c)}{a+b\operatorname{sech}^{2}(c)} & \text{for}\; d=0\\ \frac{x-\frac{\log(\tanh(c+dx)+1)}{d}}{a} & \text{for}\; b=0\\ \frac{x}{a}+\frac{\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\operatorname{sech}(c+dx)\right)}{2ad}+\frac{\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\operatorname{sech}(c+dx)\right)}{2ad}-\frac{\log(\tanh(c+dx)+1)}{ad} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x*tanh(c)/sech(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (1/(2
*b*d*sech(c + d*x)**2), Eq(a, 0)), (x*tanh(c)/(a + b*sech(c)**2), Eq(d, 0))
, ((x - log(tanh(c + d*x) + 1)/d)/a, Eq(b, 0)), (x/a + log(-I*sqrt(a)*sqrt(
1/b) + sech(c + d*x))/(2*a*d) + log(I*sqrt(a)*sqrt(1/b) + sech(c + d*x))/(2
*a*d) - log(tanh(c + d*x) + 1)/(a*d), True))
```

Giac [B] time = 1.39835, size = 76, normalized size = 3.3

$$-\frac{\frac{2\,dx}{a} - \frac{\log\left(ae^{(4\,dx+4\,c)} + 2\,ae^{(2\,dx+2\,c)} + 4\,be^{(2\,dx+2\,c)} + a\right)}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*(2*d*x/a - log(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x
+ 2*c) + a)/a)/d
```

3.143 $\int \frac{1}{a+b\,\mathbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=46

$$\frac{x}{a}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{ad\sqrt{a+b}}$$

```
[Out] x/a - (Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*Sqrt[a + b]
*d)
```

Rubi [A] time = 0.0463049, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4127, 3181, 208}

$$\frac{x}{a}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{ad\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^(-1),x]
```

```
[Out] x/a - (Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*Sqrt[a + b]
*d)
```

Rule 4127

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/a, x
] - Dist[b/a, Int[1/(b + a*Cos[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f}, x
] && NeQ[a + b, 0]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{a+b\operatorname{sech}^2(c+dx)}\,dx &= \frac{x}{a}-\frac{b\int \frac{1}{b+a\cosh^2(c+dx)}\,dx}{a}\\
&= \frac{x}{a}-\frac{b\,\text{Subst}\left(\int \frac{1}{b-(a+b)x^2}\,dx,x,\coth(c+dx)\right)}{ad}\\
&= \frac{x}{a}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+bd}}
\end{aligned}$$

Mathematica [B] time = 0.246705, size = 172, normalized size = 3.74

$$\frac{\operatorname{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b)\left(dx\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}+b(\sinh(2c)-\cosh(2c))\tanh^{-1}\left(\frac{(\cosh(2\cdots}{\cdots}\right.\right.}{2ad\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}\left(a+b\operatorname{sech}^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^(-1),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(Sqrt[a + b]*d*x*Sqrt[b*(C
osh[c] - Sinh[c])^4] + b*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2
*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c
])^4])]*(-Cosh[2*c] + Sinh[2*c])))/(2*a*Sqrt[a + b]*d*(a + b*Sech[c + d*x]^
2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])
```

Maple [B] time = 0.052, size = 152, normalized size = 3.3

$$\frac{1}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{1}{2\,da}\sqrt{b}\ln\left(\sqrt{a+b}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+2\,\tanh(1/2\,dx+c/2)\sqrt{b}+\sqrt{a+b}\right)\frac{1}{\sqrt{a+b}}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/d/a*ln(tanh(1/2*d*x+1/2*c)+1)-1/2/d*b^(1/2)/a/(a+b)^(1/2)*ln((a+b)^(1/2)*
tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/2/d/a*b^
(1/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*
c)*b^(1/2)-(a+b)^(1/2))-1/d/a*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.30798, size = 1122, normalized size = 24.39

$$\left[\frac{2\,dx+\sqrt{\frac{b}{a+b}}\log\left(\frac{a^2\cosh(dx+c)^4+4\,a^2\cosh(dx+c)\sinh(dx+c)^3+a^2\sinh(dx+c)^4+2\left(a^2+2\,ab\right)\cosh(dx+c)^2+2\left(3\,a^2\cosh(dx+c)^2+a^2+2\,ab\right)\sinh(dx+c\cdots}{a\cosh(dx+c)^4+4\,a\cosh(dx+c)\sinh(dx+c)^3+a\sinh(dx+c)^4+2\,(a+\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*d*x + sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c
)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 +
 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*
b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) +
 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)
 + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*c
osh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*
(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)
^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)))/(
a*d), (d*x - sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x
+ c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b))/(a*d
)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a + b\operatorname{sech}^2(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(1/(a + b*sech(c + d*x)**2), x)
```

Giac [A] time = 1.17124, size = 88, normalized size = 1.91

$$-\frac{b \arctan\left(\frac{a e^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}ad} + \frac{dx+c}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -b*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b -
b^2)*a*d) + (d*x + c)/(a*d)
```

3.144 $\int \frac{\coth(c+dx)}{a+b\,\mathbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=46

$$\frac{\log(\sinh(c+dx))}{d(a+b)}+\frac{b\log\left(a\cosh^2(c+dx)+b\right)}{2ad(a+b)}$$

```
[Out] (b*Log[b + a*Cosh[c + d*x]^2])/(2*a*(a + b)*d) + Log[Sinh[c + d*x]]/((a + b
)*d)
```

Rubi [A] time = 0.087361, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 446, 72}

$$\frac{\log(\sinh(c+dx))}{d(a+b)}+\frac{b\log\left(a\cosh^2(c+dx)+b\right)}{2ad(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (b*Log[b + a*Cosh[c + d*x]^2])/(2*a*(a + b)*d) + Log[Sinh[c + d*x]]/((a + b
)*d)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\coth(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{x^3}{(1-x^2)(b+ax^2)}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\operatorname{Subst}\left(\int \frac{x}{(1-x)(b+ax)}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{\operatorname{Subst}\left(\int \left(\frac{1}{(-a-b)(-1+x)} - \frac{b}{(a+b)(b+ax)}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{b\log\left(b+a\cosh^2(c+dx)\right)}{2a(a+b)d} + \frac{\log(\sinh(c+dx))}{(a+b)d}$$

Mathematica [A] time = 0.0890806, size = 42, normalized size = 0.91

$$\frac{b\log\left(a\sinh^2(c+dx)+a+b\right)+2a\log(\sinh(c+dx))}{2a^2d+2abd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (2*a*Log[Sinh[c + d*x]] + b*Log[a + b + a*Sinh[c + d*x]^2])/(2*a^2*d + 2*a*
b*d)
```

Maple [B] time = 0.059, size = 133, normalized size = 2.9

$$-\frac{1}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{b}{2da(a+b)}\ln\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4a+b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4+2\,(\tanh(1/2\,dx+c/2))^2\,a-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/d/a*ln(tanh(1/2*d*x+1/2*c)+1)+1/2/d*b/a/(a+b)*ln(tanh(1/2*d*x+1/2*c)^4*a
+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*
b+a+b)+1/d/(a+b)*ln(tanh(1/2*d*x+1/2*c))-1/d/a*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.04251, size = 135, normalized size = 2.93

$$\frac{b\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\left(a^2+ab\right)d}+\frac{dx+c}{ad}+\frac{\log\left(e^{(-dx-c)}+1\right)}{(a+b)d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{(a+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*b*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/((a^2 + a*
b)*d) + (d*x + c)/(a*d) + log(e^(-d*x - c) + 1)/((a + b)*d) + log(e^(-d*x -
 c) - 1)/((a + b)*d)
```

Fricas [B] time = 2.79042, size = 305, normalized size = 6.63

$$-\frac{2\,(a+b)dx-b\log\left(\frac{2\left(a\cosh(dx+c)^{2}+a\sinh(dx+c)^{2}+a+2\,b\right)}{\cosh(dx+c)^{2}-2\,\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^{2}}\right)-2\,a\log\left(\frac{2\,\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{2\left(a^{2}+ab\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a + b)*d*x - b*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a +
2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) -
 2*a*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/((a^2 + a*b)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\coth\left(c+dx\right)}{a+b\,\mathrm{sech}^{2}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(coth(c + d*x)/(a + b*sech(c + d*x)**2), x)
```

Giac [B] time = 1.46974, size = 131, normalized size = 2.85

$$-\frac{\frac{2\,dx}{a}-\frac{b\log\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}{a^{2}+ab}-\frac{2\,e^{(2\,c)}\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{ae^{(2\,c)}+be^{(2\,c)}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*(2*d*x/a - b*log(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*
x + 2*c) + a)/(a^2 + a*b) - 2*e^(2*c)*log(abs(e^(2*d*x + 2*c) - 1))/(a*e^(2
*c) + b*e^(2*c)))/d
```

3.145 $\int \frac{\coth^2(c+dx)}{a+b\mathbf{sech}^2(c+dx)} dx$

Optimal. Leaf size=62

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{ad(a+b)^{3/2}}-\frac{\coth(c+dx)}{d(a+b)}+\frac{x}{a}$$

```
[Out] x/a - (b^(3/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*(a + b)^(3/
2)*d) - Coth[c + d*x]/((a + b)*d)
```

Rubi [A] time = 0.179616, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4141, 1975, 480, 522, 206, 208}

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{ad(a+b)^{3/2}}-\frac{\coth(c+dx)}{d(a+b)}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] x/a - (b^(3/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*(a + b)^(3/
2)*d) - Coth[c + d*x]/((a + b)*d)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\coth^2(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1-x^2)(a+b(1-x^2))}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1-x^2)(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{\coth(c+dx)}{(a+b)d} + \frac{\text{Subst}\left(\int \frac{a+2b-bx^2}{(1-x^2)(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{(a+b)d}$$

$$= -\frac{\coth(c+dx)}{(a+b)d} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{ad} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \tanh(c\right.}{a(a+b)d}$$

$$= \frac{x}{a} - \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a(a+b)^{3/2}d} - \frac{\coth(c+dx)}{(a+b)d}$$

Mathematica [B] time = 1.1116, size = 193, normalized size = 3.11

$$\frac{\text{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b)\left(b^2(\sinh(2c)-\cosh(2c))\tanh^{-1}\left(\frac{(\cosh(2c)-\sinh(2c))\text{sech}(dx)((a+2b)\sinh(dx)-a\sinh\right.}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right.}{2ad(a+b)^{3/2}\sqrt{b(\cosh(c)-\sinh(c))^4}\left(a+bs\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(b^2*ArcTanh[(Sech[d*x]*(C
osh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a
+ b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(-Cosh[2*c] + Sinh[2*c]) + Sqrt[a + b]
*Sqrt[b*(Cosh[c] - Sinh[c])^4]*((a + b)*d*x + a*Csch[c]*Csch[c + d*x]*Sinh[
d*x])))/(2*a*(a + b)^(3/2)*d*(a + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh
[c])^4])
```

Maple [B] time = 0.072, size = 189, normalized size = 3.1

$$-\frac{1}{2d(a+b)}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{1}{2da}b^{\frac{3}{2}}\ln\left(\sqrt{a+b}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+2\tanh(1/2\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/2/d/(a+b)*tanh(1/2*d*x+1/2*c)+1/d/a*ln(tanh(1/2*d*x+1/2*c)+1)-1/2/d*b^(3
/2)/a/(a+b)^(3/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c
)*b^(1/2)+(a+b)^(1/2))+1/2/d*b^(3/2)/a/(a+b)^(3/2)*ln((a+b)^(1/2)*tanh(1/2*
d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-1/2/d/(a+b)/tanh(1/
2*d*x+1/2*c)-1/d/a*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.66152, size = 1980, normalized size = 31.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a + b)*d*x*cosh(d*x + c)^2 + 4*(a + b)*d*x*cosh(d*x + c)*sinh(d*x
+ c) + 2*(a + b)*d*x*sinh(d*x + c)^2 - 2*(a + b)*d*x + (b*cosh(d*x + c)^2 +
 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*sqrt(b/(a + b))*l
og((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*
x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2
 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 +
(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2
 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2
+ a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x +
 c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(
3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a
+ 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - 4*a)/((a^2 + a*b)*d*cosh(d*x +
c)^2 + 2*(a^2 + a*b)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*d*sinh(d*x
 + c)^2 - (a^2 + a*b)*d), ((a + b)*d*x*cosh(d*x + c)^2 + 2*(a + b)*d*x*cosh
(d*x + c)*sinh(d*x + c) + (a + b)*d*x*sinh(d*x + c)^2 - (a + b)*d*x - (b*co
sh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*sq
rt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x +
 c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b) - 2*a)/((a^2 + a*b)*
d*cosh(d*x + c)^2 + 2*(a^2 + a*b)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*
b)*d*sinh(d*x + c)^2 - (a^2 + a*b)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(c + dx)}{a + b\,\mathrm{sech}^2(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(coth(c + d*x)**2/(a + b*sech(c + d*x)**2), x)
```

Giac [A] time = 1.61348, size = 120, normalized size = 1.94

$$-\frac{\frac{b^2\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{(a^2+ab)\sqrt{-ab-b^2}}-\frac{dx}{a}+\frac{2}{(a+b)\left(e^{(2\,dx+2\,c)}-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -(b^2*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/((a^2 + a*
b)*sqrt(-a*b - b^2)) - d*x/a + 2/((a + b)*(e^(2*d*x + 2*c) - 1)))/d
```

3.146 $\int \frac{\coth^3(c+dx)}{a+b\,\mathbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{b^2\log\left(a\cosh^2(c+dx)+b\right)}{2ad(a+b)^2}-\frac{\operatorname{csch}^2(c+dx)}{2d(a+b)}+\frac{(a+2b)\log(\sinh(c+dx))}{d(a+b)^2}$$

```
[Out] -Csch[c + d*x]^2/(2*(a + b)*d) + (b^2*Log[b + a*Cosh[c + d*x]^2])/(2*a*(a +
 b)^2*d) + ((a + 2*b)*Log[Sinh[c + d*x]])/((a + b)^2*d)
```

Rubi [A] time = 0.121826, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4138, 446, 88}

$$\frac{b^2\log\left(a\cosh^2(c+dx)+b\right)}{2ad(a+b)^2}-\frac{\operatorname{csch}^2(c+dx)}{2d(a+b)}+\frac{(a+2b)\log(\sinh(c+dx))}{d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -Csch[c + d*x]^2/(2*(a + b)*d) + (b^2*Log[b + a*Cosh[c + d*x]^2])/(2*a*(a +
 b)^2*d) + ((a + 2*b)*Log[Sinh[c + d*x]])/((a + b)^2*d)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\coth^3(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(1-x^2)^2(b+ax^2)}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1-x)^2(b+ax)}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)(-1+x)^2}+\frac{a+2b}{(a+b)^2(-1+x)}+\frac{b^2}{(a+b)^2(b+ax)}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{\text{csch}^2(c+dx)}{2(a+b)d}+\frac{b^2\log\left(b+a\cosh^2(c+dx)\right)}{2a(a+b)^2d}+\frac{(a+2b)\log(\sinh(c+dx))}{(a+b)^2d}$$

Mathematica [A] time = 0.221937, size = 100, normalized size = 1.37

$$-\frac{\text{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b)\left(b^2\left(-\log\left(a\sinh^2(c+dx)+a+b\right)\right)+a(a+b)\text{csch}^2(c+dx)-2a(a+2b}{4ad(a+b)^2\left(a+b\text{sech}^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((a + 2*b + a*Cosh[2*(c + d*x)])*(a*(a + b)*Csch[c + d*x]^2 - 2*a*(a + 2*b
)*Log[Sinh[c + d*x]] - b^2*Log[a + b + a*Sinh[c + d*x]^2])*Sech[c + d*x]^2)
/(4*a*(a + b)^2*d*(a + b*Sech[c + d*x]^2))
```

Maple [B] time = 0.07, size = 199, normalized size = 2.7

$$-\frac{1}{8d(a+b)}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{1}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{b^2}{2da(a+b)^2}\ln\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4a+b\left(\tanh\left(\frac{dx}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/8/d*tanh(1/2*d*x+1/2*c)^2/(a+b)-1/d/a*ln(tanh(1/2*d*x+1/2*c)+1)+1/2/d*b^
2/a/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d
*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)-1/8/d/(a+b)/tanh(1/2*d*x+1/2*c
)^2+1/d/(a+b)^2*ln(tanh(1/2*d*x+1/2*c))*a+2/d/(a+b)^2*ln(tanh(1/2*d*x+1/2*c
))*b-1/d/a*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.20983, size = 252, normalized size = 3.45

$$\frac{b^2\log\left(2(a+2b)e^{(-2dx-2c)}+ae^{(-4dx-4c)}+a\right)}{2\left(a^3+2a^2b+ab^2\right)d}+\frac{(a+2b)\log\left(e^{(-dx-c)}+1\right)}{\left(a^2+2ab+b^2\right)d}+\frac{(a+2b)\log\left(e^{(-dx-c)}-1\right)}{\left(a^2+2ab+b^2\right)d}+\frac{dx+c}{ad}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*b^2*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/((a^3 +
2*a^2*b + a*b^2)*d) + (a + 2*b)*log(e^(-d*x - c) + 1)/((a^2 + 2*a*b + b^2)*
d) + (a + 2*b)*log(e^(-d*x - c) - 1)/((a^2 + 2*a*b + b^2)*d) + (d*x + c)/(a
*d) + 2*e^(-2*d*x - 2*c)/((2*(a + b)*e^(-2*d*x - 2*c) - (a + b)*e^(-4*d*x -
 4*c) - a - b)*d)
```

Fricas [B] time = 3.02111, size = 2144, normalized size = 29.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 8*(a^2 + 2*a*b + b^2)*d*x
*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^4
+ 2*(a^2 + 2*a*b + b^2)*d*x - 4*((a^2 + 2*a*b + b^2)*d*x - a^2 - a*b)*cosh(
d*x + c)^2 + 4*(3*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 - (a^2 + 2*a*b +
b^2)*d*x + a^2 + a*b)*sinh(d*x + c)^2 - (b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d
*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 - 2*b^2*cosh(d*x + c)^2 + 2*(
3*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x + c)^3
 - b^2*cosh(d*x + c))*sinh(d*x + c))*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x
+ c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d
*x + c)^2)) - 2*((a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (a^2 + 2*a*b)*sinh(d*x + c)^4 - 2*(a^2 + 2*a*b)*cosh(
d*x + c)^2 + 2*(3*(a^2 + 2*a*b)*cosh(d*x + c)^2 - a^2 - 2*a*b)*sinh(d*x + c
)^2 + a^2 + 2*a*b + 4*((a^2 + 2*a*b)*cosh(d*x + c)^3 - (a^2 + 2*a*b)*cosh(d
*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))
) + 8*((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 - ((a^2 + 2*a*b + b^2)*d*x -
 a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + 2*a^2*b + a*b^2)*d*cosh(d
*x + c)^4 + 4*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^
3 + 2*a^2*b + a*b^2)*d*sinh(d*x + c)^4 - 2*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d
*x + c)^2 + 2*(3*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^2 - (a^3 + 2*a^2*b
 + a*b^2)*d)*sinh(d*x + c)^2 + (a^3 + 2*a^2*b + a*b^2)*d + 4*((a^3 + 2*a^2*
b + a*b^2)*d*cosh(d*x + c)^3 - (a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c))*sin
h(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(coth(c + d*x)**3/(a + b*sech(c + d*x)**2), x)
```

Giac [B] time = 2.07635, size = 288, normalized size = 3.95

$$\frac{\frac{b^2\log\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}{a^3+2\,a^2b+ab^2}-\frac{2\,dx}{a}+\frac{2\left(ae^{(2\,c)}+2\,be^{(2\,c)}\right)\log\left(\left|-e^{(2\,dx+2\,c)}+1\right|\right)}{a^2e^{(2\,c)}+2\,abe^{(2\,c)}+b^2e^{(2\,c)}}-\frac{3\,ae^{(4\,dx+4\,c)}+6\,be^{(4\,dx+4\,c)}-2\,ae^{(2\,dx+2\,c)}-8\,be^{(2\,dx+2\,c)}+3\,a}{\left(a^2+2\,ab+b^2\right)\left(e^{(2\,dx+2\,c)}-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*log(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c)
+ a)/(a^3 + 2*a^2*b + a*b^2) - 2*d*x/a + 2*(a*e^(2*c) + 2*b*e^(2*c))*log(ab
s(-e^(2*d*x + 2*c) + 1))/(a^2*e^(2*c) + 2*a*b*e^(2*c) + b^2*e^(2*c)) - (3*a
*e^(4*d*x + 4*c) + 6*b*e^(4*d*x + 4*c) - 2*a*e^(2*d*x + 2*c) - 8*b*e^(2*d*x
 + 2*c) + 3*a + 6*b)/((a^2 + 2*a*b + b^2)*(e^(2*d*x + 2*c) - 1)^2))/d
```

3.147 $\int \frac{\coth^4(c+dx)}{a+b\mathbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=87

$$-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{ad(a+b)^{5/2}}-\frac{\coth^3(c+dx)}{3d(a+b)}-\frac{(a+2b)\coth(c+dx)}{d(a+b)^2}+\frac{x}{a}$$

```
[Out] x/a - (b^(5/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*(a + b)^(5/
2)*d) - ((a + 2*b)*Coth[c + d*x])/((a + b)^2*d) - Coth[c + d*x]^3/(3*(a + b
)*d)
```

Rubi [A] time = 0.285895, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4141, 1975, 480, 583, 522, 206, 208}

$$-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{ad(a+b)^{5/2}}-\frac{\coth^3(c+dx)}{3d(a+b)}-\frac{(a+2b)\coth(c+dx)}{d(a+b)^2}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```


```
[Out] x/a - (b^(5/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*(a + b)^(5/
2)*d) - ((a + 2*b)*Coth[c + d*x])/((a + b)^2*d) - Coth[c + d*x]^3/(3*(a + b
)*d)
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
```

```
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\coth^4(c+dx)}{a+b\,\text{sech}^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1-x^2)\left(a+b(1-x^2)\right)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^4(1-x^2)(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{\coth^3(c+dx)}{3(a+b)d} + \frac{\text{Subst}\left(\int \frac{3(a+2b)-3bx^2}{x^2(1-x^2)(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{3(a+b)d}$$

$$= -\frac{(a+2b)\coth(c+dx)}{(a+b)^2d} - \frac{\coth^3(c+dx)}{3(a+b)d} - \frac{\text{Subst}\left(\int \frac{-3(a^2+3ab+3b^2)+3b(a+2b)x^2}{(1-x^2)(a+b-bx^2)}\,dx, x, \tanh(\right.}{3(a+b)^2d}$$

$$= -\frac{(a+2b)\coth(c+dx)}{(a+b)^2d} - \frac{\coth^3(c+dx)}{3(a+b)d} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{ad} - \frac{b^3\,\text{Subs}}{}$$

$$= \frac{x}{a} - \frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a(a+b)^{5/2}d} - \frac{(a+2b)\coth(c+dx)}{(a+b)^2d} - \frac{\coth^3(c+dx)}{3(a+b)d}$$

Mathematica [B] time = 3.26882, size = 380, normalized size = 4.37

$$\frac{\text{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\frac{1}{8}\sqrt{a+b}\,\text{csch}(c)\sqrt{b(\cosh(c)-\sinh(c))^4}\,\text{csch}^3(c+dx)\left(-12a^2\sinh(2c+dx\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(3*b^3*ArcTanh[(Sech[d*x]*
(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[
a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(-Cosh[2*c] + Sinh[2*c]) + (Sqrt[a +
 b]*Csch[c]*Csch[c + d*x]^3*Sqrt[b*(Cosh[c] - Sinh[c])^4]*(9*(a + b)^2*d*x*
Cosh[d*x] - 9*(a + b)^2*d*x*Cosh[2*c + d*x] - 3*a^2*d*x*Cosh[2*c + 3*d*x] -
 6*a*b*d*x*Cosh[2*c + 3*d*x] - 3*b^2*d*x*Cosh[2*c + 3*d*x] + 3*a^2*d*x*Cosh
[4*c + 3*d*x] + 6*a*b*d*x*Cosh[4*c + 3*d*x] + 3*b^2*d*x*Cosh[4*c + 3*d*x] -
 12*a^2*Sinh[d*x] - 24*a*b*Sinh[d*x] - 12*a^2*Sinh[2*c + d*x] - 18*a*b*Sinh
[2*c + d*x] + 8*a^2*Sinh[2*c + 3*d*x] + 14*a*b*Sinh[2*c + 3*d*x]))/8))/(6*a
*(a + b)^(5/2)*d*(a + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])
```

Maple [B] time = 0.085, size = 301, normalized size = 3.5

$$-\frac{a}{24d(a+b)^2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{b}{24d(a+b)^2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{5a}{8d(a+b)^2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{9b}{8d(a+b)^2}\tanh\left(\frac{d}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^4/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/24/d/(a+b)^2*a*tanh(1/2*d*x+1/2*c)^3-1/24/d/(a+b)^2*b*tanh(1/2*d*x+1/2*c
)^3-5/8/d/(a+b)^2*a*tanh(1/2*d*x+1/2*c)-9/8/d/(a+b)^2*tanh(1/2*d*x+1/2*c)*b
+1/d/a*ln(tanh(1/2*d*x+1/2*c)+1)-1/2/d*b^(5/2)/a/(a+b)^(5/2)*ln((a+b)^(1/2)
*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/2/d*b^(
5/2)/a/(a+b)^(5/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2
*c)*b^(1/2)-(a+b)^(1/2))-1/24/d/(a+b)/tanh(1/2*d*x+1/2*c)^3-5/8/d/(a+b)^2/t
anh(1/2*d*x+1/2*c)*a-9/8/d/(a+b)^2/tanh(1/2*d*x+1/2*c)*b-1/d/a*ln(tanh(1/2*
d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.62148, size = 6730, normalized size = 77.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(6*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^6 + 36*(a^2 + 2*a*b + b^2)*d*
x*cosh(d*x + c)*sinh(d*x + c)^5 + 6*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^6
 - 6*(3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 6*a*b)*cosh(d*x + c)^4 + 6*(15*(a
^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 - 3*(a^2 + 2*a*b + b^2)*d*x - 4*a^2 -
 6*a*b)*sinh(d*x + c)^4 + 24*(5*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 - (
```

```
3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 6*a*b)*cosh(d*x + c))*sinh(d*x + c)^3 -
 6*(a^2 + 2*a*b + b^2)*d*x + 6*(3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 8*a*b)*
cosh(d*x + c)^2 + 6*(15*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 3*(a^2 +
2*a*b + b^2)*d*x - 6*(3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 6*a*b)*cosh(d*x +
 c)^2 + 4*a^2 + 8*a*b)*sinh(d*x + c)^2 + 3*(b^2*cosh(d*x + c)^6 + 6*b^2*cos
h(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(d*x + c)^6 - 3*b^2*cosh(d*x + c)^4 +
3*(5*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x + c)^4 + 3*b^2*cosh(d*x + c)^2 + 4
*(5*b^2*cosh(d*x + c)^3 - 3*b^2*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*b^2*c
osh(d*x + c)^4 - 6*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^2 - b^2 + 6*(b^
2*cosh(d*x + c)^5 - 2*b^2*cosh(d*x + c)^3 + b^2*cosh(d*x + c))*sinh(d*x + c
))*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x
+ c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*c
osh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^
2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 +
a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a
*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c
)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*c
osh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*c
osh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - 16*a^2 - 28
*a*b + 12*(3*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^5 - 2*(3*(a^2 + 2*a*b +
b^2)*d*x + 4*a^2 + 6*a*b)*cosh(d*x + c)^3 + (3*(a^2 + 2*a*b + b^2)*d*x + 4*
a^2 + 8*a*b)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + 2*a^2*b + a*b^2)*d*cosh(
d*x + c)^6 + 6*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a
^3 + 2*a^2*b + a*b^2)*d*sinh(d*x + c)^6 - 3*(a^3 + 2*a^2*b + a*b^2)*d*cosh(
d*x + c)^4 + 3*(5*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^2 - (a^3 + 2*a^2*
b + a*b^2)*d)*sinh(d*x + c)^4 + 3*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^2
 + 4*(5*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^3 - 3*(a^3 + 2*a^2*b + a*b^
2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d
*x + c)^4 - 6*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^2 + (a^3 + 2*a^2*b +
a*b^2)*d)*sinh(d*x + c)^2 - (a^3 + 2*a^2*b + a*b^2)*d + 6*((a^3 + 2*a^2*b +
 a*b^2)*d*cosh(d*x + c)^5 - 2*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^3 + (
a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)), 1/3*(3*(a^2 + 2*a*b
 + b^2)*d*x*cosh(d*x + c)^6 + 18*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)*sinh
(d*x + c)^5 + 3*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^6 - 3*(3*(a^2 + 2*a*b
 + b^2)*d*x + 4*a^2 + 6*a*b)*cosh(d*x + c)^4 + 3*(15*(a^2 + 2*a*b + b^2)*d*
x*cosh(d*x + c)^2 - 3*(a^2 + 2*a*b + b^2)*d*x - 4*a^2 - 6*a*b)*sinh(d*x + c
)^4 + 12*(5*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 - (3*(a^2 + 2*a*b + b^2
)*d*x + 4*a^2 + 6*a*b)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(a^2 + 2*a*b + b^
2)*d*x + 3*(3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 8*a*b)*cosh(d*x + c)^2 + 3*
(15*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 3*(a^2 + 2*a*b + b^2)*d*x - 6
*(3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 6*a*b)*cosh(d*x + c)^2 + 4*a^2 + 8*a*
b)*sinh(d*x + c)^2 - 3*(b^2*cosh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x
+ c)^5 + b^2*sinh(d*x + c)^6 - 3*b^2*cosh(d*x + c)^4 + 3*(5*b^2*cosh(d*x +
c)^2 - b^2)*sinh(d*x + c)^4 + 3*b^2*cosh(d*x + c)^2 + 4*(5*b^2*cosh(d*x + c
)^3 - 3*b^2*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*b^2*cosh(d*x + c)^4 - 6*b
^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^2 - b^2 + 6*(b^2*cosh(d*x + c)^5 -
2*b^2*cosh(d*x + c)^3 + b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a + b))*
arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*
x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b) - 8*a^2 - 14*a*b + 6*(3*(a^2 + 2*a*
b + b^2)*d*x*cosh(d*x + c)^5 - 2*(3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 6*a*b
)*cosh(d*x + c)^3 + (3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 8*a*b)*cosh(d*x +
c))*sinh(d*x + c))/((a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^6 + 6*(a^3 + 2*
a^2*b + a*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^3 + 2*a^2*b + a*b^2)*d*
sinh(d*x + c)^6 - 3*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^4 + 3*(5*(a^3 +
 2*a^2*b + a*b^2)*d*cosh(d*x + c)^2 - (a^3 + 2*a^2*b + a*b^2)*d)*sinh(d*x +
 c)^4 + 3*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^2 + 4*(5*(a^3 + 2*a^2*b +
 a*b^2)*d*cosh(d*x + c)^3 - 3*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c))*sinh
(d*x + c)^3 + 3*(5*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^4 - 6*(a^3 + 2*a
^2*b + a*b^2)*d*cosh(d*x + c)^2 + (a^3 + 2*a^2*b + a*b^2)*d)*sinh(d*x + c)^
```

```
2 - (a^3 + 2*a^2*b + a*b^2)*d + 6*((a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^
5 - 2*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^3 + (a^3 + 2*a^2*b + a*b^2)*d
*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**4/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(coth(c + d*x)**4/(a + b*sech(c + d*x)**2), x)
```

Giac [B] time = 2.26468, size = 217, normalized size = 2.49

$$-\frac{\frac{3\,b^3\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\left(a^3+2\,a^2b+ab^2\right)\sqrt{-ab-b^2}}-\frac{3\,dx}{a}+\frac{2\left(6\,ae^{(4\,dx+4\,c)}+9\,be^{(4\,dx+4\,c)}-6\,ae^{(2\,dx+2\,c)}-12\,be^{(2\,dx+2\,c)}+4\,a+7\,b\right)}{\left(a^2+2\,ab+b^2\right)\left(e^{(2\,dx+2\,c)}-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/3*(3*b^3*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/((a^
3 + 2*a^2*b + a*b^2)*sqrt(-a*b - b^2)) - 3*d*x/a + 2*(6*a*e^(4*d*x + 4*c) +
 9*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) - 12*b*e^(2*d*x + 2*c) + 4*a + 7
*b)/((a^2 + 2*a*b + b^2)*(e^(2*d*x + 2*c) - 1)^3))/d
```

3.148 $\int \frac{\tanh^5(c+dx)}{\left(a+b\mathbf{sech}^2(c+dx)\right)^2} dx$

Optimal. Leaf size=76

$$\frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(a\cosh^2(c+dx)+b\right)}{2d}+\frac{(a+b)^2}{2a^2bd\left(a\cosh^2(c+dx)+b\right)}+\frac{\log(\cosh(c+dx))}{b^2d}$$

```
[Out] (a + b)^2/(2*a^2*b*d*(b + a*Cosh[c + d*x]^2)) + Log[Cosh[c + d*x]]/(b^2*d)
+ ((a^(-2) - b^(-2))*Log[b + a*Cosh[c + d*x]^2])/(2*d)
```

Rubi [A] time = 0.116779, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4138, 446, 88}

$$\frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(a\cosh^2(c+dx)+b\right)}{2d}+\frac{(a+b)^2}{2a^2bd\left(a\cosh^2(c+dx)+b\right)}+\frac{\log(\cosh(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^5/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (a + b)^2/(2*a^2*b*d*(b + a*Cosh[c + d*x]^2)) + Log[Cosh[c + d*x]]/(b^2*d)
+ ((a^(-2) - b^(-2))*Log[b + a*Cosh[c + d*x]^2])/(2*d)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^2}{x(b+ax^2)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \frac{(1-x)^2}{x(b+ax)^2}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{b^2x} - \frac{(a+b)^2}{ab(b+ax)^2} + \frac{-a^2+b^2}{ab^2(b+ax)}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{(a+b)^2}{2a^2bd\left(b+a\cosh^2(c+dx)\right)} + \frac{\log(\cosh(c+dx))}{b^2d} + \frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(b+a\cosh^2(c+dx)\right)}{2d}$$

Mathematica [A] time = 0.444285, size = 109, normalized size = 1.43

$$\frac{\operatorname{sech}^4(c+dx)(a\cosh(2c+2dx)+a+2b)^2\left(\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(a\cosh^2(c+dx)+b\right)+\frac{(a+b)^2}{a^2b\left(a\cosh^2(c+dx)+b\right)}+\frac{2\log(\cosh(c+dx))}{b^2}\right)}{8d\left(a+b\operatorname{sech}^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^5/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*c + 2*d*x])^2*((a + b)^2/(a^2*b*(b + a*Cosh[c + d*x]^2
)) + (2*Log[Cosh[c + d*x]])/b^2 + (a^(-2) - b^(-2))*Log[b + a*Cosh[c + d*x]
^2])*Sech[c + d*x]^4)/(8*d*(a + b*Sech[c + d*x]^2)^2)
```

Maple [B] time = 0.083, size = 351, normalized size = 4.6

$$-\frac{1}{da^2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-2\frac{(\tanh(1/2\,dx+c/2))^2}{bd\left((\tanh(1/2\,dx+c/2))^4a+b(\tanh(1/2\,dx+c/2))^4+2(\tanh(1/2\,dx+c/2))^2a-2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/d/a^2*ln(tanh(1/2*d*x+1/2*c)+1)-2/d/b*tanh(1/2*d*x+1/2*c)^2/(tanh(1/2*d*
x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d
*x+1/2*c)^2*b+a+b)-1/2/d/b^2*ln(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*
c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)-2/d/a*tanh(1/
2*d*x+1/2*c)^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*
d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+1/2/d/a^2*ln(tanh(1/2*d*x+1/2
*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/
2*c)^2*b+a+b)-1/d/a^2*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/b^2*ln(tanh(1/2*d*x+1/2
*c)^2+1)
```

Maxima [B] time = 1.63365, size = 208, normalized size = 2.74

$$\frac{2\left(a^2+2\,ab+b^2\right)e^{(-2\,dx-2\,c)}}{\left(a^3be^{(-4\,dx-4\,c)}+a^3b+2\left(a^3b+2\,a^2b^2\right)e^{(-2\,dx-2\,c)}\right)d}+\frac{dx+c}{a^2d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{b^2d}-\frac{\left(a^2-b^2\right)\log\left(2\left(a+2\,b\right)e^{(-2\,dx-2\,c)}+\right.}{2\,a^2b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 2*(a^2 + 2*a*b + b^2)*e^(-2*d*x - 2*c)/((a^3*b*e^(-4*d*x - 4*c) + a^3*b + 2
*(a^3*b + 2*a^2*b^2)*e^(-2*d*x - 2*c))*d) + (d*x + c)/(a^2*d) + log(e^(-2*d
*x - 2*c) + 1)/(b^2*d) - 1/2*(a^2 - b^2)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) +
 a*e^(-4*d*x - 4*c) + a)/(a^2*b^2*d)
```

Fricas [B] time = 2.70322, size = 2044, normalized size = 26.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b^2*d*x*cosh(d*x + c)^4 + 8*a*b^2*d*x*cosh(d*x + c)*sinh(d*x + c)
^3 + 2*a*b^2*d*x*sinh(d*x + c)^4 + 2*a*b^2*d*x - 4*(a^2*b + 2*a*b^2 + b^3 -
 (a*b^2 + 2*b^3)*d*x)*cosh(d*x + c)^2 + 4*(3*a*b^2*d*x*cosh(d*x + c)^2 - a^
2*b - 2*a*b^2 - b^3 + (a*b^2 + 2*b^3)*d*x)*sinh(d*x + c)^2 + ((a^3 - a*b^2)
*cosh(d*x + c)^4 + 4*(a^3 - a*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 - a
*b^2)*sinh(d*x + c)^4 + a^3 - a*b^2 + 2*(a^3 + 2*a^2*b - a*b^2 - 2*b^3)*cos
h(d*x + c)^2 + 2*(a^3 + 2*a^2*b - a*b^2 - 2*b^3 + 3*(a^3 - a*b^2)*cosh(d*x
+ c)^2)*sinh(d*x + c)^2 + 4*((a^3 - a*b^2)*cosh(d*x + c)^3 + (a^3 + 2*a^2*b
 - a*b^2 - 2*b^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*(a*cosh(d*x + c)^2 +
a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x +
c) + sinh(d*x + c)^2)) - 2*(a^3*cosh(d*x + c)^4 + 4*a^3*cosh(d*x + c)*sinh(
d*x + c)^3 + a^3*sinh(d*x + c)^4 + a^3 + 2*(a^3 + 2*a^2*b)*cosh(d*x + c)^2
+ 2*(3*a^3*cosh(d*x + c)^2 + a^3 + 2*a^2*b)*sinh(d*x + c)^2 + 4*(a^3*cosh(d
*x + c)^3 + (a^3 + 2*a^2*b)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x +
c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(a*b^2*d*x*cosh(d*x + c)^3 - (a^2*b
 + 2*a*b^2 + b^3 - (a*b^2 + 2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(a^3*
b^2*d*cosh(d*x + c)^4 + 4*a^3*b^2*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*b^2
*d*sinh(d*x + c)^4 + a^3*b^2*d + 2*(a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c)^2
+ 2*(3*a^3*b^2*d*cosh(d*x + c)^2 + (a^3*b^2 + 2*a^2*b^3)*d)*sinh(d*x + c)^2
 + 4*(a^3*b^2*d*cosh(d*x + c)^3 + (a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c))*si
nh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**5/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(tanh(c + d*x)**5/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [B] time = 2.99672, size = 281, normalized size = 3.7

$$-\frac{\frac{2\,dx}{a^2}-\frac{2\,\log\left(e^{(2\,dx+2\,c)}+1\right)}{b^2}+\frac{\left(a^2-b^2\right)\log\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}{a^2b^2}-\frac{a^2e^{(4\,dx+4\,c)}-b^2e^{(4\,dx+4\,c)}+2\,a^2e^{(2\,dx+2\,c)}+8\,abe^{(2\,dx+2\,c)}+6\,b^2e^{(2\,dx+2\,c)}}{\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)ab^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*d*x/a^2 - 2*log(e^(2*d*x + 2*c) + 1)/b^2 + (a^2 - b^2)*log(a*e^(4*d
*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)/(a^2*b^2) - (a^2
*e^(4*d*x + 4*c) - b^2*e^(4*d*x + 4*c) + 2*a^2*e^(2*d*x + 2*c) + 8*a*b*e^(2
*d*x + 2*c) + 6*b^2*e^(2*d*x + 2*c) + a^2 - b^2)/((a*e^(4*d*x + 4*c) + 2*a*
e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)*a*b^2))/d
```

3.149 $\int \frac{\tanh^4(c+dx)}{\left(a+b\mathbf{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=91

$$\frac{(a-2b)\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2b^{3/2}d}+\frac{x}{a^2}-\frac{(a+b)\tanh(c+dx)}{2abd\left(a-b\tanh^2(c+dx)+b\right)}$$

```
[Out] x/a^2 + ((a - 2*b)*Sqrt[a + b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]]
)/(2*a^2*b^(3/2)*d) - ((a + b)*Tanh[c + d*x])/(2*a*b*d*(a + b - b*Tanh[c +
d*x]^2))
```

Rubi [A] time = 0.194069, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4141, 1975, 470, 522, 206, 208}

$$\frac{(a-2b)\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2b^{3/2}d}+\frac{x}{a^2}-\frac{(a+b)\tanh(c+dx)}{2abd\left(a-b\tanh^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] x/a^2 + ((a - 2*b)*Sqrt[a + b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]]
)/(2*a^2*b^(3/2)*d) - ((a + b)*Tanh[c + d*x])/(2*a*b*d*(a + b - b*Tanh[c +
d*x]^2))
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)\left(a+b(1-x^2)\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{(a+b)\tanh(c+dx)}{2abd\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{a+b+(-a+b)x^2}{(1-x^2)\left(a+b-bx^2\right)}\,dx, x, \tanh(c+dx)\right)}{2abd}$$

$$= -\frac{(a+b)\tanh(c+dx)}{2abd\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{a^2d} + \frac{((a-2b)(a+b)}{}$$

$$= \frac{x}{a^2} + \frac{(a-2b)\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2b^{3/2}d} - \frac{(a+b)\tanh(c+dx)}{2abd\left(a+b-b\tanh^2(c+dx)\right)}$$

Mathematica [B] time = 2.10837, size = 228, normalized size = 2.51

$$\frac{\operatorname{sech}^4(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\frac{\left(a^2-ab-2b^2\right)(\cosh(2c)-\sinh(2c))(a\cosh(2(c+dx))+a+2b)\tanh^{-1}\left(\frac{(\cosh(2c)-\sinh(2c))\operatorname{sech}(dx)((a+2b)\sinh}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh}}\right.}{bd\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right.}{8a^2\left(a+b\operatorname{sech}^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*(2*x*(a + 2*b + a*Cosh[2*(
c + d*x)]) + ((a^2 - a*b - 2*b^2)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c]
)*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c]
 - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])*(Cosh[2*c] - Sinh[2*c]))/(
b*Sqrt[a + b]*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + ((a + b)*Sech[2*c]*((a + 2
```

```
*b)*Sinh[2*c] - a*Sinh[2*d*x]))/(b*d)))/(8*a^2*(a + b*Sech[c + d*x]^2)^2)
```

Maple [B] time = 0.086, size = 667, normalized size = 7.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d/a^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d
*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)/b*tanh
(1/2*d*x+1/2*c)^3-1/d/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*
tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*tanh(1/2*d*x+1/2*c)^
3-1/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c
)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)/b*tanh(1/2*d*x+1/2*c)-1/d/a/(tanh(1/2*
d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2
*d*x+1/2*c)^2*b+a+b)*tanh(1/2*d*x+1/2*c)+1/4/d/b^(3/2)/(a+b)^(1/2)*ln((a+b)
^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-1/4
/d/a/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*
x+1/2*c)*b^(1/2)+(a+b)^(1/2))-1/2/d*b^(1/2)/a^2/(a+b)^(1/2)*ln((a+b)^(1/2)*
tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-1/4/d/b^(3
/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*
b^(1/2)+(a+b)^(1/2))+1/4/d/a/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*
x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/2/d*b^(1/2)/a^2/(a+
b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)
+(a+b)^(1/2))-1/d/a^2*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.38887, size = 3748, normalized size = 41.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*b*d*x*cosh(d*x + c)^4 + 16*a*b*d*x*cosh(d*x + c)*sinh(d*x + c)^3
+ 4*a*b*d*x*sinh(d*x + c)^4 + 4*a*b*d*x + 4*(2*(a*b + 2*b^2)*d*x + a^2 + 3*
a*b + 2*b^2)*cosh(d*x + c)^2 + 4*(6*a*b*d*x*cosh(d*x + c)^2 + 2*(a*b + 2*b^
2)*d*x + a^2 + 3*a*b + 2*b^2)*sinh(d*x + c)^2 - ((a^2 - 2*a*b)*cosh(d*x + c
)^4 + 4*(a^2 - 2*a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 - 2*a*b)*sinh(d*
x + c)^4 + 2*(a^2 - 4*b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 - 2*a*b)*cosh(d*x +
```

```
c)^2 + a^2 - 4*b^2)*sinh(d*x + c)^2 + a^2 - 2*a*b + 4*((a^2 - 2*a*b)*cosh(d
*x + c)^3 + (a^2 - 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a + b)/b)*log
((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x
+ c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 +
 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a
^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*cosh(d*x + c)^2 + 2*a*b*c
osh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + a*b + 2*b^2)*sqrt((a + b
)/b))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x +
 c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sin
h(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c
) + a)) + 4*a^2 + 4*a*b + 8*(2*a*b*d*x*cosh(d*x + c)^3 + (2*(a*b + 2*b^2)*d
*x + a^2 + 3*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))/(a^3*b*d*cosh(d*x +
 c)^4 + 4*a^3*b*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*b*d*sinh(d*x + c)^4 +
 a^3*b*d + 2*(a^3*b + 2*a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*a^3*b*d*cosh(d*x
+ c)^2 + (a^3*b + 2*a^2*b^2)*d)*sinh(d*x + c)^2 + 4*(a^3*b*d*cosh(d*x + c)^
3 + (a^3*b + 2*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*a*b*d*x*cos
h(d*x + c)^4 + 8*a*b*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*a*b*d*x*sinh(d*x
 + c)^4 + 2*a*b*d*x + 2*(2*(a*b + 2*b^2)*d*x + a^2 + 3*a*b + 2*b^2)*cosh(d*
x + c)^2 + 2*(6*a*b*d*x*cosh(d*x + c)^2 + 2*(a*b + 2*b^2)*d*x + a^2 + 3*a*b
 + 2*b^2)*sinh(d*x + c)^2 + ((a^2 - 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 - 2*a*b
)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 - 2*a*b)*sinh(d*x + c)^4 + 2*(a^2 -
4*b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 - 2*a*b)*cosh(d*x + c)^2 + a^2 - 4*b^2)*
sinh(d*x + c)^2 + a^2 - 2*a*b + 4*((a^2 - 2*a*b)*cosh(d*x + c)^3 + (a^2 - 4
*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-(a + b)/b)*arctan(1/2*(a*cosh(d*x
 + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sq
rt(-(a + b)/b)/(a + b)) + 2*a^2 + 2*a*b + 4*(2*a*b*d*x*cosh(d*x + c)^3 + (2
*(a*b + 2*b^2)*d*x + a^2 + 3*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))/(a^
3*b*d*cosh(d*x + c)^4 + 4*a^3*b*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*b*d*s
inh(d*x + c)^4 + a^3*b*d + 2*(a^3*b + 2*a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*a
^3*b*d*cosh(d*x + c)^2 + (a^3*b + 2*a^2*b^2)*d)*sinh(d*x + c)^2 + 4*(a^3*b*
d*cosh(d*x + c)^3 + (a^3*b + 2*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**4/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(tanh(c + d*x)**4/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [B] time = 2.5214, size = 252, normalized size = 2.77

$$\frac{\frac{2\,dx}{a^2}+\frac{\left(a^2e^{(2\,c)}-abe^{(2\,c)}-2\,b^2e^{(2\,c)}\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)e^{(-2\,c)}}{\sqrt{-ab-b^2}a^2b}+\frac{2\left(a^2e^{(2\,dx+2\,c)}+3\,abe^{(2\,dx+2\,c)}+2\,b^2e^{(2\,dx+2\,c)}+a^2+ab\right)}{\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)a^2b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*d*x/a^2 + (a^2*e^(2*c) - a*b*e^(2*c) - 2*b^2*e^(2*c))*arctan(1/2*(a*
e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))*e^(-2*c)/(sqrt(-a*b - b^2)*a^2
*b) + 2*(a^2*e^(2*d*x + 2*c) + 3*a*b*e^(2*d*x + 2*c) + 2*b^2*e^(2*d*x + 2*c
) + a^2 + a*b)/((a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2
*c) + a)*a^2*b))/d
```

3.150 $\int \frac{\tanh^3(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=51

$$\frac{a+b}{2a^2d\left(a\cosh^2(c+dx)+b\right)}+\frac{\log\left(a\cosh^2(c+dx)+b\right)}{2a^2d}$$

```
[Out] (a + b)/(2*a^2*d*(b + a*Cosh[c + d*x]^2)) + Log[b + a*Cosh[c + d*x]^2]/(2*a
^2*d)
```

Rubi [A] time = 0.0898654, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4138, 444, 43}

$$\frac{a+b}{2a^2d\left(a\cosh^2(c+dx)+b\right)}+\frac{\log\left(a\cosh^2(c+dx)+b\right)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (a + b)/(2*a^2*d*(b + a*Cosh[c + d*x]^2)) + Log[b + a*Cosh[c + d*x]^2]/(2*a
^2*d)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x(1-x^2)}{(b+ax^2)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{1-x}{(b+ax)^2}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a+b}{a(b+ax)^2} - \frac{1}{a(b+ax)}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{a+b}{2a^2d\left(b+a\cosh^2(c+dx)\right)} + \frac{\log\left(b+a\cosh^2(c+dx)\right)}{2a^2d}$$

Mathematica [A] time = 0.710901, size = 81, normalized size = 1.59

$$\frac{(a+2b)\log(a\cosh(2(c+dx))+a+2b)+a\cosh(2(c+dx))\log(a\cosh(2(c+dx))+a+2b)+2(a+b)}{2a^2d(a\cosh(2(c+dx))+a+2b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (2*(a + b) + (a + 2*b)*Log[a + 2*b + a*Cosh[2*(c + d*x)]] + a*Cosh[2*(c + d
*x)]*Log[a + 2*b + a*Cosh[2*(c + d*x)]])/(2*a^2*d*(a + 2*b + a*Cosh[2*(c +
d*x)]))
```

Maple [B] time = 0.076, size = 186, normalized size = 3.7

$$-\frac{1}{da^2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-2\frac{(\tanh(1/2\,dx+c/2))^2}{da\left((\tanh(1/2\,dx+c/2))^4a+b(\tanh(1/2\,dx+c/2))^4+2(\tanh(1/2\,dx+c/2))^2a-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/d/a^2*ln(tanh(1/2*d*x+1/2*c)+1)-2/d/a*tanh(1/2*d*x+1/2*c)^2/(tanh(1/2*d*
x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d
*x+1/2*c)^2*b+a+b)+1/2/d/a^2*ln(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*
c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)-1/d/a^2*ln(ta
nh(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.23022, size = 146, normalized size = 2.86

$$\frac{2(a+b)e^{(-2\,dx-2\,c)}}{\left(a^3e^{(-4\,dx-4\,c)}+a^3+2\left(a^3+2\,a^2b\right)e^{(-2\,dx-2\,c)}\right)d}+\frac{dx+c}{a^2d}+\frac{\log\left(2(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 2*(a + b)*e^(-2*d*x - 2*c)/((a^3*e^(-4*d*x - 4*c) + a^3 + 2*(a^3 + 2*a^2*b)
*e^(-2*d*x - 2*c))*d) + (d*x + c)/(a^2*d) + 1/2*log(2*(a + 2*b)*e^(-2*d*x -
 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^2*d)
```

Fricas [B] time = 2.2051, size = 1260, normalized size = 24.71

$$-\frac{2\,adx\cosh\left(dx+c\right)^{4}+8\,adx\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+2\,adx\sinh\left(dx+c\right)^{4}+2\,adx+4\left((a+2\,b)dx-a-b\right)\cosh\left(a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d*x*cosh(d*x + c)^4 + 8*a*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*a
*d*x*sinh(d*x + c)^4 + 2*a*d*x + 4*((a + 2*b)*d*x - a - b)*cosh(d*x + c)^2
+ 4*(3*a*d*x*cosh(d*x + c)^2 + (a + 2*b)*d*x - a - b)*sinh(d*x + c)^2 - (a*
cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2
*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c
)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)*lo
g(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*
cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 8*(a*d*x*cosh(d*x + c)^3
+ ((a + 2*b)*d*x - a - b)*cosh(d*x + c))*sinh(d*x + c))/(a^3*d*cosh(d*x + c
)^4 + 4*a^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*d*sinh(d*x + c)^4 + a^3*d
 + 2*(a^3 + 2*a^2*b)*d*cosh(d*x + c)^2 + 2*(3*a^3*d*cosh(d*x + c)^2 + (a^3
+ 2*a^2*b)*d)*sinh(d*x + c)^2 + 4*(a^3*d*cosh(d*x + c)^3 + (a^3 + 2*a^2*b)*
d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**3/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.13644, size = 163, normalized size = 3.2

$$-\frac{\frac{2\,dx}{a^{2}}+\frac{e^{(4\,dx+4\,c)}-2\,e^{(2\,dx+2\,c)}+1}{\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)a}-\frac{\log\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}{a^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*d*x/a^2 + (e^(4*d*x + 4*c) - 2*e^(2*d*x + 2*c) + 1)/((a*e^(4*d*x +
4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)*a) - log(a*e^(4*d*x +
 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)/a^2)/d
```

3.151 $\int \frac{\tanh^2(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=85

$$-\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2\sqrt{b}d\sqrt{a+b}}+\frac{x}{a^2}-\frac{\tanh(c+dx)}{2ad\left(a-b\tanh^2(c+dx)+b\right)}$$

```
[Out] x/a^2 - ((a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(2*a^2*Sqr
t[b]*Sqrt[a + b]*d) - Tanh[c + d*x]/(2*a*d*(a + b - b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.169109, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4141, 1975, 471, 522, 206, 208}

$$-\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2\sqrt{b}d\sqrt{a+b}}+\frac{x}{a^2}-\frac{\tanh(c+dx)}{2ad\left(a-b\tanh^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```


```
[Out] x/a^2 - ((a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(2*a^2*Sqr
t[b]*Sqrt[a + b]*d) - Tanh[c + d*x]/(2*a*d*(a + b - b*Tanh[c + d*x]^2))
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)\left(a+b(1-x^2)\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{\tanh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{1+x^2}{(1-x^2)(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{2ad}$$

$$= -\frac{\tanh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{a^2d} - \frac{(a+2b)\operatorname{Subst}\left(\right.}{}$$

$$= \frac{x}{a^2} - \frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2\sqrt{b}\sqrt{a+b}d} - \frac{\tanh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)}$$

Mathematica [B] time = 4.48855, size = 326, normalized size = 3.84

$$\frac{\operatorname{sech}^4(c+dx)(a\cosh(2(c+dx))+a+2b)^2\left(\frac{(a^2+8ab+8b^2)\operatorname{sech}(2c)((a+2b)\sinh(2c)-a\sinh(2dx))}{a^2bd(a+b)(a\cosh(2(c+dx))+a+2b)} + \frac{(-6a^2b+a^3-24ab^2-16b^3)(\cosh(2c)-\sinh(2}{a^2bd(}\right.}{64\left(a+b\operatorname{sech}^2(c\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tanh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]^4*((16*x)/a^2 - ((a + 2*b)
*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(b^(3/2)*(a + b)^(3/2)*d) +
((a^3 - 6*a^2*b - 24*a*b^2 - 16*b^3)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2
*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh
[c] - Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]))/(a^2*b*(a + b)^(3/2)*d*Sqrt[b*
(Cosh[c] - Sinh[c])^4]) + ((a^2 + 8*a*b + 8*b^2)*Sech[2*c]*((a + 2*b)*Sinh[
2*c] - a*Sinh[2*d*x]))/(a^2*b*(a + b)*d*(a + 2*b + a*Cosh[2*(c + d*x)])) +
(a*Sinh[2*(c + d*x)])/(b*(a + b)*d*(a + 2*b + a*Cosh[2*(c + d*x)]))))/(64*(
a + b*Sech[c + d*x]^2)^2)
```

Maple [B] time = 0.081, size = 417, normalized size = 4.9

$$\frac{1}{da^2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{1}{da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4a+b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4+2\left(\tanh(1/2\,dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d/a^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/d/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2
*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*tanh
(1/2*d*x+1/2*c)^3-1/d/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*
tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*tanh(1/2*d*x+1/2*c)-
1/4/d/a/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2
*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/4/d/a/b^(1/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2
)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2))-1/2/d*b^
(1/2)/a^2/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1
/2*c)*b^(1/2)+(a+b)^(1/2))+1/2/d/a^2*b^(1/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*ta
nh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2))-1/d/a^2*ln(t
anh(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.50604, size = 4431, normalized size = 52.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^4 + 16*(a^2*b + a*b^2)*d*x*cosh(d
*x + c)*sinh(d*x + c)^3 + 4*(a^2*b + a*b^2)*d*x*sinh(d*x + c)^4 + 4*a^2*b +
 4*a*b^2 + 4*(a^2*b + a*b^2)*d*x + 4*(a^2*b + 3*a*b^2 + 2*b^3 + 2*(a^2*b +
3*a*b^2 + 2*b^3)*d*x)*cosh(d*x + c)^2 + 4*(6*(a^2*b + a*b^2)*d*x*cosh(d*x +
 c)^2 + a^2*b + 3*a*b^2 + 2*b^3 + 2*(a^2*b + 3*a*b^2 + 2*b^3)*d*x)*sinh(d*x
 + c)^2 + ((a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b)*cosh(d*x + c)*si
nh(d*x + c)^3 + (a^2 + 2*a*b)*sinh(d*x + c)^4 + 2*(a^2 + 4*a*b + 4*b^2)*cos
h(d*x + c)^2 + 2*(3*(a^2 + 2*a*b)*cosh(d*x + c)^2 + a^2 + 4*a*b + 4*b^2)*si
nh(d*x + c)^2 + a^2 + 2*a*b + 4*((a^2 + 2*a*b)*cosh(d*x + c)^3 + (a^2 + 4*a
*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b + b^2)*log((a^2*cosh(d*x
 + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^
2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d
*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*co
```

```
sh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(
d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4
 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh
(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh
(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 8*(2*(a^2*b +
a*b^2)*d*x*cosh(d*x + c)^3 + (a^2*b + 3*a*b^2 + 2*b^3 + 2*(a^2*b + 3*a*b^2
+ 2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^4*b + a^3*b^2)*d*cosh(d*x +
 c)^4 + 4*(a^4*b + a^3*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b + a^3*
b^2)*d*sinh(d*x + c)^4 + 2*(a^4*b + 3*a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c)^
2 + 2*(3*(a^4*b + a^3*b^2)*d*cosh(d*x + c)^2 + (a^4*b + 3*a^3*b^2 + 2*a^2*b
^3)*d)*sinh(d*x + c)^2 + (a^4*b + a^3*b^2)*d + 4*((a^4*b + a^3*b^2)*d*cosh(
d*x + c)^3 + (a^4*b + 3*a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)
), 1/2*(2*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^4 + 8*(a^2*b + a*b^2)*d*x*cosh(
d*x + c)*sinh(d*x + c)^3 + 2*(a^2*b + a*b^2)*d*x*sinh(d*x + c)^4 + 2*a^2*b
+ 2*a*b^2 + 2*(a^2*b + a*b^2)*d*x + 2*(a^2*b + 3*a*b^2 + 2*b^3 + 2*(a^2*b +
 3*a*b^2 + 2*b^3)*d*x)*cosh(d*x + c)^2 + 2*(6*(a^2*b + a*b^2)*d*x*cosh(d*x
+ c)^2 + a^2*b + 3*a*b^2 + 2*b^3 + 2*(a^2*b + 3*a*b^2 + 2*b^3)*d*x)*sinh(d*
x + c)^2 - ((a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b)*cosh(d*x + c)*s
inh(d*x + c)^3 + (a^2 + 2*a*b)*sinh(d*x + c)^4 + 2*(a^2 + 4*a*b + 4*b^2)*co
sh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b)*cosh(d*x + c)^2 + a^2 + 4*a*b + 4*b^2)*s
inh(d*x + c)^2 + a^2 + 2*a*b + 4*((a^2 + 2*a*b)*cosh(d*x + c)^3 + (a^2 + 4*
a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b - b^2)*arctan(1/2*(a*c
osh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a +
2*b)*sqrt(-a*b - b^2)/(a*b + b^2)) + 4*(2*(a^2*b + a*b^2)*d*x*cosh(d*x + c)
^3 + (a^2*b + 3*a*b^2 + 2*b^3 + 2*(a^2*b + 3*a*b^2 + 2*b^3)*d*x)*cosh(d*x +
 c))*sinh(d*x + c))/((a^4*b + a^3*b^2)*d*cosh(d*x + c)^4 + 4*(a^4*b + a^3*b
^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b + a^3*b^2)*d*sinh(d*x + c)^4 +
 2*(a^4*b + 3*a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^4*b + a^3*b^
2)*d*cosh(d*x + c)^2 + (a^4*b + 3*a^3*b^2 + 2*a^2*b^3)*d)*sinh(d*x + c)^2 +
 (a^4*b + a^3*b^2)*d + 4*((a^4*b + a^3*b^2)*d*cosh(d*x + c)^3 + (a^4*b + 3*
a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(tanh(c + d*x)**2/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [A] time = 1.92285, size = 198, normalized size = 2.33

$$-\frac{\frac{\left(ae^{(2c)}+2\,be^{(2c)}\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)e^{(-2\,c)}}{\sqrt{-ab-b^2}\,a^2}-\frac{2\,dx}{a^2}-\frac{2\left(ae^{(2\,dx+2\,c)}+2\,be^{(2\,dx+2\,c)}+a\right)}{\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((a*e^(2*c) + 2*b*e^(2*c))*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sq
rt(-a*b - b^2))*e^(-2*c)/(sqrt(-a*b - b^2)*a^2) - 2*d*x/a^2 - 2*(a*e^(2*d*x
 + 2*c) + 2*b*e^(2*d*x + 2*c) + a)/((a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c
) + 4*b*e^(2*d*x + 2*c) + a)*a^2))/d
```

3.152 $\int \frac{\tanh(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=49

$$\frac{b}{2a^2d\left(a\cosh^2(c+dx)+b\right)}+\frac{\log\left(a\cosh^2(c+dx)+b\right)}{2a^2d}$$

```
[Out] b/(2*a^2*d*(b + a*Cosh[c + d*x]^2)) + Log[b + a*Cosh[c + d*x]^2]/(2*a^2*d)
```

Rubi [A] time = 0.0624705, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 266, 43}

$$\frac{b}{2a^2d\left(a\cosh^2(c+dx)+b\right)}+\frac{\log\left(a\cosh^2(c+dx)+b\right)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] b/(2*a^2*d*(b + a*Cosh[c + d*x]^2)) + Log[b + a*Cosh[c + d*x]^2]/(2*a^2*d)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\tanh(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{\left(b+ax^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{(b+ax)^2}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{b}{a(b+ax)^2}+\frac{1}{a(b+ax)}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{b}{2a^2d\left(b+a\cosh^2(c+dx)\right)}+\frac{\log\left(b+a\cosh^2(c+dx)\right)}{2a^2d}$$

Mathematica [A] time = 0.571651, size = 79, normalized size = 1.61

$$\frac{(a+2b)\log(a\cosh(2(c+dx))+a+2b)+a\cosh(2(c+dx))\log(a\cosh(2(c+dx))+a+2b)+2b}{2a^2d(a\cosh(2(c+dx))+a+2b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (2*b + (a + 2*b)*Log[a + 2*b + a*Cosh[2*(c + d*x)]] + a*Cosh[2*(c + d*x)]*L
og[a + 2*b + a*Cosh[2*(c + d*x)]])/(2*a^2*d*(a + 2*b + a*Cosh[2*(c + d*x)])
)
```

Maple [A] time = 0.025, size = 60, normalized size = 1.2

$$-\frac{1}{2\,da\left(a+b\left(\text{sech}\left(dx+c\right)\right)^2\right)}+\frac{\ln\left(a+b\left(\text{sech}\left(dx+c\right)\right)^2\right)}{2\,da^2}-\frac{\ln\left(\text{sech}\left(dx+c\right)\right)}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/2/d/a/(a+b*sech(d*x+c)^2)+1/2/d/a^2*ln(a+b*sech(d*x+c)^2)-1/d/a^2*ln(sec
h(d*x+c))
```

Maxima [B] time = 1.26886, size = 143, normalized size = 2.92

$$\frac{2\,be^{(-2\,dx-2\,c)}}{\left(a^3e^{(-4\,dx-4\,c)}+a^3+2\left(a^3+2\,a^2b\right)e^{(-2\,dx-2\,c)}\right)d}+\frac{dx+c}{a^2d}+\frac{\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 2*b*e^(-2*d*x - 2*c)/((a^3*e^(-4*d*x - 4*c) + a^3 + 2*(a^3 + 2*a^2*b)*e^(-2
*d*x - 2*c))*d) + (d*x + c)/(a^2*d) + 1/2*log(2*(a + 2*b)*e^(-2*d*x - 2*c)
```

```
+ a*e^(-4*d*x - 4*c) + a)/(a^2*d)
```

Fricas [B] time = 2.28445, size = 1243, normalized size = 25.37

$$-\frac{2\,adx\cosh\left(dx+c\right)^4+8\,adx\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+2\,adx\sinh\left(dx+c\right)^4+2\,adx+4\left(\left(a+2\,b\right)dx-b\right)\cosh\left(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d*x*cosh(d*x + c)^4 + 8*a*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*a
*d*x*sinh(d*x + c)^4 + 2*a*d*x + 4*((a + 2*b)*d*x - b)*cosh(d*x + c)^2 + 4*
(3*a*d*x*cosh(d*x + c)^2 + (a + 2*b)*d*x - b)*sinh(d*x + c)^2 - (a*cosh(d*x
 + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*
b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*
(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)*log(2*(a*c
osh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x
 + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 8*(a*d*x*cosh(d*x + c)^3 + ((a +
2*b)*d*x - b)*cosh(d*x + c))*sinh(d*x + c))/(a^3*d*cosh(d*x + c)^4 + 4*a^3*
d*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*d*sinh(d*x + c)^4 + a^3*d + 2*(a^3 +
2*a^2*b)*d*cosh(d*x + c)^2 + 2*(3*a^3*d*cosh(d*x + c)^2 + (a^3 + 2*a^2*b)*d
)*sinh(d*x + c)^2 + 4*(a^3*d*cosh(d*x + c)^3 + (a^3 + 2*a^2*b)*d*cosh(d*x +
 c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.59107, size = 163, normalized size = 3.33

$$-\frac{\frac{2\,dx}{a^2}+\frac{e^{(4\,dx+4\,c)}+2\,e^{(2\,dx+2\,c)}+1}{\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)a}-\frac{\log\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}{a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*d*x/a^2 + (e^(4*d*x + 4*c) + 2*e^(2*d*x + 2*c) + 1)/((a*e^(4*d*x +
4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)*a) - log(a*e^(4*d*x +
 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)/a^2)/d
```

3.153 $\int \frac{1}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=93

$$-\frac{\sqrt{b}(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2d(a+b)^{3/2}}+\frac{x}{a^2}-\frac{b\tanh(c+dx)}{2ad(a+b)\left(a-b\tanh^2(c+dx)+b\right)}$$

```
[Out] x/a^2 - (Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/
(2*a^2*(a + b)^(3/2)*d) - (b*Tanh[c + d*x])/(2*a*(a + b)*d*(a + b - b*Tanh[
c + d*x]^2))
```

Rubi [A] time = 0.0964635, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4128, 414, 522, 206, 208}

$$-\frac{\sqrt{b}(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2d(a+b)^{3/2}}+\frac{x}{a^2}-\frac{b\tanh(c+dx)}{2ad(a+b)\left(a-b\tanh^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^(-2),x]
```

```
[Out] x/a^2 - (Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/
(2*a^2*(a + b)^(3/2)*d) - (b*Tanh[c + d*x])/(2*a*(a + b)*d*(a + b - b*Tanh[
c + d*x]^2))
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)(a+b-bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\tanh(c+dx)}{2a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} - \frac{\operatorname{Subst}\left(\int \frac{-2a-b-bx^2}{(1-x^2)(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{2a(a+b)d}$$

$$= -\frac{b\tanh(c+dx)}{2a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{a^2d} - \frac{(b(3a+2b}{}$$

$$= \frac{x}{a^2} - \frac{\sqrt{b}(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}d} - \frac{b\tanh(c+dx)}{2a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)}$$

Mathematica [B] time = 2.02878, size = 221, normalized size = 2.38

$$\frac{\operatorname{sech}^4(c+dx)(a\cosh(2(c+dx))+a+2b)\left(2x(a\cosh(2(c+dx))+a+2b)+\frac{b\operatorname{sech}(2c)((a+2b)\sinh(2c)-a\sinh(2dx))}{d(a+b)}-\frac{b(3a+2b)(\cos}{}\right.}{8a^2\left(a+b\operatorname{sech}^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^(-2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*(2*x*(a + 2*b + a*Cosh[2*(
c + d*x)]) - (b*(3*a + 2*b)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a
+ 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sin
h[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])*(Cosh[2*c] - Sinh[2*c]))/((a + b
)^(3/2)*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (b*Sech[2*c]*((a + 2*b)*Sinh[2*c
] - a*Sinh[2*d*x]))/((a + b)*d)))/(8*a^2*(a + b*Sech[c + d*x]^2)^2)
```

Maple [B] time = 0.073, size = 423, normalized size = 4.6

$$\frac{1}{da^2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right) - \frac{b}{da(a+b)}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 a + b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 + 2\,(\tanh(1/2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d/a^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d
*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*b/a/(a
+b)*tanh(1/2*d*x+1/2*c)^3-1/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c
)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*b/a/(a+b)*tanh
(1/2*d*x+1/2*c)-3/4/d/a*b^(1/2)/(a+b)^(3/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2
*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+3/4/d/a*b^(1/2)/(a+b)^(3/2
)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^
(1/2))-1/2/d*b^(3/2)/a^2/(a+b)^(3/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2
*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/2/d*b^(3/2)/a^2/(a+b)^(3/2)*ln(
(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2)
)-1/d/a^2*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.53874, size = 4199, normalized size = 45.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^2 + a*b)*d*x*cosh(d*x + c)^4 + 16*(a^2 + a*b)*d*x*cosh(d*x + c)*
sinh(d*x + c)^3 + 4*(a^2 + a*b)*d*x*sinh(d*x + c)^4 + 4*(a^2 + a*b)*d*x + 4
*(2*(a^2 + 3*a*b + 2*b^2)*d*x + a*b + 2*b^2)*cosh(d*x + c)^2 + 4*(6*(a^2 +
a*b)*d*x*cosh(d*x + c)^2 + 2*(a^2 + 3*a*b + 2*b^2)*d*x + a*b + 2*b^2)*sinh(
d*x + c)^2 + ((3*a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(3*a^2 + 2*a*b)*cosh(d*x
+ c)*sinh(d*x + c)^3 + (3*a^2 + 2*a*b)*sinh(d*x + c)^4 + 2*(3*a^2 + 8*a*b +
 4*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a^2 + 2*a*b)*cosh(d*x + c)^2 + 3*a^2 + 8*
a*b + 4*b^2)*sinh(d*x + c)^2 + 3*a^2 + 2*a*b + 4*((3*a^2 + 2*a*b)*cosh(d*x
+ c)^3 + (3*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a +
b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*si
nh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2
+ a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)
^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x +
 c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x +
c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(
d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2
+ 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3
+ (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 4*a*b + 8*(2*(a^2 + a*b)*d
*x*cosh(d*x + c)^3 + (2*(a^2 + 3*a*b + 2*b^2)*d*x + a*b + 2*b^2)*cosh(d*x +
 c))*sinh(d*x + c))/((a^4 + a^3*b)*d*cosh(d*x + c)^4 + 4*(a^4 + a^3*b)*d*co
sh(d*x + c)*sinh(d*x + c)^3 + (a^4 + a^3*b)*d*sinh(d*x + c)^4 + 2*(a^4 + 3*
a^3*b + 2*a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^4 + a^3*b)*d*cosh(d*x + c)^2
 + (a^4 + 3*a^3*b + 2*a^2*b^2)*d)*sinh(d*x + c)^2 + (a^4 + a^3*b)*d + 4*((a
^4 + a^3*b)*d*cosh(d*x + c)^3 + (a^4 + 3*a^3*b + 2*a^2*b^2)*d*cosh(d*x + c)
)*sinh(d*x + c)), 1/2*(2*(a^2 + a*b)*d*x*cosh(d*x + c)^4 + 8*(a^2 + a*b)*d*
```

```
x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^2 + a*b)*d*x*sinh(d*x + c)^4 + 2*(a^
2 + a*b)*d*x + 2*(2*(a^2 + 3*a*b + 2*b^2)*d*x + a*b + 2*b^2)*cosh(d*x + c)^
2 + 2*(6*(a^2 + a*b)*d*x*cosh(d*x + c)^2 + 2*(a^2 + 3*a*b + 2*b^2)*d*x + a*
b + 2*b^2)*sinh(d*x + c)^2 - ((3*a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(3*a^2 +
2*a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 + 2*a*b)*sinh(d*x + c)^4 + 2*
(3*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a^2 + 2*a*b)*cosh(d*x + c
)^2 + 3*a^2 + 8*a*b + 4*b^2)*sinh(d*x + c)^2 + 3*a^2 + 2*a*b + 4*((3*a^2 +
2*a*b)*cosh(d*x + c)^3 + (3*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x +
c))*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh
(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b) + 2*a*b + 4*(2
*(a^2 + a*b)*d*x*cosh(d*x + c)^3 + (2*(a^2 + 3*a*b + 2*b^2)*d*x + a*b + 2*b
^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + a^3*b)*d*cosh(d*x + c)^4 + 4*(a^4
 + a^3*b)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + a^3*b)*d*sinh(d*x + c)^4
 + 2*(a^4 + 3*a^3*b + 2*a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^4 + a^3*b)*d*c
osh(d*x + c)^2 + (a^4 + 3*a^3*b + 2*a^2*b^2)*d)*sinh(d*x + c)^2 + (a^4 + a^
3*b)*d + 4*((a^4 + a^3*b)*d*cosh(d*x + c)^3 + (a^4 + 3*a^3*b + 2*a^2*b^2)*d
*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + b\operatorname{sech}^2\left(c + dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**(-2), x)
```

Giac [A] time = 1.14321, size = 224, normalized size = 2.41

$$-\frac{\left(3\,ab + 2\,b^2\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{2\,\left(a^3d + a^2bd\right)\sqrt{-ab - b^2}} + \frac{abe^{(2\,dx+2\,c)} + 2\,b^2e^{(2\,dx+2\,c)} + ab}{\left(a^3d + a^2bd\right)\left(ae^{(4\,dx+4\,c)} + 2\,ae^{(2\,dx+2\,c)} + 4\,be^{(2\,dx+2\,c)} + a\right)} + \frac{dx + c}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*a*b + 2*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b
^2))/((a^3*d + a^2*b*d)*sqrt(-a*b - b^2)) + (a*b*e^(2*d*x + 2*c) + 2*b^2*e^
(2*d*x + 2*c) + a*b)/((a^3*d + a^2*b*d)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x +
 2*c) + 4*b*e^(2*d*x + 2*c) + a)) + (d*x + c)/(a^2*d)
```

3.154 $\int \frac{\coth(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=83

$$\frac{b^2}{2a^2d(a+b)\left(a\cosh^2(c+dx)+b\right)}+\frac{b(2a+b)\log\left(a\cosh^2(c+dx)+b\right)}{2a^2d(a+b)^2}+\frac{\log(\sinh(c+dx))}{d(a+b)^2}$$

```
[Out] b^2/(2*a^2*(a + b)*d*(b + a*Cosh[c + d*x]^2)) + (b*(2*a + b)*Log[b + a*Cosh
[c + d*x]^2])/(2*a^2*(a + b)^2*d) + Log[Sinh[c + d*x]]/((a + b)^2*d)
```

Rubi [A] time = 0.128867, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 446, 88}

$$\frac{b^2}{2a^2d(a+b)\left(a\cosh^2(c+dx)+b\right)}+\frac{b(2a+b)\log\left(a\cosh^2(c+dx)+b\right)}{2a^2d(a+b)^2}+\frac{\log(\sinh(c+dx))}{d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```


```
[Out] b^2/(2*a^2*(a + b)*d*(b + a*Cosh[c + d*x]^2)) + (b*(2*a + b)*Log[b + a*Cosh
[c + d*x]^2])/(2*a^2*(a + b)^2*d) + Log[Sinh[c + d*x]]/((a + b)^2*d)
```


Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\coth(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x^5}{(1-x^2)(b+ax^2)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{x^2}{(1-x)(b+ax)^2}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^2(-1+x)} + \frac{b^2}{a(a+b)(b+ax)^2} - \frac{b(2a+b)}{a(a+b)^2(b+ax)}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{b^2}{2a^2(a+b)d\left(b+a\cosh^2(c+dx)\right)} + \frac{b(2a+b)\log\left(b+a\cosh^2(c+dx)\right)}{2a^2(a+b)^2d} + \frac{\log(\sinh(c+a}{(a+b)^2d}$$

Mathematica [A] time = 0.287691, size = 115, normalized size = 1.39

$$\frac{a\sinh^2(c+dx)\left(2a^2\log(\sinh(c+dx))+b(2a+b)\log\left(a\sinh^2(c+dx)+a+b\right)\right)+(a+b)\left(2a^2\log(\sinh(c+dx))+b\left((2a\right.\right.}{a^2d(a+b)^2(a\cosh(2(c+dx))+a+2b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + b)*(2*a^2*Log[Sinh[c + d*x]] + b*(b + (2*a + b)*Log[a + b + a*Sinh[c
+ d*x]^2])) + a*(2*a^2*Log[Sinh[c + d*x]] + b*(2*a + b)*Log[a + b + a*Sinh[
c + d*x]^2])*Sinh[c + d*x]^2)/(a^2*(a + b)^2*d*(a + 2*b + a*Cosh[2*(c + d*x
)]))
```

Maple [B] time = 0.084, size = 292, normalized size = 3.5

$$-\frac{1}{da^2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-2\frac{b^2(\tanh(1/2\,dx+c/2))^2}{d(a+b)^2\left((\tanh(1/2\,dx+c/2))^4a+b(\tanh(1/2\,dx+c/2))^4+2(\tanh(1/2\,dx+c/2))^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/d/a^2*ln(tanh(1/2*d*x+1/2*c)+1)-2/d*b^2/(a+b)^2/(tanh(1/2*d*x+1/2*c)^4*a
+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*
b+a+b)/a*tanh(1/2*d*x+1/2*c)^2+1/d*b/a/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)^4*a+b
*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+
a+b)+1/2/d*b^2/a^2/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)
^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+1/d/(a+b)^2*ln(
tanh(1/2*d*x+1/2*c))-1/d/a^2*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.37354, size = 282, normalized size = 3.4

$$\frac{2\,b^2e^{(-2\,dx-2\,c)}}{\left(a^4+a^3b+2\left(a^4+3\,a^3b+2\,a^2b^2\right)e^{(-2\,dx-2\,c)}+\left(a^4+a^3b\right)e^{(-4\,dx-4\,c)}\right)d}+\frac{\left(2\,ab+b^2\right)\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c}\right.}{2\left(a^4+2\,a^3b+a^2b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 2*b^2*e^(-2*d*x - 2*c)/((a^4 + a^3*b + 2*(a^4 + 3*a^3*b + 2*a^2*b^2)*e^(-2*
d*x - 2*c) + (a^4 + a^3*b)*e^(-4*d*x - 4*c))*d) + 1/2*(2*a*b + b^2)*log(2*(
a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/((a^4 + 2*a^3*b + a^2*b
^2)*d) + log(e^(-d*x - c) + 1)/((a^2 + 2*a*b + b^2)*d) + log(e^(-d*x - c) -
 1)/((a^2 + 2*a*b + b^2)*d) + (d*x + c)/(a^2*d)
```

Fricas [B] time = 3.30547, size = 2457, normalized size = 29.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^4 + 8*(a^3 + 2*a^2*b + a*
b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^3 + 2*a^2*b + a*b^2)*d*x*sinh
(d*x + c)^4 + 2*(a^3 + 2*a^2*b + a*b^2)*d*x - 4*(a*b^2 + b^3 - (a^3 + 4*a^2
*b + 5*a*b^2 + 2*b^3)*d*x)*cosh(d*x + c)^2 + 4*(3*(a^3 + 2*a^2*b + a*b^2)*d
*x*cosh(d*x + c)^2 - a*b^2 - b^3 + (a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*d*x)*s
inh(d*x + c)^2 - ((2*a^2*b + a*b^2)*cosh(d*x + c)^4 + 4*(2*a^2*b + a*b^2)*c
osh(d*x + c)*sinh(d*x + c)^3 + (2*a^2*b + a*b^2)*sinh(d*x + c)^4 + 2*a^2*b
+ a*b^2 + 2*(2*a^2*b + 5*a*b^2 + 2*b^3)*cosh(d*x + c)^2 + 2*(2*a^2*b + 5*a*
b^2 + 2*b^3 + 3*(2*a^2*b + a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((2*
a^2*b + a*b^2)*cosh(d*x + c)^3 + (2*a^2*b + 5*a*b^2 + 2*b^3)*cosh(d*x + c))
*sinh(d*x + c))*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(co
sh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 2*(a^3*
cosh(d*x + c)^4 + 4*a^3*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*sinh(d*x + c)^4
 + a^3 + 2*(a^3 + 2*a^2*b)*cosh(d*x + c)^2 + 2*(3*a^3*cosh(d*x + c)^2 + a^3
 + 2*a^2*b)*sinh(d*x + c)^2 + 4*(a^3*cosh(d*x + c)^3 + (a^3 + 2*a^2*b)*cosh
(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c
))) + 8*((a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^3 - (a*b^2 + b^3 - (a^3
+ 4*a^2*b + 5*a*b^2 + 2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + 2*a
^4*b + a^3*b^2)*d*cosh(d*x + c)^4 + 4*(a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x
+ c)*sinh(d*x + c)^3 + (a^5 + 2*a^4*b + a^3*b^2)*d*sinh(d*x + c)^4 + 2*(a^5
 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^5 + 2*a^4*b
 + a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*d)*
sinh(d*x + c)^2 + (a^5 + 2*a^4*b + a^3*b^2)*d + 4*((a^5 + 2*a^4*b + a^3*b^2
)*d*cosh(d*x + c)^3 + (a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x +
c))*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(c + dx)}{\left(a + b\operatorname{sech}^2(c + dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(coth(c + d*x)/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [B] time = 1.69404, size = 332, normalized size = 4.

$$\frac{\frac{\left(2\,ab+b^2\right)\log\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}{a^4+2\,a^3b+a^2b^2}+\frac{2\,e^{(2\,c)}\log\left(\left|-e^{(2\,dx+2\,c)}+1\right|\right)}{a^2e^{(2\,c)}+2\,abe^{(2\,c)}+b^2e^{(2\,c)}}-\frac{2\,dx}{a^2}-\frac{2\,abe^{(4\,dx+4\,c)}+b^2e^{(4\,dx+4\,c)}+4\,abe^{(2\,dx+2\,c)}+6\,b^2e^{(2\,dx+2\,c)}+2\,ab\cdots}{\left(a^3+2\,a^2b+ab^2\right)\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((2*a*b + b^2)*log(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d
*x + 2*c) + a)/(a^4 + 2*a^3*b + a^2*b^2) + 2*e^(2*c)*log(abs(-e^(2*d*x + 2*
c) + 1))/(a^2*e^(2*c) + 2*a*b*e^(2*c) + b^2*e^(2*c)) - 2*d*x/a^2 - (2*a*b*e
^(4*d*x + 4*c) + b^2*e^(4*d*x + 4*c) + 4*a*b*e^(2*d*x + 2*c) + 6*b^2*e^(2*d
*x + 2*c) + 2*a*b + b^2)/((a^3 + 2*a^2*b + a*b^2)*(a*e^(4*d*x + 4*c) + 2*a*
e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)))/d
```

3.155 $\int \frac{\coth^2(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=121

$$-\frac{b^{3/2}(5a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2d(a+b)^{5/2}}+\frac{x}{a^2}-\frac{(2a-b)\coth(c+dx)}{2ad(a+b)^2}-\frac{b\coth(c+dx)}{2ad(a+b)\left(a-b\tanh^2(c+dx)+b\right)}$$

```
[Out] x/a^2 - (b^(3/2)*(5*a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/
(2*a^2*(a + b)^(5/2)*d) - ((2*a - b)*Coth[c + d*x])/(2*a*(a + b)^2*d) - (b*
Coth[c + d*x])/(2*a*(a + b)*d*(a + b - b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.284105, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4141, 1975, 472, 583, 522, 206, 208}

$$-\frac{b^{3/2}(5a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2d(a+b)^{5/2}}+\frac{x}{a^2}-\frac{(2a-b)\coth(c+dx)}{2ad(a+b)^2}-\frac{b\coth(c+dx)}{2ad(a+b)\left(a-b\tanh^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] x/a^2 - (b^(3/2)*(5*a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/
(2*a^2*(a + b)^(5/2)*d) - ((2*a - b)*Coth[c + d*x])/(2*a*(a + b)^2*d) - (b*
Coth[c + d*x])/(2*a*(a + b)*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\coth^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1-x^2)\left(a+b(1-x^2)\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1-x^2)\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\coth(c+dx)}{2a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-2a+b-3bx^2}{x^2(1-x^2)\left(a+b-bx^2\right)}\,dx, x, \tanh(c+dx)\right)}{2a(a+b)d}$$

$$= -\frac{(2a-b)\coth(c+dx)}{2a(a+b)^2d} - \frac{b\coth(c+dx)}{2a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{2a^2+6ab+b^2-(2a\cdots}{(1-x^2)\left(a+b-b\cdots\right.}\cdots\right.}{2a(a\cdots}$$

$$= -\frac{(2a-b)\coth(c+dx)}{2a(a+b)^2d} - \frac{b\coth(c+dx)}{2a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tan\cdots\right.}{a^2d}$$

$$= \frac{x}{a^2} - \frac{b^{3/2}(5a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{5/2}d} - \frac{(2a-b)\coth(c+dx)}{2a(a+b)^2d} - \frac{b\coth(c+\cdots}{2a(a+b)d\left(a+b-b\cdots\right.}$$

Mathematica [B] time = 2.84748, size = 268, normalized size = 2.21

$$\frac{\operatorname{sech}^4(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\frac{b^2\operatorname{sech}(2c)((a+2b)\sinh(2c)-a\sinh(2dx))}{a^2d(a+b)^2} - \frac{b^2(5a+2b)(\cosh(2c)-\sinh(2c))(a\cosh(2(c+dx))+a+2b)\,t\cdots}{a^2d(a+b)^{5/2}\sqrt{b\cdots}}\cdots\right.}{8\left(a+b\operatorname{sech}^2(\cdots\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*((2*x*(a + 2*b + a*Cosh[2*
(c + d*x)]))/a^2 - (b^2*(5*a + 2*b)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*
c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[
c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])*(Cosh[2*c] - Sinh[2*c]))
/(a^2*(a + b)^(5/2)*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (2*(a + 2*b + a*Cosh
[2*(c + d*x)])*Csch[c]*Csch[c + d*x]*Sinh[d*x])/((a + b)^2*d) + (b^2*Sech[2
*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/(a^2*(a + b)^2*d)))/(8*(a + b*Se
ch[c + d*x]^2)^2)
```

Maple [B] time = 0.099, size = 481, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/2/d/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)+1/d/a^2*ln(tanh(1/2*d*x+1/2*c)+1
)-1/d*b^2/a/(a+b)^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh
(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*tanh(1/2*d*x+1/2*c)^3-1/
d*b^2/a/(a+b)^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2
*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*tanh(1/2*d*x+1/2*c)-5/4/d*b^
(3/2)/a/(a+b)^(5/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2
*c)*b^(1/2)+(a+b)^(1/2))+5/4/d*b^(3/2)/a/(a+b)^(5/2)*ln(-(a+b)^(1/2)*tanh(1
/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2))-1/2/d*b^(5/2)/a^
2/(a+b)^(5/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^
(1/2)+(a+b)^(1/2))+1/2/d*b^(5/2)/a^2/(a+b)^(5/2)*ln(-(a+b)^(1/2)*tanh(1/2*d
*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2))-1/2/d/(a+b)^2/tanh(1
/2*d*x+1/2*c)-1/d/a^2*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.82252, size = 8483, normalized size = 70.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^6 + 24*(a^3 + 2*a^2*b + a
*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^5 + 4*(a^3 + 2*a^2*b + a*b^2)*d*x*sin
h(d*x + c)^6 - 4*(2*a^3 - a*b^2 - 2*b^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)
*d*x)*cosh(d*x + c)^4 + 4*(15*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^2 -
 2*a^3 + a*b^2 + 2*b^3 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)*sinh(d*x +
c)^4 + 16*(5*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^3 - (2*a^3 - a*b^2 -
 2*b^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c
)^3 - 8*a^3 - 4*a*b^2 - 4*(a^3 + 2*a^2*b + a*b^2)*d*x - 4*(4*a^3 + 8*a^2*b
+ 2*b^3 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)*cosh(d*x + c)^2 + 4*(15*(a
^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^4 - 4*a^3 - 8*a^2*b - 2*b^3 - (a^3
+ 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x - 6*(2*a^3 - a*b^2 - 2*b^3 - (a^3 + 6*a^2*
b + 9*a*b^2 + 4*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((5*a^2*b + 2*
a*b^2)*cosh(d*x + c)^6 + 6*(5*a^2*b + 2*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^
5 + (5*a^2*b + 2*a*b^2)*sinh(d*x + c)^6 + (5*a^2*b + 22*a*b^2 + 8*b^3)*cosh
(d*x + c)^4 + (5*a^2*b + 22*a*b^2 + 8*b^3 + 15*(5*a^2*b + 2*a*b^2)*cosh(d*x
 + c)^2)*sinh(d*x + c)^4 + 4*(5*(5*a^2*b + 2*a*b^2)*cosh(d*x + c)^3 + (5*a^
2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 5*a^2*b - 2*a*b^2
- (5*a^2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c)^2 + (15*(5*a^2*b + 2*a*b^2)*co
sh(d*x + c)^4 - 5*a^2*b - 22*a*b^2 - 8*b^3 + 6*(5*a^2*b + 22*a*b^2 + 8*b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(5*a^2*b + 2*a*b^2)*cosh(d*x + c)^
5 + 2*(5*a^2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c)^3 - (5*a^2*b + 22*a*b^2 +
8*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)
^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2
*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x +
c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*
x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh
(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2
)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 +
 a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 +
 a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))
*sinh(d*x + c) + a)) + 8*(3*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^5 - 2
*(2*a^3 - a*b^2 - 2*b^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)*cosh(d*x +
 c)^3 - (4*a^3 + 8*a^2*b + 2*b^3 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)*c
osh(d*x + c))*sinh(d*x + c))/((a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^6 +
 6*(a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^5 + 2*a^4
*b + a^3*b^2)*d*sinh(d*x + c)^6 + (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d
*cosh(d*x + c)^4 + (15*(a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 +
 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d)*sinh(d*x + c)^4 - (a^5 + 6*a^4*b + 9*a
^3*b^2 + 4*a^2*b^3)*d*cosh(d*x + c)^2 + 4*(5*(a^5 + 2*a^4*b + a^3*b^2)*d*co
sh(d*x + c)^3 + (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d*cosh(d*x + c))*si
nh(d*x + c)^3 + (15*(a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^4 + 6*(a^5 +
6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d*cosh(d*x + c)^2 - (a^5 + 6*a^4*b + 9*a^3
*b^2 + 4*a^2*b^3)*d)*sinh(d*x + c)^2 - (a^5 + 2*a^4*b + a^3*b^2)*d + 2*(3*(
a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^5 + 2*(a^5 + 6*a^4*b + 9*a^3*b^2 +
 4*a^2*b^3)*d*cosh(d*x + c)^3 - (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d*c
osh(d*x + c))*sinh(d*x + c)), 1/2*(2*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x +
 c)^6 + 12*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(a
^3 + 2*a^2*b + a*b^2)*d*x*sinh(d*x + c)^6 - 2*(2*a^3 - a*b^2 - 2*b^3 - (a^3
 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)*cosh(d*x + c)^4 + 2*(15*(a^3 + 2*a^2*b +
 a*b^2)*d*x*cosh(d*x + c)^2 - 2*a^3 + a*b^2 + 2*b^3 + (a^3 + 6*a^2*b + 9*a*
b^2 + 4*b^3)*d*x)*sinh(d*x + c)^4 + 8*(5*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d
*x + c)^3 - (2*a^3 - a*b^2 - 2*b^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)
*cosh(d*x + c))*sinh(d*x + c)^3 - 4*a^3 - 2*a*b^2 - 2*(a^3 + 2*a^2*b + a*b^
2)*d*x - 2*(4*a^3 + 8*a^2*b + 2*b^3 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x
)*cosh(d*x + c)^2 + 2*(15*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^4 - 4*a
^3 - 8*a^2*b - 2*b^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x - 6*(2*a^3 - a
*b^2 - 2*b^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)*cosh(d*x + c)^2)*sinh
(d*x + c)^2 - ((5*a^2*b + 2*a*b^2)*cosh(d*x + c)^6 + 6*(5*a^2*b + 2*a*b^2)*
cosh(d*x + c)*sinh(d*x + c)^5 + (5*a^2*b + 2*a*b^2)*sinh(d*x + c)^6 + (5*a^
```

```
2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c)^4 + (5*a^2*b + 22*a*b^2 + 8*b^3 + 15*
(5*a^2*b + 2*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(5*a^2*b + 2*a*
b^2)*cosh(d*x + c)^3 + (5*a^2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c))*sinh(d*x
 + c)^3 - 5*a^2*b - 2*a*b^2 - (5*a^2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c)^2
+ (15*(5*a^2*b + 2*a*b^2)*cosh(d*x + c)^4 - 5*a^2*b - 22*a*b^2 - 8*b^3 + 6*
(5*a^2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(5*a^2
*b + 2*a*b^2)*cosh(d*x + c)^5 + 2*(5*a^2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c
)^3 - (5*a^2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a
 + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*
sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b) + 4*(3*(a^3 + 2*a^2*b + a*b^
2)*d*x*cosh(d*x + c)^5 - 2*(2*a^3 - a*b^2 - 2*b^3 - (a^3 + 6*a^2*b + 9*a*b^
2 + 4*b^3)*d*x)*cosh(d*x + c)^3 - (4*a^3 + 8*a^2*b + 2*b^3 + (a^3 + 6*a^2*b
 + 9*a*b^2 + 4*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + 2*a^4*b + a^
3*b^2)*d*cosh(d*x + c)^6 + 6*(a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)*sinh
(d*x + c)^5 + (a^5 + 2*a^4*b + a^3*b^2)*d*sinh(d*x + c)^6 + (a^5 + 6*a^4*b
+ 9*a^3*b^2 + 4*a^2*b^3)*d*cosh(d*x + c)^4 + (15*(a^5 + 2*a^4*b + a^3*b^2)*
d*cosh(d*x + c)^2 + (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d)*sinh(d*x + c
)^4 - (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d*cosh(d*x + c)^2 + 4*(5*(a^5
 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^3 + (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^
2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (15*(a^5 + 2*a^4*b + a^3*b^2)*d*c
osh(d*x + c)^4 + 6*(a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d*cosh(d*x + c)^
2 - (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d)*sinh(d*x + c)^2 - (a^5 + 2*a
^4*b + a^3*b^2)*d + 2*(3*(a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^5 + 2*(a
^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d*cosh(d*x + c)^3 - (a^5 + 6*a^4*b +
9*a^3*b^2 + 4*a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(coth(c + d*x)**2/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [B] time = 2.16753, size = 381, normalized size = 3.15

$$\frac{\frac{\left(5ab^2e^{(2c)}+2b^3e^{(2c)}\right)\arctan\left(-\frac{ae^{(2dx+2c)}+a+2b}{2\sqrt{-ab-b^2}}\right)e^{(-2c)}}{\left(a^4+2a^3b+a^2b^2\right)\sqrt{-ab-b^2}}+\frac{2dx}{a^2}-\frac{2\left(2a^3e^{(4dx+4c)}-ab^2e^{(4dx+4c)}-2b^3e^{(4dx+4c)}+4a^3e^{(2dx+2c)}+8a^2be^{(2dx+2c)}+2b^3e^{(2dx+2c}\cdots\right.}{\left(a^4+2a^3b+a^2b^2\right)\left(ae^{(6dx+6c)}+ae^{(4dx+4c)}+4be^{(4dx+4c)}-ae^{(2dx+2c)}-4be^{(2dx+2c)}\cdots\right.}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((5*a*b^2*e^(2*c) + 2*b^3*e^(2*c))*arctan(-1/2*(a*e^(2*d*x + 2*c) + a +
 2*b)/sqrt(-a*b - b^2))*e^(-2*c)/((a^4 + 2*a^3*b + a^2*b^2)*sqrt(-a*b - b^2
)) + 2*d*x/a^2 - 2*(2*a^3*e^(4*d*x + 4*c) - a*b^2*e^(4*d*x + 4*c) - 2*b^3*e
^(4*d*x + 4*c) + 4*a^3*e^(2*d*x + 2*c) + 8*a^2*b*e^(2*d*x + 2*c) + 2*b^3*e^
(2*d*x + 2*c) + 2*a^3 + a*b^2)/((a^4 + 2*a^3*b + a^2*b^2)*(a*e^(6*d*x + 6*c
) + a*e^(4*d*x + 4*c) + 4*b*e^(4*d*x + 4*c) - a*e^(2*d*x + 2*c) - 4*b*e^(2*
d*x + 2*c) - a)))/d
```

3.156 $\int \frac{\coth^3(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=110

$$\frac{b^3}{2a^2d(a+b)^2\left(a\cosh^2(c+dx)+b\right)}+\frac{b^2(3a+b)\log\left(a\cosh^2(c+dx)+b\right)}{2a^2d(a+b)^3}-\frac{\operatorname{csch}^2(c+dx)}{2d(a+b)^2}+\frac{(a+3b)\log(\sinh(c+dx))}{d(a+b)^3}$$

```
[Out] b^3/(2*a^2*(a + b)^2*d*(b + a*Cosh[c + d*x]^2)) - Csch[c + d*x]^2/(2*(a + b
)^2*d) + (b^2*(3*a + b)*Log[b + a*Cosh[c + d*x]^2])/(2*a^2*(a + b)^3*d) + (
(a + 3*b)*Log[Sinh[c + d*x]])/((a + b)^3*d)
```

Rubi [A] time = 0.174502, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4138, 446, 88}

$$\frac{b^3}{2a^2d(a+b)^2\left(a\cosh^2(c+dx)+b\right)}+\frac{b^2(3a+b)\log\left(a\cosh^2(c+dx)+b\right)}{2a^2d(a+b)^3}-\frac{\operatorname{csch}^2(c+dx)}{2d(a+b)^2}+\frac{(a+3b)\log(\sinh(c+dx))}{d(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] b^3/(2*a^2*(a + b)^2*d*(b + a*Cosh[c + d*x]^2)) - Csch[c + d*x]^2/(2*(a + b
)^2*d) + (b^2*(3*a + b)*Log[b + a*Cosh[c + d*x]^2])/(2*a^2*(a + b)^3*d) + (
(a + 3*b)*Log[Sinh[c + d*x]])/((a + b)^3*d)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\coth^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^7}{\left(1-x^2\right)^2\left(b+ax^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^3}{(1-x)^2(b+ax)^2}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)^2(-1+x)^2} + \frac{a+3b}{(a+b)^3(-1+x)} - \frac{b^3}{a(a+b)^2(b+ax)^2} + \frac{b^2(3a+b)}{a(a+b)^3(b+ax)}\right) dx, x, \cosh^2(c+dx\right.}{2d}$$

$$= \frac{b^3}{2a^2(a+b)^2d\left(b+a\cosh^2(c+dx)\right)} - \frac{\text{csch}^2(c+dx)}{2(a+b)^2d} + \frac{b^2(3a+b)\log\left(b+a\cosh^2(c+\right.}{2a^2(a+b)^3d}$$

Mathematica [A] time = 1.29486, size = 130, normalized size = 1.18

$$\frac{\text{sech}^4(c+dx)(a\cosh(2(c+dx))+a+2b)^2\left(\frac{b^3(a+b)}{a^2\left(a\sinh^2(c+dx)+a+b\right)} + \frac{b^2(3a+b)\log\left(a\sinh^2(c+dx)+a+b\right)}{a^2} - (a+b)\text{csch}^2(c+dx) + \right.}{8d(a+b)^3\left(a+b\text{sech}^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]^4*(-((a + b)*Csch[c + d*x]
^2) + 2*(a + 3*b)*Log[Sinh[c + d*x]] + (b^2*(3*a + b)*Log[a + b + a*Sinh[c
+ d*x]^2])/a^2 + (b^3*(a + b))/(a^2*(a + b + a*Sinh[c + d*x]^2))))/(8*(a +
b)^3*d*(a + b*Sech[c + d*x]^2)^2)
```

Maple [B] time = 0.096, size = 367, normalized size = 3.3

$$-\frac{1}{8\,d\left(a^2+2\,ab+b^2\right)}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 - \frac{1}{da^2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right) - 2\,\frac{}{da\,(a+b)^3\left((\tanh(1/2\,dx+c/2))^4\,a+b\,(\mathrm{t}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/8/d*tanh(1/2*d*x+1/2*c)^2/(a^2+2*a*b+b^2)-1/d/a^2*ln(tanh(1/2*d*x+1/2*c)
+1)-2/d*b^3/a/(a+b)^3*tanh(1/2*d*x+1/2*c)^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh
(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+
3/2/d*b^2/a/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*ta
nh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+1/2/d*b^3/a^2/(a+b)^3*
ln(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*
a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)-1/8/d/(a+b)^2/tanh(1/2*d*x+1/2*c)^2+1/d/(a
+b)^3*ln(tanh(1/2*d*x+1/2*c))*a+3/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c))*b-1/d/a
^2*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.26792, size = 518, normalized size = 4.71

$$\frac{\left(3\,ab^2+b^3\right)\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\left(a^5+3\,a^4b+3\,a^3b^2+a^2b^3\right)d} + \frac{(a+3\,b)\log\left(e^{(-dx-c)}+1\right)}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)d} + \frac{(a+3\,b)\log\left(e^{(-dx-c)}-1\right)}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)d} - $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(3*a*b^2 + b^3)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) +
 a)/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d) + (a + 3*b)*log(e^(-d*x - c)
+ 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + (a + 3*b)*log(e^(-d*x - c) - 1)/
((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 2*((a^3 - b^3)*e^(-2*d*x - 2*c) + 2*(
a^3 + 2*a^2*b + b^3)*e^(-4*d*x - 4*c) + (a^3 - b^3)*e^(-6*d*x - 6*c))/((a^5
 + 2*a^4*b + a^3*b^2 + 4*(a^4*b + 2*a^3*b^2 + a^2*b^3)*e^(-2*d*x - 2*c) - 2
*(a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*e^(-4*d*x - 4*c) + 4*(a^4*b + 2*a^
3*b^2 + a^2*b^3)*e^(-6*d*x - 6*c) + (a^5 + 2*a^4*b + a^3*b^2)*e^(-8*d*x - 8
*c))*d) + (d*x + c)/(a^2*d)
```

Fricas [B] time = 4.7792, size = 8284, normalized size = 75.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^8 + 16*(a^4 +
 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(a^4 +
3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*sinh(d*x + c)^8 + 4*(a^4 + a^3*b - a*b^3 -
 b^4 + 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*x)*cosh(d*x + c)^6 + 4*(14*(
a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^2 + a^4 + a^3*b - a*b^
3 - b^4 + 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*x)*sinh(d*x + c)^6 + 8*(1
4*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^3 + 3*(a^4 + a^3*b
- a*b^3 - b^4 + 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*x)*cosh(d*x + c))*s
inh(d*x + c)^5 + 4*(2*a^4 + 6*a^3*b + 4*a^2*b^2 + 2*a*b^3 + 2*b^4 - (a^4 +
7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d*x)*cosh(d*x + c)^4 + 4*(35*(a^4
+ 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^4 + 2*a^4 + 6*a^3*b + 4*a^
2*b^2 + 2*a*b^3 + 2*b^4 - (a^4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d
*x + 15*(a^4 + a^3*b - a*b^3 - b^4 + 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*
d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 +
a*b^3)*d*x*cosh(d*x + c)^5 + 5*(a^4 + a^3*b - a*b^3 - b^4 + 2*(a^3*b + 3*a^
2*b^2 + 3*a*b^3 + b^4)*d*x)*cosh(d*x + c)^3 + (2*a^4 + 6*a^3*b + 4*a^2*b^2
+ 2*a*b^3 + 2*b^4 - (a^4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d*x)*co
sh(d*x + c))*sinh(d*x + c)^3 + 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x +
4*(a^4 + a^3*b - a*b^3 - b^4 + 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*x)*c
osh(d*x + c)^2 + 4*(14*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c
)^6 + 15*(a^4 + a^3*b - a*b^3 - b^4 + 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)
*d*x)*cosh(d*x + c)^4 + a^4 + a^3*b - a*b^3 - b^4 + 2*(a^3*b + 3*a^2*b^2 +
3*a*b^3 + b^4)*d*x + 6*(2*a^4 + 6*a^3*b + 4*a^2*b^2 + 2*a*b^3 + 2*b^4 - (a^
4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x
 + c)^2 - ((3*a^2*b^2 + a*b^3)*cosh(d*x + c)^8 + 8*(3*a^2*b^2 + a*b^3)*cosh
(d*x + c)*sinh(d*x + c)^7 + (3*a^2*b^2 + a*b^3)*sinh(d*x + c)^8 + 4*(3*a*b^
3 + b^4)*cosh(d*x + c)^6 + 4*(3*a*b^3 + b^4 + 7*(3*a^2*b^2 + a*b^3)*cosh(d*
x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2*b^2 + a*b^3)*cosh(d*x + c)^3 + 3*(3
*a*b^3 + b^4)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(3*a^2*b^2 + 13*a*b^3 + 4*
b^4)*cosh(d*x + c)^4 + 2*(35*(3*a^2*b^2 + a*b^3)*cosh(d*x + c)^4 - 3*a^2*b^
2 - 13*a*b^3 - 4*b^4 + 30*(3*a*b^3 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4
+ 3*a^2*b^2 + a*b^3 + 8*(7*(3*a^2*b^2 + a*b^3)*cosh(d*x + c)^5 + 10*(3*a*b^
3 + b^4)*cosh(d*x + c)^3 - (3*a^2*b^2 + 13*a*b^3 + 4*b^4)*cosh(d*x + c))*si
nh(d*x + c)^3 + 4*(3*a*b^3 + b^4)*cosh(d*x + c)^2 + 4*(7*(3*a^2*b^2 + a*b^3
)*cosh(d*x + c)^6 + 15*(3*a*b^3 + b^4)*cosh(d*x + c)^4 + 3*a*b^3 + b^4 - 3*
```

```
(3*a^2*b^2 + 13*a*b^3 + 4*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*a^2
*b^2 + a*b^3)*cosh(d*x + c)^7 + 3*(3*a*b^3 + b^4)*cosh(d*x + c)^5 - (3*a^2*
b^2 + 13*a*b^3 + 4*b^4)*cosh(d*x + c)^3 + (3*a*b^3 + b^4)*cosh(d*x + c))*si
nh(d*x + c))*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(
d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 2*((a^4 +
3*a^3*b)*cosh(d*x + c)^8 + 8*(a^4 + 3*a^3*b)*cosh(d*x + c)*sinh(d*x + c)^7
+ (a^4 + 3*a^3*b)*sinh(d*x + c)^8 + 4*(a^3*b + 3*a^2*b^2)*cosh(d*x + c)^6 +
 4*(a^3*b + 3*a^2*b^2 + 7*(a^4 + 3*a^3*b)*cosh(d*x + c)^2)*sinh(d*x + c)^6
+ 8*(7*(a^4 + 3*a^3*b)*cosh(d*x + c)^3 + 3*(a^3*b + 3*a^2*b^2)*cosh(d*x + c
))*sinh(d*x + c)^5 - 2*(a^4 + 7*a^3*b + 12*a^2*b^2)*cosh(d*x + c)^4 + 2*(35
*(a^4 + 3*a^3*b)*cosh(d*x + c)^4 - a^4 - 7*a^3*b - 12*a^2*b^2 + 30*(a^3*b +
 3*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + a^4 + 3*a^3*b + 8*(7*(a^4 +
3*a^3*b)*cosh(d*x + c)^5 + 10*(a^3*b + 3*a^2*b^2)*cosh(d*x + c)^3 - (a^4 +
7*a^3*b + 12*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^3*b + 3*a^2*b^2
)*cosh(d*x + c)^2 + 4*(7*(a^4 + 3*a^3*b)*cosh(d*x + c)^6 + 15*(a^3*b + 3*a^
2*b^2)*cosh(d*x + c)^4 + a^3*b + 3*a^2*b^2 - 3*(a^4 + 7*a^3*b + 12*a^2*b^2)
*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^4 + 3*a^3*b)*cosh(d*x + c)^7 + 3*
(a^3*b + 3*a^2*b^2)*cosh(d*x + c)^5 - (a^4 + 7*a^3*b + 12*a^2*b^2)*cosh(d*x
 + c)^3 + (a^3*b + 3*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x
+ c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a
*b^3)*d*x*cosh(d*x + c)^7 + 3*(a^4 + a^3*b - a*b^3 - b^4 + 2*(a^3*b + 3*a^2
*b^2 + 3*a*b^3 + b^4)*d*x)*cosh(d*x + c)^5 + 2*(2*a^4 + 6*a^3*b + 4*a^2*b^2
 + 2*a*b^3 + 2*b^4 - (a^4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d*x)*c
osh(d*x + c)^3 + (a^4 + a^3*b - a*b^3 - b^4 + 2*(a^3*b + 3*a^2*b^2 + 3*a*b^
3 + b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a
^3*b^3)*d*cosh(d*x + c)^8 + 8*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(
d*x + c)*sinh(d*x + c)^7 + (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*sinh(d*x
 + c)^8 + 4*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^6 + 4
*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^2 + (a^5*b + 3*a^
4*b^2 + 3*a^3*b^3 + a^2*b^4)*d)*sinh(d*x + c)^6 - 2*(a^6 + 7*a^5*b + 15*a^4
*b^2 + 13*a^3*b^3 + 4*a^2*b^4)*d*cosh(d*x + c)^4 + 8*(7*(a^6 + 3*a^5*b + 3*
a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^3 + 3*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a
^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^6 + 3*a^5*b + 3*a^4*b^2
 + a^3*b^3)*d*cosh(d*x + c)^4 + 30*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4
)*d*cosh(d*x + c)^2 - (a^6 + 7*a^5*b + 15*a^4*b^2 + 13*a^3*b^3 + 4*a^2*b^4)
*d)*sinh(d*x + c)^4 + 4*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*d*cosh(d*
x + c)^2 + 8*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^5 + 1
0*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^3 - (a^6 + 7*a^
5*b + 15*a^4*b^2 + 13*a^3*b^3 + 4*a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3
 + 4*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^6 + 15*(a^5*b
 + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^4 - 3*(a^6 + 7*a^5*b +
15*a^4*b^2 + 13*a^3*b^3 + 4*a^2*b^4)*d*cosh(d*x + c)^2 + (a^5*b + 3*a^4*b^2
 + 3*a^3*b^3 + a^2*b^4)*d)*sinh(d*x + c)^2 + (a^6 + 3*a^5*b + 3*a^4*b^2 + a
^3*b^3)*d + 8*((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^7 + 3*
(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^5 - (a^6 + 7*a^5*
b + 15*a^4*b^2 + 13*a^3*b^3 + 4*a^2*b^4)*d*cosh(d*x + c)^3 + (a^5*b + 3*a^4
*b^2 + 3*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(coth(c + d*x)**3/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [B] time = 2.69152, size = 517, normalized size = 4.7

$$\frac{\frac{\left(3\,ab^2+b^3\right)\log\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}{a^5+3\,a^4b+3\,a^3b^2+a^2b^3}+\frac{2\left(ae^{(2\,c)}+3\,be^{(2\,c)}\right)\log\left(\left|-e^{(2\,dx+2\,c)}+1\right|\right)}{a^3e^{(2\,c)}+3\,a^2be^{(2\,c)}+3\,ab^2e^{(2\,c)}+b^3e^{(2\,c)}}-\frac{2\,dx}{a^2}-\frac{3\,ab^2e^{(4\,dx+4\,c)}+b^3e^{(4\,dx+4\,c)}+6\,ab^2e^{(2\,dx+2\,c)}+10\,b}{\left(a^4+3\,a^3b+3\,a^2b^2+ab^3\right)\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2}\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((3*a*b^2 + b^3)*log(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2
*d*x + 2*c) + a)/(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3) + 2*(a*e^(2*c) + 3*b
*e^(2*c))*log(abs(-e^(2*d*x + 2*c) + 1))/(a^3*e^(2*c) + 3*a^2*b*e^(2*c) + 3
*a*b^2*e^(2*c) + b^3*e^(2*c)) - 2*d*x/a^2 - (3*a*b^2*e^(4*d*x + 4*c) + b^3*
e^(4*d*x + 4*c) + 6*a*b^2*e^(2*d*x + 2*c) + 10*b^3*e^(2*d*x + 2*c) + 3*a*b^
2 + b^3)/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*(a*e^(4*d*x + 4*c) + 2*a*e^(2
*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)) - (3*a*e^(4*d*x + 4*c) + 9*b*e^(4*d
*x + 4*c) - 2*a*e^(2*d*x + 2*c) - 14*b*e^(2*d*x + 2*c) + 3*a + 9*b)/((a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*(e^(2*d*x + 2*c) - 1)^2))/d
```

3.157 $\int \frac{\coth^4(c+dx)}{(a+b\,\mathrm{sech}^2(c+dx))^2}\,dx$

Optimal. Leaf size=161

$$-\frac{b^{5/2}(7a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2d(a+b)^{7/2}}-\frac{(2a^2+6ab-b^2)\coth(c+dx)}{2ad(a+b)^3}+\frac{x}{a^2}-\frac{(2a-3b)\coth^3(c+dx)}{6ad(a+b)^2}-\frac{b\,\cdots}{2ad(a+b)(\cdots}$$

```
[Out] x/a^2 - (b^(5/2)*(7*a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/
(2*a^2*(a + b)^(7/2)*d) - ((2*a^2 + 6*a*b - b^2)*Coth[c + d*x])/(2*a*(a + b
)^3*d) - ((2*a - 3*b)*Coth[c + d*x]^3)/(6*a*(a + b)^2*d) - (b*Coth[c + d*x]
^3)/(2*a*(a + b)*d*(a + b - b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.411326, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4141, 1975, 472, 583, 522, 206, 208}

$$-\frac{b^{5/2}(7a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2d(a+b)^{7/2}}-\frac{(2a^2+6ab-b^2)\coth(c+dx)}{2ad(a+b)^3}+\frac{x}{a^2}-\frac{(2a-3b)\coth^3(c+dx)}{6ad(a+b)^2}-\frac{b\,\cdots}{2ad(a+b)(\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] x/a^2 - (b^(5/2)*(7*a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/
(2*a^2*(a + b)^(7/2)*d) - ((2*a^2 + 6*a*b - b^2)*Coth[c + d*x])/(2*a*(a + b
)^3*d) - ((2*a - 3*b)*Coth[c + d*x]^3)/(6*a*(a + b)^2*d) - (b*Coth[c + d*x]
^3)/(2*a*(a + b)*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\coth^4(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx = \frac{\mathrm{Subst}\left(\int \frac{1}{x^4(1-x^2)\left(a+b(1-x^2)\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\mathrm{Subst}\left(\int \frac{1}{x^4(1-x^2)\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\coth^3(c+dx)}{2a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} - \frac{\mathrm{Subst}\left(\int \frac{-2a+3b-5bx^2}{x^4(1-x^2)\left(a+b-bx^2\right)}\,dx, x, \tanh(c+dx)\right)}{2a(a+b)d}$$

$$= -\frac{(2a-3b)\coth^3(c+dx)}{6a(a+b)^2d} - \frac{b\coth^3(c+dx)}{2a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\mathrm{Subst}\left(\int \frac{3\left(2a^2+6ab-b^2\right.}{x^2(1-x^2)(}\right.}{6}$$

$$= -\frac{\left(2a^2+6ab-b^2\right)\coth(c+dx)}{2a(a+b)^3d} - \frac{(2a-3b)\coth^3(c+dx)}{6a(a+b)^2d} - \frac{b\coth^3(c+dx)}{2a(a+b)d\left(a+b-b\tanh^2(\right.}$$

$$= -\frac{\left(2a^2+6ab-b^2\right)\coth(c+dx)}{2a(a+b)^3d} - \frac{(2a-3b)\coth^3(c+dx)}{6a(a+b)^2d} - \frac{b\coth^3(c+dx)}{2a(a+b)d\left(a+b-b\tanh^2(\right.}$$

$$= \frac{x}{a^2} - \frac{b^{5/2}(7a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{7/2}d} - \frac{\left(2a^2+6ab-b^2\right)\coth(c+dx)}{2a(a+b)^3d} - \frac{(2a-3b)\mathrm{co}}{6a(a+}$$

Mathematica [B] time = 5.22686, size = 350, normalized size = 2.17

$$\frac{\operatorname{sech}^4(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\frac{3b^3\operatorname{sech}(2c)((a+2b)\sinh(2c)-a\sinh(2dx))}{a^2d(a+b)^3}-\frac{3b^3(7a+2b)(\cosh(2c)-\sinh(2c))(a\cosh(2(c+dx))+a\cdots}{a^2d(a+b\cdots}\right.\cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*((6*x*(a + 2*b + a*Cosh[2*
(c + d*x)]))/a^2 - (2*(a + 2*b + a*Cosh[2*(c + d*x)])*Coth[c]*Csch[c + d*x]
^2)/((a + b)^2*d) - (3*b^3*(7*a + 2*b)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh
[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Co
sh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])*(Cosh[2*c] - Sinh[2*c
]))/(a^2*(a + b)^(7/2)*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (4*(2*a + 5*b)*(a
 + 2*b + a*Cosh[2*(c + d*x)])*Csch[c]*Csch[c + d*x]*Sinh[d*x])/((a + b)^3*d
) + (2*(a + 2*b + a*Cosh[2*(c + d*x)])*Csch[c]*Csch[c + d*x]^3*Sinh[d*x])/(
(a + b)^2*d) + (3*b^3*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/(a^2
*(a + b)^3*d)))/(24*(a + b*Sech[c + d*x]^2)^2)
```

Maple [B] time = 0.109, size = 634, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/24/d/(a+b)/(a^2+2*a*b+b^2)*a*tanh(1/2*d*x+1/2*c)^3-1/24/d/(a+b)/(a^2+2*a
*b+b^2)*b*tanh(1/2*d*x+1/2*c)^3-5/8/d/(a+b)/(a^2+2*a*b+b^2)*a*tanh(1/2*d*x+
1/2*c)-13/8/d/(a+b)/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)*b+1/d/a^2*ln(tanh(1
/2*d*x+1/2*c)+1)-1/d*b^3/a/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+
1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*tanh(1/2*
d*x+1/2*c)^3-1/d*b^3/a/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*
c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*tanh(1/2*d*x+
1/2*c)-7/4/d*b^(5/2)/a/(a+b)^(7/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*t
anh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+7/4/d*b^(5/2)/a/(a+b)^(7/2)*ln(-(a+
b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2))-1
/2/d*b^(7/2)/a^2/(a+b)^(7/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/
2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/2/d*b^(7/2)/a^2/(a+b)^(7/2)*ln(-(a+b)^(
1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2))-1/24/
d/(a+b)^2/tanh(1/2*d*x+1/2*c)^3-5/8/d/(a+b)^3/tanh(1/2*d*x+1/2*c)*a-13/8/d/
(a+b)^3/tanh(1/2*d*x+1/2*c)*b-1/d/a^2*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.60531, size = 22652, normalized size = 140.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(12*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^10 + 120*(a
^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^9 + 12*(a
^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*sinh(d*x + c)^10 - 12*(4*a^4 + 8*a^3*
b - a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(
d*x + c)^8 + 12*(45*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^2
 - 4*a^4 - 8*a^3*b + a*b^3 + 2*b^4 - (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 -
4*b^4)*d*x)*sinh(d*x + c)^8 + 96*(15*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*
x*cosh(d*x + c)^3 - (4*a^4 + 8*a^3*b - a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2
*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 - 24*(2*a^4 +
10*a^3*b + 16*a^2*b^2 + a*b^3 + 3*b^4 + (a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*
b^3 + 6*b^4)*d*x)*cosh(d*x + c)^6 + 24*(105*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*
b^3)*d*x*cosh(d*x + c)^4 - 2*a^4 - 10*a^3*b - 16*a^2*b^2 - a*b^3 - 3*b^4 -
(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x - 14*(4*a^4 + 8*a^3*b -
 a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x
 + c)^2)*sinh(d*x + c)^6 + 48*(63*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*c
osh(d*x + c)^5 - 14*(4*a^4 + 8*a^3*b - a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2
*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x + c)^3 - 3*(2*a^4 + 10*a^3*b + 16*a^
2*b^2 + a*b^3 + 3*b^4 + (a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x
)*cosh(d*x + c))*sinh(d*x + c)^5 + 8*(2*a^4 + 38*a^3*b + 72*a^2*b^2 + 9*b^4
 + 3*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x)*cosh(d*x + c)^4 +
 8*(315*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^6 - 105*(4*a^
4 + 8*a^3*b - a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*
d*x)*cosh(d*x + c)^4 + 2*a^4 + 38*a^3*b + 72*a^2*b^2 + 9*b^4 + 3*(a^4 + 9*a
^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x - 45*(2*a^4 + 10*a^3*b + 16*a^2*b
^2 + a*b^3 + 3*b^4 + (a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x)*c
osh(d*x + c)^2)*sinh(d*x + c)^4 - 32*a^4 - 80*a^3*b - 12*a*b^3 + 32*(45*(a^
4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^7 - 21*(4*a^4 + 8*a^3*b
- a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*
x + c)^5 - 15*(2*a^4 + 10*a^3*b + 16*a^2*b^2 + a*b^3 + 3*b^4 + (a^4 + 9*a^3
*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x)*cosh(d*x + c)^3 + (2*a^4 + 38*a^3*
b + 72*a^2*b^2 + 9*b^4 + 3*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*
d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 12*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3
)*d*x - 4*(4*a^4 + 36*a^3*b + 80*a^2*b^2 - 6*a*b^3 + 6*b^4 - 3*(a^4 - a^3*b
 - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x + c)^2 + 4*(135*(a^4 + 3*a^3
*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^8 - 84*(4*a^4 + 8*a^3*b - a*b^3 -
 2*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x + c)^6
- 90*(2*a^4 + 10*a^3*b + 16*a^2*b^2 + a*b^3 + 3*b^4 + (a^4 + 9*a^3*b + 21*a
^2*b^2 + 19*a*b^3 + 6*b^4)*d*x)*cosh(d*x + c)^4 - 4*a^4 - 36*a^3*b - 80*a^2
*b^2 + 6*a*b^3 - 6*b^4 + 3*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x
 + 12*(2*a^4 + 38*a^3*b + 72*a^2*b^2 + 9*b^4 + 3*(a^4 + 9*a^3*b + 21*a^2*b^
2 + 19*a*b^3 + 6*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*((7*a^2*b^2
 + 2*a*b^3)*cosh(d*x + c)^10 + 10*(7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)*sinh(
d*x + c)^9 + (7*a^2*b^2 + 2*a*b^3)*sinh(d*x + c)^10 - (7*a^2*b^2 - 26*a*b^3
 - 8*b^4)*cosh(d*x + c)^8 - (7*a^2*b^2 - 26*a*b^3 - 8*b^4 - 45*(7*a^2*b^2 +
 2*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(7*a^2*b^2 + 2*a*b^3)*co
sh(d*x + c)^3 - (7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh(d*x + c))*sinh(d*x + c)
^7 - 2*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*cosh(d*x + c)^6 + 2*(105*(7*a^2*b^2
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+ 2*a*b^3)*cosh(d*x + c)^4 - 7*a^2*b^2 - 44*a*b^3 - 12*b^4 - 14*(7*a^2*b^2
- 26*a*b^3 - 8*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(7*a^2*b^2 + 2
*a*b^3)*cosh(d*x + c)^5 - 14*(7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh(d*x + c)^3
 - 3*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(7*
a^2*b^2 + 44*a*b^3 + 12*b^4)*cosh(d*x + c)^4 + 2*(105*(7*a^2*b^2 + 2*a*b^3)
*cosh(d*x + c)^6 - 35*(7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh(d*x + c)^4 + 7*a^
2*b^2 + 44*a*b^3 + 12*b^4 - 15*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*cosh(d*x + c
)^2)*sinh(d*x + c)^4 - 7*a^2*b^2 - 2*a*b^3 + 8*(15*(7*a^2*b^2 + 2*a*b^3)*co
sh(d*x + c)^7 - 7*(7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh(d*x + c)^5 - 5*(7*a^2
*b^2 + 44*a*b^3 + 12*b^4)*cosh(d*x + c)^3 + (7*a^2*b^2 + 44*a*b^3 + 12*b^4)
*cosh(d*x + c))*sinh(d*x + c)^3 + (7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh(d*x +
 c)^2 + (45*(7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^8 - 28*(7*a^2*b^2 - 26*a*b^
3 - 8*b^4)*cosh(d*x + c)^6 - 30*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*cosh(d*x +
c)^4 + 7*a^2*b^2 - 26*a*b^3 - 8*b^4 + 12*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*co
sh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*(7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^9
 - 4*(7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh(d*x + c)^7 - 6*(7*a^2*b^2 + 44*a*b
^3 + 12*b^4)*cosh(d*x + c)^5 + 4*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*cosh(d*x +
 c)^3 + (7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b
/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 +
a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x +
 c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*
x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh
(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(
d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a
*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x +
 c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x +
 c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 8*(15*(a^4 + 3*a^3*b
 + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^9 - 12*(4*a^4 + 8*a^3*b - a*b^3 - 2
*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x + c)^7 -
18*(2*a^4 + 10*a^3*b + 16*a^2*b^2 + a*b^3 + 3*b^4 + (a^4 + 9*a^3*b + 21*a^2
*b^2 + 19*a*b^3 + 6*b^4)*d*x)*cosh(d*x + c)^5 + 4*(2*a^4 + 38*a^3*b + 72*a^
2*b^2 + 9*b^4 + 3*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x)*cosh
(d*x + c)^3 - (4*a^4 + 36*a^3*b + 80*a^2*b^2 - 6*a*b^3 + 6*b^4 - 3*(a^4 - a
^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a
^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^10 + 10*(a^6 + 3*a^5*b
+ 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^6 + 3*a^5*b + 3
*a^4*b^2 + a^3*b^3)*d*sinh(d*x + c)^10 - (a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*
b^3 - 4*a^2*b^4)*d*cosh(d*x + c)^8 + (45*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b
^3)*d*cosh(d*x + c)^2 - (a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*
d)*sinh(d*x + c)^8 - 2*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4
)*d*cosh(d*x + c)^6 + 8*(15*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*
x + c)^3 - (a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*d*cosh(d*x +
c))*sinh(d*x + c)^7 + 2*(105*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d
*x + c)^4 - 14*(a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*d*cosh(d*
x + c)^2 - (a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*d)*sinh(d*
x + c)^6 + 2*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*d*cosh(d
*x + c)^4 + 4*(63*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^5 -
 14*(a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*d*cosh(d*x + c)^3 -
3*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*d*cosh(d*x + c))*si
nh(d*x + c)^5 + 2*(105*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c
)^6 - 35*(a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*d*cosh(d*x + c)
^4 - 15*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*d*cosh(d*x +
c)^2 + (a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*d)*sinh(d*x +
c)^4 + (a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*d*cosh(d*x + c)^2
 + 8*(15*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^7 - 7*(a^6 -
 a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*d*cosh(d*x + c)^5 - 5*(a^6 + 9
*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*d*cosh(d*x + c)^3 + (a^6 + 9*
a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)
^3 + (45*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^8 - 28*(a^6
```

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- a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*d*cosh(d*x + c)^6 - 30*(a^6 +
 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*d*cosh(d*x + c)^4 + 12*(a^6
 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*d*cosh(d*x + c)^2 + (a^6
- a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*d)*sinh(d*x + c)^2 - (a^6 + 3
*a^5*b + 3*a^4*b^2 + a^3*b^3)*d + 2*(5*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3
)*d*cosh(d*x + c)^9 - 4*(a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*
d*cosh(d*x + c)^7 - 6*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)
*d*cosh(d*x + c)^5 + 4*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4
)*d*cosh(d*x + c)^3 + (a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*d*
cosh(d*x + c))*sinh(d*x + c)), 1/6*(6*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d
*x*cosh(d*x + c)^10 + 60*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x +
 c)*sinh(d*x + c)^9 + 6*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*sinh(d*x +
c)^10 - 6*(4*a^4 + 8*a^3*b - a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*
a*b^3 - 4*b^4)*d*x)*cosh(d*x + c)^8 + 6*(45*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*
b^3)*d*x*cosh(d*x + c)^2 - 4*a^4 - 8*a^3*b + a*b^3 + 2*b^4 - (a^4 - a^3*b -
 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*sinh(d*x + c)^8 + 48*(15*(a^4 + 3*a^3*b
 + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^3 - (4*a^4 + 8*a^3*b - a*b^3 - 2*b^
4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x + c))*sinh(d
*x + c)^7 - 12*(2*a^4 + 10*a^3*b + 16*a^2*b^2 + a*b^3 + 3*b^4 + (a^4 + 9*a^
3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x)*cosh(d*x + c)^6 + 12*(105*(a^4 +
3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^4 - 2*a^4 - 10*a^3*b - 16*a^
2*b^2 - a*b^3 - 3*b^4 - (a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x
 - 14*(4*a^4 + 8*a^3*b - a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^
3 - 4*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 24*(63*(a^4 + 3*a^3*b +
3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^5 - 14*(4*a^4 + 8*a^3*b - a*b^3 - 2*b^
4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x + c)^3 - 3*(
2*a^4 + 10*a^3*b + 16*a^2*b^2 + a*b^3 + 3*b^4 + (a^4 + 9*a^3*b + 21*a^2*b^2
 + 19*a*b^3 + 6*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*(2*a^4 + 38*a^
3*b + 72*a^2*b^2 + 9*b^4 + 3*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4
)*d*x)*cosh(d*x + c)^4 + 4*(315*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cos
h(d*x + c)^6 - 105*(4*a^4 + 8*a^3*b - a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2*
b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x + c)^4 + 2*a^4 + 38*a^3*b + 72*a^2*b^
2 + 9*b^4 + 3*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x - 45*(2*a
^4 + 10*a^3*b + 16*a^2*b^2 + a*b^3 + 3*b^4 + (a^4 + 9*a^3*b + 21*a^2*b^2 +
19*a*b^3 + 6*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 16*a^4 - 40*a^3*b
 - 6*a*b^3 + 16*(45*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^7
 - 21*(4*a^4 + 8*a^3*b - a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^
3 - 4*b^4)*d*x)*cosh(d*x + c)^5 - 15*(2*a^4 + 10*a^3*b + 16*a^2*b^2 + a*b^3
 + 3*b^4 + (a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x)*cosh(d*x +
c)^3 + (2*a^4 + 38*a^3*b + 72*a^2*b^2 + 9*b^4 + 3*(a^4 + 9*a^3*b + 21*a^2*b
^2 + 19*a*b^3 + 6*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 6*(a^4 + 3*a^3
*b + 3*a^2*b^2 + a*b^3)*d*x - 2*(4*a^4 + 36*a^3*b + 80*a^2*b^2 - 6*a*b^3 +
6*b^4 - 3*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x + c)^2
 + 2*(135*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^8 - 84*(4*a
^4 + 8*a^3*b - a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)
*d*x)*cosh(d*x + c)^6 - 90*(2*a^4 + 10*a^3*b + 16*a^2*b^2 + a*b^3 + 3*b^4 +
 (a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x)*cosh(d*x + c)^4 - 4*a
^4 - 36*a^3*b - 80*a^2*b^2 + 6*a*b^3 - 6*b^4 + 3*(a^4 - a^3*b - 9*a^2*b^2 -
 11*a*b^3 - 4*b^4)*d*x + 12*(2*a^4 + 38*a^3*b + 72*a^2*b^2 + 9*b^4 + 3*(a^4
 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x
+ c)^2 - 3*((7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^10 + 10*(7*a^2*b^2 + 2*a*b^
3)*cosh(d*x + c)*sinh(d*x + c)^9 + (7*a^2*b^2 + 2*a*b^3)*sinh(d*x + c)^10 -
 (7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh(d*x + c)^8 - (7*a^2*b^2 - 26*a*b^3 - 8
*b^4 - 45*(7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(7
*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^3 - (7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh(d
*x + c))*sinh(d*x + c)^7 - 2*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*cosh(d*x + c)^
6 + 2*(105*(7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^4 - 7*a^2*b^2 - 44*a*b^3 - 1
2*b^4 - 14*(7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6
+ 4*(63*(7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^5 - 14*(7*a^2*b^2 - 26*a*b^3 -
```

```
8*b^4)*cosh(d*x + c)^3 - 3*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*cosh(d*x + c))*s
inh(d*x + c)^5 + 2*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*cosh(d*x + c)^4 + 2*(105
*(7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^6 - 35*(7*a^2*b^2 - 26*a*b^3 - 8*b^4)*
cosh(d*x + c)^4 + 7*a^2*b^2 + 44*a*b^3 + 12*b^4 - 15*(7*a^2*b^2 + 44*a*b^3
+ 12*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 7*a^2*b^2 - 2*a*b^3 + 8*(15*(7
*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^7 - 7*(7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh
(d*x + c)^5 - 5*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*cosh(d*x + c)^3 + (7*a^2*b^
2 + 44*a*b^3 + 12*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + (7*a^2*b^2 - 26*a*b
^3 - 8*b^4)*cosh(d*x + c)^2 + (45*(7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^8 - 2
8*(7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh(d*x + c)^6 - 30*(7*a^2*b^2 + 44*a*b^3
 + 12*b^4)*cosh(d*x + c)^4 + 7*a^2*b^2 - 26*a*b^3 - 8*b^4 + 12*(7*a^2*b^2 +
 44*a*b^3 + 12*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*(7*a^2*b^2 + 2*
a*b^3)*cosh(d*x + c)^9 - 4*(7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh(d*x + c)^7 -
 6*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*cosh(d*x + c)^5 + 4*(7*a^2*b^2 + 44*a*b^
3 + 12*b^4)*cosh(d*x + c)^3 + (7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh(d*x + c))
*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d
*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b) +
4*(15*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^9 - 12*(4*a^4 +
 8*a^3*b - a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x
)*cosh(d*x + c)^7 - 18*(2*a^4 + 10*a^3*b + 16*a^2*b^2 + a*b^3 + 3*b^4 + (a^
4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x)*cosh(d*x + c)^5 + 4*(2*a^
4 + 38*a^3*b + 72*a^2*b^2 + 9*b^4 + 3*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^
3 + 6*b^4)*d*x)*cosh(d*x + c)^3 - (4*a^4 + 36*a^3*b + 80*a^2*b^2 - 6*a*b^3
+ 6*b^4 - 3*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x + c)
)*sinh(d*x + c))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^10
+ 10*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^9
+ (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*sinh(d*x + c)^10 - (a^6 - a^5*b -
 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*d*cosh(d*x + c)^8 + (45*(a^6 + 3*a^5*b
 + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^2 - (a^6 - a^5*b - 9*a^4*b^2 - 11*a
^3*b^3 - 4*a^2*b^4)*d)*sinh(d*x + c)^8 - 2*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19
*a^3*b^3 + 6*a^2*b^4)*d*cosh(d*x + c)^6 + 8*(15*(a^6 + 3*a^5*b + 3*a^4*b^2
+ a^3*b^3)*d*cosh(d*x + c)^3 - (a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^
2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^6 + 3*a^5*b + 3*a^4*b^2
 + a^3*b^3)*d*cosh(d*x + c)^4 - 14*(a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 -
4*a^2*b^4)*d*cosh(d*x + c)^2 - (a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6
*a^2*b^4)*d)*sinh(d*x + c)^6 + 2*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 +
 6*a^2*b^4)*d*cosh(d*x + c)^4 + 4*(63*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)
*d*cosh(d*x + c)^5 - 14*(a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*
d*cosh(d*x + c)^3 - 3*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)
*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3
*b^3)*d*cosh(d*x + c)^6 - 35*(a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*
b^4)*d*cosh(d*x + c)^4 - 15*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^
2*b^4)*d*cosh(d*x + c)^2 + (a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2
*b^4)*d)*sinh(d*x + c)^4 + (a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^
4)*d*cosh(d*x + c)^2 + 8*(15*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d
*x + c)^7 - 7*(a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*d*cosh(d*x
 + c)^5 - 5*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*d*cosh(d*
x + c)^3 + (a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*d*cosh(d*x
 + c))*sinh(d*x + c)^3 + (45*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d
*x + c)^8 - 28*(a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*d*cosh(d*
x + c)^6 - 30*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*d*cosh(
d*x + c)^4 + 12*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*d*cos
h(d*x + c)^2 + (a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*d)*sinh(d
*x + c)^2 - (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d + 2*(5*(a^6 + 3*a^5*b +
 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^9 - 4*(a^6 - a^5*b - 9*a^4*b^2 - 11*a
^3*b^3 - 4*a^2*b^4)*d*cosh(d*x + c)^7 - 6*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19*
a^3*b^3 + 6*a^2*b^4)*d*cosh(d*x + c)^5 + 4*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19
*a^3*b^3 + 6*a^2*b^4)*d*cosh(d*x + c)^3 + (a^6 - a^5*b - 9*a^4*b^2 - 11*a^3
*b^3 - 4*a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**4/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(coth(c + d*x)**4/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [B] time = 3.03384, size = 408, normalized size = 2.53

$$-\frac{\frac{3\left(7\,ab^3e^{(2\,c)}+2\,b^4e^{(2\,c)}\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)e^{(-2\,c)}}{\left(a^5+3\,a^4b+3\,a^3b^2+a^2b^3\right)\sqrt{-ab-b^2}}-\frac{6\,dx}{a^2}-\frac{6\left(ab^3e^{(2\,dx+2\,c)}+2\,b^4e^{(2\,dx+2\,c)}+ab^3\right)}{\left(a^5+3\,a^4b+3\,a^3b^2+a^2b^3\right)\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}+\frac{8\left(3\,ae^{(4\,dx+4\,c)}+6\,be\cdots\right.}{\left(a^3+3\cdots\right.}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/6*(3*(7*a*b^3*e^(2*c) + 2*b^4*e^(2*c))*arctan(1/2*(a*e^(2*d*x + 2*c) + a
 + 2*b)/sqrt(-a*b - b^2))*e^(-2*c)/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*s
qrt(-a*b - b^2)) - 6*d*x/a^2 - 6*(a*b^3*e^(2*d*x + 2*c) + 2*b^4*e^(2*d*x +
2*c) + a*b^3)/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*(a*e^(4*d*x + 4*c) + 2
*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)) + 8*(3*a*e^(4*d*x + 4*c) + 6
*b*e^(4*d*x + 4*c) - 3*a*e^(2*d*x + 2*c) - 9*b*e^(2*d*x + 2*c) + 2*a + 5*b)
/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(e^(2*d*x + 2*c) - 1)^3))/d
```

3.158 $\int \frac{\tanh^6(c+dx)}{\left(a+b\mathbf{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=148

$$\frac{(3a-4b)(a+b)\tanh(c+dx)}{8a^2b^2d\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\sqrt{a+b}\left(3a^2-4ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3b^{5/2}d}+\frac{x}{a^3}-\frac{(a+b)\tanh^3(c+dx)}{4abd\left(a-b\tanh^2(c+dx)+b\right)^2}$$

```
[Out] x/a^3 - (Sqrt[a + b]*(3*a^2 - 4*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x]
)/Sqrt[a + b]])/(8*a^3*b^(5/2)*d) - ((a + b)*Tanh[c + d*x]^3)/(4*a*b*d*(a +
 b - b*Tanh[c + d*x]^2)^2) + ((3*a - 4*b)*(a + b)*Tanh[c + d*x])/(8*a^2*b^2
*d*(a + b - b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.319562, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4141, 1975, 470, 578, 522, 206, 208}

$$\frac{(3a-4b)(a+b)\tanh(c+dx)}{8a^2b^2d\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\sqrt{a+b}\left(3a^2-4ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3b^{5/2}d}+\frac{x}{a^3}-\frac{(a+b)\tanh^3(c+dx)}{4abd\left(a-b\tanh^2(c+dx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^6/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] x/a^3 - (Sqrt[a + b]*(3*a^2 - 4*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x]
)/Sqrt[a + b]])/(8*a^3*b^(5/2)*d) - ((a + b)*Tanh[c + d*x]^3)/(4*a*b*d*(a +
 b - b*Tanh[c + d*x]^2)^2) + ((3*a - 4*b)*(a + b)*Tanh[c + d*x])/(8*a^2*b^2
*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
```

```
p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tanh^6(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3}\,dx = \frac{\mathrm{Subst}\left(\int \frac{x^6}{(1-x^2)\left(a+b(1-x^2)\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\mathrm{Subst}\left(\int \frac{x^6}{(1-x^2)\left(a+b-bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{(a+b)\tanh^3(c+dx)}{4abd\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{\mathrm{Subst}\left(\int \frac{x^2\left(3(a+b)+(-3a+b)x^2\right)}{(1-x^2)\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{4abd}$$

$$= -\frac{(a+b)\tanh^3(c+dx)}{4abd\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{(3a-4b)(a+b)\tanh(c+dx)}{8a^2b^2d\left(a+b-b\tanh^2(c+dx)\right)} - \frac{\mathrm{Subst}\left(\int \frac{(3a-4b)(\cdots}{(\cdots}\right.}{\cdots}$$

$$= -\frac{(a+b)\tanh^3(c+dx)}{4abd\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{(3a-4b)(a+b)\tanh(c+dx)}{8a^2b^2d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx \cdots\right.}{a\cdots}$$

$$= \frac{x}{a^3} - \frac{\sqrt{a+b}\left(3a^2-4ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3b^{5/2}d} - \frac{(a+b)\tanh^3(c+dx)}{4abd\left(a+b-b\tanh^2(c+dx)\right)^2} \cdots$$

Mathematica [B] time = 6.0547, size = 515, normalized size = 3.48

$$\frac{\operatorname{sech}^6(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\operatorname{sech}(2c)\left(-6a^2b^2\sinh(2(c+2dx))+20a^2b^2\sinh(4c+2dx)+4a^2b^2dx\cos\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tanh[c + d*x]^6/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^6*((-2*(3*a^3 - a^2*b + 4*a*
b^2 + 8*b^3)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x
] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(a +
 2*b + a*Cosh[2*(c + d*x)])^2*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*
(Cosh[c] - Sinh[c])^4]) + Sech[2*c]*(8*b^2*(3*a^2 + 8*a*b + 8*b^2)*d*x*Cosh
[2*c] + 16*a*b^2*(a + 2*b)*d*x*Cosh[2*d*x] + 4*a^2*b^2*d*x*Cosh[2*(c + 2*d*
x)] + 16*a^2*b^2*d*x*Cosh[4*c + 2*d*x] + 32*a*b^3*d*x*Cosh[4*c + 2*d*x] + 4
*a^2*b^2*d*x*Cosh[6*c + 4*d*x] - 9*a^4*Sinh[2*c] - 15*a^3*b*Sinh[2*c] + 18*
a^2*b^2*Sinh[2*c] + 72*a*b^3*Sinh[2*c] + 48*b^4*Sinh[2*c] + 9*a^4*Sinh[2*d*
x] + 13*a^3*b*Sinh[2*d*x] - 28*a^2*b^2*Sinh[2*d*x] - 32*a*b^3*Sinh[2*d*x] +
 3*a^4*Sinh[2*(c + 2*d*x)] - 3*a^3*b*Sinh[2*(c + 2*d*x)] - 6*a^2*b^2*Sinh[2
*(c + 2*d*x)] - 3*a^4*Sinh[4*c + 2*d*x] + a^3*b*Sinh[4*c + 2*d*x] + 20*a^2*
b^2*Sinh[4*c + 2*d*x] + 16*a*b^3*Sinh[4*c + 2*d*x])))/(128*a^3*b^2*d*(a + b
*Sech[c + d*x]^2)^3)
```

Maple [B] time = 0.106, size = 1713, normalized size = 11.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 3/16/d/b^(5/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*
d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-5/4/d/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2
*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*ta
nh(1/2*d*x+1/2*c)^7-19/4/d/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)
^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+
1/2*c)^5-19/4/d/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1
/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^3-5/
4/d/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c
)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)+1/2/d/(tanh(1/2*
d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2
*d*x+1/2*c)^2*b+a+b)^2/b*tanh(1/2*d*x+1/2*c)^7-7/2/d/(tanh(1/2*d*x+1/2*c)^4
*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^
2*b+a+b)^2/b*tanh(1/2*d*x+1/2*c)^5-7/2/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/
2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/b
*tanh(1/2*d*x+1/2*c)^3+1/2/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)
^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/b*tanh(1/2*d*
x+1/2*c)-3/16/d/b^(5/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*
tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/d*b/a^2/(tanh(1/2*d*x+1/2*c)^4*a
+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*
b+a+b)^2*tanh(1/2*d*x+1/2*c)^3-1/d*b/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/
2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*t
```

```
anh(1/2*d*x+1/2*c)-1/2/d*b^(1/2)/a^3/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*
x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+3/4/d*a/(tanh(1/2*d*x
+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*
x+1/2*c)^2*b+a+b)^2/b^2*tanh(1/2*d*x+1/2*c)^7+9/4/d*a/(tanh(1/2*d*x+1/2*c)^
4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)
^2*b+a+b)^2/b^2*tanh(1/2*d*x+1/2*c)^5+1/16/d/a/b^(3/2)/(a+b)^(1/2)*ln((a+b)
^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-1/4
/d/a^2/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*
d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-1/16/d/a/b^(3/2)/(a+b)^(1/2)*ln((a+b)^(1/2)
*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/4/d/a^2
/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/
2*c)*b^(1/2)+(a+b)^(1/2))+9/4/d*a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1
/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/b^2*tanh
(1/2*d*x+1/2*c)^3+3/4/d*a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+
2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/b^2*tanh(1/2*d*x
+1/2*c)-1/d/a^3*ln(tanh(1/2*d*x+1/2*c)-1)+1/d*b/a^2/(tanh(1/2*d*x+1/2*c)^4*
a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2
*b+a+b)^2*tanh(1/2*d*x+1/2*c)^5+1/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)+1/2/d/a^3
*b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/
2*c)*b^(1/2)+(a+b)^(1/2))-1/d*b/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x
+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1
/2*d*x+1/2*c)^7
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 2.89494, size = 12740, normalized size = 86.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [1/16*(16*a^2*b^2*d*x*cosh(d*x + c)^8 + 128*a^2*b^2*d*x*cosh(d*x + c)*sinh(
d*x + c)^7 + 16*a^2*b^2*d*x*sinh(d*x + c)^8 - 4*(3*a^4 - a^3*b - 20*a^2*b^2
 - 16*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^6 + 4*(112*a^2*b^2*
d*x*cosh(d*x + c)^2 - 3*a^4 + a^3*b + 20*a^2*b^2 + 16*a*b^3 + 16*(a^2*b^2 +
 2*a*b^3)*d*x)*sinh(d*x + c)^6 + 16*a^2*b^2*d*x + 8*(112*a^2*b^2*d*x*cosh(d
*x + c)^3 - 3*(3*a^4 - a^3*b - 20*a^2*b^2 - 16*a*b^3 - 16*(a^2*b^2 + 2*a*b^
3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 4*(9*a^4 + 15*a^3*b - 18*a^2*b^2 -
 72*a*b^3 - 48*b^4 - 8*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*d*x)*cosh(d*x + c)^4 +
 4*(280*a^2*b^2*d*x*cosh(d*x + c)^4 - 9*a^4 - 15*a^3*b + 18*a^2*b^2 + 72*a*
b^3 + 48*b^4 + 8*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*d*x - 15*(3*a^4 - a^3*b - 20
*a^2*b^2 - 16*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x
 + c)^4 - 12*a^4 + 12*a^3*b + 24*a^2*b^2 + 16*(56*a^2*b^2*d*x*cosh(d*x + c)
^5 - 5*(3*a^4 - a^3*b - 20*a^2*b^2 - 16*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x)
*cosh(d*x + c)^3 - (9*a^4 + 15*a^3*b - 18*a^2*b^2 - 72*a*b^3 - 48*b^4 - 8*(
```

```
3*a^2*b^2 + 8*a*b^3 + 8*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(9*a^4
 + 13*a^3*b - 28*a^2*b^2 - 32*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x
+ c)^2 + 4*(112*a^2*b^2*d*x*cosh(d*x + c)^6 - 15*(3*a^4 - a^3*b - 20*a^2*b^
2 - 16*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^4 - 9*a^4 - 13*a^3
*b + 28*a^2*b^2 + 32*a*b^3 + 16*(a^2*b^2 + 2*a*b^3)*d*x - 6*(9*a^4 + 15*a^3
*b - 18*a^2*b^2 - 72*a*b^3 - 48*b^4 - 8*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*d*x)*
cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((3*a^4 - 4*a^3*b + 8*a^2*b^2)*cosh(d*x
+ c)^8 + 8*(3*a^4 - 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (3
*a^4 - 4*a^3*b + 8*a^2*b^2)*sinh(d*x + c)^8 + 4*(3*a^4 + 2*a^3*b + 16*a*b^3
)*cosh(d*x + c)^6 + 4*(3*a^4 + 2*a^3*b + 16*a*b^3 + 7*(3*a^4 - 4*a^3*b + 8*
a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^4 - 4*a^3*b + 8*a^2*b
^2)*cosh(d*x + c)^3 + 3*(3*a^4 + 2*a^3*b + 16*a*b^3)*cosh(d*x + c))*sinh(d*
x + c)^5 + 2*(9*a^4 + 12*a^3*b + 16*a^2*b^2 + 32*a*b^3 + 64*b^4)*cosh(d*x +
 c)^4 + 2*(35*(3*a^4 - 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + 9*a^4 + 12*a^
3*b + 16*a^2*b^2 + 32*a*b^3 + 64*b^4 + 30*(3*a^4 + 2*a^3*b + 16*a*b^3)*cosh
(d*x + c)^2)*sinh(d*x + c)^4 + 3*a^4 - 4*a^3*b + 8*a^2*b^2 + 8*(7*(3*a^4 -
4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^5 + 10*(3*a^4 + 2*a^3*b + 16*a*b^3)*cosh
(d*x + c)^3 + (9*a^4 + 12*a^3*b + 16*a^2*b^2 + 32*a*b^3 + 64*b^4)*cosh(d*x
+ c))*sinh(d*x + c)^3 + 4*(3*a^4 + 2*a^3*b + 16*a*b^3)*cosh(d*x + c)^2 + 4*
(7*(3*a^4 - 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^6 + 15*(3*a^4 + 2*a^3*b + 16
*a*b^3)*cosh(d*x + c)^4 + 3*a^4 + 2*a^3*b + 16*a*b^3 + 3*(9*a^4 + 12*a^3*b
+ 16*a^2*b^2 + 32*a*b^3 + 64*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*
a^4 - 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^7 + 3*(3*a^4 + 2*a^3*b + 16*a*b^3)
*cosh(d*x + c)^5 + (9*a^4 + 12*a^3*b + 16*a^2*b^2 + 32*a*b^3 + 64*b^4)*cosh
(d*x + c)^3 + (3*a^4 + 2*a^3*b + 16*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sq
rt((a + b)/b)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^
3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d
*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cos
h(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*cosh(d*x
 + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + a*b + 2
*b^2)*sqrt((a + b)/b))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)
^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)
^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x +
 c))*sinh(d*x + c) + a)) + 8*(16*a^2*b^2*d*x*cosh(d*x + c)^7 - 3*(3*a^4 - a
^3*b - 20*a^2*b^2 - 16*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^5
- 2*(9*a^4 + 15*a^3*b - 18*a^2*b^2 - 72*a*b^3 - 48*b^4 - 8*(3*a^2*b^2 + 8*a
*b^3 + 8*b^4)*d*x)*cosh(d*x + c)^3 - (9*a^4 + 13*a^3*b - 28*a^2*b^2 - 32*a*
b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(a^5*b^2*d*
cosh(d*x + c)^8 + 8*a^5*b^2*d*cosh(d*x + c)*sinh(d*x + c)^7 + a^5*b^2*d*sin
h(d*x + c)^8 + a^5*b^2*d + 4*(a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^6 + 4*(7
*a^5*b^2*d*cosh(d*x + c)^2 + (a^5*b^2 + 2*a^4*b^3)*d)*sinh(d*x + c)^6 + 2*(
3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)^4 + 8*(7*a^5*b^2*d*cosh(
d*x + c)^3 + 3*(a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(
35*a^5*b^2*d*cosh(d*x + c)^4 + 30*(a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^2 +
 (3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*d)*sinh(d*x + c)^4 + 4*(a^5*b^2 + 2*a^
4*b^3)*d*cosh(d*x + c)^2 + 8*(7*a^5*b^2*d*cosh(d*x + c)^5 + 10*(a^5*b^2 + 2
*a^4*b^3)*d*cosh(d*x + c)^3 + (3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*
x + c))*sinh(d*x + c)^3 + 4*(7*a^5*b^2*d*cosh(d*x + c)^6 + 15*(a^5*b^2 + 2*
a^4*b^3)*d*cosh(d*x + c)^4 + 3*(3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*d*cosh(d
*x + c)^2 + (a^5*b^2 + 2*a^4*b^3)*d)*sinh(d*x + c)^2 + 8*(a^5*b^2*d*cosh(d*
x + c)^7 + 3*(a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^5 + (3*a^5*b^2 + 8*a^4*b
^3 + 8*a^3*b^4)*d*cosh(d*x + c)^3 + (a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c))*
sinh(d*x + c)), 1/8*(8*a^2*b^2*d*x*cosh(d*x + c)^8 + 64*a^2*b^2*d*x*cosh(d*
x + c)*sinh(d*x + c)^7 + 8*a^2*b^2*d*x*sinh(d*x + c)^8 - 2*(3*a^4 - a^3*b -
 20*a^2*b^2 - 16*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^6 + 2*(1
12*a^2*b^2*d*x*cosh(d*x + c)^2 - 3*a^4 + a^3*b + 20*a^2*b^2 + 16*a*b^3 + 16
*(a^2*b^2 + 2*a*b^3)*d*x)*sinh(d*x + c)^6 + 8*a^2*b^2*d*x + 4*(112*a^2*b^2*
d*x*cosh(d*x + c)^3 - 3*(3*a^4 - a^3*b - 20*a^2*b^2 - 16*a*b^3 - 16*(a^2*b^
2 + 2*a*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(9*a^4 + 15*a^3*b - 18
```

```
*a^2*b^2 - 72*a*b^3 - 48*b^4 - 8*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*d*x)*cosh(d*
x + c)^4 + 2*(280*a^2*b^2*d*x*cosh(d*x + c)^4 - 9*a^4 - 15*a^3*b + 18*a^2*b
^2 + 72*a*b^3 + 48*b^4 + 8*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*d*x - 15*(3*a^4 -
a^3*b - 20*a^2*b^2 - 16*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^2
)*sinh(d*x + c)^4 - 6*a^4 + 6*a^3*b + 12*a^2*b^2 + 8*(56*a^2*b^2*d*x*cosh(d
*x + c)^5 - 5*(3*a^4 - a^3*b - 20*a^2*b^2 - 16*a*b^3 - 16*(a^2*b^2 + 2*a*b^
3)*d*x)*cosh(d*x + c)^3 - (9*a^4 + 15*a^3*b - 18*a^2*b^2 - 72*a*b^3 - 48*b^
4 - 8*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 2
*(9*a^4 + 13*a^3*b - 28*a^2*b^2 - 32*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x)*co
sh(d*x + c)^2 + 2*(112*a^2*b^2*d*x*cosh(d*x + c)^6 - 15*(3*a^4 - a^3*b - 20
*a^2*b^2 - 16*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^4 - 9*a^4 -
 13*a^3*b + 28*a^2*b^2 + 32*a*b^3 + 16*(a^2*b^2 + 2*a*b^3)*d*x - 6*(9*a^4 +
 15*a^3*b - 18*a^2*b^2 - 72*a*b^3 - 48*b^4 - 8*(3*a^2*b^2 + 8*a*b^3 + 8*b^4
)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((3*a^4 - 4*a^3*b + 8*a^2*b^2)*co
sh(d*x + c)^8 + 8*(3*a^4 - 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)
^7 + (3*a^4 - 4*a^3*b + 8*a^2*b^2)*sinh(d*x + c)^8 + 4*(3*a^4 + 2*a^3*b + 1
6*a*b^3)*cosh(d*x + c)^6 + 4*(3*a^4 + 2*a^3*b + 16*a*b^3 + 7*(3*a^4 - 4*a^3
*b + 8*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^4 - 4*a^3*b +
8*a^2*b^2)*cosh(d*x + c)^3 + 3*(3*a^4 + 2*a^3*b + 16*a*b^3)*cosh(d*x + c))*
sinh(d*x + c)^5 + 2*(9*a^4 + 12*a^3*b + 16*a^2*b^2 + 32*a*b^3 + 64*b^4)*cos
h(d*x + c)^4 + 2*(35*(3*a^4 - 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + 9*a^4
+ 12*a^3*b + 16*a^2*b^2 + 32*a*b^3 + 64*b^4 + 30*(3*a^4 + 2*a^3*b + 16*a*b^
3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 3*a^4 - 4*a^3*b + 8*a^2*b^2 + 8*(7*(3
*a^4 - 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^5 + 10*(3*a^4 + 2*a^3*b + 16*a*b^
3)*cosh(d*x + c)^3 + (9*a^4 + 12*a^3*b + 16*a^2*b^2 + 32*a*b^3 + 64*b^4)*co
sh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a^4 + 2*a^3*b + 16*a*b^3)*cosh(d*x + c)
^2 + 4*(7*(3*a^4 - 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^6 + 15*(3*a^4 + 2*a^3
*b + 16*a*b^3)*cosh(d*x + c)^4 + 3*a^4 + 2*a^3*b + 16*a*b^3 + 3*(9*a^4 + 12
*a^3*b + 16*a^2*b^2 + 32*a*b^3 + 64*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
 8*((3*a^4 - 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^7 + 3*(3*a^4 + 2*a^3*b + 16
*a*b^3)*cosh(d*x + c)^5 + (9*a^4 + 12*a^3*b + 16*a^2*b^2 + 32*a*b^3 + 64*b^
4)*cosh(d*x + c)^3 + (3*a^4 + 2*a^3*b + 16*a*b^3)*cosh(d*x + c))*sinh(d*x +
 c))*sqrt(-(a + b)/b)*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sin
h(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-(a + b)/b)/(a + b)) + 4*(16
*a^2*b^2*d*x*cosh(d*x + c)^7 - 3*(3*a^4 - a^3*b - 20*a^2*b^2 - 16*a*b^3 - 1
6*(a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^5 - 2*(9*a^4 + 15*a^3*b - 18*a^2*b
^2 - 72*a*b^3 - 48*b^4 - 8*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*d*x)*cosh(d*x + c)
^3 - (9*a^4 + 13*a^3*b - 28*a^2*b^2 - 32*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x
)*cosh(d*x + c))*sinh(d*x + c))/(a^5*b^2*d*cosh(d*x + c)^8 + 8*a^5*b^2*d*co
sh(d*x + c)*sinh(d*x + c)^7 + a^5*b^2*d*sinh(d*x + c)^8 + a^5*b^2*d + 4*(a^
5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^6 + 4*(7*a^5*b^2*d*cosh(d*x + c)^2 + (a^
5*b^2 + 2*a^4*b^3)*d)*sinh(d*x + c)^6 + 2*(3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^
4)*d*cosh(d*x + c)^4 + 8*(7*a^5*b^2*d*cosh(d*x + c)^3 + 3*(a^5*b^2 + 2*a^4*
b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^5*b^2*d*cosh(d*x + c)^4 + 3
0*(a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^2 + (3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*
b^4)*d)*sinh(d*x + c)^4 + 4*(a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^2 + 8*(7*
a^5*b^2*d*cosh(d*x + c)^5 + 10*(a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^3 + (3
*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a
^5*b^2*d*cosh(d*x + c)^6 + 15*(a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^4 + 3*(
3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)^2 + (a^5*b^2 + 2*a^4*b^3
)*d)*sinh(d*x + c)^2 + 8*(a^5*b^2*d*cosh(d*x + c)^7 + 3*(a^5*b^2 + 2*a^4*b^
3)*d*cosh(d*x + c)^5 + (3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)^
3 + (a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**6/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 4.31904, size = 501, normalized size = 3.39

$$\frac{8\,dx}{a^3} - \frac{\left(3\,a^3e^{(2\,c)} - a^2be^{(2\,c)} + 4\,ab^2e^{(2\,c)} + 8\,b^3e^{(2\,c)}\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)} + a + 2\,b}{2\,\sqrt{-ab-b^2}}\right)e^{(-2\,c)}}{\sqrt{-ab-b^2}a^3b^2} - \frac{2\left(3\,a^4e^{(6\,dx+6\,c)} - a^3be^{(6\,dx+6\,c)} - 20\,a^2b^2e^{(6\,dx+6\,c)} - 16\,ab^3e^{(6\,dx+6\,c)} + 9\,a^4e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(8*d*x/a^3 - (3*a^3*e^(2*c) - a^2*b*e^(2*c) + 4*a*b^2*e^(2*c) + 8*b^3*e
^(2*c))*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))*e^(-2*c)
/(sqrt(-a*b - b^2)*a^3*b^2) - 2*(3*a^4*e^(6*d*x + 6*c) - a^3*b*e^(6*d*x + 6
*c) - 20*a^2*b^2*e^(6*d*x + 6*c) - 16*a*b^3*e^(6*d*x + 6*c) + 9*a^4*e^(4*d*
x + 4*c) + 15*a^3*b*e^(4*d*x + 4*c) - 18*a^2*b^2*e^(4*d*x + 4*c) - 72*a*b^3
*e^(4*d*x + 4*c) - 48*b^4*e^(4*d*x + 4*c) + 9*a^4*e^(2*d*x + 2*c) + 13*a^3*
b*e^(2*d*x + 2*c) - 28*a^2*b^2*e^(2*d*x + 2*c) - 32*a*b^3*e^(2*d*x + 2*c) +
 3*a^4 - 3*a^3*b - 6*a^2*b^2)/((a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4
*b*e^(2*d*x + 2*c) + a)^2*a^3*b^2))/d
```

3.159 $\int \frac{\tanh^5(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=77

$$-\frac{(a+b)^2}{4a^3d\left(a\cosh^2(c+dx)+b\right)^2}+\frac{a+b}{a^3d\left(a\cosh^2(c+dx)+b\right)}+\frac{\log\left(a\cosh^2(c+dx)+b\right)}{2a^3d}$$

```
[Out] -(a + b)^2/(4*a^3*d*(b + a*Cosh[c + d*x]^2)^2) + (a + b)/(a^3*d*(b + a*Cosh
[c + d*x]^2)) + Log[b + a*Cosh[c + d*x]^2]/(2*a^3*d)
```

Rubi [A] time = 0.119592, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4138, 444, 43}

$$-\frac{(a+b)^2}{4a^3d\left(a\cosh^2(c+dx)+b\right)^2}+\frac{a+b}{a^3d\left(a\cosh^2(c+dx)+b\right)}+\frac{\log\left(a\cosh^2(c+dx)+b\right)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^5/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -(a + b)^2/(4*a^3*d*(b + a*Cosh[c + d*x]^2)^2) + (a + b)/(a^3*d*(b + a*Cosh
[c + d*x]^2)) + Log[b + a*Cosh[c + d*x]^2]/(2*a^3*d)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x\left(1-x^2\right)^2}{\left(b+ax^2\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{(1-x)^2}{(b+ax)^3}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{(a+b)^2}{a^2(b+ax)^3} - \frac{2(a+b)}{a^2(b+ax)^2} + \frac{1}{a^2(b+ax)}\right) dx, x, \cosh^2(c+dx)\right)}{2d}$$

$$= -\frac{(a+b)^2}{4a^3d\left(b+a\cosh^2(c+dx)\right)^2} + \frac{a+b}{a^3d\left(b+a\cosh^2(c+dx)\right)} + \frac{\log\left(b+a\cosh^2(c+dx)\right)}{2a^3d}$$

Mathematica [A] time = 2.07362, size = 136, normalized size = 1.77

$$\frac{2\left(a^2+4ab+3b^2\right)+a^2\cosh^2(2(c+dx))\log(a\cosh(2(c+dx))+a+2b)+(a+2b)^2\log(a\cosh(2(c+dx))+a+2b)\ \cdot}{2a^3d(a\cosh(2(c+dx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^5/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (2*(a^2 + 4*a*b + 3*b^2) + (a + 2*b)^2*Log[a + 2*b + a*Cosh[2*(c + d*x)]] +
 a^2*Cosh[2*(c + d*x)]^2*Log[a + 2*b + a*Cosh[2*(c + d*x)]] + 2*a*Cosh[2*(c
 + d*x)]*(2*(a + b) + (a + 2*b)*Log[a + 2*b + a*Cosh[2*(c + d*x)]]))/(2*a^3
*d*(a + 2*b + a*Cosh[2*(c + d*x)])^2)
```

Maple [B] time = 0.093, size = 579, normalized size = 7.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -1/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)-2/d/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/
2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*t
anh(1/2*d*x+1/2*c)^6-2/d*b/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*
c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*
x+1/2*c)^6-8/d/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/
2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^4+4/d
/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)
^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^4*b-2/d/a/(tanh(1
/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(
1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^2-2/d*b/a^2/(tanh(1/2*d*x+1/2
*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/
2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^2+1/2/d/a^3*ln(tanh(1/2*d*x+1/2*c)^4*a+
b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b
+a+b)-1/d/a^3*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.22144, size = 278, normalized size = 3.61

$$\frac{4\left(\left(a^2+ab\right)e^{(-2\,dx-2\,c)}+\left(a^2+4\,ab+3\,b^2\right)e^{(-4\,dx-4\,c)}+\left(a^2+ab\right)e^{(-6\,dx-6\,c)}\right)}{\left(a^5e^{(-8\,dx-8\,c)}+a^5+4\left(a^5+2\,a^4b\right)e^{(-2\,dx-2\,c)}+2\left(3\,a^5+8\,a^4b+8\,a^3b^2\right)e^{(-4\,dx-4\,c)}+4\left(a^5+2\,a^4b\right)e^{(-6\,dx-6\,c)}\right)d}+\frac{dx+c}{a^3d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 4*((a^2 + a*b)*e^(-2*d*x - 2*c) + (a^2 + 4*a*b + 3*b^2)*e^(-4*d*x - 4*c) +
(a^2 + a*b)*e^(-6*d*x - 6*c))/((a^5*e^(-8*d*x - 8*c) + a^5 + 4*(a^5 + 2*a^4
*b)*e^(-2*d*x - 2*c) + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*e^(-4*d*x - 4*c) + 4
*(a^5 + 2*a^4*b)*e^(-6*d*x - 6*c))*d) + (d*x + c)/(a^3*d) + 1/2*log(2*(a +
2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^3*d)
```

Fricas [B] time = 2.39798, size = 4238, normalized size = 55.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*d*x*cosh(d*x + c)^8 + 16*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^7
+ 2*a^2*d*x*sinh(d*x + c)^8 + 8*((a^2 + 2*a*b)*d*x - a^2 - a*b)*cosh(d*x +
c)^6 + 8*(7*a^2*d*x*cosh(d*x + c)^2 + (a^2 + 2*a*b)*d*x - a^2 - a*b)*sinh(d
*x + c)^6 + 16*(7*a^2*d*x*cosh(d*x + c)^3 + 3*((a^2 + 2*a*b)*d*x - a^2 - a*
b)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*((3*a^2 + 8*a*b + 8*b^2)*d*x - 2*a^2
- 8*a*b - 6*b^2)*cosh(d*x + c)^4 + 4*(35*a^2*d*x*cosh(d*x + c)^4 + (3*a^2 +
 8*a*b + 8*b^2)*d*x + 30*((a^2 + 2*a*b)*d*x - a^2 - a*b)*cosh(d*x + c)^2 -
2*a^2 - 8*a*b - 6*b^2)*sinh(d*x + c)^4 + 2*a^2*d*x + 16*(7*a^2*d*x*cosh(d*x
 + c)^5 + 10*((a^2 + 2*a*b)*d*x - a^2 - a*b)*cosh(d*x + c)^3 + ((3*a^2 + 8*
a*b + 8*b^2)*d*x - 2*a^2 - 8*a*b - 6*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 +
8*((a^2 + 2*a*b)*d*x - a^2 - a*b)*cosh(d*x + c)^2 + 8*(7*a^2*d*x*cosh(d*x +
 c)^6 + 15*((a^2 + 2*a*b)*d*x - a^2 - a*b)*cosh(d*x + c)^4 + (a^2 + 2*a*b)*
d*x + 3*((3*a^2 + 8*a*b + 8*b^2)*d*x - 2*a^2 - 8*a*b - 6*b^2)*cosh(d*x + c)
^2 - a^2 - a*b)*sinh(d*x + c)^2 - (a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c
)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^6 +
 4*(7*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^6 + 8*(7*a^2*cosh(d*
x + c)^3 + 3*(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^2 + 8*a*
b + 8*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*cosh(d*x + c)^4 + 30*(a^2 + 2*a*b)*c
osh(d*x + c)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(d*x + c)^4 + 8*(7*a^2*cosh(d*x
 + c)^5 + 10*(a^2 + 2*a*b)*cosh(d*x + c)^3 + (3*a^2 + 8*a*b + 8*b^2)*cosh(d
*x + c))*sinh(d*x + c)^3 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 4*(7*a^2*cosh(
d*x + c)^6 + 15*(a^2 + 2*a*b)*cosh(d*x + c)^4 + 3*(3*a^2 + 8*a*b + 8*b^2)*c
osh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*(a^2*cosh(d*x + c)^
7 + 3*(a^2 + 2*a*b)*cosh(d*x + c)^5 + (3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)
^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))*log(2*(a*cosh(d*x + c)^2 +
 a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x +
 c) + sinh(d*x + c)^2)) + 16*(a^2*d*x*cosh(d*x + c)^7 + 3*((a^2 + 2*a*b)*d*
x - a^2 - a*b)*cosh(d*x + c)^5 + ((3*a^2 + 8*a*b + 8*b^2)*d*x - 2*a^2 - 8*a
*b - 6*b^2)*cosh(d*x + c)^3 + ((a^2 + 2*a*b)*d*x - a^2 - a*b)*cosh(d*x + c)
)*sinh(d*x + c))/(a^5*d*cosh(d*x + c)^8 + 8*a^5*d*cosh(d*x + c)*sinh(d*x +
c)^7 + a^5*d*sinh(d*x + c)^8 + 4*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^6 + 4*(7*a
^5*d*cosh(d*x + c)^2 + (a^5 + 2*a^4*b)*d)*sinh(d*x + c)^6 + a^5*d + 2*(3*a^
5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c)^4 + 8*(7*a^5*d*cosh(d*x + c)^3 + 3
```

```
*(a^5 + 2*a^4*b)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^5*d*cosh(d*x +
c)^4 + 30*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^2 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)
*d)*sinh(d*x + c)^4 + 4*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^2 + 8*(7*a^5*d*cosh
(d*x + c)^5 + 10*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^3 + (3*a^5 + 8*a^4*b + 8*a
^3*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a^5*d*cosh(d*x + c)^6 + 15*
(a^5 + 2*a^4*b)*d*cosh(d*x + c)^4 + 3*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(
d*x + c)^2 + (a^5 + 2*a^4*b)*d)*sinh(d*x + c)^2 + 8*(a^5*d*cosh(d*x + c)^7
+ 3*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^5 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cos
h(d*x + c)^3 + (a^5 + 2*a^4*b)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**5/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.53196, size = 252, normalized size = 3.27

$$-\frac{\frac{4\,dx}{a^3}-\frac{2\,\log\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}{a^3}+\frac{3\,ae^{(8\,dx+8\,c)}-4\,ae^{(6\,dx+6\,c)}+8\,be^{(6\,dx+6\,c)}+2\,ae^{(4\,dx+4\,c)}-16\,be^{(4\,dx+4\,c)}-4\,ae^{(2\,dx+2\,c)}+8\,be^{(2\,dx+2\,c)}}{\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)^2a^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/4*(4*d*x/a^3 - 2*log(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*
d*x + 2*c) + a)/a^3 + (3*a*e^(8*d*x + 8*c) - 4*a*e^(6*d*x + 6*c) + 8*b*e^(6
*d*x + 6*c) + 2*a*e^(4*d*x + 4*c) - 16*b*e^(4*d*x + 4*c) - 4*a*e^(2*d*x + 2
*c) + 8*b*e^(2*d*x + 2*c) + 3*a)/((a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c)
+ 4*b*e^(2*d*x + 2*c) + a)^2*a^2))/d
```

3.160 $\int \frac{\tanh^4(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=139

$$\frac{\left(a^2-4ab-8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3b^{3/2}d\sqrt{a+b}}+\frac{(a-4b)\tanh(c+dx)}{8a^2bd\left(a-b\tanh^2(c+dx)+b\right)}+\frac{x}{a^3}-\frac{(a+b)\tanh(c+dx)}{4abd\left(a-b\tanh^2(c+dx)+b\right)^2}$$

```
[Out] x/a^3 + ((a^2 - 4*a*b - 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]]
)/(8*a^3*b^(3/2)*Sqrt[a + b]*d) - ((a + b)*Tanh[c + d*x])/(4*a*b*d*(a + b -
 b*Tanh[c + d*x]^2)^2) + ((a - 4*b)*Tanh[c + d*x])/(8*a^2*b*d*(a + b - b*Ta
nh[c + d*x]^2))
```

Rubi [A] time = 0.294107, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4141, 1975, 470, 527, 522, 206, 208}

$$\frac{\left(a^2-4ab-8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3b^{3/2}d\sqrt{a+b}}+\frac{(a-4b)\tanh(c+dx)}{8a^2bd\left(a-b\tanh^2(c+dx)+b\right)}+\frac{x}{a^3}-\frac{(a+b)\tanh(c+dx)}{4abd\left(a-b\tanh^2(c+dx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] x/a^3 + ((a^2 - 4*a*b - 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]]
)/(8*a^3*b^(3/2)*Sqrt[a + b]*d) - ((a + b)*Tanh[c + d*x])/(4*a*b*d*(a + b -
 b*Tanh[c + d*x]^2)^2) + ((a - 4*b)*Tanh[c + d*x])/(8*a^2*b*d*(a + b - b*Ta
nh[c + d*x]^2))
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
```

```
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3}\,dx = \frac{\mathrm{Subst}\left(\int \frac{x^4}{(1-x^2)\left(a+b(1-x^2)\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\mathrm{Subst}\left(\int \frac{x^4}{(1-x^2)\left(a+b-bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{(a+b)\tanh(c+dx)}{4abd\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{\mathrm{Subst}\left(\int \frac{a+b+(-a+3b)x^2}{(1-x^2)\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{4abd}$$

$$= -\frac{(a+b)\tanh(c+dx)}{4abd\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{(a-4b)\tanh(c+dx)}{8a^2bd\left(a+b-b\tanh^2(c+dx)\right)} - \frac{\mathrm{Subst}\left(\int \frac{-(a+}{}\right.}{}$$

$$= -\frac{(a+b)\tanh(c+dx)}{4abd\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{(a-4b)\tanh(c+dx)}{8a^2bd\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\mathrm{Subst}\left(\int \frac{1}{1-x^2}\right.}{}$$

$$= \frac{x}{a^3} + \frac{\left(a^2-4ab-8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3b^{3/2}\sqrt{a+b}\,d} - \frac{(a+b)\tanh(c+dx)}{4abd\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{}{8a^2}$$

Mathematica [B] time = 14.2781, size = 1317, normalized size = 9.47

$$(\cosh(2(c+dx))a+a+2b)^3\text{sech}^6(c+dx)\left(\frac{6a(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}+\frac{4\left(3a^2+8ba+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}-\frac{4a\sqrt{b}\left(3a^2+16ba+3\right.}{(a+}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tanh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^3*Sech[c + d*x]^6*((6*a*(a + 2*b)*ArcTanh[
(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/2) + (4*(3*a^2 + 8*a*b + 8
*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/2) - (4*a*Sq
rt[b]*(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cosh[2*(c + d*x)])*Sinh[2*(c
 + d*x)])/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2) - (2*Sqrt[b]*(3*a^3
 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*b^2)*Cosh[2*(c + d*x
)])*Sinh[2*(c + d*x)])/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2) + (Sqr
t[b]*((-2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3 + 640*a*b^4 + 256*b^
5)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh
[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(Cosh[2*c] - S
inh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (Sech[2*c]*(256*b^
2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*d*x*Cosh[2*c] + 512*a*b^2*(a + b)^2*(a
+ 2*b)*d*x*Cosh[2*d*x] + 128*a^4*b^2*d*x*Cosh[2*(c + 2*d*x)] + 256*a^3*b^3*
d*x*Cosh[2*(c + 2*d*x)] + 128*a^2*b^4*d*x*Cosh[2*(c + 2*d*x)] + 512*a^4*b^2
*d*x*Cosh[4*c + 2*d*x] + 2048*a^3*b^3*d*x*Cosh[4*c + 2*d*x] + 2560*a^2*b^4*
d*x*Cosh[4*c + 2*d*x] + 1024*a*b^5*d*x*Cosh[4*c + 2*d*x] + 128*a^4*b^2*d*x*
Cosh[6*c + 4*d*x] + 256*a^3*b^3*d*x*Cosh[6*c + 4*d*x] + 128*a^2*b^4*d*x*Cos
h[6*c + 4*d*x] - 9*a^6*Sinh[2*c] + 12*a^5*b*Sinh[2*c] + 684*a^4*b^2*Sinh[2*
c] + 2880*a^3*b^3*Sinh[2*c] + 5280*a^2*b^4*Sinh[2*c] + 4608*a*b^5*Sinh[2*c]
 + 1536*b^6*Sinh[2*c] + 9*a^6*Sinh[2*d*x] - 14*a^5*b*Sinh[2*d*x] - 608*a^4*
b^2*Sinh[2*d*x] - 2112*a^3*b^3*Sinh[2*d*x] - 2560*a^2*b^4*Sinh[2*d*x] - 102
4*a*b^5*Sinh[2*d*x] + 3*a^6*Sinh[2*(c + 2*d*x)] - 12*a^5*b*Sinh[2*(c + 2*d*
x)] - 204*a^4*b^2*Sinh[2*(c + 2*d*x)] - 384*a^3*b^3*Sinh[2*(c + 2*d*x)] - 1
92*a^2*b^4*Sinh[2*(c + 2*d*x)] - 3*a^6*Sinh[4*c + 2*d*x] + 10*a^5*b*Sinh[4*
c + 2*d*x] + 304*a^4*b^2*Sinh[4*c + 2*d*x] + 1056*a^3*b^3*Sinh[4*c + 2*d*x]
 + 1280*a^2*b^4*Sinh[4*c + 2*d*x] + 512*a*b^5*Sinh[4*c + 2*d*x]))/(a + 2*b
+ a*Cosh[2*(c + d*x)])^2))/(a^3*(a + b)^2) - (2*Sqrt[b]*((6*a^2*ArcTanh[(Se
ch[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/
(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]))/(Sq
rt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (a*Sech[2*c]*((-9*a^4 - 16*a^3*b
 + 48*a^2*b^2 + 128*a*b^3 + 64*b^4)*Sinh[2*d*x] + a*(-3*a^3 + 2*a^2*b + 24*
a*b^2 + 16*b^3)*Sinh[2*(c + 2*d*x)] + (3*a^4 - 64*a^2*b^2 - 128*a*b^3 - 64*
b^4)*Sinh[4*c + 2*d*x]) + (9*a^5 + 18*a^4*b - 64*a^3*b^2 - 256*a^2*b^3 - 32
0*a*b^4 - 128*b^5)*Tanh[2*c])/(a^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2)))/(a
+ b)^2))/(4096*b^(5/2)*d*(a + b*Sech[c + d*x]^2)^3)
```

Maple [B] time = 0.104, size = 1306, normalized size = 9.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)-1/4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2
*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/b*
tanh(1/2*d*x+1/2*c)^7-5/4/d/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c
)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x
+1/2*c)^7-1/d*b/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh
(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7-
3/4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c
)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/b*tanh(1/2*d*x+1/2*c)^5-19/4/d/a/(ta
nh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*t
anh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^5+1/d*b/a^2/(tanh(1/2*d*x
+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*
x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^5-3/4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*
tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a
+b)^2/b*tanh(1/2*d*x+1/2*c)^3-19/4/d/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*
d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tan
h(1/2*d*x+1/2*c)^3+1/d*b/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)
^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+
1/2*c)^3-1/4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*
d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/b*tanh(1/2*d*x+1/2*c)-5/4/d
/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2
*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)-1/d*b/a^2/(tanh(1/2
*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/
2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)+1/16/d/a/b^(3/2)/(a+b)^(1/2)*ln
((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2
))-1/4/d/a^2/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tan
h(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-1/2/d*b^(1/2)/a^3/(a+b)^(1/2)*ln((a+b
)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-1/
16/d/a/b^(3/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2
*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2))+1/4/d/a^2/b^(1/2)/(a+b)^(1/2)*ln(-(a+b)^(1
/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2))+1/2/d*
b^(1/2)/a^3/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*
x+1/2*c)*b^(1/2)-(a+b)^(1/2))-1/d/a^3*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.12074, size = 14841, normalized size = 106.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*(a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^8 + 128*(a^3*b^2 + a^2*b^3)
*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 16*(a^3*b^2 + a^2*b^3)*d*x*sinh(d*x +
```

c)^8 + 4*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 16*(a^3*b^2 + 3*a^2*
b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^6 + 4*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 +
16*a*b^4 + 112*(a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^2 + 16*(a^3*b^2 + 3*a^
2*b^3 + 2*a*b^4)*d*x)*sinh(d*x + c)^6 + 8*(112*(a^3*b^2 + a^2*b^3)*d*x*cosh
(d*x + c)^3 + 3*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 16*(a^3*b^2 +
3*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*a^4*b + 28*a^
3*b^2 + 24*a^2*b^3 + 4*(3*a^4*b + 29*a^3*b^2 + 82*a^2*b^3 + 104*a*b^4 + 48*
b^5 + 8*(3*a^3*b^2 + 11*a^2*b^3 + 16*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^4 +
4*(280*(a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^4 + 3*a^4*b + 29*a^3*b^2 + 82*
a^2*b^3 + 104*a*b^4 + 48*b^5 + 8*(3*a^3*b^2 + 11*a^2*b^3 + 16*a*b^4 + 8*b^5
)*d*x + 15*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 16*(a^3*b^2 + 3*a^
2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(56*(a^3*b^2 +
a^2*b^3)*d*x*cosh(d*x + c)^5 + 5*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 + 16*a*b^
4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^3 + (3*a^4*b + 29
*a^3*b^2 + 82*a^2*b^3 + 104*a*b^4 + 48*b^5 + 8*(3*a^3*b^2 + 11*a^2*b^3 + 16
*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 16*(a^3*b^2 + a^2*b^3
)*d*x + 4*(3*a^4*b + 23*a^3*b^2 + 52*a^2*b^3 + 32*a*b^4 + 16*(a^3*b^2 + 3*a
^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^2 + 4*(112*(a^3*b^2 + a^2*b^3)*d*x*cos
h(d*x + c)^6 + 3*a^4*b + 23*a^3*b^2 + 52*a^2*b^3 + 32*a*b^4 + 15*(a^4*b + 1
3*a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)
*cosh(d*x + c)^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x + 6*(3*a^4*b + 29
*a^3*b^2 + 82*a^2*b^3 + 104*a*b^4 + 48*b^5 + 8*(3*a^3*b^2 + 11*a^2*b^3 + 16
*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((a^4 - 4*a^3*b - 8
*a^2*b^2)*cosh(d*x + c)^8 + 8*(a^4 - 4*a^3*b - 8*a^2*b^2)*cosh(d*x + c)*sin
h(d*x + c)^7 + (a^4 - 4*a^3*b - 8*a^2*b^2)*sinh(d*x + c)^8 + 4*(a^4 - 2*a^3
*b - 16*a^2*b^2 - 16*a*b^3)*cosh(d*x + c)^6 + 4*(a^4 - 2*a^3*b - 16*a^2*b^2
- 16*a*b^3 + 7*(a^4 - 4*a^3*b - 8*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^
6 + 8*(7*(a^4 - 4*a^3*b - 8*a^2*b^2)*cosh(d*x + c)^3 + 3*(a^4 - 2*a^3*b - 1
6*a^2*b^2 - 16*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^4 - 4*a^3*b -
48*a^2*b^2 - 96*a*b^3 - 64*b^4)*cosh(d*x + c)^4 + 2*(35*(a^4 - 4*a^3*b - 8
*a^2*b^2)*cosh(d*x + c)^4 + 3*a^4 - 4*a^3*b - 48*a^2*b^2 - 96*a*b^3 - 64*b^
4 + 30*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*cosh(d*x + c)^2)*sinh(d*x +
c)^4 + a^4 - 4*a^3*b - 8*a^2*b^2 + 8*(7*(a^4 - 4*a^3*b - 8*a^2*b^2)*cosh(d*
x + c)^5 + 10*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*cosh(d*x + c)^3 + (3*
a^4 - 4*a^3*b - 48*a^2*b^2 - 96*a*b^3 - 64*b^4)*cosh(d*x + c))*sinh(d*x + c
)^3 + 4*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*cosh(d*x + c)^2 + 4*(7*(a^4
- 4*a^3*b - 8*a^2*b^2)*cosh(d*x + c)^6 + 15*(a^4 - 2*a^3*b - 16*a^2*b^2 -
16*a*b^3)*cosh(d*x + c)^4 + a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3 + 3*(3*a^
4 - 4*a^3*b - 48*a^2*b^2 - 96*a*b^3 - 64*b^4)*cosh(d*x + c)^2)*sinh(d*x + c
)^2 + 8*((a^4 - 4*a^3*b - 8*a^2*b^2)*cosh(d*x + c)^7 + 3*(a^4 - 2*a^3*b - 1
6*a^2*b^2 - 16*a*b^3)*cosh(d*x + c)^5 + (3*a^4 - 4*a^3*b - 48*a^2*b^2 - 96*
a*b^3 - 64*b^4)*cosh(d*x + c)^3 + (a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*c
osh(d*x + c))*sinh(d*x + c))*sqrt(a*b + b^2)*log((a^2*cosh(d*x + c)^4 + 4*a
^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*co
sh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a
^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*
sinh(d*x + c) + 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*
sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d
*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 +
2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 +
(a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 8*(16*(a^3*b^2 + a^2*b^3)*d
*x*cosh(d*x + c)^7 + 3*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 16*(a^
3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^5 + 2*(3*a^4*b + 29*a^3*b^2
+ 82*a^2*b^3 + 104*a*b^4 + 48*b^5 + 8*(3*a^3*b^2 + 11*a^2*b^3 + 16*a*b^4 +
8*b^5)*d*x)*cosh(d*x + c)^3 + (3*a^4*b + 23*a^3*b^2 + 52*a^2*b^3 + 32*a*b^
4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(
(a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^8 + 8*(a^6*b^2 + a^5*b^3)*d*cosh(d*x +
c)*sinh(d*x + c)^7 + (a^6*b^2 + a^5*b^3)*d*sinh(d*x + c)^8 + 4*(a^6*b^2 + 3
*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^6*b^2 + a^5*b^3)*d*cosh(d

```
*x + c)^2 + (a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d)*sinh(d*x + c)^6 + 2*(3*a^6
*b^2 + 11*a^5*b^3 + 16*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^6*b
^2 + a^5*b^3)*d*cosh(d*x + c)^3 + 3*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d*cos
h(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^4 +
 30*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^2 + (3*a^6*b^2 + 11*a
^5*b^3 + 16*a^4*b^4 + 8*a^3*b^5)*d)*sinh(d*x + c)^4 + 4*(a^6*b^2 + 3*a^5*b^
3 + 2*a^4*b^4)*d*cosh(d*x + c)^2 + 8*(7*(a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)
^5 + 10*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^3 + (3*a^6*b^2 +
11*a^5*b^3 + 16*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(
7*(a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^6 + 15*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b
^4)*d*cosh(d*x + c)^4 + 3*(3*a^6*b^2 + 11*a^5*b^3 + 16*a^4*b^4 + 8*a^3*b^5)
*d*cosh(d*x + c)^2 + (a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d)*sinh(d*x + c)^2 +
 (a^6*b^2 + a^5*b^3)*d + 8*((a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^7 + 3*(a^6*
b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^5 + (3*a^6*b^2 + 11*a^5*b^3 +
16*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^3 + (a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^
4)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*(8*(a^3*b^2 + a^2*b^3)*d*x*cosh(d*x
 + c)^8 + 64*(a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 8*(a^3
*b^2 + a^2*b^3)*d*x*sinh(d*x + c)^8 + 2*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 +
16*a*b^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^6 + 2*(a^4
*b + 13*a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 112*(a^3*b^2 + a^2*b^3)*d*x*cosh(
d*x + c)^2 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*sinh(d*x + c)^6 + 4*(1
12*(a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^3 + 3*(a^4*b + 13*a^3*b^2 + 28*a^2
*b^3 + 16*a*b^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c))*si
nh(d*x + c)^5 + 2*a^4*b + 14*a^3*b^2 + 12*a^2*b^3 + 2*(3*a^4*b + 29*a^3*b^2
 + 82*a^2*b^3 + 104*a*b^4 + 48*b^5 + 8*(3*a^3*b^2 + 11*a^2*b^3 + 16*a*b^4 +
 8*b^5)*d*x)*cosh(d*x + c)^4 + 2*(280*(a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)
^4 + 3*a^4*b + 29*a^3*b^2 + 82*a^2*b^3 + 104*a*b^4 + 48*b^5 + 8*(3*a^3*b^2
+ 11*a^2*b^3 + 16*a*b^4 + 8*b^5)*d*x + 15*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3
+ 16*a*b^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^2)*sinh(
d*x + c)^4 + 8*(56*(a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^5 + 5*(a^4*b + 13*
a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*c
osh(d*x + c)^3 + (3*a^4*b + 29*a^3*b^2 + 82*a^2*b^3 + 104*a*b^4 + 48*b^5 +
8*(3*a^3*b^2 + 11*a^2*b^3 + 16*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c))*sinh(d*x
+ c)^3 + 8*(a^3*b^2 + a^2*b^3)*d*x + 2*(3*a^4*b + 23*a^3*b^2 + 52*a^2*b^3 +
 32*a*b^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^2 + 2*(11
2*(a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^6 + 3*a^4*b + 23*a^3*b^2 + 52*a^2*b
^3 + 32*a*b^4 + 15*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 16*(a^3*b^
2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2
*a*b^4)*d*x + 6*(3*a^4*b + 29*a^3*b^2 + 82*a^2*b^3 + 104*a*b^4 + 48*b^5 + 8
*(3*a^3*b^2 + 11*a^2*b^3 + 16*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^2)*sinh(d*x
 + c)^2 + ((a^4 - 4*a^3*b - 8*a^2*b^2)*cosh(d*x + c)^8 + 8*(a^4 - 4*a^3*b -
 8*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^4 - 4*a^3*b - 8*a^2*b^2)*sin
h(d*x + c)^8 + 4*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*cosh(d*x + c)^6 +
4*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3 + 7*(a^4 - 4*a^3*b - 8*a^2*b^2)*co
sh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^4 - 4*a^3*b - 8*a^2*b^2)*cosh(d*x
+ c)^3 + 3*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*cosh(d*x + c))*sinh(d*x
+ c)^5 + 2*(3*a^4 - 4*a^3*b - 48*a^2*b^2 - 96*a*b^3 - 64*b^4)*cosh(d*x + c)
^4 + 2*(35*(a^4 - 4*a^3*b - 8*a^2*b^2)*cosh(d*x + c)^4 + 3*a^4 - 4*a^3*b -
48*a^2*b^2 - 96*a*b^3 - 64*b^4 + 30*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^4 + a^4 - 4*a^3*b - 8*a^2*b^2 + 8*(7*(a^4 -
 4*a^3*b - 8*a^2*b^2)*cosh(d*x + c)^5 + 10*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16
*a*b^3)*cosh(d*x + c)^3 + (3*a^4 - 4*a^3*b - 48*a^2*b^2 - 96*a*b^3 - 64*b^4
)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3
)*cosh(d*x + c)^2 + 4*(7*(a^4 - 4*a^3*b - 8*a^2*b^2)*cosh(d*x + c)^6 + 15*(
a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*cosh(d*x + c)^4 + a^4 - 2*a^3*b - 16
*a^2*b^2 - 16*a*b^3 + 3*(3*a^4 - 4*a^3*b - 48*a^2*b^2 - 96*a*b^3 - 64*b^4)*
cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^4 - 4*a^3*b - 8*a^2*b^2)*cosh(d*x
+ c)^7 + 3*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*cosh(d*x + c)^5 + (3*a^4
 - 4*a^3*b - 48*a^2*b^2 - 96*a*b^3 - 64*b^4)*cosh(d*x + c)^3 + (a^4 - 2*a^3
```

```
*b - 16*a^2*b^2 - 16*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b - b^2)*
arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*
x + c)^2 + a + 2*b)*sqrt(-a*b - b^2)/(a*b + b^2)) + 4*(16*(a^3*b^2 + a^2*b^
3)*d*x*cosh(d*x + c)^7 + 3*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 16
*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^5 + 2*(3*a^4*b + 29*a^3
*b^2 + 82*a^2*b^3 + 104*a*b^4 + 48*b^5 + 8*(3*a^3*b^2 + 11*a^2*b^3 + 16*a*b
^4 + 8*b^5)*d*x)*cosh(d*x + c)^3 + (3*a^4*b + 23*a^3*b^2 + 52*a^2*b^3 + 32*
a*b^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c
))/((a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^8 + 8*(a^6*b^2 + a^5*b^3)*d*cosh(d*
x + c)*sinh(d*x + c)^7 + (a^6*b^2 + a^5*b^3)*d*sinh(d*x + c)^8 + 4*(a^6*b^2
 + 3*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^6*b^2 + a^5*b^3)*d*co
sh(d*x + c)^2 + (a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d)*sinh(d*x + c)^6 + 2*(3
*a^6*b^2 + 11*a^5*b^3 + 16*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a
^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^3 + 3*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d
*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)
^4 + 30*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^2 + (3*a^6*b^2 +
11*a^5*b^3 + 16*a^4*b^4 + 8*a^3*b^5)*d)*sinh(d*x + c)^4 + 4*(a^6*b^2 + 3*a^
5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^2 + 8*(7*(a^6*b^2 + a^5*b^3)*d*cosh(d*x
+ c)^5 + 10*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^3 + (3*a^6*b^
2 + 11*a^5*b^3 + 16*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 +
 4*(7*(a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^6 + 15*(a^6*b^2 + 3*a^5*b^3 + 2*a
^4*b^4)*d*cosh(d*x + c)^4 + 3*(3*a^6*b^2 + 11*a^5*b^3 + 16*a^4*b^4 + 8*a^3*
b^5)*d*cosh(d*x + c)^2 + (a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d)*sinh(d*x + c)
^2 + (a^6*b^2 + a^5*b^3)*d + 8*((a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^7 + 3*(
a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^5 + (3*a^6*b^2 + 11*a^5*b^
3 + 16*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^3 + (a^6*b^2 + 3*a^5*b^3 + 2*a^
4*b^4)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**4/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.18911, size = 398, normalized size = 2.86

$$\frac{\frac{8\,dx}{a^3} + \frac{\left(a^2 e^{(2\,c)} - 4\,abe^{(2\,c)} - 8\,b^2 e^{(2\,c)}\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)e^{(-2\,c)}}{\sqrt{-ab-b^2}a^3 b} + \frac{2\left(a^3 e^{(6\,dx+6\,c)}+12\,a^2 be^{(6\,dx+6\,c)}+16\,ab^2 e^{(6\,dx+6\,c)}+3\,a^3 e^{(4\,dx+4\,c)}+26\,a^2 be^{(4\,dx+4\,c)}+56\,ab^2 \cdots\right.}{\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c} \cdots\right.}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(8*d*x/a^3 + (a^2*e^(2*c) - 4*a*b*e^(2*c) - 8*b^2*e^(2*c))*arctan(1/2*(
a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))*e^(-2*c)/(sqrt(-a*b - b^2)*a
^3*b) + 2*(a^3*e^(6*d*x + 6*c) + 12*a^2*b*e^(6*d*x + 6*c) + 16*a*b^2*e^(6*d
*x + 6*c) + 3*a^3*e^(4*d*x + 4*c) + 26*a^2*b*e^(4*d*x + 4*c) + 56*a*b^2*e^(
4*d*x + 4*c) + 48*b^3*e^(4*d*x + 4*c) + 3*a^3*e^(2*d*x + 2*c) + 20*a^2*b*e^
(2*d*x + 2*c) + 32*a*b^2*e^(2*d*x + 2*c) + a^3 + 6*a^2*b)/((a*e^(4*d*x + 4*
```

```
c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)^2*a^3*b))/d
```

3.161 $\int \frac{\tanh^3(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=81

$$-\frac{b(a+b)}{4a^3d\left(a\cosh^2(c+dx)+b\right)^2}+\frac{a+2b}{2a^3d\left(a\cosh^2(c+dx)+b\right)}+\frac{\log\left(a\cosh^2(c+dx)+b\right)}{2a^3d}$$

```
[Out] -(b*(a + b))/(4*a^3*d*(b + a*Cosh[c + d*x]^2)^2) + (a + 2*b)/(2*a^3*d*(b +
a*Cosh[c + d*x]^2)) + Log[b + a*Cosh[c + d*x]^2]/(2*a^3*d)
```

Rubi [A] time = 0.126136, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4138, 446, 77}

$$-\frac{b(a+b)}{4a^3d\left(a\cosh^2(c+dx)+b\right)^2}+\frac{a+2b}{2a^3d\left(a\cosh^2(c+dx)+b\right)}+\frac{\log\left(a\cosh^2(c+dx)+b\right)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -(b*(a + b))/(4*a^3*d*(b + a*Cosh[c + d*x]^2)^2) + (a + 2*b)/(2*a^3*d*(b +
a*Cosh[c + d*x]^2)) + Log[b + a*Cosh[c + d*x]^2]/(2*a^3*d)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^3(1-x^2)}{\left(b+ax^2\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{(1-x)x}{(b+ax)^3}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{b(a+b)}{a^2(b+ax)^3} + \frac{a+2b}{a^2(b+ax)^2} - \frac{1}{a^2(b+ax)}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$

$$= -\frac{b(a+b)}{4a^3d\left(b+a\cosh^2(c+dx)\right)^2} + \frac{a+2b}{2a^3d\left(b+a\cosh^2(c+dx)\right)} + \frac{\log\left(b+a\cosh^2(c+dx)\right)}{2a^3d}$$

Mathematica [A] time = 1.39039, size = 131, normalized size = 1.62

$$\frac{2\left(a^2+3ab+3b^2\right)+a^2\cosh^2(2(c+dx))\log(a\cosh(2(c+dx))+a+2b)+(a+2b)^2\log(a\cosh(2(c+dx))+a+2b)+\ldots}{2a^3d(a\cosh(2(c+dx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (2*(a^2 + 3*a*b + 3*b^2) + (a + 2*b)^2*Log[a + 2*b + a*Cosh[2*(c + d*x)]] +
 a^2*Cosh[2*(c + d*x)]^2*Log[a + 2*b + a*Cosh[2*(c + d*x)]] + 2*a*(a + 2*b)
*Cosh[2*(c + d*x)]*(1 + Log[a + 2*b + a*Cosh[2*(c + d*x)]]))/(2*a^3*d*(a +
2*b + a*Cosh[2*(c + d*x)])^2)
```

Maple [B] time = 0.093, size = 672, normalized size = 8.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -1/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)-2/d/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/
2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*t
anh(1/2*d*x+1/2*c)^6-2/d*b/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*
c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*
x+1/2*c)^6-4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*
d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^4
-4/d*b/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/
2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^4+4/d*b
^2/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*
c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^4-2/d/a/(
tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2
*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^2-2/d*b/a^2/(tanh(1/2*d
*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*
d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^2+1/2/d/a^3*ln(tanh(1/2*d*x+1/2*c
)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*
c)^2*b+a+b)-1/d/a^3*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.29911, size = 282, normalized size = 3.48

$$\frac{2\left(\left(a^2+2\,ab\right)e^{(-2\,dx-2\,c)}+2\left(a^2+3\,ab+3\,b^2\right)e^{(-4\,dx-4\,c)}+\left(a^2+2\,ab\right)e^{(-6\,dx-6\,c)}\right)}{\left(a^5e^{(-8\,dx-8\,c)}+a^5+4\left(a^5+2\,a^4b\right)e^{(-2\,dx-2\,c)}+2\left(3\,a^5+8\,a^4b+8\,a^3b^2\right)e^{(-4\,dx-4\,c)}+4\left(a^5+2\,a^4b\right)e^{(-6\,dx-6\,c)}\right)d}+\frac{dx+c}{a^3d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 2*((a^2 + 2*a*b)*e^(-2*d*x - 2*c) + 2*(a^2 + 3*a*b + 3*b^2)*e^(-4*d*x - 4*c
) + (a^2 + 2*a*b)*e^(-6*d*x - 6*c))/((a^5*e^(-8*d*x - 8*c) + a^5 + 4*(a^5 +
 2*a^4*b)*e^(-2*d*x - 2*c) + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*e^(-4*d*x - 4*
c) + 4*(a^5 + 2*a^4*b)*e^(-6*d*x - 6*c))*d) + (d*x + c)/(a^3*d) + 1/2*log(2
*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^3*d)
```

Fricas [B] time = 2.4993, size = 4297, normalized size = 53.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*d*x*cosh(d*x + c)^8 + 16*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^7
+ 2*a^2*d*x*sinh(d*x + c)^8 + 4*(2*(a^2 + 2*a*b)*d*x - a^2 - 2*a*b)*cosh(d*
x + c)^6 + 4*(14*a^2*d*x*cosh(d*x + c)^2 + 2*(a^2 + 2*a*b)*d*x - a^2 - 2*a*
b)*sinh(d*x + c)^6 + 8*(14*a^2*d*x*cosh(d*x + c)^3 + 3*(2*(a^2 + 2*a*b)*d*x
 - a^2 - 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*((3*a^2 + 8*a*b + 8*b^2)
*d*x - 2*a^2 - 6*a*b - 6*b^2)*cosh(d*x + c)^4 + 4*(35*a^2*d*x*cosh(d*x + c)
^4 + (3*a^2 + 8*a*b + 8*b^2)*d*x + 15*(2*(a^2 + 2*a*b)*d*x - a^2 - 2*a*b)*c
osh(d*x + c)^2 - 2*a^2 - 6*a*b - 6*b^2)*sinh(d*x + c)^4 + 2*a^2*d*x + 16*(7
*a^2*d*x*cosh(d*x + c)^5 + 5*(2*(a^2 + 2*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x +
 c)^3 + ((3*a^2 + 8*a*b + 8*b^2)*d*x - 2*a^2 - 6*a*b - 6*b^2)*cosh(d*x + c)
)*sinh(d*x + c)^3 + 4*(2*(a^2 + 2*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^2 +
 4*(14*a^2*d*x*cosh(d*x + c)^6 + 15*(2*(a^2 + 2*a*b)*d*x - a^2 - 2*a*b)*cos
h(d*x + c)^4 + 2*(a^2 + 2*a*b)*d*x + 6*((3*a^2 + 8*a*b + 8*b^2)*d*x - 2*a^2
 - 6*a*b - 6*b^2)*cosh(d*x + c)^2 - a^2 - 2*a*b)*sinh(d*x + c)^2 - (a^2*cos
h(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 +
4*(a^2 + 2*a*b)*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*s
inh(d*x + c)^6 + 8*(7*a^2*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b)*cosh(d*x + c))*
sinh(d*x + c)^5 + 2*(3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*cos
h(d*x + c)^4 + 30*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 3*a^2 + 8*a*b + 8*b^2)*si
nh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^5 + 10*(a^2 + 2*a*b)*cosh(d*x + c)^3
 + (3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^2 + 2*a*b)
*cosh(d*x + c)^2 + 4*(7*a^2*cosh(d*x + c)^6 + 15*(a^2 + 2*a*b)*cosh(d*x + c
)^4 + 3*(3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c
)^2 + a^2 + 8*(a^2*cosh(d*x + c)^7 + 3*(a^2 + 2*a*b)*cosh(d*x + c)^5 + (3*a
^2 + 8*a*b + 8*b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x
 + c))*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x +
c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 8*(2*a^2*d*x*cos
h(d*x + c)^7 + 3*(2*(a^2 + 2*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^5 + 2*((
3*a^2 + 8*a*b + 8*b^2)*d*x - 2*a^2 - 6*a*b - 6*b^2)*cosh(d*x + c)^3 + (2*(a
^2 + 2*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c))*sinh(d*x + c))/(a^5*d*cosh(d*
x + c)^8 + 8*a^5*d*cosh(d*x + c)*sinh(d*x + c)^7 + a^5*d*sinh(d*x + c)^8 +
4*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^6 + 4*(7*a^5*d*cosh(d*x + c)^2 + (a^5 + 2
*a^4*b)*d)*sinh(d*x + c)^6 + a^5*d + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh
```

```
(d*x + c)^4 + 8*(7*a^5*d*cosh(d*x + c)^3 + 3*(a^5 + 2*a^4*b)*d*cosh(d*x + c
))*sinh(d*x + c)^5 + 2*(35*a^5*d*cosh(d*x + c)^4 + 30*(a^5 + 2*a^4*b)*d*cos
h(d*x + c)^2 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*d)*sinh(d*x + c)^4 + 4*(a^5 +
2*a^4*b)*d*cosh(d*x + c)^2 + 8*(7*a^5*d*cosh(d*x + c)^5 + 10*(a^5 + 2*a^4*b
)*d*cosh(d*x + c)^3 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c))*sinh(d
*x + c)^3 + 4*(7*a^5*d*cosh(d*x + c)^6 + 15*(a^5 + 2*a^4*b)*d*cosh(d*x + c)
^4 + 3*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 + 2*a^4*b)*d)
*sinh(d*x + c)^2 + 8*(a^5*d*cosh(d*x + c)^7 + 3*(a^5 + 2*a^4*b)*d*cosh(d*x
+ c)^5 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c)^3 + (a^5 + 2*a^4*b)*
d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**3/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.75303, size = 236, normalized size = 2.91

$$-\frac{\frac{4\,dx}{a^3} - \frac{2\,\log\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}{a^3} + \frac{3\,ae^{(8\,dx+8\,c)}+4\,ae^{(6\,dx+6\,c)}+8\,be^{(6\,dx+6\,c)}+2\,ae^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}+8\,be^{(2\,dx+2\,c)}+3\,a}{\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)^2 a^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/4*(4*d*x/a^3 - 2*log(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*
d*x + 2*c) + a)/a^3 + (3*a*e^(8*d*x + 8*c) + 4*a*e^(6*d*x + 6*c) + 8*b*e^(6
*d*x + 6*c) + 2*a*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) + 8*b*e^(2*d*x + 2*
c) + 3*a)/((a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) +
 a)^2*a^2))/d
```

3.162 $\int \frac{\tanh^2(c+dx)}{\left(a+b\mathbf{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=139

$$-\frac{\left(3a^2+12ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3\sqrt{b}d(a+b)^{3/2}}-\frac{(3a+4b)\tanh(c+dx)}{8a^2d(a+b)\left(a-b\tanh^2(c+dx)+b\right)}+\frac{x}{a^3}-\frac{\tanh(c+dx)}{4ad\left(a-b\tanh^2(c+dx)+b\right.}$$

```
[Out] x/a^3 - ((3*a^2 + 12*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a +
b]])/(8*a^3*Sqrt[b]*(a + b)^(3/2)*d) - Tanh[c + d*x]/(4*a*d*(a + b - b*Tanh
[c + d*x]^2)^2) - ((3*a + 4*b)*Tanh[c + d*x])/(8*a^2*(a + b)*d*(a + b - b*T
anh[c + d*x]^2))
```

Rubi [A] time = 0.246268, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4141, 1975, 471, 527, 522, 206, 208}

$$-\frac{\left(3a^2+12ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3\sqrt{b}d(a+b)^{3/2}}-\frac{(3a+4b)\tanh(c+dx)}{8a^2d(a+b)\left(a-b\tanh^2(c+dx)+b\right)}+\frac{x}{a^3}-\frac{\tanh(c+dx)}{4ad\left(a-b\tanh^2(c+dx)+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```


```
[Out] x/a^3 - ((3*a^2 + 12*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a +
b]])/(8*a^3*Sqrt[b]*(a + b)^(3/2)*d) - Tanh[c + d*x]/(4*a*d*(a + b - b*Tanh
[c + d*x]^2)^2) - ((3*a + 4*b)*Tanh[c + d*x])/(8*a^2*(a + b)*d*(a + b - b*T
anh[c + d*x]^2))
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)\left(a+b(1-x^2)\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)\left(a+b-bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{\tanh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{\operatorname{Subst}\left(\int \frac{1+3x^2}{(1-x^2)\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{4ad}$$

$$= -\frac{\tanh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{(3a+4b)\tanh(c+dx)}{8a^2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} - \frac{\operatorname{Subst}\left(\int\right.}{}$$

$$= -\frac{\tanh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{(3a+4b)\tanh(c+dx)}{8a^2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int\right.}{}$$

$$= \frac{x}{a^3} - \frac{\left(3a^2+12ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3\sqrt{b}(a+b)^{3/2}d} - \frac{\tanh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{}{8a}$$

Mathematica [B] time = 14.5523, size = 1317, normalized size = 9.47

$$(\cosh(2(c+dx))a+a+2b)^3\operatorname{sech}^6(c+dx)\left(\frac{6a(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}-\frac{4(3a^2+8ba+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}+\frac{4a\sqrt{b}(3a^2+16ba+3\cdots}{(a+\cdots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tanh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^3*Sech[c + d*x]^6*((6*a*(a + 2*b)*ArcTanh[
(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/2) - (4*(3*a^2 + 8*a*b + 8
*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/2) + (4*a*Sq
rt[b]*(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cosh[2*(c + d*x)])*Sinh[2*(c
 + d*x)])/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2) - (2*Sqrt[b]*(3*a^3
 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*b^2)*Cosh[2*(c + d*x
)])*Sinh[2*(c + d*x)])/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2) + (Sqr
t[b]*((-2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3 + 640*a*b^4 + 256*b^
5)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh
[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(Cosh[2*c] - S
inh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (Sech[2*c]*(256*b^
2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*d*x*Cosh[2*c] + 512*a*b^2*(a + b)^2*(a
+ 2*b)*d*x*Cosh[2*d*x] + 128*a^4*b^2*d*x*Cosh[2*(c + 2*d*x)] + 256*a^3*b^3*
d*x*Cosh[2*(c + 2*d*x)] + 128*a^2*b^4*d*x*Cosh[2*(c + 2*d*x)] + 512*a^4*b^2
*d*x*Cosh[4*c + 2*d*x] + 2048*a^3*b^3*d*x*Cosh[4*c + 2*d*x] + 2560*a^2*b^4*
d*x*Cosh[4*c + 2*d*x] + 1024*a*b^5*d*x*Cosh[4*c + 2*d*x] + 128*a^4*b^2*d*x*
Cosh[6*c + 4*d*x] + 256*a^3*b^3*d*x*Cosh[6*c + 4*d*x] + 128*a^2*b^4*d*x*Cos
h[6*c + 4*d*x] - 9*a^6*Sinh[2*c] + 12*a^5*b*Sinh[2*c] + 684*a^4*b^2*Sinh[2*
c] + 2880*a^3*b^3*Sinh[2*c] + 5280*a^2*b^4*Sinh[2*c] + 4608*a*b^5*Sinh[2*c]
 + 1536*b^6*Sinh[2*c] + 9*a^6*Sinh[2*d*x] - 14*a^5*b*Sinh[2*d*x] - 608*a^4*
b^2*Sinh[2*d*x] - 2112*a^3*b^3*Sinh[2*d*x] - 2560*a^2*b^4*Sinh[2*d*x] - 102
4*a*b^5*Sinh[2*d*x] + 3*a^6*Sinh[2*(c + 2*d*x)] - 12*a^5*b*Sinh[2*(c + 2*d*
x)] - 204*a^4*b^2*Sinh[2*(c + 2*d*x)] - 384*a^3*b^3*Sinh[2*(c + 2*d*x)] - 1
92*a^2*b^4*Sinh[2*(c + 2*d*x)] - 3*a^6*Sinh[4*c + 2*d*x] + 10*a^5*b*Sinh[4*
c + 2*d*x] + 304*a^4*b^2*Sinh[4*c + 2*d*x] + 1056*a^3*b^3*Sinh[4*c + 2*d*x]
 + 1280*a^2*b^4*Sinh[4*c + 2*d*x] + 512*a*b^5*Sinh[4*c + 2*d*x]))/(a + 2*b
+ a*Cosh[2*(c + d*x)])^2))/(a^3*(a + b)^2) + (2*Sqrt[b]*((6*a^2*ArcTanh[(Se
ch[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/
(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]))/(Sq
rt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (a*Sech[2*c]*((-9*a^4 - 16*a^3*b
 + 48*a^2*b^2 + 128*a*b^3 + 64*b^4)*Sinh[2*d*x] + a*(-3*a^3 + 2*a^2*b + 24*
a*b^2 + 16*b^3)*Sinh[2*(c + 2*d*x)] + (3*a^4 - 64*a^2*b^2 - 128*a*b^3 - 64*
b^4)*Sinh[4*c + 2*d*x]) + (9*a^5 + 18*a^4*b - 64*a^3*b^2 - 256*a^2*b^3 - 32
0*a*b^4 - 128*b^5)*Tanh[2*c])/(a^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2)))/(a
+ b)^2))/(4096*b^(5/2)*d*(a + b*Sech[c + d*x]^2)^3)
```

Maple [B] time = 0.104, size = 1173, normalized size = 8.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)-5/4/d/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1
/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*
tanh(1/2*d*x+1/2*c)^7-1/d*b/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2
*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d
*x+1/2*c)^7-15/4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(
1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*
c)^5-15/4/d*b/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2
*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^
5+1/d*b^2/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d
*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^5-
15/4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*
c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^3-15/4/d*
b/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^
2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^3+1/d*b^2/a^
2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*
a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^3-5/4/d/a/(tan
h(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*ta
nh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)-1/d*b/a^2/(tanh(1/2*d*x+1/
2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1
/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)-3/16/d/a/(a+b)^(3/2)/b^(1/2)*ln((a+b)^
(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-3/4/
d*b^(1/2)/a^2/(a+b)^(3/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d
*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-1/2/d*b^(3/2)/a^3/(a+b)^(3/2)*ln((a+b)^(1/2)
*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+3/16/d/a/
(a+b)^(3/2)/b^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2
*c)*b^(1/2)+(a+b)^(1/2))+3/4/d*b^(1/2)/a^2/(a+b)^(3/2)*ln((a+b)^(1/2)*tanh(
1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/2/d*b^(3/2)/a
^3/(a+b)^(3/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b
^(1/2)+(a+b)^(1/2))-1/d/a^3*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.31096, size = 16496, normalized size = 118.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^8 + 128*(a^4*b +
2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 16*(a^4*b + 2*a^3*
b^2 + a^2*b^3)*d*x*sinh(d*x + c)^8 + 4*(5*a^4*b + 25*a^3*b^2 + 36*a^2*b^3 +
 16*a*b^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)
```

```
^6 + 4*(5*a^4*b + 25*a^3*b^2 + 36*a^2*b^3 + 16*a*b^4 + 112*(a^4*b + 2*a^3*b
^2 + a^2*b^3)*d*x*cosh(d*x + c)^2 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a
*b^4)*d*x)*sinh(d*x + c)^6 + 8*(112*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(
d*x + c)^3 + 3*(5*a^4*b + 25*a^3*b^2 + 36*a^2*b^3 + 16*a*b^4 + 16*(a^4*b +
4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 20*a
^4*b + 44*a^3*b^2 + 24*a^2*b^3 + 4*(15*a^4*b + 73*a^3*b^2 + 146*a^2*b^3 + 1
36*a*b^4 + 48*b^5 + 8*(3*a^4*b + 14*a^3*b^2 + 27*a^2*b^3 + 24*a*b^4 + 8*b^5
)*d*x)*cosh(d*x + c)^4 + 4*(280*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x
+ c)^4 + 15*a^4*b + 73*a^3*b^2 + 146*a^2*b^3 + 136*a*b^4 + 48*b^5 + 8*(3*a^
4*b + 14*a^3*b^2 + 27*a^2*b^3 + 24*a*b^4 + 8*b^5)*d*x + 15*(5*a^4*b + 25*a^
3*b^2 + 36*a^2*b^3 + 16*a*b^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4
)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(56*(a^4*b + 2*a^3*b^2 + a^2*b
^3)*d*x*cosh(d*x + c)^5 + 5*(5*a^4*b + 25*a^3*b^2 + 36*a^2*b^3 + 16*a*b^4 +
 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^3 + (15*a^
4*b + 73*a^3*b^2 + 146*a^2*b^3 + 136*a*b^4 + 48*b^5 + 8*(3*a^4*b + 14*a^3*b
^2 + 27*a^2*b^3 + 24*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 1
6*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x + 4*(15*a^4*b + 59*a^3*b^2 + 76*a^2*b^3
 + 32*a*b^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x +
c)^2 + 4*(112*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^6 + 15*a^4*b
+ 59*a^3*b^2 + 76*a^2*b^3 + 32*a*b^4 + 15*(5*a^4*b + 25*a^3*b^2 + 36*a^2*b^
3 + 16*a*b^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x +
 c)^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x + 6*(15*a^4*b + 73
*a^3*b^2 + 146*a^2*b^3 + 136*a*b^4 + 48*b^5 + 8*(3*a^4*b + 14*a^3*b^2 + 27*
a^2*b^3 + 24*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((3*a^4
 + 12*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^8 + 8*(3*a^4 + 12*a^3*b + 8*a^2*b^2)
*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^4 + 12*a^3*b + 8*a^2*b^2)*sinh(d*x +
c)^8 + 4*(3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^6 + 4*(3*
a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3 + 7*(3*a^4 + 12*a^3*b + 8*a^2*b^2)*c
osh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^4 + 12*a^3*b + 8*a^2*b^2)*cosh(
d*x + c)^3 + 3*(3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c))*si
nh(d*x + c)^5 + 2*(9*a^4 + 60*a^3*b + 144*a^2*b^2 + 160*a*b^3 + 64*b^4)*cos
h(d*x + c)^4 + 2*(35*(3*a^4 + 12*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + 9*a^4
 + 60*a^3*b + 144*a^2*b^2 + 160*a*b^3 + 64*b^4 + 30*(3*a^4 + 18*a^3*b + 32*
a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 3*a^4 + 12*a^3*b + 8
*a^2*b^2 + 8*(7*(3*a^4 + 12*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^5 + 10*(3*a^4
+ 18*a^3*b + 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^3 + (9*a^4 + 60*a^3*b + 1
44*a^2*b^2 + 160*a*b^3 + 64*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a^4
+ 18*a^3*b + 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^2 + 4*(7*(3*a^4 + 12*a^3*
b + 8*a^2*b^2)*cosh(d*x + c)^6 + 15*(3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b
^3)*cosh(d*x + c)^4 + 3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3 + 3*(9*a^4 +
 60*a^3*b + 144*a^2*b^2 + 160*a*b^3 + 64*b^4)*cosh(d*x + c)^2)*sinh(d*x + c
)^2 + 8*((3*a^4 + 12*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^7 + 3*(3*a^4 + 18*a^3
*b + 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^5 + (9*a^4 + 60*a^3*b + 144*a^2*b
^2 + 160*a*b^3 + 64*b^4)*cosh(d*x + c)^3 + (3*a^4 + 18*a^3*b + 32*a^2*b^2 +
 16*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b + b^2)*log((a^2*cosh(d*x
+ c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2
 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*
x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cos
h(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d
*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4
+ 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(
d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(
d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 8*(16*(a^4*b +
2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^7 + 3*(5*a^4*b + 25*a^3*b^2 + 36*a^2
*b^3 + 16*a*b^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*
x + c)^5 + 2*(15*a^4*b + 73*a^3*b^2 + 146*a^2*b^3 + 136*a*b^4 + 48*b^5 + 8*
(3*a^4*b + 14*a^3*b^2 + 27*a^2*b^3 + 24*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^3
 + (15*a^4*b + 59*a^3*b^2 + 76*a^2*b^3 + 32*a*b^4 + 16*(a^4*b + 4*a^3*b^2 +
 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^7*b + 2*a^6*b^
```

```
2 + a^5*b^3)*d*cosh(d*x + c)^8 + 8*(a^7*b + 2*a^6*b^2 + a^5*b^3)*d*cosh(d*x
 + c)*sinh(d*x + c)^7 + (a^7*b + 2*a^6*b^2 + a^5*b^3)*d*sinh(d*x + c)^8 + 4
*(a^7*b + 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^7*
b + 2*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^2 + (a^7*b + 4*a^6*b^2 + 5*a^5*b^3
 + 2*a^4*b^4)*d)*sinh(d*x + c)^6 + 2*(3*a^7*b + 14*a^6*b^2 + 27*a^5*b^3 + 2
4*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^7*b + 2*a^6*b^2 + a^5*b^
3)*d*cosh(d*x + c)^3 + 3*(a^7*b + 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*d*cosh
(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^7*b + 2*a^6*b^2 + a^5*b^3)*d*cosh(d*x
 + c)^4 + 30*(a^7*b + 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^2
+ (3*a^7*b + 14*a^6*b^2 + 27*a^5*b^3 + 24*a^4*b^4 + 8*a^3*b^5)*d)*sinh(d*x
+ c)^4 + 4*(a^7*b + 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^2 +
8*(7*(a^7*b + 2*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^5 + 10*(a^7*b + 4*a^6*b^
2 + 5*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^3 + (3*a^7*b + 14*a^6*b^2 + 27*a
^5*b^3 + 24*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a
^7*b + 2*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^6 + 15*(a^7*b + 4*a^6*b^2 + 5*a
^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^4 + 3*(3*a^7*b + 14*a^6*b^2 + 27*a^5*b^
3 + 24*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^2 + (a^7*b + 4*a^6*b^2 + 5*a^5*
b^3 + 2*a^4*b^4)*d)*sinh(d*x + c)^2 + (a^7*b + 2*a^6*b^2 + a^5*b^3)*d + 8*(
(a^7*b + 2*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^7 + 3*(a^7*b + 4*a^6*b^2 + 5*
a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^5 + (3*a^7*b + 14*a^6*b^2 + 27*a^5*b^3
 + 24*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^3 + (a^7*b + 4*a^6*b^2 + 5*a^5*b
^3 + 2*a^4*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*(8*(a^4*b + 2*a^3*b^2
+ a^2*b^3)*d*x*cosh(d*x + c)^8 + 64*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(
d*x + c)*sinh(d*x + c)^7 + 8*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*sinh(d*x + c
)^8 + 2*(5*a^4*b + 25*a^3*b^2 + 36*a^2*b^3 + 16*a*b^4 + 16*(a^4*b + 4*a^3*b
^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^6 + 2*(5*a^4*b + 25*a^3*b^2 +
36*a^2*b^3 + 16*a*b^4 + 112*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)
^2 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*sinh(d*x + c)^6 + 4*
(112*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^3 + 3*(5*a^4*b + 25*a^
3*b^2 + 36*a^2*b^3 + 16*a*b^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4
)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 10*a^4*b + 22*a^3*b^2 + 12*a^2*b^3
+ 2*(15*a^4*b + 73*a^3*b^2 + 146*a^2*b^3 + 136*a*b^4 + 48*b^5 + 8*(3*a^4*b
+ 14*a^3*b^2 + 27*a^2*b^3 + 24*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^4 + 2*(280
*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^4 + 15*a^4*b + 73*a^3*b^2
+ 146*a^2*b^3 + 136*a*b^4 + 48*b^5 + 8*(3*a^4*b + 14*a^3*b^2 + 27*a^2*b^3 +
 24*a*b^4 + 8*b^5)*d*x + 15*(5*a^4*b + 25*a^3*b^2 + 36*a^2*b^3 + 16*a*b^4 +
 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*
x + c)^4 + 8*(56*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^5 + 5*(5*a
^4*b + 25*a^3*b^2 + 36*a^2*b^3 + 16*a*b^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b
^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^3 + (15*a^4*b + 73*a^3*b^2 + 146*a^2*b^3 +
 136*a*b^4 + 48*b^5 + 8*(3*a^4*b + 14*a^3*b^2 + 27*a^2*b^3 + 24*a*b^4 + 8*b
^5)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 8*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d
*x + 2*(15*a^4*b + 59*a^3*b^2 + 76*a^2*b^3 + 32*a*b^4 + 16*(a^4*b + 4*a^3*b
^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^2 + 2*(112*(a^4*b + 2*a^3*b^2
+ a^2*b^3)*d*x*cosh(d*x + c)^6 + 15*a^4*b + 59*a^3*b^2 + 76*a^2*b^3 + 32*a*
b^4 + 15*(5*a^4*b + 25*a^3*b^2 + 36*a^2*b^3 + 16*a*b^4 + 16*(a^4*b + 4*a^3*
b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^4 + 16*(a^4*b + 4*a^3*b^2 + 5
*a^2*b^3 + 2*a*b^4)*d*x + 6*(15*a^4*b + 73*a^3*b^2 + 146*a^2*b^3 + 136*a*b^
4 + 48*b^5 + 8*(3*a^4*b + 14*a^3*b^2 + 27*a^2*b^3 + 24*a*b^4 + 8*b^5)*d*x)*
cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((3*a^4 + 12*a^3*b + 8*a^2*b^2)*cosh(d*x
 + c)^8 + 8*(3*a^4 + 12*a^3*b + 8*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 +
(3*a^4 + 12*a^3*b + 8*a^2*b^2)*sinh(d*x + c)^8 + 4*(3*a^4 + 18*a^3*b + 32*a
^2*b^2 + 16*a*b^3)*cosh(d*x + c)^6 + 4*(3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*
a*b^3 + 7*(3*a^4 + 12*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 +
 8*(7*(3*a^4 + 12*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 + 3*(3*a^4 + 18*a^3*b
+ 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(9*a^4 + 60*a^3
*b + 144*a^2*b^2 + 160*a*b^3 + 64*b^4)*cosh(d*x + c)^4 + 2*(35*(3*a^4 + 12*
a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + 9*a^4 + 60*a^3*b + 144*a^2*b^2 + 160*a
*b^3 + 64*b^4 + 30*(3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)
```

```
^2)*sinh(d*x + c)^4 + 3*a^4 + 12*a^3*b + 8*a^2*b^2 + 8*(7*(3*a^4 + 12*a^3*b
 + 8*a^2*b^2)*cosh(d*x + c)^5 + 10*(3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^
3)*cosh(d*x + c)^3 + (9*a^4 + 60*a^3*b + 144*a^2*b^2 + 160*a*b^3 + 64*b^4)*
cosh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^
3)*cosh(d*x + c)^2 + 4*(7*(3*a^4 + 12*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^6 +
15*(3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^4 + 3*a^4 + 18*
a^3*b + 32*a^2*b^2 + 16*a*b^3 + 3*(9*a^4 + 60*a^3*b + 144*a^2*b^2 + 160*a*b
^3 + 64*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*a^4 + 12*a^3*b + 8*a^
2*b^2)*cosh(d*x + c)^7 + 3*(3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3)*cosh(
d*x + c)^5 + (9*a^4 + 60*a^3*b + 144*a^2*b^2 + 160*a*b^3 + 64*b^4)*cosh(d*x
 + c)^3 + (3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c))*sinh(d*
x + c))*sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*
sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-a*b - b^2)/(a*b + b^2))
+ 4*(16*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^7 + 3*(5*a^4*b + 25
*a^3*b^2 + 36*a^2*b^3 + 16*a*b^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*
b^4)*d*x)*cosh(d*x + c)^5 + 2*(15*a^4*b + 73*a^3*b^2 + 146*a^2*b^3 + 136*a*
b^4 + 48*b^5 + 8*(3*a^4*b + 14*a^3*b^2 + 27*a^2*b^3 + 24*a*b^4 + 8*b^5)*d*x
)*cosh(d*x + c)^3 + (15*a^4*b + 59*a^3*b^2 + 76*a^2*b^3 + 32*a*b^4 + 16*(a^
4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(
(a^7*b + 2*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^8 + 8*(a^7*b + 2*a^6*b^2 + a^
5*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^7*b + 2*a^6*b^2 + a^5*b^3)*d*si
nh(d*x + c)^8 + 4*(a^7*b + 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x +
c)^6 + 4*(7*(a^7*b + 2*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^2 + (a^7*b + 4*a^
6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*d)*sinh(d*x + c)^6 + 2*(3*a^7*b + 14*a^6*b^2
 + 27*a^5*b^3 + 24*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^7*b + 2
*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^3 + 3*(a^7*b + 4*a^6*b^2 + 5*a^5*b^3 +
2*a^4*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^7*b + 2*a^6*b^2 + a^
5*b^3)*d*cosh(d*x + c)^4 + 30*(a^7*b + 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*d
*cosh(d*x + c)^2 + (3*a^7*b + 14*a^6*b^2 + 27*a^5*b^3 + 24*a^4*b^4 + 8*a^3*
b^5)*d)*sinh(d*x + c)^4 + 4*(a^7*b + 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*d*c
osh(d*x + c)^2 + 8*(7*(a^7*b + 2*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^5 + 10*
(a^7*b + 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^3 + (3*a^7*b +
14*a^6*b^2 + 27*a^5*b^3 + 24*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x
 + c)^3 + 4*(7*(a^7*b + 2*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^6 + 15*(a^7*b
+ 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^4 + 3*(3*a^7*b + 14*a^
6*b^2 + 27*a^5*b^3 + 24*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^2 + (a^7*b + 4
*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*d)*sinh(d*x + c)^2 + (a^7*b + 2*a^6*b^2 +
 a^5*b^3)*d + 8*((a^7*b + 2*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^7 + 3*(a^7*b
 + 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^5 + (3*a^7*b + 14*a^6
*b^2 + 27*a^5*b^3 + 24*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^3 + (a^7*b + 4*
a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.40493, size = 417, normalized size = 3.

$$-\frac{\frac{\left(3\,a^{2}e^{(2\,c)}+12\,abe^{(2\,c)}+8\,b^{2}e^{(2\,c)}\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^{2}}}\right)e^{(-2\,c)}}{\left(a^{4}+a^{3}b\right)\sqrt{-ab-b^{2}}}-\frac{8\,dx}{a^{3}}-\frac{2\left(5\,a^{3}e^{(6\,dx+6\,c)}+20\,a^{2}be^{(6\,dx+6\,c)}+16\,ab^{2}e^{(6\,dx+6\,c)}+15\,a^{3}e^{(4\,dx+4\,c)}+58\,a^{2}be^{(4\,dx}\cdots\right.}{\left(a^{4}+a^{3}b\right)\left(ae^{(4\,d}\cdots\right.}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((3*a^2*e^(2*c) + 12*a*b*e^(2*c) + 8*b^2*e^(2*c))*arctan(1/2*(a*e^(2*d
*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))*e^(-2*c)/((a^4 + a^3*b)*sqrt(-a*b -
b^2)) - 8*d*x/a^3 - 2*(5*a^3*e^(6*d*x + 6*c) + 20*a^2*b*e^(6*d*x + 6*c) + 1
6*a*b^2*e^(6*d*x + 6*c) + 15*a^3*e^(4*d*x + 4*c) + 58*a^2*b*e^(4*d*x + 4*c)
 + 88*a*b^2*e^(4*d*x + 4*c) + 48*b^3*e^(4*d*x + 4*c) + 15*a^3*e^(2*d*x + 2*
c) + 44*a^2*b*e^(2*d*x + 2*c) + 32*a*b^2*e^(2*d*x + 2*c) + 5*a^3 + 6*a^2*b)
/((a^4 + a^3*b)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2
*c) + a)^2))/d
```

3.163 $\int \frac{\tanh(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=73

$$-\frac{b^2}{4a^3d\left(a\cosh^2(c+dx)+b\right)^2}+\frac{b}{a^3d\left(a\cosh^2(c+dx)+b\right)}+\frac{\log\left(a\cosh^2(c+dx)+b\right)}{2a^3d}$$

```
[Out] -b^2/(4*a^3*d*(b + a*Cosh[c + d*x]^2)^2) + b/(a^3*d*(b + a*Cosh[c + d*x]^2)
) + Log[b + a*Cosh[c + d*x]^2]/(2*a^3*d)
```

Rubi [A] time = 0.082045, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 266, 43}

$$-\frac{b^2}{4a^3d\left(a\cosh^2(c+dx)+b\right)^2}+\frac{b}{a^3d\left(a\cosh^2(c+dx)+b\right)}+\frac{\log\left(a\cosh^2(c+dx)+b\right)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -b^2/(4*a^3*d*(b + a*Cosh[c + d*x]^2)^2) + b/(a^3*d*(b + a*Cosh[c + d*x]^2)
) + Log[b + a*Cosh[c + d*x]^2]/(2*a^3*d)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\tanh(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{\left(b+ax^2\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(b+ax)^3}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{b^2}{a^2(b+ax)^3} - \frac{2b}{a^2(b+ax)^2} + \frac{1}{a^2(b+ax)}\right) dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{b^2}{4a^3d\left(b+a\cosh^2(c+dx)\right)^2} + \frac{b}{a^3d\left(b+a\cosh^2(c+dx)\right)} + \frac{\log\left(b+a\cosh^2(c+dx)\right)}{2a^3d}$$

Mathematica [A] time = 1.89572, size = 129, normalized size = 1.77

$$\frac{a^2\cosh^2(2(c+dx))\log(a\cosh(2(c+dx))+a+2b)+(a+2b)^2\log(a\cosh(2(c+dx))+a+2b)+2a\cosh(2(c+dx))(}{2a^3d(a\cosh(2(c+dx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (2*b*(2*a + 3*b) + (a + 2*b)^2*Log[a + 2*b + a*Cosh[2*(c + d*x)]] + a^2*Cos
h[2*(c + d*x)]^2*Log[a + 2*b + a*Cosh[2*(c + d*x)]] + 2*a*Cosh[2*(c + d*x)]
*(2*b + (a + 2*b)*Log[a + 2*b + a*Cosh[2*(c + d*x)]]))/(2*a^3*d*(a + 2*b +
a*Cosh[2*(c + d*x)])^2)
```

Maple [A] time = 0.027, size = 82, normalized size = 1.1

$$-\frac{1}{2\,da^2\left(a+b\,(\text{sech}\,(dx+c))^2\right)} - \frac{1}{4\,da\left(a+b\,(\text{sech}\,(dx+c))^2\right)^2} + \frac{\ln\left(a+b\,(\text{sech}\,(dx+c))^2\right)}{2\,da^3} - \frac{\ln(\text{sech}\,(dx+c))}{da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -1/2/d/a^2/(a+b*sech(d*x+c)^2)-1/4/d/a/(a+b*sech(d*x+c)^2)^2+1/2/d/a^3*ln(a
+b*sech(d*x+c)^2)-1/d/a^3*ln(sech(d*x+c))
```

Maxima [B] time = 1.21352, size = 261, normalized size = 3.58

$$\frac{4\left(abe^{(-2\,dx-2\,c)}+abe^{(-6\,dx-6\,c)}+\left(2\,ab+3\,b^2\right)e^{(-4\,dx-4\,c)}\right)}{\left(a^5e^{(-8\,dx-8\,c)}+a^5+4\left(a^5+2\,a^4b\right)e^{(-2\,dx-2\,c)}+2\left(3\,a^5+8\,a^4b+8\,a^3b^2\right)e^{(-4\,dx-4\,c)}+4\left(a^5+2\,a^4b\right)e^{(-6\,dx-6\,c)}\right)d} + \frac{dx+c}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 4*(a*b*e^(-2*d*x - 2*c) + a*b*e^(-6*d*x - 6*c) + (2*a*b + 3*b^2)*e^(-4*d*x
 - 4*c))/((a^5*e^(-8*d*x - 8*c) + a^5 + 4*(a^5 + 2*a^4*b)*e^(-2*d*x - 2*c) +
```

```
 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*e^(-4*d*x - 4*c) + 4*(a^5 + 2*a^4*b)*e^(-6
*d*x - 6*c))*d) + (d*x + c)/(a^3*d) + 1/2*log(2*(a + 2*b)*e^(-2*d*x - 2*c)
+ a*e^(-4*d*x - 4*c) + a)/(a^3*d)
```

Fricas [B] time = 2.46631, size = 4103, normalized size = 56.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*d*x*cosh(d*x + c)^8 + 16*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^7
+ 2*a^2*d*x*sinh(d*x + c)^8 + 8*((a^2 + 2*a*b)*d*x - a*b)*cosh(d*x + c)^6 +
 8*(7*a^2*d*x*cosh(d*x + c)^2 + (a^2 + 2*a*b)*d*x - a*b)*sinh(d*x + c)^6 +
16*(7*a^2*d*x*cosh(d*x + c)^3 + 3*((a^2 + 2*a*b)*d*x - a*b)*cosh(d*x + c))*
sinh(d*x + c)^5 + 4*((3*a^2 + 8*a*b + 8*b^2)*d*x - 4*a*b - 6*b^2)*cosh(d*x
+ c)^4 + 4*(35*a^2*d*x*cosh(d*x + c)^4 + (3*a^2 + 8*a*b + 8*b^2)*d*x + 30*(
(a^2 + 2*a*b)*d*x - a*b)*cosh(d*x + c)^2 - 4*a*b - 6*b^2)*sinh(d*x + c)^4 +
 2*a^2*d*x + 16*(7*a^2*d*x*cosh(d*x + c)^5 + 10*((a^2 + 2*a*b)*d*x - a*b)*c
osh(d*x + c)^3 + ((3*a^2 + 8*a*b + 8*b^2)*d*x - 4*a*b - 6*b^2)*cosh(d*x + c
))*sinh(d*x + c)^3 + 8*((a^2 + 2*a*b)*d*x - a*b)*cosh(d*x + c)^2 + 8*(7*a^2
*d*x*cosh(d*x + c)^6 + 15*((a^2 + 2*a*b)*d*x - a*b)*cosh(d*x + c)^4 + (a^2
+ 2*a*b)*d*x + 3*((3*a^2 + 8*a*b + 8*b^2)*d*x - 4*a*b - 6*b^2)*cosh(d*x + c
)^2 - a*b)*sinh(d*x + c)^2 - (a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sin
h(d*x + c)^7 + a^2*sinh(d*x + c)^8 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^6 + 4*(7
*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^6 + 8*(7*a^2*cosh(d*x + c
)^3 + 3*(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^2 + 8*a*b + 8
*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*cosh(d*x + c)^4 + 30*(a^2 + 2*a*b)*cosh(d
*x + c)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)
^5 + 10*(a^2 + 2*a*b)*cosh(d*x + c)^3 + (3*a^2 + 8*a*b + 8*b^2)*cosh(d*x +
c))*sinh(d*x + c)^3 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 4*(7*a^2*cosh(d*x +
 c)^6 + 15*(a^2 + 2*a*b)*cosh(d*x + c)^4 + 3*(3*a^2 + 8*a*b + 8*b^2)*cosh(d
*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*(a^2*cosh(d*x + c)^7 + 3
*(a^2 + 2*a*b)*cosh(d*x + c)^5 + (3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)^3 +
(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))*log(2*(a*cosh(d*x + c)^2 + a*si
nh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2)) + 16*(a^2*d*x*cosh(d*x + c)^7 + 3*((a^2 + 2*a*b)*d*x - a
*b)*cosh(d*x + c)^5 + ((3*a^2 + 8*a*b + 8*b^2)*d*x - 4*a*b - 6*b^2)*cosh(d*
x + c)^3 + ((a^2 + 2*a*b)*d*x - a*b)*cosh(d*x + c))*sinh(d*x + c))/(a^5*d*c
osh(d*x + c)^8 + 8*a^5*d*cosh(d*x + c)*sinh(d*x + c)^7 + a^5*d*sinh(d*x + c
)^8 + 4*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^6 + 4*(7*a^5*d*cosh(d*x + c)^2 + (a
^5 + 2*a^4*b)*d)*sinh(d*x + c)^6 + a^5*d + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*
d*cosh(d*x + c)^4 + 8*(7*a^5*d*cosh(d*x + c)^3 + 3*(a^5 + 2*a^4*b)*d*cosh(d
*x + c))*sinh(d*x + c)^5 + 2*(35*a^5*d*cosh(d*x + c)^4 + 30*(a^5 + 2*a^4*b)
*d*cosh(d*x + c)^2 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*d)*sinh(d*x + c)^4 + 4*(
a^5 + 2*a^4*b)*d*cosh(d*x + c)^2 + 8*(7*a^5*d*cosh(d*x + c)^5 + 10*(a^5 + 2
*a^4*b)*d*cosh(d*x + c)^3 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c))*
sinh(d*x + c)^3 + 4*(7*a^5*d*cosh(d*x + c)^6 + 15*(a^5 + 2*a^4*b)*d*cosh(d*
x + c)^4 + 3*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 + 2*a^4
*b)*d)*sinh(d*x + c)^2 + 8*(a^5*d*cosh(d*x + c)^7 + 3*(a^5 + 2*a^4*b)*d*cos
h(d*x + c)^5 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c)^3 + (a^5 + 2*a
^4*b)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.8185, size = 252, normalized size = 3.45

$$-\frac{\frac{4\,dx}{a^3}-\frac{2\,\log\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}{a^3}+\frac{3\,ae^{(8\,dx+8\,c)}+12\,ae^{(6\,dx+6\,c)}+8\,be^{(6\,dx+6\,c)}+18\,ae^{(4\,dx+4\,c)}+16\,be^{(4\,dx+4\,c)}+12\,ae^{(2\,dx+2\,c)}+8\,be^{(2\,dx}}{\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)^2a^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/4*(4*d*x/a^3 - 2*log(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*
d*x + 2*c) + a)/a^3 + (3*a*e^(8*d*x + 8*c) + 12*a*e^(6*d*x + 6*c) + 8*b*e^(
6*d*x + 6*c) + 18*a*e^(4*d*x + 4*c) + 16*b*e^(4*d*x + 4*c) + 12*a*e^(2*d*x
+ 2*c) + 8*b*e^(2*d*x + 2*c) + 3*a)/((a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*
c) + 4*b*e^(2*d*x + 2*c) + a)^2*a^2))/d
```

3.164 $\int \frac{1}{\left(a+b\,\text{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=146

$$-\frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3d(a+b)^{5/2}}-\frac{b(7a+4b)\tanh(c+dx)}{8a^2d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)}+\frac{x}{a^3}-\frac{b\tanh(c+}{4ad(a+b)\left(a-b\tanh\right.}$$

```
[Out] x/a^3 - (Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/
Sqrt[a + b]])/(8*a^3*(a + b)^(5/2)*d) - (b*Tanh[c + d*x])/(4*a*(a + b)*d*(a
 + b - b*Tanh[c + d*x]^2)^2) - (b*(7*a + 4*b)*Tanh[c + d*x])/(8*a^2*(a + b)
^2*d*(a + b - b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.181249, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4128, 414, 527, 522, 206, 208}

$$-\frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3d(a+b)^{5/2}}-\frac{b(7a+4b)\tanh(c+dx)}{8a^2d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)}+\frac{x}{a^3}-\frac{b\tanh(c+}{4ad(a+b)\left(a-b\tanh\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^(-3),x]
```


```
[Out] x/a^3 - (Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/
Sqrt[a + b]])/(8*a^3*(a + b)^(5/2)*d) - (b*Tanh[c + d*x])/(4*a*(a + b)*d*(a
 + b - b*Tanh[c + d*x]^2)^2) - (b*(7*a + 4*b)*Tanh[c + d*x])/(8*a^2*(a + b)
^2*d*(a + b - b*Tanh[c + d*x]^2))
```


Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx=\frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\left(a+b-bx^2\right)^3}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$=-\frac{b\tanh(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2}-\frac{\operatorname{Subst}\left(\int \frac{-4a-b-3bx^2}{(1-x^2)\left(a+b-bx^2\right)^2}\,dx,x,\tanh(c+dx)\right)}{4a(a+b)d}$$

$$=-\frac{b\tanh(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2}-\frac{b(7a+4b)\tanh(c+dx)}{8a^2(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)}+$$

$$=-\frac{b\tanh(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2}-\frac{b(7a+4b)\tanh(c+dx)}{8a^2(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)}+$$

$$=\frac{x}{a^3}-\frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}d}-\frac{b\tanh(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+\right.}$$

Mathematica [B] time = 6.11276, size = 301, normalized size = 2.06

$$\frac{\operatorname{sech}^6(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\frac{b\operatorname{sech}(2c)\left(\left(9a^2+28ab+16b^2\right)\sinh(2c)-3a(3a+2b)\sinh(2dx)\right)(a\cosh(2(c+dx))+a+2b)}{d(a+b)^2}-\frac{b\left(15a^2+\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^(-3),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^6*(8*x*(a + 2*b + a*Cosh[2*(
c + d*x)])^2 - (b*(15*a^2 + 20*a*b + 8*b^2)*ArcTanh[(Sech[d*x]*(Cosh[2*c] -
```

```
 Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[
b*(Cosh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*(Cosh[2*c] - S
inh[2*c]))/((a + b)^(5/2)*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) - (4*b^2*Sech[2*
c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/((a + b)*d) + (b*(a + 2*b + a*Cos
h[2*(c + d*x)])*Sech[2*c]*((9*a^2 + 28*a*b + 16*b^2)*Sinh[2*c] - 3*a*(3*a +
 2*b)*Sinh[2*d*x]))/((a + b)^2*d)))/(64*a^3*(a + b*Sech[c + d*x]^2)^3)
```

Maple [B] time = 0.093, size = 1292, normalized size = 8.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)-9/4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2
*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*b/
(a+b)/a*tanh(1/2*d*x+1/2*c)^7-1/d*b^2/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1
/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/
(a+b)*tanh(1/2*d*x+1/2*c)^7-27/4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+
1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*b/(a+b)
^2*tanh(1/2*d*x+1/2*c)^5-11/4/d*b^2/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d
*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b
)^2*tanh(1/2*d*x+1/2*c)^5+1/d*b^3/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d
*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b
)^2*tanh(1/2*d*x+1/2*c)^5-27/4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/
2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*b/(a+b)^2
*tanh(1/2*d*x+1/2*c)^3-11/4/d*b^2/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x
+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^
2*tanh(1/2*d*x+1/2*c)^3+1/d*b^3/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x
+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^
2*tanh(1/2*d*x+1/2*c)^3-9/4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c
)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*b/(a+b)/a*ta
nh(1/2*d*x+1/2*c)-1/d*b^2/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c
)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1
/2*d*x+1/2*c)-15/16/d*b^(1/2)/a/(a^2+2*a*b+b^2)/(a+b)^(1/2)*ln((a+b)^(1/2)*
tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-5/4/d*b^(3
/2)/a^2/(a^2+2*a*b+b^2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*
tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-1/2/d*b^(5/2)/a^3/(a^2+2*a*b+b^2)/
(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1
/2)+(a+b)^(1/2))+15/16/d*b^(1/2)/a/(a^2+2*a*b+b^2)/(a+b)^(1/2)*ln(-(a+b)^(1
/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2))+5/4/d*
b^(3/2)/a^2/(a^2+2*a*b+b^2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)
^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2))+1/2/d*b^(5/2)/a^3/(a^2+2*a*b+
b^2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c
)*b^(1/2)-(a+b)^(1/2))-1/d/a^3*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.32923, size = 15505, normalized size = 106.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^8 + 128*(a^4 + 2*a^3*
b + a^2*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 16*(a^4 + 2*a^3*b + a^2*b^
2)*d*x*sinh(d*x + c)^8 + 4*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4 + 4*a
^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^6 + 4*(112*(a^4 + 2*a^3*b +
a^2*b^2)*d*x*cosh(d*x + c)^2 + 9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4 +
4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*sinh(d*x + c)^6 + 8*(112*(a^4 + 2*a^3*b
 + a^2*b^2)*d*x*cosh(d*x + c)^3 + 3*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(
a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 +
4*(27*a^3*b + 90*a^2*b^2 + 120*a*b^3 + 48*b^4 + 8*(3*a^4 + 14*a^3*b + 27*a^
2*b^2 + 24*a*b^3 + 8*b^4)*d*x)*cosh(d*x + c)^4 + 4*(280*(a^4 + 2*a^3*b + a^
2*b^2)*d*x*cosh(d*x + c)^4 + 27*a^3*b + 90*a^2*b^2 + 120*a*b^3 + 48*b^4 + 8
*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*d*x + 15*(9*a^3*b + 28*
a^2*b^2 + 16*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x
 + c)^2)*sinh(d*x + c)^4 + 36*a^3*b + 24*a^2*b^2 + 16*(56*(a^4 + 2*a^3*b +
a^2*b^2)*d*x*cosh(d*x + c)^5 + 5*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4
 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^3 + (27*a^3*b + 90*a^2
*b^2 + 120*a*b^3 + 48*b^4 + 8*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8
*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 16*(a^4 + 2*a^3*b + a^2*b^2)*d*
x + 4*(27*a^3*b + 68*a^2*b^2 + 32*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2
*a*b^3)*d*x)*cosh(d*x + c)^2 + 4*(112*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*
x + c)^6 + 15*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*
b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^4 + 27*a^3*b + 68*a^2*b^2 + 32*a*b^3 + 16
*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x + 6*(27*a^3*b + 90*a^2*b^2 + 120
*a*b^3 + 48*b^4 + 8*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*d*x)
*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d
*x + c)^8 + 8*(15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^7
 + (15*a^4 + 20*a^3*b + 8*a^2*b^2)*sinh(d*x + c)^8 + 4*(15*a^4 + 50*a^3*b +
 48*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^6 + 4*(15*a^4 + 50*a^3*b + 48*a^2*b^2
 + 16*a*b^3 + 7*(15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x +
 c)^6 + 8*(7*(15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 + 3*(15*a^4 +
50*a^3*b + 48*a^2*b^2 + 16*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(45*a^
4 + 180*a^3*b + 304*a^2*b^2 + 224*a*b^3 + 64*b^4)*cosh(d*x + c)^4 + 2*(35*(
15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + 45*a^4 + 180*a^3*b + 304*a
^2*b^2 + 224*a*b^3 + 64*b^4 + 30*(15*a^4 + 50*a^3*b + 48*a^2*b^2 + 16*a*b^3
)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 15*a^4 + 20*a^3*b + 8*a^2*b^2 + 8*(7*(
15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^5 + 10*(15*a^4 + 50*a^3*b + 48
*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^3 + (45*a^4 + 180*a^3*b + 304*a^2*b^2 +
224*a*b^3 + 64*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(15*a^4 + 50*a^3*b +
 48*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^2 + 4*(7*(15*a^4 + 20*a^3*b + 8*a^2*b
^2)*cosh(d*x + c)^6 + 15*(15*a^4 + 50*a^3*b + 48*a^2*b^2 + 16*a*b^3)*cosh(d
*x + c)^4 + 15*a^4 + 50*a^3*b + 48*a^2*b^2 + 16*a*b^3 + 3*(45*a^4 + 180*a^3
*b + 304*a^2*b^2 + 224*a*b^3 + 64*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8
*((15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^7 + 3*(15*a^4 + 50*a^3*b +
48*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^5 + (45*a^4 + 180*a^3*b + 304*a^2*b^2
+ 224*a*b^3 + 64*b^4)*cosh(d*x + c)^3 + (15*a^4 + 50*a^3*b + 48*a^2*b^2 + 1
6*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a + b))*log((a^2*cosh(d*x +
c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 +
```

2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 8*(16*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^7 + 3*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^5 + 2*(27*a^3*b + 90*a^2*b^2 + 120*a*b^3 + 48*b^4 + 8*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*d*x)*cosh(d*x + c)^3 + (27*a^3*b + 68*a^2*b^2 + 32*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^8 + 8*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^7 + 2*a^6*b + a^5*b^2)*d*sinh(d*x + c)^8 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^6 + 4*(7*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^2 + (a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d)*sinh(d*x + c)^6 + 2*(3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)^4 + 8*(7*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^3 + 3*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^4 + 30*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^2 + (3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*d)*sinh(d*x + c)^4 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^2 + 8*(7*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^5 + 10*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^3 + (3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^6 + 15*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^4 + 3*(3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)^2 + (a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d)*sinh(d*x + c)^2 + (a^7 + 2*a^6*b + a^5*b^2)*d + 8*((a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^7 + 3*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^5 + (3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)^3 + (a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*(8*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^8 + 64*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 8*(a^4 + 2*a^3*b + a^2*b^2)*d*x*sinh(d*x + c)^8 + 2*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^6 + 2*(112*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^2 + 9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*sinh(d*x + c)^6 + 4*(112*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^3 + 3*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(27*a^3*b + 90*a^2*b^2 + 120*a*b^3 + 48*b^4 + 8*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*d*x)*cosh(d*x + c)^4 + 2*(280*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^4 + 27*a^3*b + 90*a^2*b^2 + 120*a*b^3 + 48*b^4 + 8*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*d*x + 15*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 18*a^3*b + 12*a^2*b^2 + 8*(56*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^5 + 5*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^3 + (27*a^3*b + 90*a^2*b^2 + 120*a*b^3 + 48*b^4 + 8*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 8*(a^4 + 2*a^3*b + a^2*b^2)*d*x + 2*(27*a^3*b + 68*a^2*b^2 + 32*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^2 + 2*(112*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^6 + 15*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^4 + 27*a^3*b + 68*a^2*b^2 + 32*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x + 6*(27*a^3*b + 90*a^2*b^2 + 120*a*b^3 + 48*b^4 + 8*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^8 + 8*(15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^

```
7 + (15*a^4 + 20*a^3*b + 8*a^2*b^2)*sinh(d*x + c)^8 + 4*(15*a^4 + 50*a^3*b
+ 48*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^6 + 4*(15*a^4 + 50*a^3*b + 48*a^2*b^
2 + 16*a*b^3 + 7*(15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x
+ c)^6 + 8*(7*(15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 + 3*(15*a^4 +
 50*a^3*b + 48*a^2*b^2 + 16*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(45*a
^4 + 180*a^3*b + 304*a^2*b^2 + 224*a*b^3 + 64*b^4)*cosh(d*x + c)^4 + 2*(35*
(15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + 45*a^4 + 180*a^3*b + 304*
a^2*b^2 + 224*a*b^3 + 64*b^4 + 30*(15*a^4 + 50*a^3*b + 48*a^2*b^2 + 16*a*b^
3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 15*a^4 + 20*a^3*b + 8*a^2*b^2 + 8*(7*
(15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^5 + 10*(15*a^4 + 50*a^3*b + 4
8*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^3 + (45*a^4 + 180*a^3*b + 304*a^2*b^2 +
 224*a*b^3 + 64*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(15*a^4 + 50*a^3*b
+ 48*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^2 + 4*(7*(15*a^4 + 20*a^3*b + 8*a^2*
b^2)*cosh(d*x + c)^6 + 15*(15*a^4 + 50*a^3*b + 48*a^2*b^2 + 16*a*b^3)*cosh(
d*x + c)^4 + 15*a^4 + 50*a^3*b + 48*a^2*b^2 + 16*a*b^3 + 3*(45*a^4 + 180*a^
3*b + 304*a^2*b^2 + 224*a*b^3 + 64*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
8*((15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^7 + 3*(15*a^4 + 50*a^3*b +
 48*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^5 + (45*a^4 + 180*a^3*b + 304*a^2*b^2
 + 224*a*b^3 + 64*b^4)*cosh(d*x + c)^3 + (15*a^4 + 50*a^3*b + 48*a^2*b^2 +
16*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*(a*cosh
(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b
)*sqrt(-b/(a + b))/b) + 4*(16*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^7
 + 3*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a
*b^3)*d*x)*cosh(d*x + c)^5 + 2*(27*a^3*b + 90*a^2*b^2 + 120*a*b^3 + 48*b^4
+ 8*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*d*x)*cosh(d*x + c)^3
 + (27*a^3*b + 68*a^2*b^2 + 32*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*
b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d
*x + c)^8 + 8*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (
a^7 + 2*a^6*b + a^5*b^2)*d*sinh(d*x + c)^8 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 +
 2*a^4*b^3)*d*cosh(d*x + c)^6 + 4*(7*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x +
 c)^2 + (a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d)*sinh(d*x + c)^6 + 2*(3*a
^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)^4 + 8*
(7*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^3 + 3*(a^7 + 4*a^6*b + 5*a^5*b
^2 + 2*a^4*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^7 + 2*a^6*b + a
^5*b^2)*d*cosh(d*x + c)^4 + 30*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*co
sh(d*x + c)^2 + (3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*d)
*sinh(d*x + c)^4 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c
)^2 + 8*(7*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^5 + 10*(a^7 + 4*a^6*b
+ 5*a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^3 + (3*a^7 + 14*a^6*b + 27*a^5*b^2
 + 24*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^7 + 2
*a^6*b + a^5*b^2)*d*cosh(d*x + c)^6 + 15*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4
*b^3)*d*cosh(d*x + c)^4 + 3*(3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8
*a^3*b^4)*d*cosh(d*x + c)^2 + (a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d)*si
nh(d*x + c)^2 + (a^7 + 2*a^6*b + a^5*b^2)*d + 8*((a^7 + 2*a^6*b + a^5*b^2)*
d*cosh(d*x + c)^7 + 3*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x +
c)^5 + (3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x
+ c)^3 + (a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c))*sinh(d*x
+ c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.1767, size = 450, normalized size = 3.08

$$-\frac{\left(15\,a^2b+20\,ab^2+8\,b^3\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{8\,\left(a^5d+2\,a^4bd+a^3b^2d\right)\sqrt{-ab-b^2}}+\frac{9\,a^3be^{(6\,dx+6\,c)}+28\,a^2b^2e^{(6\,dx+6\,c)}+16\,ab^3e^{(6\,dx+6\,c)}+27\,a^3be^{(4\,dx+4\,c)}}{4\,\left(a^5d+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(15*a^2*b + 20*a*b^2 + 8*b^3)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)
/sqrt(-a*b - b^2))/((a^5*d + 2*a^4*b*d + a^3*b^2*d)*sqrt(-a*b - b^2)) + 1/4
*(9*a^3*b*e^(6*d*x + 6*c) + 28*a^2*b^2*e^(6*d*x + 6*c) + 16*a*b^3*e^(6*d*x
+ 6*c) + 27*a^3*b*e^(4*d*x + 4*c) + 90*a^2*b^2*e^(4*d*x + 4*c) + 120*a*b^3*
e^(4*d*x + 4*c) + 48*b^4*e^(4*d*x + 4*c) + 27*a^3*b*e^(2*d*x + 2*c) + 68*a^
2*b^2*e^(2*d*x + 2*c) + 32*a*b^3*e^(2*d*x + 2*c) + 9*a^3*b + 6*a^2*b^2)/((a
^5*d + 2*a^4*b*d + a^3*b^2*d)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*
b*e^(2*d*x + 2*c) + a)^2) + (d*x + c)/(a^3*d)
```

3.165 $\int \frac{\coth(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=130

$$-\frac{b^3}{4a^3d(a+b)\left(a\cosh^2(c+dx)+b\right)^2}+\frac{b^2(3a+2b)}{2a^3d(a+b)^2\left(a\cosh^2(c+dx)+b\right)}+\frac{b\left(3a^2+3ab+b^2\right)\log\left(a\cosh^2(c+dx)+\right.}{2a^3d(a+b)^3}$$

```
[Out] -b^3/(4*a^3*(a + b)*d*(b + a*Cosh[c + d*x]^2)^2) + (b^2*(3*a + 2*b))/(2*a^3
*(a + b)^2*d*(b + a*Cosh[c + d*x]^2)) + (b*(3*a^2 + 3*a*b + b^2)*Log[b + a*
Cosh[c + d*x]^2])/(2*a^3*(a + b)^3*d) + Log[Sinh[c + d*x]]/((a + b)^3*d)
```

Rubi [A] time = 0.185543, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 446, 88}

$$-\frac{b^3}{4a^3d(a+b)\left(a\cosh^2(c+dx)+b\right)^2}+\frac{b^2(3a+2b)}{2a^3d(a+b)^2\left(a\cosh^2(c+dx)+b\right)}+\frac{b\left(3a^2+3ab+b^2\right)\log\left(a\cosh^2(c+dx)+\right.}{2a^3d(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -b^3/(4*a^3*(a + b)*d*(b + a*Cosh[c + d*x]^2)^2) + (b^2*(3*a + 2*b))/(2*a^3
*(a + b)^2*d*(b + a*Cosh[c + d*x]^2)) + (b*(3*a^2 + 3*a*b + b^2)*Log[b + a*
Cosh[c + d*x]^2])/(2*a^3*(a + b)^3*d) + Log[Sinh[c + d*x]]/((a + b)^3*d)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\coth(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^7}{(1-x^2)\left(b+ax^2\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^3}{(1-x)(b+ax)^3}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^3(-1+x)} - \frac{b^3}{a^2(a+b)(b+ax)^3} + \frac{b^2(3a+2b)}{a^2(a+b)^2(b+ax)^2} - \frac{b\left(3a^2+3ab+b^2\right)}{a^2(a+b)^3(b+ax)}\right) dx, x, \cosh^2(c+ \right.}{2d}$$

$$= -\frac{b^3}{4a^3(a+b)d\left(b+a\cosh^2(c+dx)\right)^2} + \frac{b^2(3a+2b)}{2a^3(a+b)^2d\left(b+a\cosh^2(c+dx)\right)} + \frac{b\left(3a^2+3ab\right.}{}$$

Mathematica [A] time = 1.07267, size = 155, normalized size = 1.19

$$\frac{\text{sech}^6(c+dx)(a\cosh(2(c+dx))+a+2b)^3\left(-\frac{b^3(a+b)^2}{a^3\left(a\sinh^2(c+dx)+a+b\right)^2} + \frac{2b^2(a+b)(3a+2b)}{a^3\left(a\sinh^2(c+dx)+a+b\right)} + \frac{2b\left(3a^2+3ab+b^2\right)\log\left(a\sinh^2(c+dx)+a+b\right.}{a^3}\right.}{32d(a+b)^3\left(a+b\text{sech}^2(c+dx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^3*Sech[c + d*x]^6*(4*Log[Sinh[c + d*x]] +
(2*b*(3*a^2 + 3*a*b + b^2)*Log[a + b + a*Sinh[c + d*x]^2])/a^3 - (b^3*(a +
b)^2)/(a^3*(a + b + a*Sinh[c + d*x]^2)^2) + (2*b^2*(a + b)*(3*a + 2*b))/(a^
3*(a + b + a*Sinh[c + d*x]^2))))/(32*(a + b)^3*d*(a + b*Sech[c + d*x]^2)^3)
```

Maple [B] time = 0.101, size = 1046, normalized size = 8.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -1/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)-6/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a
+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*
b+a+b)^2*tanh(1/2*d*x+1/2*c)^6-8/d*b^3/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*t
anh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+
b)^2/a*tanh(1/2*d*x+1/2*c)^6-2/d*b^4/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tan
h(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)
^2/a^2*tanh(1/2*d*x+1/2*c)^6-12/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*ta
nh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b
)^2*tanh(1/2*d*x+1/2*c)^4+4/d*b^3/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1
/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/
a*tanh(1/2*d*x+1/2*c)^4+4/d*b^4/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2
*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a^
2*tanh(1/2*d*x+1/2*c)^4-6/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2
*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*ta
nh(1/2*d*x+1/2*c)^2-8/d*b^3/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x
+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a*tanh
```

```
(1/2*d*x+1/2*c)^2-2/d*b^4/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1
/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a^2*tanh
(1/2*d*x+1/2*c)^2+3/2/d*b/a/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d
*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+3/2/d*
b^2/a^2/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1
/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+1/2/d*b^3/a^3/(a+b)^3*ln(t
anh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*
tanh(1/2*d*x+1/2*c)^2*b+a+b)+1/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c))-1/d/a^3*ln
(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.37454, size = 566, normalized size = 4.35

$$\frac{\left(3\,a^2b+3\,ab^2+b^3\right)\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\left(a^6+3\,a^5b+3\,a^4b^2+a^3b^3\right)d}+\frac{}{\left(a^7+2\,a^6b+a^5b^2+4\left(a^7+4\,a^6b+5\,a^5b^2+2\,a^4b^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(3*a^2*b + 3*a*b^2 + b^3)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*
x - 4*c) + a)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d) + 2*((3*a^2*b^2 + 2
*a*b^3)*e^(-2*d*x - 2*c) + 2*(3*a^2*b^2 + 7*a*b^3 + 3*b^4)*e^(-4*d*x - 4*c)
 + (3*a^2*b^2 + 2*a*b^3)*e^(-6*d*x - 6*c))/((a^7 + 2*a^6*b + a^5*b^2 + 4*(a
^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^(-2*d*x - 2*c) + 2*(3*a^7 + 14*a^6*
b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*e^(-4*d*x - 4*c) + 4*(a^7 + 4*a^6*
b + 5*a^5*b^2 + 2*a^4*b^3)*e^(-6*d*x - 6*c) + (a^7 + 2*a^6*b + a^5*b^2)*e^(
-8*d*x - 8*c))*d) + log(e^(-d*x - c) + 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*
d) + log(e^(-d*x - c) - 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + (d*x + c)/
(a^3*d)
```

Fricas [B] time = 5.79157, size = 9400, normalized size = 72.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^8 + 16*(a^5
 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(a^
5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*sinh(d*x + c)^8 - 4*(3*a^3*b^2 + 5*a
^2*b^3 + 2*a*b^4 - 2*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*d*x)
*cosh(d*x + c)^6 - 4*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4 - 14*(a^5 + 3*a^4*b +
 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^2 - 2*(a^5 + 5*a^4*b + 9*a^3*b^2 +
7*a^2*b^3 + 2*a*b^4)*d*x)*sinh(d*x + c)^6 + 8*(14*(a^5 + 3*a^4*b + 3*a^3*b^
2 + a^2*b^3)*d*x*cosh(d*x + c)^3 - 3*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4 - 2*(
a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c))*sinh(d
*x + c)^5 - 4*(6*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 6*b^5 - (3*a^5 + 17*a^4*
b + 41*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^4 + 4*(3
5*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^4 - 6*a^3*b^2 - 2
0*a^2*b^3 - 20*a*b^4 - 6*b^5 + (3*a^5 + 17*a^4*b + 41*a^3*b^2 + 51*a^2*b^3
+ 32*a*b^4 + 8*b^5)*d*x - 15*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4 - 2*(a^5 + 5*
a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c
)^4 + 16*(7*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^5 - 5*(
```

3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4 - 2*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3
+ 2*a*b^4)*d*x)*cosh(d*x + c)^3 - (6*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 6*b^
5 - (3*a^5 + 17*a^4*b + 41*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5)*d*x)*co
sh(d*x + c))*sinh(d*x + c)^3 + 2*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x
- 4*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4 - 2*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2
*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^2 + 4*(14*(a^5 + 3*a^4*b + 3*a^3*b^2 + a
^2*b^3)*d*x*cosh(d*x + c)^6 - 3*a^3*b^2 - 5*a^2*b^3 - 2*a*b^4 - 15*(3*a^3*b
^2 + 5*a^2*b^3 + 2*a*b^4 - 2*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b
^4)*d*x)*cosh(d*x + c)^4 + 2*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b
^4)*d*x - 6*(6*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 6*b^5 - (3*a^5 + 17*a^4*b
+ 41*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^2)*sinh(d*
x + c)^2 - ((3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^8 + 8*(3*a^4*b +
3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^4*b + 3*a^3*b^2 +
a^2*b^3)*sinh(d*x + c)^8 + 4*(3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*c
osh(d*x + c)^6 + 4*(3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4 + 7*(3*a^4*b
+ 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^4*b + 3
*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^3 + 3*(3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 +
2*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 + 2
*(9*a^4*b + 33*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5)*cosh(d*x + c)^4 + 2
*(9*a^4*b + 33*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5 + 35*(3*a^4*b + 3*a^
3*b^2 + a^2*b^3)*cosh(d*x + c)^4 + 30*(3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*
a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(3*a^4*b + 3*a^3*b^2 + a^2*b
^3)*cosh(d*x + c)^5 + 10*(3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*cosh(d
*x + c)^3 + (9*a^4*b + 33*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5)*cosh(d*x
+ c))*sinh(d*x + c)^3 + 4*(3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*cosh
(d*x + c)^2 + 4*(7*(3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^6 + 3*a^4*
b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4 + 15*(3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 +
2*a*b^4)*cosh(d*x + c)^4 + 3*(9*a^4*b + 33*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4
+ 8*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*a^4*b + 3*a^3*b^2 + a^2*
b^3)*cosh(d*x + c)^7 + 3*(3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*cosh(d
*x + c)^5 + (9*a^4*b + 33*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5)*cosh(d*x
+ c)^3 + (3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*cosh(d*x + c))*sinh(d
*x + c))*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x
+ c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 2*(a^5*cosh(d*
x + c)^8 + 8*a^5*cosh(d*x + c)*sinh(d*x + c)^7 + a^5*sinh(d*x + c)^8 + 4*(a
^5 + 2*a^4*b)*cosh(d*x + c)^6 + 4*(7*a^5*cosh(d*x + c)^2 + a^5 + 2*a^4*b)*s
inh(d*x + c)^6 + 8*(7*a^5*cosh(d*x + c)^3 + 3*(a^5 + 2*a^4*b)*cosh(d*x + c)
)*sinh(d*x + c)^5 + a^5 + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(d*x + c)^4 +
2*(35*a^5*cosh(d*x + c)^4 + 3*a^5 + 8*a^4*b + 8*a^3*b^2 + 30*(a^5 + 2*a^4*
b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*a^5*cosh(d*x + c)^5 + 10*(a^5 +
2*a^4*b)*cosh(d*x + c)^3 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(d*x + c))*sin
h(d*x + c)^3 + 4*(a^5 + 2*a^4*b)*cosh(d*x + c)^2 + 4*(7*a^5*cosh(d*x + c)^6
+ a^5 + 2*a^4*b + 15*(a^5 + 2*a^4*b)*cosh(d*x + c)^4 + 3*(3*a^5 + 8*a^4*b
+ 8*a^3*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(a^5*cosh(d*x + c)^7 + 3*
(a^5 + 2*a^4*b)*cosh(d*x + c)^5 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(d*x +
c)^3 + (a^5 + 2*a^4*b)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(c
osh(d*x + c) - sinh(d*x + c))) + 8*(2*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)
*d*x*cosh(d*x + c)^7 - 3*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4 - 2*(a^5 + 5*a^4*
b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^5 - 2*(6*a^3*b^2 +
20*a^2*b^3 + 20*a*b^4 + 6*b^5 - (3*a^5 + 17*a^4*b + 41*a^3*b^2 + 51*a^2*b^3
+ 32*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^3 - (3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^
4 - 2*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c))
*sinh(d*x + c))/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^8 +
8*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (
a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*sinh(d*x + c)^8 + 4*(a^8 + 5*a^7*b +
9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^8 + 3*a^7*b
+ 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^2 + (a^8 + 5*a^7*b + 9*a^6*b^2 + 7*
a^5*b^3 + 2*a^4*b^4)*d)*sinh(d*x + c)^6 + 2*(3*a^8 + 17*a^7*b + 41*a^6*b^2
+ 51*a^5*b^3 + 32*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^8 + 3*a^

```
7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^3 + 3*(a^8 + 5*a^7*b + 9*a^6*b^2
 + 7*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^8 + 3
*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^4 + 30*(a^8 + 5*a^7*b + 9*a^6
*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^2 + (3*a^8 + 17*a^7*b + 41*a^
6*b^2 + 51*a^5*b^3 + 32*a^4*b^4 + 8*a^3*b^5)*d)*sinh(d*x + c)^4 + 4*(a^8 +
5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^2 + 8*(7*(a^8
+ 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^5 + 10*(a^8 + 5*a^7*b + 9*
a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^3 + (3*a^8 + 17*a^7*b + 41
*a^6*b^2 + 51*a^5*b^3 + 32*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x +
 c)^3 + 4*(7*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^6 + 15*(
a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^4 + 3*(3
*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b^3 + 32*a^4*b^4 + 8*a^3*b^5)*d*cosh(
d*x + c)^2 + (a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d)*sinh(d*
x + c)^2 + (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d + 8*((a^8 + 3*a^7*b + 3*
a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^7 + 3*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5
*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^5 + (3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a
^5*b^3 + 32*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^3 + (a^8 + 5*a^7*b + 9*a^6
*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.93199, size = 641, normalized size = 4.93

$$\frac{2\left(3\,a^2b+3\,ab^2+b^3\right)\log\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}{a^6+3\,a^5b+3\,a^4b^2+a^3b^3}+\frac{4\,e^{(2\,c)}\log\left(\left|-e^{(2\,dx+2\,c)}+1\right|\right)}{a^3e^{(2\,c)}+3\,a^2be^{(2\,c)}+3\,ab^2e^{(2\,c)}+b^3e^{(2\,c)}}-\frac{4\,dx}{a^3}-\frac{9\,a^3be^{(8\,dx+8\,c)}+9\,a^2b^2e^{(8\,dx+8\,c)}+3\,ab^3 \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*(3*a^2*b + 3*a*b^2 + b^3)*log(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c
) + 4*b*e^(2*d*x + 2*c) + a)/(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3) + 4*e^(2
*c)*log(abs(-e^(2*d*x + 2*c) + 1))/(a^3*e^(2*c) + 3*a^2*b*e^(2*c) + 3*a*b^2
*e^(2*c) + b^3*e^(2*c)) - 4*d*x/a^3 - (9*a^3*b*e^(8*d*x + 8*c) + 9*a^2*b^2*
e^(8*d*x + 8*c) + 3*a*b^3*e^(8*d*x + 8*c) + 36*a^3*b*e^(6*d*x + 6*c) + 84*a
^2*b^2*e^(6*d*x + 6*c) + 44*a*b^3*e^(6*d*x + 6*c) + 8*b^4*e^(6*d*x + 6*c) +
 54*a^3*b*e^(4*d*x + 4*c) + 150*a^2*b^2*e^(4*d*x + 4*c) + 146*a*b^3*e^(4*d*
x + 4*c) + 32*b^4*e^(4*d*x + 4*c) + 36*a^3*b*e^(2*d*x + 2*c) + 84*a^2*b^2*e
^(2*d*x + 2*c) + 44*a*b^3*e^(2*d*x + 2*c) + 8*b^4*e^(2*d*x + 2*c) + 9*a^3*b
 + 9*a^2*b^2 + 3*a*b^3)/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*(a*e^(4*d*x
+ 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)^2))/d
```

3.166 $\int \frac{\coth^2(c+dx)}{\left(a+b\mathbf{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=182

$$-\frac{b^{3/2}\left(35a^2+28ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3d(a+b)^{7/2}}-\frac{\left(8a^2-11ab-4b^2\right)\coth(c+dx)}{8a^2d(a+b)^3}-\frac{b(9a+4b)\coth(c+dx)}{8a^2d(a+b)^2\left(a-b\tanh^2(c+dx)\right)}$$

```
[Out] x/a^3 - (b^(3/2)*(35*a^2 + 28*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/
Sqrt[a + b]])/(8*a^3*(a + b)^(7/2)*d) - ((8*a^2 - 11*a*b - 4*b^2)*Coth[c +
d*x])/(8*a^2*(a + b)^3*d) - (b*Coth[c + d*x])/(4*a*(a + b)*d*(a + b - b*Tan
h[c + d*x]^2)^2) - (b*(9*a + 4*b)*Coth[c + d*x])/(8*a^2*(a + b)^2*d*(a + b
- b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.403154, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4141, 1975, 472, 579, 583, 522, 206, 208}

$$-\frac{b^{3/2}\left(35a^2+28ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3d(a+b)^{7/2}}-\frac{\left(8a^2-11ab-4b^2\right)\coth(c+dx)}{8a^2d(a+b)^3}-\frac{b(9a+4b)\coth(c+dx)}{8a^2d(a+b)^2\left(a-b\tanh^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```


```
[Out] x/a^3 - (b^(3/2)*(35*a^2 + 28*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/
Sqrt[a + b]])/(8*a^3*(a + b)^(7/2)*d) - ((8*a^2 - 11*a*b - 4*b^2)*Coth[c +
d*x])/(8*a^2*(a + b)^3*d) - (b*Coth[c + d*x])/(4*a*(a + b)*d*(a + b - b*Tan
h[c + d*x]^2)^2) - (b*(9*a + 4*b)*Coth[c + d*x])/(8*a^2*(a + b)^2*d*(a + b
- b*Tanh[c + d*x]^2))
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
```

```
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\coth^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1-x^2)\left(a+b(1-x^2)\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1-x^2)\left(a+b-bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\coth(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-4a+b-5bx^2}{x^2(1-x^2)\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{4a(a+b)d}$$

$$= -\frac{b\coth(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{b(9a+4b)\coth(c+dx)}{8a^2(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\text{Subs}}{}$$

$$= -\frac{\left(8a^2-11ab-4b^2\right)\coth(c+dx)}{8a^2(a+b)^3d} - \frac{b\coth(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{b(9a}{8a^2(a+b)^2d}$$

$$= -\frac{\left(8a^2-11ab-4b^2\right)\coth(c+dx)}{8a^2(a+b)^3d} - \frac{b\coth(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{b(9a}{8a^2(a+b)^2d}$$

$$= \frac{x}{a^3} - \frac{b^{3/2}\left(35a^2+28ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{7/2}d} - \frac{\left(8a^2-11ab-4b^2\right)\coth(c+dx)}{8a^2(a+b)^3d} -$$

Mathematica [C] time = 7.03088, size = 2083, normalized size = 11.45

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((35*a^2 + 28*a*b + 8*b^2)*(a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^
6*(((I/64)*b^2*ArcTan[Sech[d*x]*(((-I/2)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b*Cos
h[4*c] - b*Sinh[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] -
b*Sinh[4*c]]))*(-(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x])]*Cosh[2
*c])/(a^3*Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) - ((I/64)*b^2*ArcT
an[Sech[d*x]*(((-I/2)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c
]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]))*(-(a
*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x])]*Sinh[2*c])/(a^3*Sqrt[a +
b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]])))/((a + b)^3*(a + b*Sech[c + d*x]^2)^
3) + ((a + 2*b + a*Cosh[2*c + 2*d*x])*Csch[c]*Csch[c + d*x]*Sech[2*c]*Sech[
c + d*x]^6*(8*a^5*d*x*Cosh[d*x] + 56*a^4*b*d*x*Cosh[d*x] + 184*a^3*b^2*d*x*
Cosh[d*x] + 296*a^2*b^3*d*x*Cosh[d*x] + 224*a*b^4*d*x*Cosh[d*x] + 64*b^5*d*
x*Cosh[d*x] - 12*a^5*d*x*Cosh[3*d*x] - 68*a^4*b*d*x*Cosh[3*d*x] - 132*a^3*b
^2*d*x*Cosh[3*d*x] - 108*a^2*b^3*d*x*Cosh[3*d*x] - 32*a*b^4*d*x*Cosh[3*d*x]
 - 8*a^5*d*x*Cosh[2*c - d*x] - 56*a^4*b*d*x*Cosh[2*c - d*x] - 184*a^3*b^2*d
*x*Cosh[2*c - d*x] - 296*a^2*b^3*d*x*Cosh[2*c - d*x] - 224*a*b^4*d*x*Cosh[2
*c - d*x] - 64*b^5*d*x*Cosh[2*c - d*x] - 8*a^5*d*x*Cosh[2*c + d*x] - 56*a^4
*b*d*x*Cosh[2*c + d*x] - 184*a^3*b^2*d*x*Cosh[2*c + d*x] - 296*a^2*b^3*d*x*
Cosh[2*c + d*x] - 224*a*b^4*d*x*Cosh[2*c + d*x] - 64*b^5*d*x*Cosh[2*c + d*x
] + 8*a^5*d*x*Cosh[4*c + d*x] + 56*a^4*b*d*x*Cosh[4*c + d*x] + 184*a^3*b^2*
d*x*Cosh[4*c + d*x] + 296*a^2*b^3*d*x*Cosh[4*c + d*x] + 224*a*b^4*d*x*Cosh[
```

```
4*c + d*x] + 64*b^5*d*x*Cosh[4*c + d*x] + 12*a^5*d*x*Cosh[2*c + 3*d*x] + 68
*a^4*b*d*x*Cosh[2*c + 3*d*x] + 132*a^3*b^2*d*x*Cosh[2*c + 3*d*x] + 108*a^2*
b^3*d*x*Cosh[2*c + 3*d*x] + 32*a*b^4*d*x*Cosh[2*c + 3*d*x] - 12*a^5*d*x*Cos
h[4*c + 3*d*x] - 68*a^4*b*d*x*Cosh[4*c + 3*d*x] - 132*a^3*b^2*d*x*Cosh[4*c
+ 3*d*x] - 108*a^2*b^3*d*x*Cosh[4*c + 3*d*x] - 32*a*b^4*d*x*Cosh[4*c + 3*d*
x] + 12*a^5*d*x*Cosh[6*c + 3*d*x] + 68*a^4*b*d*x*Cosh[6*c + 3*d*x] + 132*a^
3*b^2*d*x*Cosh[6*c + 3*d*x] + 108*a^2*b^3*d*x*Cosh[6*c + 3*d*x] + 32*a*b^4*
d*x*Cosh[6*c + 3*d*x] - 4*a^5*d*x*Cosh[2*c + 5*d*x] - 12*a^4*b*d*x*Cosh[2*c
 + 5*d*x] - 12*a^3*b^2*d*x*Cosh[2*c + 5*d*x] - 4*a^2*b^3*d*x*Cosh[2*c + 5*d
*x] + 4*a^5*d*x*Cosh[4*c + 5*d*x] + 12*a^4*b*d*x*Cosh[4*c + 5*d*x] + 12*a^3
*b^2*d*x*Cosh[4*c + 5*d*x] + 4*a^2*b^3*d*x*Cosh[4*c + 5*d*x] - 4*a^5*d*x*Co
sh[6*c + 5*d*x] - 12*a^4*b*d*x*Cosh[6*c + 5*d*x] - 12*a^3*b^2*d*x*Cosh[6*c
+ 5*d*x] - 4*a^2*b^3*d*x*Cosh[6*c + 5*d*x] + 4*a^5*d*x*Cosh[8*c + 5*d*x] +
12*a^4*b*d*x*Cosh[8*c + 5*d*x] + 12*a^3*b^2*d*x*Cosh[8*c + 5*d*x] + 4*a^2*b
^3*d*x*Cosh[8*c + 5*d*x] - 32*a^5*Sinh[d*x] - 64*a^4*b*Sinh[d*x] - 30*a^2*b
^3*Sinh[d*x] - 120*a*b^4*Sinh[d*x] - 48*b^5*Sinh[d*x] + 32*a^5*Sinh[3*d*x]
+ 64*a^4*b*Sinh[3*d*x] + 26*a^3*b^2*Sinh[3*d*x] + 86*a^2*b^3*Sinh[3*d*x] +
32*a*b^4*Sinh[3*d*x] - 48*a^5*Sinh[2*c - d*x] - 128*a^4*b*Sinh[2*c - d*x] -
 128*a^3*b^2*Sinh[2*c - d*x] - 30*a^2*b^3*Sinh[2*c - d*x] - 120*a*b^4*Sinh[
2*c - d*x] - 48*b^5*Sinh[2*c - d*x] + 48*a^5*Sinh[2*c + d*x] + 128*a^4*b*Si
nh[2*c + d*x] + 102*a^3*b^2*Sinh[2*c + d*x] - 86*a^2*b^3*Sinh[2*c + d*x] -
136*a*b^4*Sinh[2*c + d*x] - 48*b^5*Sinh[2*c + d*x] - 32*a^5*Sinh[4*c + d*x]
 - 64*a^4*b*Sinh[4*c + d*x] + 26*a^3*b^2*Sinh[4*c + d*x] + 86*a^2*b^3*Sinh[
4*c + d*x] + 136*a*b^4*Sinh[4*c + d*x] + 48*b^5*Sinh[4*c + d*x] - 8*a^5*Sin
h[2*c + 3*d*x] - 26*a^3*b^2*Sinh[2*c + 3*d*x] - 86*a^2*b^3*Sinh[2*c + 3*d*x
] - 32*a*b^4*Sinh[2*c + 3*d*x] + 32*a^5*Sinh[4*c + 3*d*x] + 64*a^4*b*Sinh[4
*c + 3*d*x] - 13*a^3*b^2*Sinh[4*c + 3*d*x] - 36*a^2*b^3*Sinh[4*c + 3*d*x] -
 16*a*b^4*Sinh[4*c + 3*d*x] - 8*a^5*Sinh[6*c + 3*d*x] + 13*a^3*b^2*Sinh[6*c
 + 3*d*x] + 36*a^2*b^3*Sinh[6*c + 3*d*x] + 16*a*b^4*Sinh[6*c + 3*d*x] + 8*a
^5*Sinh[2*c + 5*d*x] + 13*a^3*b^2*Sinh[2*c + 5*d*x] + 6*a^2*b^3*Sinh[2*c +
5*d*x] - 13*a^3*b^2*Sinh[4*c + 5*d*x] - 6*a^2*b^3*Sinh[4*c + 5*d*x] + 8*a^5
*Sinh[6*c + 5*d*x]))/(512*a^3*(a + b)^3*d*(a + b*Sech[c + d*x]^2)^3)
```

Maple [B] time = 0.12, size = 1442, normalized size = 7.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -1/2/d/(a^3+3*a^2*b+3*a*b^2+b^3)*tanh(1/2*d*x+1/2*c)+1/d/a^3*ln(tanh(1/2*d*
x+1/2*c)+1)-13/4/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*
c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*
x+1/2*c)^7-17/4/d*b^3/a/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2
*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d
*x+1/2*c)^7-1/d*b^4/a^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2
*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d
*x+1/2*c)^7-39/4/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*
c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*
x+1/2*c)^5-7/4/d*b^3/a/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*
c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*
x+1/2*c)^5+1/d*b^4/a^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*
c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*
x+1/2*c)^5-39/4/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c
)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x
+1/2*c)^3-7/4/d*b^3/a/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c
```

```
)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x
+1/2*c)^3+1/d*b^4/a^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c
)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x
+1/2*c)^3-13/4/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)
^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+
1/2*c)-17/4/d*b^3/a/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^
4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1
/2*c)-1/d*b^4/a^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+
2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2
*c)-35/16/d*b^(3/2)/a/(a+b)^(7/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*ta
nh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+35/16/d*b^(3/2)/a/(a+b)^(7/2)*ln(-(a
+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2))-
7/4/d*b^(5/2)/a^2/(a+b)^(7/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1
/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+7/4/d*b^(5/2)/a^2/(a+b)^(7/2)*ln(-(a+b)^
(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2))-1/2/
d*b^(7/2)/a^3/(a+b)^(7/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d
*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/2/d*b^(7/2)/a^3/(a+b)^(7/2)*ln(-(a+b)^(1/2
)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2))-1/2/d/(a
+b)^3/tanh(1/2*d*x+1/2*c)-1/d/a^3*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.81232, size = 27173, normalized size = 149.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^10 + 160*
(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^9 + 1
6*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*sinh(d*x + c)^10 - 4*(8*a^5 - 1
3*a^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a
^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c)^8 - 4*(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3
 - 16*a*b^4 - 180*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^2
 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*sinh(d*x +
 c)^8 + 32*(60*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^3 -
(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*
b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 - 8*(16*a^5
 + 32*a^4*b - 13*a^3*b^2 - 43*a^2*b^3 - 68*a*b^4 - 24*b^5 - 4*(a^5 + 7*a^4*
b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^6 + 8*(4
20*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^4 - 16*a^5 - 32*
a^4*b + 13*a^3*b^2 + 43*a^2*b^3 + 68*a*b^4 + 24*b^5 + 4*(a^5 + 7*a^4*b + 23
*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x - 14*(8*a^5 - 13*a^3*b^2 - 36
*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b
^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 16*(252*(a^5 + 3*a^4*b + 3*a^3*
```

b^2 + a^2*b^3)*d*x*cosh(d*x + c)^5 - 14*(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c)^3 - 3*(16*a^5 + 32*a^4*b - 13*a^3*b^2 - 43*a^2*b^3 - 68*a*b^4 - 24*b^5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 32*a^5 - 52*a^3*b^2 - 24*a^2*b^3 - 8*(24*a^5 + 64*a^4*b + 64*a^3*b^2 + 15*a^2*b^3 + 60*a*b^4 + 24*b^5 + 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^4 + 8*(420*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^6 - 24*a^5 - 64*a^4*b - 64*a^3*b^2 - 15*a^2*b^3 - 60*a*b^4 - 24*b^5 - 35*(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c)^4 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x - 15*(16*a^5 + 32*a^4*b - 13*a^3*b^2 - 43*a^2*b^3 - 68*a*b^4 - 24*b^5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 32*(60*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^7 - 7*(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c)^5 - 5*(16*a^5 + 32*a^4*b - 13*a^3*b^2 - 43*a^2*b^3 - 68*a*b^4 - 24*b^5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^3 - (24*a^5 + 64*a^4*b + 64*a^3*b^2 + 15*a^2*b^3 + 60*a*b^4 + 24*b^5 + 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 16*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x - 8*(16*a^5 + 32*a^4*b + 13*a^3*b^2 + 43*a^2*b^3 + 16*a*b^4 + 2*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c)^2 + 8*(90*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^8 - 14*(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c)^6 - 16*a^5 - 32*a^4*b - 13*a^3*b^2 - 43*a^2*b^3 - 16*a*b^4 - 15*(16*a^5 + 32*a^4*b - 13*a^3*b^2 - 43*a^2*b^3 - 68*a*b^4 - 24*b^5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^4 - 2*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x - 6*(24*a^5 + 64*a^4*b + 64*a^3*b^2 + 15*a^2*b^3 + 60*a*b^4 + 24*b^5 + 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cosh(d*x + c)^10 + 10*(35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^9 + (35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*sinh(d*x + c)^10 + (105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*cosh(d*x + c)^8 + (105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4 + 45*(35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cosh(d*x + c)^3 + (105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*cosh(d*x + c)^6 + 2*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5 + 105*(35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cosh(d*x + c)^4 + 14*(105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cosh(d*x + c)^5 + 14*(105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*cosh(d*x + c)^3 + 3*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*cosh(d*x + c))*sinh(d*x + c)^5 - 35*a^4*b - 28*a^3*b^2 - 8*a^2*b^3 - 2*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*cosh(d*x + c)^4 + 2*(105*(35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cosh(d*x + c)^6 - 35*a^4*b - 168*a^3*b^2 - 400*a^2*b^3 - 256*a*b^4 - 64*b^5 + 35*(105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*cosh(d*x + c)^4 + 15*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*(35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cosh(d*x + c)^7 + 7*(105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*cosh(d*x + c)^5 + 5*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*cosh(d*x + c)^3 - (35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 - (105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*cosh(d*x + c)^2 + (45*(35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cosh(d*x + c)^8 + 28*(105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*cosh(d*x + c)^6 - 105*a^4*b - 364*a^3*b^2 - 248*a^2*b^3 - 64*a*b^4 + 30*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a *b^4 + 64*b^5)*cosh(d*x + c)^4 - 12*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*(35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cosh(d*x + c)^9 + 4*(105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*cosh(d*x + c)^7 + 6*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*cosh(d*x + c)^5 - 4*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*cosh(d*x + c)^3 - (105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 16*(10*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^9 - 2*(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c)^7 - 3*(16*a^5 + 32*a^4*b - 13*a^3*b^2 - 43*a^2*b^3 - 68*a*b^4 - 24*b^5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^5 - 2*(24*a^5 + 64*a^4*b + 64*a^3*b^2 + 15*a^2*b^3 + 60*a*b^4 + 24*b^5 + 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^3 - (16*a^5 + 32*a^4*b + 13*a^3*b^2 + 43*a^2*b^3 + 16*a*b^4 + 2*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^10 + 10*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*sinh(d*x + c)^10 + (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^8 + (45*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^2 + (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d)*sinh(d*x + c)^8 + 2*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^6 + 8*(15*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^3 + (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^4 + 14*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^2 + (a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d)*sinh(d*x + c)^6 - 2*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^4 + 4*(63*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^5 + 14*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^3 + 3*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^6 + 35*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^4 + 15*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^2 - (a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d)*sinh(d*x + c)^4 - (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^2 + 8*(15*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^7 + 7*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^5 + 5*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^3 - (a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (45*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^8 + 28*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^6 + 30*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^4 - 12*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^2 - (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d)*sinh(d*x + c)^2 - (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d + 2*(5*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^9 + 4*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^7 + 6*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3

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*b^5)*d*cosh(d*x + c)^5 - 4*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a
^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^3 - (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27
*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*(8*(a^5 + 3*a^4*
b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^10 + 80*(a^5 + 3*a^4*b + 3*a^3*b
^2 + a^2*b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^9 + 8*(a^5 + 3*a^4*b + 3*a^3*
b^2 + a^2*b^3)*d*x*sinh(d*x + c)^10 - 2*(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 -
16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*co
sh(d*x + c)^8 - 2*(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 180*(a^5 +
3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^2 - 4*(3*a^5 + 17*a^4*b +
33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*sinh(d*x + c)^8 + 16*(60*(a^5 + 3*a
^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^3 - (8*a^5 - 13*a^3*b^2 - 36*
a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^
4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(16*a^5 + 32*a^4*b - 13*a^3*b^2
- 43*a^2*b^3 - 68*a*b^4 - 24*b^5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b
^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^6 + 4*(420*(a^5 + 3*a^4*b + 3*a^3
*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^4 - 16*a^5 - 32*a^4*b + 13*a^3*b^2 + 43*a
^2*b^3 + 68*a*b^4 + 24*b^5 + 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 2
8*a*b^4 + 8*b^5)*d*x - 14*(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 4*(
3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c)^2)
*sinh(d*x + c)^6 + 8*(252*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*
x + c)^5 - 14*(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a
^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c)^3 - 3*(16*a^5
+ 32*a^4*b - 13*a^3*b^2 - 43*a^2*b^3 - 68*a*b^4 - 24*b^5 - 4*(a^5 + 7*a^4*b
 + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c))*sinh(d*x
 + c)^5 - 16*a^5 - 26*a^3*b^2 - 12*a^2*b^3 - 4*(24*a^5 + 64*a^4*b + 64*a^3*
b^2 + 15*a^2*b^3 + 60*a*b^4 + 24*b^5 + 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a
^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^4 + 4*(420*(a^5 + 3*a^4*b + 3
*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^6 - 24*a^5 - 64*a^4*b - 64*a^3*b^2 -
15*a^2*b^3 - 60*a*b^4 - 24*b^5 - 35*(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 - 16*a
*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d
*x + c)^4 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*
d*x - 15*(16*a^5 + 32*a^4*b - 13*a^3*b^2 - 43*a^2*b^3 - 68*a*b^4 - 24*b^5 -
 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d
*x + c)^2)*sinh(d*x + c)^4 + 16*(60*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d
*x*cosh(d*x + c)^7 - 7*(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 4*(3*a
^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c)^5 - 5
*(16*a^5 + 32*a^4*b - 13*a^3*b^2 - 43*a^2*b^3 - 68*a*b^4 - 24*b^5 - 4*(a^5
+ 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^
3 - (24*a^5 + 64*a^4*b + 64*a^3*b^2 + 15*a^2*b^3 + 60*a*b^4 + 24*b^5 + 4*(a
^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x +
c))*sinh(d*x + c)^3 - 8*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x - 4*(16*a
^5 + 32*a^4*b + 13*a^3*b^2 + 43*a^2*b^3 + 16*a*b^4 + 2*(3*a^5 + 17*a^4*b +
33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c)^2 + 4*(90*(a^5 + 3*a^
4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^8 - 14*(8*a^5 - 13*a^3*b^2 - 3
6*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*
b^4)*d*x)*cosh(d*x + c)^6 - 16*a^5 - 32*a^4*b - 13*a^3*b^2 - 43*a^2*b^3 - 1
6*a*b^4 - 15*(16*a^5 + 32*a^4*b - 13*a^3*b^2 - 43*a^2*b^3 - 68*a*b^4 - 24*b
^5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*co
sh(d*x + c)^4 - 2*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*
x - 6*(24*a^5 + 64*a^4*b + 64*a^3*b^2 + 15*a^2*b^3 + 60*a*b^4 + 24*b^5 + 4*
(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x
+ c)^2)*sinh(d*x + c)^2 - ((35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cosh(d*x + c
)^10 + 10*(35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^9
 + (35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*sinh(d*x + c)^10 + (105*a^4*b + 364*
a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*cosh(d*x + c)^8 + (105*a^4*b + 364*a^3*b^
2 + 248*a^2*b^3 + 64*a*b^4 + 45*(35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cosh(d*
x + c)^2)*sinh(d*x + c)^8 + 8*(15*(35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cosh(
d*x + c)^3 + (105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*cosh(d*x +
c))*sinh(d*x + c)^7 + 2*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 +
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64*b^5)*cosh(d*x + c)^6 + 2*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*
b^4 + 64*b^5 + 105*(35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cosh(d*x + c)^4 + 14
*(105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*cosh(d*x + c)^2)*sinh(d
x + c)^6 + 4(63*(35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cosh(d*x + c)^5 + 14*
(105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*cosh(d*x + c)^3 + 3*(35*
a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*cosh(d*x + c))*sinh
(d*x + c)^5 - 35*a^4*b - 28*a^3*b^2 - 8*a^2*b^3 - 2*(35*a^4*b + 168*a^3*b^2
 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*cosh(d*x + c)^4 + 2*(105*(35*a^4*b + 2
8*a^3*b^2 + 8*a^2*b^3)*cosh(d*x + c)^6 - 35*a^4*b - 168*a^3*b^2 - 400*a^2*b
^3 - 256*a*b^4 - 64*b^5 + 35*(105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*
b^4)*cosh(d*x + c)^4 + 15*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4
 + 64*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*(35*a^4*b + 28*a^3*b^2
+ 8*a^2*b^3)*cosh(d*x + c)^7 + 7*(105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 6
4*a*b^4)*cosh(d*x + c)^5 + 5*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*
b^4 + 64*b^5)*cosh(d*x + c)^3 - (35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256
*a*b^4 + 64*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 - (105*a^4*b + 364*a^3*b^2
+ 248*a^2*b^3 + 64*a*b^4)*cosh(d*x + c)^2 + (45*(35*a^4*b + 28*a^3*b^2 + 8*
a^2*b^3)*cosh(d*x + c)^8 + 28*(105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a
*b^4)*cosh(d*x + c)^6 - 105*a^4*b - 364*a^3*b^2 - 248*a^2*b^3 - 64*a*b^4 +
30*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*cosh(d*x + c
)^4 - 12*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*cosh(d
*x + c)^2)*sinh(d*x + c)^2 + 2*(5*(35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cosh(
d*x + c)^9 + 4*(105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*cosh(d*x
+ c)^7 + 6*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*cosh
(d*x + c)^5 - 4*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)
*cosh(d*x + c)^3 - (105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*cosh(
d*x + c))*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2
*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a +
b))/b) + 8*(10*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^9 -
2*(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^
3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c)^7 - 3*(16*a^5 + 32*a^4*b -
 13*a^3*b^2 - 43*a^2*b^3 - 68*a*b^4 - 24*b^5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^
2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^5 - 2*(24*a^5 + 64*a^
4*b + 64*a^3*b^2 + 15*a^2*b^3 + 60*a*b^4 + 24*b^5 + 4*(a^5 + 7*a^4*b + 23*a
^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^3 - (16*a^5 + 32
*a^4*b + 13*a^3*b^2 + 43*a^2*b^3 + 16*a*b^4 + 2*(3*a^5 + 17*a^4*b + 33*a^3*
b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^8 + 3*a^
7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^10 + 10*(a^8 + 3*a^7*b + 3*a^6*b
^2 + a^5*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^8 + 3*a^7*b + 3*a^6*b^2
+ a^5*b^3)*d*sinh(d*x + c)^10 + (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3
 + 8*a^4*b^4)*d*cosh(d*x + c)^8 + (45*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)
*d*cosh(d*x + c)^2 + (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^
4)*d)*sinh(d*x + c)^8 + 2*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4
*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^6 + 8*(15*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^
5*b^3)*d*cosh(d*x + c)^3 + (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*
a^4*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^8 + 3*a^7*b + 3*a^6*b
^2 + a^5*b^3)*d*cosh(d*x + c)^4 + 14*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^
5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^2 + (a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5
*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d)*sinh(d*x + c)^6 - 2*(a^8 + 7*a^7*b + 23*a
^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^4 + 4*(63*(a^
8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^5 + 14*(3*a^8 + 17*a^7*b
 + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^3 + 3*(a^8 + 7*a^7*
b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c))*sinh
(d*x + c)^5 + 2*(105*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^
6 + 35*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x
+ c)^4 + 15*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b
^5)*d*cosh(d*x + c)^2 - (a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b
^4 + 8*a^3*b^5)*d)*sinh(d*x + c)^4 - (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^
5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^2 + 8*(15*(a^8 + 3*a^7*b + 3*a^6*b^2 + a

```
^5*b^3)*d*cosh(d*x + c)^7 + 7*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 +
 8*a^4*b^4)*d*cosh(d*x + c)^5 + 5*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3
+ 28*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^3 - (a^8 + 7*a^7*b + 23*a^6*b^2 +
 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (4
5*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^8 + 28*(3*a^8 + 17*
a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^6 + 30*(a^8 +
7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)
^4 - 12*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*
d*cosh(d*x + c)^2 - (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4
)*d)*sinh(d*x + c)^2 - (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d + 2*(5*(a^8
+ 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^9 + 4*(3*a^8 + 17*a^7*b +
33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^7 + 6*(a^8 + 7*a^7*b +
 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^5 - 4*(a
^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x
 + c)^3 - (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*cosh(d
*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.58015, size = 543, normalized size = 2.98

$$-\frac{\frac{\left(35\,a^2b^2e^{(2\,c)}+28\,ab^3e^{(2\,c)}+8\,b^4e^{(2\,c)}\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)e^{(-2\,c)}}{\left(a^6+3\,a^5b+3\,a^4b^2+a^3b^3\right)\sqrt{-ab-b^2}}-\frac{8\,dx}{a^3}-\frac{2\left(13\,a^3b^2e^{(6\,dx+6\,c)}+36\,a^2b^3e^{(6\,dx+6\,c)}+16\,ab^4e^{(6\,dx+6\,c)}+39\,a^3b^2e^{(4\,dx+4\,c)}+\cdots\right.}{\left(a^6+3\,a^5\cdots\right.}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((35*a^2*b^2*e^(2*c) + 28*a*b^3*e^(2*c) + 8*b^4*e^(2*c))*arctan(1/2*(a
*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))*e^(-2*c)/((a^6 + 3*a^5*b + 3*
a^4*b^2 + a^3*b^3)*sqrt(-a*b - b^2)) - 8*d*x/a^3 - 2*(13*a^3*b^2*e^(6*d*x +
 6*c) + 36*a^2*b^3*e^(6*d*x + 6*c) + 16*a*b^4*e^(6*d*x + 6*c) + 39*a^3*b^2*
e^(4*d*x + 4*c) + 122*a^2*b^3*e^(4*d*x + 4*c) + 152*a*b^4*e^(4*d*x + 4*c) +
 48*b^5*e^(4*d*x + 4*c) + 39*a^3*b^2*e^(2*d*x + 2*c) + 92*a^2*b^3*e^(2*d*x
+ 2*c) + 32*a*b^4*e^(2*d*x + 2*c) + 13*a^3*b^2 + 6*a^2*b^3)/((a^6 + 3*a^5*b
 + 3*a^4*b^2 + a^3*b^3)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2
*d*x + 2*c) + a)^2) + 16/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(e^(2*d*x + 2*c)
- 1)))/d
```

3.167 $\int \frac{\coth^3(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=152

$$-\frac{b^4}{4a^3d(a+b)^2\left(a\cosh^2(c+dx)+b\right)^2}+\frac{b^3(2a+b)}{a^3d(a+b)^3\left(a\cosh^2(c+dx)+b\right)}+\frac{b^2\left(6a^2+4ab+b^2\right)\log\left(a\cosh^2(c+dx)+b\right.}{2a^3d(a+b)^4}$$

```
[Out] -b^4/(4*a^3*(a + b)^2*d*(b + a*Cosh[c + d*x]^2)^2) + (b^3*(2*a + b))/(a^3*(
a + b)^3*d*(b + a*Cosh[c + d*x]^2)) - Csch[c + d*x]^2/(2*(a + b)^3*d) + (b^
2*(6*a^2 + 4*a*b + b^2)*Log[b + a*Cosh[c + d*x]^2])/(2*a^3*(a + b)^4*d) + (
(a + 4*b)*Log[Sinh[c + d*x]])/((a + b)^4*d)
```

Rubi [A] time = 0.242991, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4138, 446, 88}

$$-\frac{b^4}{4a^3d(a+b)^2\left(a\cosh^2(c+dx)+b\right)^2}+\frac{b^3(2a+b)}{a^3d(a+b)^3\left(a\cosh^2(c+dx)+b\right)}+\frac{b^2\left(6a^2+4ab+b^2\right)\log\left(a\cosh^2(c+dx)+b\right.}{2a^3d(a+b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -b^4/(4*a^3*(a + b)^2*d*(b + a*Cosh[c + d*x]^2)^2) + (b^3*(2*a + b))/(a^3*(
a + b)^3*d*(b + a*Cosh[c + d*x]^2)) - Csch[c + d*x]^2/(2*(a + b)^3*d) + (b^
2*(6*a^2 + 4*a*b + b^2)*Log[b + a*Cosh[c + d*x]^2])/(2*a^3*(a + b)^4*d) + (
(a + 4*b)*Log[Sinh[c + d*x]])/((a + b)^4*d)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\coth^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^9}{\left(1-x^2\right)^2\left(b+ax^2\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4}{(1-x)^2(b+ax)^3}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)^3(-1+x)^2} + \frac{a+4b}{(a+b)^4(-1+x)} + \frac{b^4}{a^2(a+b)^2(b+ax)^3} - \frac{2b^3(2a+b)}{a^2(a+b)^3(b+ax)^2} + \frac{b^2\left(6a^2+4ab+b^2\right)}{a^2(a+b)^4(b+ax)}\right)\right.}{2d}$$

$$= -\frac{b^4}{4a^3(a+b)^2d\left(b+a\cosh^2(c+dx)\right)^2} + \frac{b^3(2a+b)}{a^3(a+b)^3d\left(b+a\cosh^2(c+dx)\right)} - \frac{\text{csch}^2(c+}{2(a+b)}$$

Mathematica [A] time = 1.93658, size = 172, normalized size = 1.13

$$-\frac{\text{sech}^6(c+dx)(a\cosh(2(c+dx))+a+2b)^3\left(\frac{b^4(a+b)^2}{a^3\left(a\sinh^2(c+dx)+a+b\right)^2} - \frac{4b^3(a+b)(2a+b)}{a^3\left(a\sinh^2(c+dx)+a+b\right)} - \frac{2b^2\left(6a^2+4ab+b^2\right)\log\left(a\sinh^2(c+dx)+\right.}{a^3}\right.}{32d(a+b)^4\left(a+b\text{sech}^2(c+dx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -((a + 2*b + a*Cosh[2*(c + d*x)])^3*Sech[c + d*x]^6*(2*(a + b)*Csch[c + d*x
]^2 - 4*(a + 4*b)*Log[Sinh[c + d*x]] - (2*b^2*(6*a^2 + 4*a*b + b^2)*Log[a +
 b + a*Sinh[c + d*x]^2])/a^3 + (b^4*(a + b)^2)/(a^3*(a + b + a*Sinh[c + d*x
]^2)^2) - (4*b^3*(a + b)*(2*a + b))/(a^3*(a + b + a*Sinh[c + d*x]^2))))/(32
*(a + b)^4*d*(a + b*Sech[c + d*x]^2)^3)
```

Maple [B] time = 0.123, size = 1128, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -1/8/d*tanh(1/2*d*x+1/2*c)^2/(a^3+3*a^2*b+3*a*b^2+b^3)-1/d/a^3*ln(tanh(1/2*
d*x+1/2*c)+1)-8/d*b^3/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c
)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x
+1/2*c)^6-10/d*b^4/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4
+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a*tanh(1/2*d*x+
1/2*c)^6-2/d*b^5/(a+b)^4/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)
^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+
1/2*c)^6-16/d*b^3/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+
2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2
*c)^4+8/d*b^4/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*ta
nh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a*tanh(1/2*d*x+1/2*c
)^4+4/d*b^5/(a+b)^4/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*
tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c
)^4-8/d*b^3/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh
(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^2-
```

```
10/d*b^4/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/
2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a*tanh(1/2*d*x+1/2*c)^2-2
/d*b^5/(a+b)^4/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(
1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^2+3
/d*b^2/(a+b)^4/a*ln(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(
1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+2/d*b^3/(a+b)^4/a^2*ln(ta
nh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*t
anh(1/2*d*x+1/2*c)^2*b+a+b)+1/2/d*b^4/(a+b)^4/a^3*ln(tanh(1/2*d*x+1/2*c)^4*
a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2
*b+a+b)-1/8/d/(a+b)^3/tanh(1/2*d*x+1/2*c)^2+1/d/(a+b)^4*ln(tanh(1/2*d*x+1/2
*c))*a+4/d/(a+b)^4*ln(tanh(1/2*d*x+1/2*c))*b-1/d/a^3*ln(tanh(1/2*d*x+1/2*c)
-1)
```

Maxima [B] time = 1.46772, size = 934, normalized size = 6.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(6*a^2*b^2 + 4*a*b^3 + b^4)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*
d*x - 4*c) + a)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d) + (a
+ 4*b)*log(e^(-d*x - c) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d
) + (a + 4*b)*log(e^(-d*x - c) - 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
 b^4)*d) - 2*((a^5 - 4*a^2*b^3 - 2*a*b^4)*e^(-2*d*x - 2*c) + 2*(2*a^5 + 4*a
^4*b - 7*a*b^4 - 3*b^5)*e^(-4*d*x - 4*c) + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2 +
 4*a^2*b^3 + 16*a*b^4 + 6*b^5)*e^(-6*d*x - 6*c) + 2*(2*a^5 + 4*a^4*b - 7*a*
b^4 - 3*b^5)*e^(-8*d*x - 8*c) + (a^5 - 4*a^2*b^3 - 2*a*b^4)*e^(-10*d*x - 10
*c))/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3 + 2*(a^8 + 7*a^7*b + 15*a^6*b^2
+ 13*a^5*b^3 + 4*a^4*b^4)*e^(-2*d*x - 2*c) - (a^8 + 3*a^7*b - 13*a^6*b^2 -
47*a^5*b^3 - 48*a^4*b^4 - 16*a^3*b^5)*e^(-4*d*x - 4*c) - 4*(a^8 + 7*a^7*b +
 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*e^(-6*d*x - 6*c) - (a^8
+ 3*a^7*b - 13*a^6*b^2 - 47*a^5*b^3 - 48*a^4*b^4 - 16*a^3*b^5)*e^(-8*d*x -
8*c) + 2*(a^8 + 7*a^7*b + 15*a^6*b^2 + 13*a^5*b^3 + 4*a^4*b^4)*e^(-10*d*x -
 10*c) + (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*e^(-12*d*x - 12*c))*d) + (d*
x + c)/(a^3*d)
```

Fricas [B] time = 9.42447, size = 23458, normalized size = 154.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x + c)
^12 + 24*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x + c
)*sinh(d*x + c)^11 + 2*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*
x*sinh(d*x + c)^12 + 4*(a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + (a^
6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*cosh(d*x
 + c)^10 + 4*(a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + 33*(a^6 + 4*a
^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x + c)^2 + (a^6 + 8*a^5*
b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*sinh(d*x + c)^10 +
 40*(11*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x + c)
```

^3 + (a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + (a^6 + 8*a^5*b + 22*a
^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*cosh(d*x + c))*sinh(d*x +
c)^9 + 2*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a*b^5 - 12*b^6 -
(a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*
d*x)*cosh(d*x + c)^8 + 2*(495*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*
b^4)*d*x*cosh(d*x + c)^4 + 8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*
a*b^5 - 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64
*a*b^5 - 16*b^6)*d*x + 90*(a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 +
(a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*cosh(
d*x + c)^2)*sinh(d*x + c)^8 + 16*(99*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3
 + a^2*b^4)*d*x*cosh(d*x + c)^5 + 30*(a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 -
 2*a*b^5 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)
*d*x)*cosh(d*x + c)^3 + (8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a*
b^5 - 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a
*b^5 - 16*b^6)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 + 8*(3*a^6 + 11*a^5*b +
16*a^4*b^2 + 12*a^3*b^3 + 20*a^2*b^4 + 22*a*b^5 + 6*b^6 - (a^6 + 8*a^5*b +
30*a^4*b^2 + 60*a^3*b^3 + 65*a^2*b^4 + 36*a*b^5 + 8*b^6)*d*x)*cosh(d*x + c)
^6 + 8*(231*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x
+ c)^6 + 3*a^6 + 11*a^5*b + 16*a^4*b^2 + 12*a^3*b^3 + 20*a^2*b^4 + 22*a*b^5
 + 6*b^6 + 105*(a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + (a^6 + 8*a^
5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*cosh(d*x + c)^4
- (a^6 + 8*a^5*b + 30*a^4*b^2 + 60*a^3*b^3 + 65*a^2*b^4 + 36*a*b^5 + 8*b^6)
*d*x + 7*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a*b^5 - 12*b^6 -
(a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*
d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 16*(99*(a^6 + 4*a^5*b + 6*a^4*b^2 +
 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x + c)^7 + 63*(a^6 + a^5*b - 4*a^3*b^3 - 6
*a^2*b^4 - 2*a*b^5 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4
+ 4*a*b^5)*d*x)*cosh(d*x + c)^5 + 7*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2
*b^4 - 40*a*b^5 - 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^
2*b^4 - 64*a*b^5 - 16*b^6)*d*x)*cosh(d*x + c)^3 + 3*(3*a^6 + 11*a^5*b + 16*
a^4*b^2 + 12*a^3*b^3 + 20*a^2*b^4 + 22*a*b^5 + 6*b^6 - (a^6 + 8*a^5*b + 30*
a^4*b^2 + 60*a^3*b^3 + 65*a^2*b^4 + 36*a*b^5 + 8*b^6)*d*x)*cosh(d*x + c))*s
inh(d*x + c)^5 + 2*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a*b^5 -
 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5
- 16*b^6)*d*x)*cosh(d*x + c)^4 + 2*(495*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*
b^3 + a^2*b^4)*d*x*cosh(d*x + c)^8 + 420*(a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b
^4 - 2*a*b^5 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*
b^5)*d*x)*cosh(d*x + c)^6 + 8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40
*a*b^5 - 12*b^6 + 70*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a*b^5
 - 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a*b^
5 - 16*b^6)*d*x)*cosh(d*x + c)^4 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3
 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*d*x + 60*(3*a^6 + 11*a^5*b + 16*a^4*b^2
+ 12*a^3*b^3 + 20*a^2*b^4 + 22*a*b^5 + 6*b^6 - (a^6 + 8*a^5*b + 30*a^4*b^2
+ 60*a^3*b^3 + 65*a^2*b^4 + 36*a*b^5 + 8*b^6)*d*x)*cosh(d*x + c)^2)*sinh(d*
x + c)^4 + 8*(55*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh
(d*x + c)^9 + 60*(a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + (a^6 + 8*
a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*cosh(d*x + c)^
7 + 14*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a*b^5 - 12*b^6 - (a
^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*d*
x)*cosh(d*x + c)^5 + 20*(3*a^6 + 11*a^5*b + 16*a^4*b^2 + 12*a^3*b^3 + 20*a^
2*b^4 + 22*a*b^5 + 6*b^6 - (a^6 + 8*a^5*b + 30*a^4*b^2 + 60*a^3*b^3 + 65*a^
2*b^4 + 36*a*b^5 + 8*b^6)*d*x)*cosh(d*x + c)^3 + (8*a^6 + 24*a^5*b + 16*a^4
*b^2 - 28*a^2*b^4 - 40*a*b^5 - 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^
3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3
 + 2*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x + 4*(a^6 + a^5*b
 - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b
^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*cosh(d*x + c)^2 + 4*(33*(a^6 + 4*a^5*b + 6*
a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x + c)^10 + 45*(a^6 + a^5*b - 4*a
^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 1

7*a^2*b^4 + 4*a*b^5)*d*x)*cosh(d*x + c)^8 + 14*(8*a^6 + 24*a^5*b + 16*a^4*b
^2 - 28*a^2*b^4 - 40*a*b^5 - 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*
b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*d*x)*cosh(d*x + c)^6 + a^6 + a^5*b -
4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + 30*(3*a^6 + 11*a^5*b + 16*a^4*b^2 + 12*a^
3*b^3 + 20*a^2*b^4 + 22*a*b^5 + 6*b^6 - (a^6 + 8*a^5*b + 30*a^4*b^2 + 60*a^
3*b^3 + 65*a^2*b^4 + 36*a*b^5 + 8*b^6)*d*x)*cosh(d*x + c)^4 + (a^6 + 8*a^5*
b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x + 3*(8*a^6 + 24*a^5
*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a*b^5 - 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*
b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*d*x)*cosh(d*x + c)^2)*si
nh(d*x + c)^2 - ((6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cosh(d*x + c)^12 + 12*(6
*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cosh(d*x + c)*sinh(d*x + c)^11 + (6*a^4*b^2
 + 4*a^3*b^3 + a^2*b^4)*sinh(d*x + c)^12 + 2*(6*a^4*b^2 + 28*a^3*b^3 + 17*a
^2*b^4 + 4*a*b^5)*cosh(d*x + c)^10 + 2*(6*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4
 + 4*a*b^5 + 33*(6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cosh(d*x + c)^2)*sinh(d*x
 + c)^10 + 20*(11*(6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cosh(d*x + c)^3 + (6*a^
4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^9 -
 (6*a^4*b^2 + 4*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*cosh(d*x + c)^8 -
 (6*a^4*b^2 + 4*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6 - 495*(6*a^4*b^2 +
 4*a^3*b^3 + a^2*b^4)*cosh(d*x + c)^4 - 90*(6*a^4*b^2 + 28*a^3*b^3 + 17*a^2
*b^4 + 4*a*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(99*(6*a^4*b^2 + 4*a^3
*b^3 + a^2*b^4)*cosh(d*x + c)^5 + 30*(6*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 +
 4*a*b^5)*cosh(d*x + c)^3 - (6*a^4*b^2 + 4*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5
- 16*b^6)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(6*a^4*b^2 + 28*a^3*b^3 + 65*a
^2*b^4 + 36*a*b^5 + 8*b^6)*cosh(d*x + c)^6 + 4*(231*(6*a^4*b^2 + 4*a^3*b^3
+ a^2*b^4)*cosh(d*x + c)^6 - 6*a^4*b^2 - 28*a^3*b^3 - 65*a^2*b^4 - 36*a*b^5
 - 8*b^6 + 105*(6*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*cosh(d*x + c
)^4 - 7*(6*a^4*b^2 + 4*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*cosh(d*x +
 c)^2)*sinh(d*x + c)^6 + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4 + 8*(99*(6*a^4*b^2
 + 4*a^3*b^3 + a^2*b^4)*cosh(d*x + c)^7 + 63*(6*a^4*b^2 + 28*a^3*b^3 + 17*a
^2*b^4 + 4*a*b^5)*cosh(d*x + c)^5 - 7*(6*a^4*b^2 + 4*a^3*b^3 - 95*a^2*b^4 -
 64*a*b^5 - 16*b^6)*cosh(d*x + c)^3 - 3*(6*a^4*b^2 + 28*a^3*b^3 + 65*a^2*b^
4 + 36*a*b^5 + 8*b^6)*cosh(d*x + c))*sinh(d*x + c)^5 - (6*a^4*b^2 + 4*a^3*b
^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*cosh(d*x + c)^4 + (495*(6*a^4*b^2 + 4*
a^3*b^3 + a^2*b^4)*cosh(d*x + c)^8 + 420*(6*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b
^4 + 4*a*b^5)*cosh(d*x + c)^6 - 6*a^4*b^2 - 4*a^3*b^3 + 95*a^2*b^4 + 64*a*b
^5 + 16*b^6 - 70*(6*a^4*b^2 + 4*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*c
osh(d*x + c)^4 - 60*(6*a^4*b^2 + 28*a^3*b^3 + 65*a^2*b^4 + 36*a*b^5 + 8*b^6
)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(55*(6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4
)*cosh(d*x + c)^9 + 60*(6*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*cosh
(d*x + c)^7 - 14*(6*a^4*b^2 + 4*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*c
osh(d*x + c)^5 - 20*(6*a^4*b^2 + 28*a^3*b^3 + 65*a^2*b^4 + 36*a*b^5 + 8*b^6
)*cosh(d*x + c)^3 - (6*a^4*b^2 + 4*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6
)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(6*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 +
 4*a*b^5)*cosh(d*x + c)^2 + 2*(33*(6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cosh(d*
x + c)^10 + 45*(6*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*cosh(d*x + c
)^8 - 14*(6*a^4*b^2 + 4*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*cosh(d*x
+ c)^6 + 6*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5 - 30*(6*a^4*b^2 + 28
*a^3*b^3 + 65*a^2*b^4 + 36*a*b^5 + 8*b^6)*cosh(d*x + c)^4 - 3*(6*a^4*b^2 +
4*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*cosh(d*x + c)^2)*sinh(d*x + c)^
2 + 4*(3*(6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cosh(d*x + c)^11 + 5*(6*a^4*b^2
+ 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*cosh(d*x + c)^9 - 2*(6*a^4*b^2 + 4*a^3
*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*cosh(d*x + c)^7 - 6*(6*a^4*b^2 + 28*
a^3*b^3 + 65*a^2*b^4 + 36*a*b^5 + 8*b^6)*cosh(d*x + c)^5 - (6*a^4*b^2 + 4*a
^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*cosh(d*x + c)^3 + (6*a^4*b^2 + 28*
a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*cosh(d*x + c))*sinh(d*x + c))*log(2*(a*cosh
(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x +
c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 2*((a^6 + 4*a^5*b)*cosh(d*x + c)^12
+ 12*(a^6 + 4*a^5*b)*cosh(d*x + c)*sinh(d*x + c)^11 + (a^6 + 4*a^5*b)*sinh(
d*x + c)^12 + 2*(a^6 + 8*a^5*b + 16*a^4*b^2)*cosh(d*x + c)^10 + 2*(a^6 + 8*

a^5*b + 16*a^4*b^2 + 33*(a^6 + 4*a^5*b)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 20*(11*(a^6 + 4*a^5*b)*cosh(d*x + c)^3 + (a^6 + 8*a^5*b + 16*a^4*b^2)*cosh (d*x + c))*sinh(d*x + c)^9 - (a^6 + 4*a^5*b - 16*a^4*b^2 - 64*a^3*b^3)*cosh (d*x + c)^8 - (a^6 + 4*a^5*b - 16*a^4*b^2 - 64*a^3*b^3 - 495*(a^6 + 4*a^5*b)*cosh(d*x + c)^4 - 90*(a^6 + 8*a^5*b + 16*a^4*b^2)*cosh(d*x + c)^2)*sinh(d *x + c)^8 + 8*(99*(a^6 + 4*a^5*b)*cosh(d*x + c)^5 + 30*(a^6 + 8*a^5*b + 16* a^4*b^2)*cosh(d*x + c)^3 - (a^6 + 4*a^5*b - 16*a^4*b^2 - 64*a^3*b^3)*cosh(d *x + c))*sinh(d*x + c)^7 - 4*(a^6 + 8*a^5*b + 24*a^4*b^2 + 32*a^3*b^3)*cosh (d*x + c)^6 + 4*(231*(a^6 + 4*a^5*b)*cosh(d*x + c)^6 - a^6 - 8*a^5*b - 24*a ^4*b^2 - 32*a^3*b^3 + 105*(a^6 + 8*a^5*b + 16*a^4*b^2)*cosh(d*x + c)^4 - 7* (a^6 + 4*a^5*b - 16*a^4*b^2 - 64*a^3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + a^6 + 4*a^5*b + 8*(99*(a^6 + 4*a^5*b)*cosh(d*x + c)^7 + 63*(a^6 + 8*a^5*b + 16*a^4*b^2)*cosh(d*x + c)^5 - 7*(a^6 + 4*a^5*b - 16*a^4*b^2 - 64*a^3*b^3)*cosh(d*x + c)^3 - 3*(a^6 + 8*a^5*b + 24*a^4*b^2 + 32*a^3*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - (a^6 + 4*a^5*b - 16*a^4*b^2 - 64*a^3*b^3)*cosh(d*x + c)^4 + (495*(a^6 + 4*a^5*b)*cosh(d*x + c)^8 + 420*(a^6 + 8*a^5*b + 16*a^4*b ^2)*cosh(d*x + c)^6 - a^6 - 4*a^5*b + 16*a^4*b^2 + 64*a^3*b^3 - 70*(a^6 + 4 *a^5*b - 16*a^4*b^2 - 64*a^3*b^3)*cosh(d*x + c)^4 - 60*(a^6 + 8*a^5*b + 24* a^4*b^2 + 32*a^3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(55*(a^6 + 4*a^5 *b)*cosh(d*x + c)^9 + 60*(a^6 + 8*a^5*b + 16*a^4*b^2)*cosh(d*x + c)^7 - 14* (a^6 + 4*a^5*b - 16*a^4*b^2 - 64*a^3*b^3)*cosh(d*x + c)^5 - 20*(a^6 + 8*a^5 *b + 24*a^4*b^2 + 32*a^3*b^3)*cosh(d*x + c)^3 - (a^6 + 4*a^5*b - 16*a^4*b^2 - 64*a^3*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(a^6 + 8*a^5*b + 16*a^4*b ^2)*cosh(d*x + c)^2 + 2*(33*(a^6 + 4*a^5*b)*cosh(d*x + c)^10 + 45*(a^6 + 8* a^5*b + 16*a^4*b^2)*cosh(d*x + c)^8 - 14*(a^6 + 4*a^5*b - 16*a^4*b^2 - 64*a ^3*b^3)*cosh(d*x + c)^6 + a^6 + 8*a^5*b + 16*a^4*b^2 - 30*(a^6 + 8*a^5*b + 24*a^4*b^2 + 32*a^3*b^3)*cosh(d*x + c)^4 - 3*(a^6 + 4*a^5*b - 16*a^4*b^2 - 64*a^3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(3*(a^6 + 4*a^5*b)*cosh(d* x + c)^11 + 5*(a^6 + 8*a^5*b + 16*a^4*b^2)*cosh(d*x + c)^9 - 2*(a^6 + 4*a^5 *b - 16*a^4*b^2 - 64*a^3*b^3)*cosh(d*x + c)^7 - 6*(a^6 + 8*a^5*b + 24*a^4*b ^2 + 32*a^3*b^3)*cosh(d*x + c)^5 - (a^6 + 4*a^5*b - 16*a^4*b^2 - 64*a^3*b^3)*cosh(d*x + c)^3 + (a^6 + 8*a^5*b + 16*a^4*b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(3*(a^6 + 4*a^ 5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x + c)^11 + 5*(a^6 + a^5* b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3* b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*cosh(d*x + c)^9 + 2*(8*a^6 + 24*a^5*b + 16 *a^4*b^2 - 28*a^2*b^4 - 40*a*b^5 - 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 6 0*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*d*x)*cosh(d*x + c)^7 + 6*(3*a^6 + 11*a^5*b + 16*a^4*b^2 + 12*a^3*b^3 + 20*a^2*b^4 + 22*a*b^5 + 6*b^6 - (a^ 6 + 8*a^5*b + 30*a^4*b^2 + 60*a^3*b^3 + 65*a^2*b^4 + 36*a*b^5 + 8*b^6)*d*x) *cosh(d*x + c)^5 + (8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a*b^5 - 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*d*x)*cosh(d*x + c)^3 + (a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a *b^5 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a ^5*b^4)*d*cosh(d*x + c)^12 + 12*(a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^ 5*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^11 + (a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^ 6*b^3 + a^5*b^4)*d*sinh(d*x + c)^12 + 2*(a^9 + 8*a^8*b + 22*a^7*b^2 + 28*a^ 6*b^3 + 17*a^5*b^4 + 4*a^4*b^5)*d*cosh(d*x + c)^10 + 2*(33*(a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*d*cosh(d*x + c)^2 + (a^9 + 8*a^8*b + 22*a ^7*b^2 + 28*a^6*b^3 + 17*a^5*b^4 + 4*a^4*b^5)*d)*sinh(d*x + c)^10 - (a^9 + 4*a^8*b - 10*a^7*b^2 - 60*a^6*b^3 - 95*a^5*b^4 - 64*a^4*b^5 - 16*a^3*b^6)*d *cosh(d*x + c)^8 + 20*(11*(a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4) *d*cosh(d*x + c)^3 + (a^9 + 8*a^8*b + 22*a^7*b^2 + 28*a^6*b^3 + 17*a^5*b^4 + 4*a^4*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^9 + (495*(a^9 + 4*a^8*b + 6*a^7 *b^2 + 4*a^6*b^3 + a^5*b^4)*d*cosh(d*x + c)^4 + 90*(a^9 + 8*a^8*b + 22*a^7* b^2 + 28*a^6*b^3 + 17*a^5*b^4 + 4*a^4*b^5)*d*cosh(d*x + c)^2 - (a^9 + 4*a^8 *b - 10*a^7*b^2 - 60*a^6*b^3 - 95*a^5*b^4 - 64*a^4*b^5 - 16*a^3*b^6)*d)*sin h(d*x + c)^8 - 4*(a^9 + 8*a^8*b + 30*a^7*b^2 + 60*a^6*b^3 + 65*a^5*b^4 + 36

```
*a^4*b^5 + 8*a^3*b^6)*d*cosh(d*x + c)^6 + 8*(99*(a^9 + 4*a^8*b + 6*a^7*b^2
+ 4*a^6*b^3 + a^5*b^4)*d*cosh(d*x + c)^5 + 30*(a^9 + 8*a^8*b + 22*a^7*b^2 +
 28*a^6*b^3 + 17*a^5*b^4 + 4*a^4*b^5)*d*cosh(d*x + c)^3 - (a^9 + 4*a^8*b -
10*a^7*b^2 - 60*a^6*b^3 - 95*a^5*b^4 - 64*a^4*b^5 - 16*a^3*b^6)*d*cosh(d*x
+ c))*sinh(d*x + c)^7 + 4*(231*(a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5
*b^4)*d*cosh(d*x + c)^6 + 105*(a^9 + 8*a^8*b + 22*a^7*b^2 + 28*a^6*b^3 + 17
*a^5*b^4 + 4*a^4*b^5)*d*cosh(d*x + c)^4 - 7*(a^9 + 4*a^8*b - 10*a^7*b^2 - 6
0*a^6*b^3 - 95*a^5*b^4 - 64*a^4*b^5 - 16*a^3*b^6)*d*cosh(d*x + c)^2 - (a^9
+ 8*a^8*b + 30*a^7*b^2 + 60*a^6*b^3 + 65*a^5*b^4 + 36*a^4*b^5 + 8*a^3*b^6)*
d)*sinh(d*x + c)^6 - (a^9 + 4*a^8*b - 10*a^7*b^2 - 60*a^6*b^3 - 95*a^5*b^4
- 64*a^4*b^5 - 16*a^3*b^6)*d*cosh(d*x + c)^4 + 8*(99*(a^9 + 4*a^8*b + 6*a^7
*b^2 + 4*a^6*b^3 + a^5*b^4)*d*cosh(d*x + c)^7 + 63*(a^9 + 8*a^8*b + 22*a^7*
b^2 + 28*a^6*b^3 + 17*a^5*b^4 + 4*a^4*b^5)*d*cosh(d*x + c)^5 - 7*(a^9 + 4*a
^8*b - 10*a^7*b^2 - 60*a^6*b^3 - 95*a^5*b^4 - 64*a^4*b^5 - 16*a^3*b^6)*d*co
sh(d*x + c)^3 - 3*(a^9 + 8*a^8*b + 30*a^7*b^2 + 60*a^6*b^3 + 65*a^5*b^4 + 3
6*a^4*b^5 + 8*a^3*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^5 + (495*(a^9 + 4*a^8
*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*d*cosh(d*x + c)^8 + 420*(a^9 + 8*a^8*
b + 22*a^7*b^2 + 28*a^6*b^3 + 17*a^5*b^4 + 4*a^4*b^5)*d*cosh(d*x + c)^6 - 7
0*(a^9 + 4*a^8*b - 10*a^7*b^2 - 60*a^6*b^3 - 95*a^5*b^4 - 64*a^4*b^5 - 16*a
^3*b^6)*d*cosh(d*x + c)^4 - 60*(a^9 + 8*a^8*b + 30*a^7*b^2 + 60*a^6*b^3 + 6
5*a^5*b^4 + 36*a^4*b^5 + 8*a^3*b^6)*d*cosh(d*x + c)^2 - (a^9 + 4*a^8*b - 10
*a^7*b^2 - 60*a^6*b^3 - 95*a^5*b^4 - 64*a^4*b^5 - 16*a^3*b^6)*d)*sinh(d*x +
 c)^4 + 2*(a^9 + 8*a^8*b + 22*a^7*b^2 + 28*a^6*b^3 + 17*a^5*b^4 + 4*a^4*b^5
)*d*cosh(d*x + c)^2 + 4*(55*(a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^
4)*d*cosh(d*x + c)^9 + 60*(a^9 + 8*a^8*b + 22*a^7*b^2 + 28*a^6*b^3 + 17*a^5
*b^4 + 4*a^4*b^5)*d*cosh(d*x + c)^7 - 14*(a^9 + 4*a^8*b - 10*a^7*b^2 - 60*a
^6*b^3 - 95*a^5*b^4 - 64*a^4*b^5 - 16*a^3*b^6)*d*cosh(d*x + c)^5 - 20*(a^9
+ 8*a^8*b + 30*a^7*b^2 + 60*a^6*b^3 + 65*a^5*b^4 + 36*a^4*b^5 + 8*a^3*b^6)*
d*cosh(d*x + c)^3 - (a^9 + 4*a^8*b - 10*a^7*b^2 - 60*a^6*b^3 - 95*a^5*b^4 -
 64*a^4*b^5 - 16*a^3*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(33*(a^9 + 4
*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*d*cosh(d*x + c)^10 + 45*(a^9 + 8*
a^8*b + 22*a^7*b^2 + 28*a^6*b^3 + 17*a^5*b^4 + 4*a^4*b^5)*d*cosh(d*x + c)^8
 - 14*(a^9 + 4*a^8*b - 10*a^7*b^2 - 60*a^6*b^3 - 95*a^5*b^4 - 64*a^4*b^5 -
16*a^3*b^6)*d*cosh(d*x + c)^6 - 30*(a^9 + 8*a^8*b + 30*a^7*b^2 + 60*a^6*b^3
 + 65*a^5*b^4 + 36*a^4*b^5 + 8*a^3*b^6)*d*cosh(d*x + c)^4 - 3*(a^9 + 4*a^8*
b - 10*a^7*b^2 - 60*a^6*b^3 - 95*a^5*b^4 - 64*a^4*b^5 - 16*a^3*b^6)*d*cosh(
d*x + c)^2 + (a^9 + 8*a^8*b + 22*a^7*b^2 + 28*a^6*b^3 + 17*a^5*b^4 + 4*a^4*
b^5)*d)*sinh(d*x + c)^2 + (a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)
*d + 4*(3*(a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*d*cosh(d*x + c)
^11 + 5*(a^9 + 8*a^8*b + 22*a^7*b^2 + 28*a^6*b^3 + 17*a^5*b^4 + 4*a^4*b^5)*
d*cosh(d*x + c)^9 - 2*(a^9 + 4*a^8*b - 10*a^7*b^2 - 60*a^6*b^3 - 95*a^5*b^4
 - 64*a^4*b^5 - 16*a^3*b^6)*d*cosh(d*x + c)^7 - 6*(a^9 + 8*a^8*b + 30*a^7*b
^2 + 60*a^6*b^3 + 65*a^5*b^4 + 36*a^4*b^5 + 8*a^3*b^6)*d*cosh(d*x + c)^5 -
(a^9 + 4*a^8*b - 10*a^7*b^2 - 60*a^6*b^3 - 95*a^5*b^4 - 64*a^4*b^5 - 16*a^3
*b^6)*d*cosh(d*x + c)^3 + (a^9 + 8*a^8*b + 22*a^7*b^2 + 28*a^6*b^3 + 17*a^5
*b^4 + 4*a^4*b^5)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.53496, size = 1034, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*((6*a^2*b^2 + 4*a*b^3 + b^4)*log(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c
) + 4*b*e^(2*d*x + 2*c) + a)/(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b
^4) + 2*(a*e^(2*c) + 4*b*e^(2*c))*log(abs(e^(2*d*x + 2*c) - 1))/(a^4*e^(2*c
) + 4*a^3*b*e^(2*c) + 6*a^2*b^2*e^(2*c) + 4*a*b^3*e^(2*c) + b^4*e^(2*c)) -
2*d*x/a^3 - (a^5*e^(12*d*x + 12*c) + 3*a^4*b*e^(12*d*x + 12*c) + 3*a^3*b^2*
e^(12*d*x + 12*c) + a^2*b^3*e^(12*d*x + 12*c) + 6*a^5*e^(10*d*x + 10*c) + 1
4*a^4*b*e^(10*d*x + 10*c) + 30*a^3*b^2*e^(10*d*x + 10*c) + 10*a^2*b^3*e^(10
*d*x + 10*c) + 15*a^5*e^(8*d*x + 8*c) + 29*a^4*b*e^(8*d*x + 8*c) + 13*a^3*b
^2*e^(8*d*x + 8*c) + 47*a^2*b^3*e^(8*d*x + 8*c) - 8*a*b^4*e^(8*d*x + 8*c) -
 8*b^5*e^(8*d*x + 8*c) + 20*a^5*e^(6*d*x + 6*c) + 36*a^4*b*e^(6*d*x + 6*c)
- 28*a^3*b^2*e^(6*d*x + 6*c) - 116*a^2*b^3*e^(6*d*x + 6*c) + 16*a*b^4*e^(6*
d*x + 6*c) + 16*b^5*e^(6*d*x + 6*c) + 15*a^5*e^(4*d*x + 4*c) + 29*a^4*b*e^(
4*d*x + 4*c) + 13*a^3*b^2*e^(4*d*x + 4*c) + 47*a^2*b^3*e^(4*d*x + 4*c) - 8*
a*b^4*e^(4*d*x + 4*c) - 8*b^5*e^(4*d*x + 4*c) + 6*a^5*e^(2*d*x + 2*c) + 14*
a^4*b*e^(2*d*x + 2*c) + 30*a^3*b^2*e^(2*d*x + 2*c) + 10*a^2*b^3*e^(2*d*x +
2*c) + a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a
^3*b^3)*(a*e^(6*d*x + 6*c) + a*e^(4*d*x + 4*c) + 4*b*e^(4*d*x + 4*c) - a*e^
(2*d*x + 2*c) - 4*b*e^(2*d*x + 2*c) - a)^2))/d
```

3.168 $\int \frac{\coth^4(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=232

$$-\frac{b^{5/2}\left(63a^2+36ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3d(a+b)^{9/2}}-\frac{\left(8a^2-39ab-12b^2\right)\coth^3(c+dx)}{24a^2d(a+b)^3}-\frac{\left(32a^2b+8a^3-15ab^2-4b^3\right)\text{co}}{8a^2d(a+b)^4}$$

```
[Out] x/a^3 - (b^(5/2)*(63*a^2 + 36*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/
Sqrt[a + b]])/(8*a^3*(a + b)^(9/2)*d) - ((8*a^3 + 32*a^2*b - 15*a*b^2 - 4*b
^3)*Coth[c + d*x])/(8*a^2*(a + b)^4*d) - ((8*a^2 - 39*a*b - 12*b^2)*Coth[c
+ d*x]^3)/(24*a^2*(a + b)^3*d) - (b*Coth[c + d*x]^3)/(4*a*(a + b)*d*(a + b
- b*Tanh[c + d*x]^2)^2) - (b*(11*a + 4*b)*Coth[c + d*x]^3)/(8*a^2*(a + b)^2
*d*(a + b - b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.513631, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4141, 1975, 472, 579, 583, 522, 206, 208}

$$-\frac{b^{5/2}\left(63a^2+36ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3d(a+b)^{9/2}}-\frac{\left(8a^2-39ab-12b^2\right)\coth^3(c+dx)}{24a^2d(a+b)^3}-\frac{\left(32a^2b+8a^3-15ab^2-4b^3\right)\text{co}}{8a^2d(a+b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```


```
[Out] x/a^3 - (b^(5/2)*(63*a^2 + 36*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/
Sqrt[a + b]])/(8*a^3*(a + b)^(9/2)*d) - ((8*a^3 + 32*a^2*b - 15*a*b^2 - 4*b
^3)*Coth[c + d*x])/(8*a^2*(a + b)^4*d) - ((8*a^2 - 39*a*b - 12*b^2)*Coth[c
+ d*x]^3)/(24*a^2*(a + b)^3*d) - (b*Coth[c + d*x]^3)/(4*a*(a + b)*d*(a + b
- b*Tanh[c + d*x]^2)^2) - (b*(11*a + 4*b)*Coth[c + d*x]^3)/(8*a^2*(a + b)^2
*d*(a + b - b*Tanh[c + d*x]^2))
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
```

```
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\coth^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx=\frac{\text{Subst}\left(\int \frac{1}{x^4(1-x^2)\left(a+b(1-x^2)\right)^3}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$=\frac{\text{Subst}\left(\int \frac{1}{x^4(1-x^2)(a+b-bx^2)^3}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$=-\frac{b\coth^3(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2}-\frac{\text{Subst}\left(\int \frac{-4a+3b-7bx^2}{x^4(1-x^2)(a+b-bx^2)^2}\,dx,x,\tanh(c+dx)\right)}{4a(a+b)d}$$

$$=-\frac{b\coth^3(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2}-\frac{b(11a+4b)\coth^3(c+dx)}{8a^2(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)}+\frac{\text{Subs}}{}$$

$$=-\frac{\left(8a^2-39ab-12b^2\right)\coth^3(c+dx)}{24a^2(a+b)^3d}-\frac{b\coth^3(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2}-\frac{b(1}{8a^2(a+b}$$

$$=-\frac{\left(8a^3+32a^2b-15ab^2-4b^3\right)\coth(c+dx)}{8a^2(a+b)^4d}-\frac{\left(8a^2-39ab-12b^2\right)\coth^3(c+dx)}{24a^2(a+b)^3d}-\frac{}{4a(a}$$

$$=-\frac{\left(8a^3+32a^2b-15ab^2-4b^3\right)\coth(c+dx)}{8a^2(a+b)^4d}-\frac{\left(8a^2-39ab-12b^2\right)\coth^3(c+dx)}{24a^2(a+b)^3d}-\frac{}{4a(a}$$

$$=\frac{x}{a^3}-\frac{b^{5/2}\left(63a^2+36ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{9/2}d}-\frac{\left(8a^3+32a^2b-15ab^2-4b^3\right)\text{coth}}{8a^2(a+b)^4d}$$

<u>Mathematica</u> [C] time = 7.52759, size = 3334, normalized size = 14.37

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((63*a^2 + 36*a*b + 8*b^2)*(a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^
6*(((I/64)*b^3*ArcTan[Sech[d*x]*(((-I/2)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b*Cos
h[4*c] - b*Sinh[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] -
b*Sinh[4*c]]))*(-(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x])]*Cosh[2
*c])/(a^3*Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) - ((I/64)*b^3*ArcT
an[Sech[d*x]*(((-I/2)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c
]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]))*(-(a
*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x])]*Sinh[2*c])/(a^3*Sqrt[a +
b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]])))/((a + b)^4*(a + b*Sech[c + d*x]^2)^
3) + ((a + 2*b + a*Cosh[2*c + 2*d*x])*Csch[c]*Csch[c + d*x]^3*Sech[2*c]*Sec
h[c + d*x]^6*(-36*a^6*d*x*Cosh[d*x] - 336*a^5*b*d*x*Cosh[d*x] - 1560*a^4*b^
2*d*x*Cosh[d*x] - 3600*a^3*b^3*d*x*Cosh[d*x] - 4260*a^2*b^4*d*x*Cosh[d*x] -
 2496*a*b^5*d*x*Cosh[d*x] - 576*b^6*d*x*Cosh[d*x] + 36*a^6*d*x*Cosh[3*d*x]
+ 240*a^5*b*d*x*Cosh[3*d*x] + 408*a^4*b^2*d*x*Cosh[3*d*x] - 48*a^3*b^3*d*x*
Cosh[3*d*x] - 732*a^2*b^4*d*x*Cosh[3*d*x] - 672*a*b^5*d*x*Cosh[3*d*x] - 192
*b^6*d*x*Cosh[3*d*x] + 36*a^6*d*x*Cosh[2*c - d*x] + 336*a^5*b*d*x*Cosh[2*c
- d*x] + 1560*a^4*b^2*d*x*Cosh[2*c - d*x] + 3600*a^3*b^3*d*x*Cosh[2*c - d*x
] + 4260*a^2*b^4*d*x*Cosh[2*c - d*x] + 2496*a*b^5*d*x*Cosh[2*c - d*x] + 576
```

*b^6*d*x*Cosh[2*c - d*x] + 36*a^6*d*x*Cosh[2*c + d*x] + 336*a^5*b*d*x*Cosh[
2*c + d*x] + 1560*a^4*b^2*d*x*Cosh[2*c + d*x] + 3600*a^3*b^3*d*x*Cosh[2*c +
d*x] + 4260*a^2*b^4*d*x*Cosh[2*c + d*x] + 2496*a*b^5*d*x*Cosh[2*c + d*x] +
576*b^6*d*x*Cosh[2*c + d*x] - 36*a^6*d*x*Cosh[4*c + d*x] - 336*a^5*b*d*x*C
osh[4*c + d*x] - 1560*a^4*b^2*d*x*Cosh[4*c + d*x] - 3600*a^3*b^3*d*x*Cosh[4
*c + d*x] - 4260*a^2*b^4*d*x*Cosh[4*c + d*x] - 2496*a*b^5*d*x*Cosh[4*c + d*
x] - 576*b^6*d*x*Cosh[4*c + d*x] - 36*a^6*d*x*Cosh[2*c + 3*d*x] - 240*a^5*b
*d*x*Cosh[2*c + 3*d*x] - 408*a^4*b^2*d*x*Cosh[2*c + 3*d*x] + 48*a^3*b^3*d*x
*Cosh[2*c + 3*d*x] + 732*a^2*b^4*d*x*Cosh[2*c + 3*d*x] + 672*a*b^5*d*x*Cosh
[2*c + 3*d*x] + 192*b^6*d*x*Cosh[2*c + 3*d*x] + 36*a^6*d*x*Cosh[4*c + 3*d*x
] + 240*a^5*b*d*x*Cosh[4*c + 3*d*x] + 408*a^4*b^2*d*x*Cosh[4*c + 3*d*x] - 4
8*a^3*b^3*d*x*Cosh[4*c + 3*d*x] - 732*a^2*b^4*d*x*Cosh[4*c + 3*d*x] - 672*a
*b^5*d*x*Cosh[4*c + 3*d*x] - 192*b^6*d*x*Cosh[4*c + 3*d*x] - 36*a^6*d*x*Cos
h[6*c + 3*d*x] - 240*a^5*b*d*x*Cosh[6*c + 3*d*x] - 408*a^4*b^2*d*x*Cosh[6*c
+ 3*d*x] + 48*a^3*b^3*d*x*Cosh[6*c + 3*d*x] + 732*a^2*b^4*d*x*Cosh[6*c + 3
*d*x] + 672*a*b^5*d*x*Cosh[6*c + 3*d*x] + 192*b^6*d*x*Cosh[6*c + 3*d*x] - 1
2*a^6*d*x*Cosh[2*c + 5*d*x] - 144*a^5*b*d*x*Cosh[2*c + 5*d*x] - 456*a^4*b^2
*d*x*Cosh[2*c + 5*d*x] - 624*a^3*b^3*d*x*Cosh[2*c + 5*d*x] - 396*a^2*b^4*d*
x*Cosh[2*c + 5*d*x] - 96*a*b^5*d*x*Cosh[2*c + 5*d*x] + 12*a^6*d*x*Cosh[4*c
+ 5*d*x] + 144*a^5*b*d*x*Cosh[4*c + 5*d*x] + 456*a^4*b^2*d*x*Cosh[4*c + 5*d
*x] + 624*a^3*b^3*d*x*Cosh[4*c + 5*d*x] + 396*a^2*b^4*d*x*Cosh[4*c + 5*d*x]
+ 96*a*b^5*d*x*Cosh[4*c + 5*d*x] - 12*a^6*d*x*Cosh[6*c + 5*d*x] - 144*a^5*
b*d*x*Cosh[6*c + 5*d*x] - 456*a^4*b^2*d*x*Cosh[6*c + 5*d*x] - 624*a^3*b^3*d
*x*Cosh[6*c + 5*d*x] - 396*a^2*b^4*d*x*Cosh[6*c + 5*d*x] - 96*a*b^5*d*x*Cos
h[6*c + 5*d*x] + 12*a^6*d*x*Cosh[8*c + 5*d*x] + 144*a^5*b*d*x*Cosh[8*c + 5*
d*x] + 456*a^4*b^2*d*x*Cosh[8*c + 5*d*x] + 624*a^3*b^3*d*x*Cosh[8*c + 5*d*x
] + 396*a^2*b^4*d*x*Cosh[8*c + 5*d*x] + 96*a*b^5*d*x*Cosh[8*c + 5*d*x] - 12
*a^6*d*x*Cosh[4*c + 7*d*x] - 48*a^5*b*d*x*Cosh[4*c + 7*d*x] - 72*a^4*b^2*d*
x*Cosh[4*c + 7*d*x] - 48*a^3*b^3*d*x*Cosh[4*c + 7*d*x] - 12*a^2*b^4*d*x*Cos
h[4*c + 7*d*x] + 12*a^6*d*x*Cosh[6*c + 7*d*x] + 48*a^5*b*d*x*Cosh[6*c + 7*d
*x] + 72*a^4*b^2*d*x*Cosh[6*c + 7*d*x] + 48*a^3*b^3*d*x*Cosh[6*c + 7*d*x] +
12*a^2*b^4*d*x*Cosh[6*c + 7*d*x] - 12*a^6*d*x*Cosh[8*c + 7*d*x] - 48*a^5*b
*d*x*Cosh[8*c + 7*d*x] - 72*a^4*b^2*d*x*Cosh[8*c + 7*d*x] - 48*a^3*b^3*d*x*
Cosh[8*c + 7*d*x] - 12*a^2*b^4*d*x*Cosh[8*c + 7*d*x] + 12*a^6*d*x*Cosh[10*c
+ 7*d*x] + 48*a^5*b*d*x*Cosh[10*c + 7*d*x] + 72*a^4*b^2*d*x*Cosh[10*c + 7*
d*x] + 48*a^3*b^3*d*x*Cosh[10*c + 7*d*x] + 12*a^2*b^4*d*x*Cosh[10*c + 7*d*x
] - 128*a^6*Sinh[d*x] - 440*a^5*b*Sinh[d*x] - 1152*a^4*b^2*Sinh[d*x] - 1920
*a^3*b^3*Sinh[d*x] + 228*a^2*b^4*Sinh[d*x] + 1320*a*b^5*Sinh[d*x] + 432*b^6
*Sinh[d*x] + 48*a^6*Sinh[3*d*x] + 104*a^5*b*Sinh[3*d*x] + 640*a^4*b^2*Sinh[
3*d*x] + 1511*a^3*b^3*Sinh[3*d*x] - 528*a^2*b^4*Sinh[3*d*x] + 264*a*b^5*Sin
h[3*d*x] + 144*b^6*Sinh[3*d*x] - 32*a^6*Sinh[2*c - d*x] + 384*a^5*b*Sinh[2*
c - d*x] + 2048*a^4*b^2*Sinh[2*c - d*x] + 3072*a^3*b^3*Sinh[2*c - d*x] + 22
8*a^2*b^4*Sinh[2*c - d*x] + 1320*a*b^5*Sinh[2*c - d*x] + 432*b^6*Sinh[2*c -
d*x] + 32*a^6*Sinh[2*c + d*x] - 384*a^5*b*Sinh[2*c + d*x] - 2048*a^4*b^2*S
inh[2*c + d*x] - 2919*a^3*b^3*Sinh[2*c + d*x] + 642*a^2*b^4*Sinh[2*c + d*x]
+ 1416*a*b^5*Sinh[2*c + d*x] + 432*b^6*Sinh[2*c + d*x] - 128*a^6*Sinh[4*c
+ d*x] - 440*a^5*b*Sinh[4*c + d*x] - 1152*a^4*b^2*Sinh[4*c + d*x] - 2073*a^
3*b^3*Sinh[4*c + d*x] - 642*a^2*b^4*Sinh[4*c + d*x] - 1416*a*b^5*Sinh[4*c +
d*x] - 432*b^6*Sinh[4*c + d*x] - 144*a^6*Sinh[2*c + 3*d*x] - 672*a^5*b*Sin
h[2*c + 3*d*x] - 960*a^4*b^2*Sinh[2*c + 3*d*x] + 153*a^3*b^3*Sinh[2*c + 3*d
*x] + 528*a^2*b^4*Sinh[2*c + 3*d*x] - 264*a*b^5*Sinh[2*c + 3*d*x] - 144*b^6
*Sinh[2*c + 3*d*x] + 48*a^6*Sinh[4*c + 3*d*x] + 104*a^5*b*Sinh[4*c + 3*d*x]
+ 640*a^4*b^2*Sinh[4*c + 3*d*x] + 1664*a^3*b^3*Sinh[4*c + 3*d*x] - 66*a^2*
b^4*Sinh[4*c + 3*d*x] - 408*a*b^5*Sinh[4*c + 3*d*x] - 144*b^6*Sinh[4*c + 3*
d*x] - 144*a^6*Sinh[6*c + 3*d*x] - 672*a^5*b*Sinh[6*c + 3*d*x] - 960*a^4*b^
2*Sinh[6*c + 3*d*x] + 66*a^2*b^4*Sinh[6*c + 3*d*x] + 408*a*b^5*Sinh[6*c + 3
*d*x] + 144*b^6*Sinh[6*c + 3*d*x] + 80*a^6*Sinh[2*c + 5*d*x] + 480*a^5*b*Si
nh[2*c + 5*d*x] + 832*a^4*b^2*Sinh[2*c + 5*d*x] + 294*a^2*b^4*Sinh[2*c + 5*
d*x] + 96*a*b^5*Sinh[2*c + 5*d*x] - 48*a^6*Sinh[4*c + 5*d*x] - 120*a^5*b*Si

```
nh[4*c + 5*d*x] - 294*a^2*b^4*Sinh[4*c + 5*d*x] - 96*a*b^5*Sinh[4*c + 5*d*x
] + 80*a^6*Sinh[6*c + 5*d*x] + 480*a^5*b*Sinh[6*c + 5*d*x] + 832*a^4*b^2*Si
nh[6*c + 5*d*x] - 51*a^3*b^3*Sinh[6*c + 5*d*x] - 132*a^2*b^4*Sinh[6*c + 5*d
*x] - 48*a*b^5*Sinh[6*c + 5*d*x] - 48*a^6*Sinh[8*c + 5*d*x] - 120*a^5*b*Sin
h[8*c + 5*d*x] + 51*a^3*b^3*Sinh[8*c + 5*d*x] + 132*a^2*b^4*Sinh[8*c + 5*d*
x] + 48*a*b^5*Sinh[8*c + 5*d*x] + 32*a^6*Sinh[4*c + 7*d*x] + 104*a^5*b*Sinh
[4*c + 7*d*x] + 51*a^3*b^3*Sinh[4*c + 7*d*x] + 18*a^2*b^4*Sinh[4*c + 7*d*x]
 - 51*a^3*b^3*Sinh[6*c + 7*d*x] - 18*a^2*b^4*Sinh[6*c + 7*d*x] + 32*a^6*Sin
h[8*c + 7*d*x] + 104*a^5*b*Sinh[8*c + 7*d*x]))/(6144*a^3*(a + b)^4*d*(a + b
*Sech[c + d*x]^2)^3)
```

Maple [B] time = 0.134, size = 1610, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/2/d*b^(9/2)/(a+b)^(9/2)/a^3*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1
/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-1/2/d*b^(9/2)/(a+b)^(9/2)/a^3*ln((a+b)^(
1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-63/16
/d*b^(5/2)/(a+b)^(9/2)/a*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*
x+1/2*c)*b^(1/2)+(a+b)^(1/2))+9/4/d*b^(7/2)/(a+b)^(9/2)/a^2*ln((a+b)^(1/2)*
tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+63/16/d*b^
(5/2)/(a+b)^(9/2)/a*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2
*c)*b^(1/2)+(a+b)^(1/2))-5/8/d/(a+b)^4/tanh(1/2*d*x+1/2*c)*a-17/8/d/(a+b)^4
/tanh(1/2*d*x+1/2*c)*b-9/4/d*b^(7/2)/(a+b)^(9/2)/a^2*ln((a+b)^(1/2)*tanh(1/
2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-1/24/d/(a+b)^3/ta
nh(1/2*d*x+1/2*c)^3-1/24/d/(a+b)/(a^3+3*a^2*b+3*a*b^2+b^3)*a*tanh(1/2*d*x+1
/2*c)^3-1/d/a^3*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)
-1/24/d/(a+b)/(a^3+3*a^2*b+3*a*b^2+b^3)*b*tanh(1/2*d*x+1/2*c)^3-5/8/d/(a+b)
/(a^3+3*a^2*b+3*a*b^2+b^3)*a*tanh(1/2*d*x+1/2*c)-17/8/d/(a+b)/(a^3+3*a^2*b+
3*a*b^2+b^3)*tanh(1/2*d*x+1/2*c)*b-17/4/d*b^3/(a+b)^4/(tanh(1/2*d*x+1/2*c)^
4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)
^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7-51/4/d*b^3/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4
*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^
2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^5-51/4/d*b^3/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*
a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2
*b+a+b)^2*tanh(1/2*d*x+1/2*c)^3-17/4/d*b^3/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a
+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*
b+a+b)^2*tanh(1/2*d*x+1/2*c)-21/4/d*b^4/(a+b)^4/a/(tanh(1/2*d*x+1/2*c)^4*a+
b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b
+a+b)^2*tanh(1/2*d*x+1/2*c)^7-1/d*b^5/(a+b)^4/a^2/(tanh(1/2*d*x+1/2*c)^4*a+
b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b
+a+b)^2*tanh(1/2*d*x+1/2*c)^7-3/4/d*b^4/(a+b)^4/a/(tanh(1/2*d*x+1/2*c)^4*a+
b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b
+a+b)^2*tanh(1/2*d*x+1/2*c)^5+1/d*b^5/(a+b)^4/a^2/(tanh(1/2*d*x+1/2*c)^4*a+
b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b
+a+b)^2*tanh(1/2*d*x+1/2*c)^5-3/4/d*b^4/(a+b)^4/a/(tanh(1/2*d*x+1/2*c)^4*a+
b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b
+a+b)^2*tanh(1/2*d*x+1/2*c)^3+1/d*b^5/(a+b)^4/a^2/(tanh(1/2*d*x+1/2*c)^4*a+
b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b
+a+b)^2*tanh(1/2*d*x+1/2*c)^3-21/4/d*b^4/(a+b)^4/a/(tanh(1/2*d*x+1/2*c)^4*a
+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*
b+a+b)^2*tanh(1/2*d*x+1/2*c)-1/d*b^5/(a+b)^4/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b
*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+
```

```
a+b)^2*tanh(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**4/(a+b*sech(d*x+c)**2)**3,x)

[Out] Timed out
```

Giac [B] time = 3.9256, size = 651, normalized size = 2.81

$$-\frac{\frac{3\left(63\,a^{2}b^{3}e^{(2\,c)}+36\,ab^{4}e^{(2\,c)}+8\,b^{5}e^{(2\,c)}\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^{2}}}\right)e^{(-2\,c)}}{\left(a^{7}+4\,a^{6}b+6\,a^{5}b^{2}+4\,a^{4}b^{3}+a^{3}b^{4}\right)\sqrt{-ab-b^{2}}}-\frac{24\,dx}{a^{3}}-\frac{6\left(17\,a^{3}b^{3}e^{(6\,dx+6\,c)}+44\,a^{2}b^{4}e^{(6\,dx+6\,c)}+16\,ab^{5}e^{(6\,dx+6\,c)}+51\,a^{3}b^{3}e^{(4\,dx+4}\right.}{\left(a^{7}+4\,a^{6}b\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")

[Out] -1/24*(3*(63*a^2*b^3*e^(2*c) + 36*a*b^4*e^(2*c) + 8*b^5*e^(2*c))*arctan(1/2
*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))*e^(-2*c)/((a^7 + 4*a^6*b +
 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*sqrt(-a*b - b^2)) - 24*d*x/a^3 - 6*(17*a^
3*b^3*e^(6*d*x + 6*c) + 44*a^2*b^4*e^(6*d*x + 6*c) + 16*a*b^5*e^(6*d*x + 6*
c) + 51*a^3*b^3*e^(4*d*x + 4*c) + 154*a^2*b^4*e^(4*d*x + 4*c) + 184*a*b^5*e
^(4*d*x + 4*c) + 48*b^6*e^(4*d*x + 4*c) + 51*a^3*b^3*e^(2*d*x + 2*c) + 116*
a^2*b^4*e^(2*d*x + 2*c) + 32*a*b^5*e^(2*d*x + 2*c) + 17*a^3*b^3 + 6*a^2*b^4
```

```
)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*(a*e^(4*d*x + 4*c) + 2
*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)^2) + 16*(6*a*e^(4*d*x + 4*c)
+ 15*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) - 24*b*e^(2*d*x + 2*c) + 4*a +
 13*b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(e^(2*d*x + 2*c) - 1)^3
))/d
```

3.169 $\int \frac{1}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^4}\,dx$

Optimal. Leaf size=207

$$-\frac{\sqrt{b}\left(70a^2b+35a^3+56ab^2+16b^3\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{16a^4d(a+b)^{7/2}}-\frac{b\left(19a^2+22ab+8b^2\right)\tanh(c+dx)}{16a^3d(a+b)^3\left(a-b\tanh^2(c+dx)+b\right)}-\frac{b(11a}{24a^2d(a+b)^2}$$

```
[Out] x/a^4 - (Sqrt[b]*(35*a^3 + 70*a^2*b + 56*a*b^2 + 16*b^3)*ArcTanh[(Sqrt[b]*T
anh[c + d*x])/Sqrt[a + b]])/(16*a^4*(a + b)^(7/2)*d) - (b*Tanh[c + d*x])/(6
*a*(a + b)*d*(a + b - b*Tanh[c + d*x]^2)^3) - (b*(11*a + 6*b)*Tanh[c + d*x]
)/(24*a^2*(a + b)^2*d*(a + b - b*Tanh[c + d*x]^2)^2) - (b*(19*a^2 + 22*a*b
+ 8*b^2)*Tanh[c + d*x])/(16*a^3*(a + b)^3*d*(a + b - b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.345628, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4128, 414, 527, 522, 206, 208}

$$-\frac{\sqrt{b}\left(70a^2b+35a^3+56ab^2+16b^3\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{16a^4d(a+b)^{7/2}}-\frac{b\left(19a^2+22ab+8b^2\right)\tanh(c+dx)}{16a^3d(a+b)^3\left(a-b\tanh^2(c+dx)+b\right)}-\frac{b(11a}{24a^2d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^(-4),x]
```


```
[Out] x/a^4 - (Sqrt[b]*(35*a^3 + 70*a^2*b + 56*a*b^2 + 16*b^3)*ArcTanh[(Sqrt[b]*T
anh[c + d*x])/Sqrt[a + b]])/(16*a^4*(a + b)^(7/2)*d) - (b*Tanh[c + d*x])/(6
*a*(a + b)*d*(a + b - b*Tanh[c + d*x]^2)^3) - (b*(11*a + 6*b)*Tanh[c + d*x]
)/(24*a^2*(a + b)^2*d*(a + b - b*Tanh[c + d*x]^2)^2) - (b*(19*a^2 + 22*a*b
+ 8*b^2)*Tanh[c + d*x])/(16*a^3*(a + b)^3*d*(a + b - b*Tanh[c + d*x]^2))
```


Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
```

```
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\operatorname{sech}^2(c+dx)\right)^4}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\left(a+b-bx^2\right)^4}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\tanh(c+dx)}{6a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^3} - \frac{\operatorname{Subst}\left(\int \frac{-6a-b-5bx^2}{(1-x^2)\left(a+b-bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{6a(a+b)d}$$

$$= -\frac{b\tanh(c+dx)}{6a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^3} - \frac{b(11a+6b)\tanh(c+dx)}{24a^2(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{\text{Su}}{}$$

$$= -\frac{b\tanh(c+dx)}{6a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^3} - \frac{b(11a+6b)\tanh(c+dx)}{24a^2(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{b}{16a}$$

$$= -\frac{b\tanh(c+dx)}{6a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^3} - \frac{b(11a+6b)\tanh(c+dx)}{24a^2(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{b}{16a}$$

$$= \frac{x}{a^4} - \frac{\sqrt{b}\left(35a^3+70a^2b+56ab^2+16b^3\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{16a^4(a+b)^{7/2}d} - \frac{b\tanh(c+}{6a(a+b)d\left(a+b-b\tanh\right.}$$

Mathematica [C] time = 6.85889, size = 1405, normalized size = 6.79

$$\left(35a^3+70ba^2+56b^2a+16b^3\right)\left(\cosh(2c+2dx)a+a+2b\right)^4\left(\frac{ib\tan^{-1}\left(\operatorname{sech}(dx)\left(\frac{i\sinh(2c)}{2\sqrt{a+b}\sqrt{b\cosh(4c)-b\sinh(4c)}}-\frac{i\cosh(2c)}{2\sqrt{a+b}\sqrt{b\cosh(4c)-b\sinh(4c)}}\right)(-\right.}{256a^4\sqrt{a+b}d\sqrt{b\cosh(4c)-b\sinh}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^(-4),x]
```

```
[Out] ((35*a^3 + 70*a^2*b + 56*a*b^2 + 16*b^3)*(a + 2*b + a*Cosh[2*c + 2*d*x])^4*
Sech[c + d*x]^8*(((I/256)*b*ArcTan[Sech[d*x]*(((-I/2)*Cosh[2*c])/(Sqrt[a +
b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b
*Cosh[4*c] - b*Sinh[4*c]]))*(-(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c +
d*x])]*Cosh[2*c])/(a^4*Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) - ((I
/256)*b*ArcTan[Sech[d*x]*(((-I/2)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c]
- b*Sinh[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[
4*c]]))*(-(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x])]*Sinh[2*c])/(a
^4*Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]])))/((a + b)^3*(a + b*Sech[
c + d*x]^2)^4) + ((a + 2*b + a*Cosh[2*c + 2*d*x])*Sech[2*c]*Sech[c + d*x]^8
*(480*a^6*d*x*Cosh[2*c] + 3168*a^5*b*d*x*Cosh[2*c] + 8928*a^4*b^2*d*x*Cosh[
2*c] + 14112*a^3*b^3*d*x*Cosh[2*c] + 13248*a^2*b^4*d*x*Cosh[2*c] + 6912*a*b
^5*d*x*Cosh[2*c] + 1536*b^6*d*x*Cosh[2*c] + 360*a^6*d*x*Cosh[2*d*x] + 2232*
a^5*b*d*x*Cosh[2*d*x] + 5688*a^4*b^2*d*x*Cosh[2*d*x] + 7272*a^3*b^3*d*x*Cos
h[2*d*x] + 4608*a^2*b^4*d*x*Cosh[2*d*x] + 1152*a*b^5*d*x*Cosh[2*d*x] + 360*
a^6*d*x*Cosh[4*c + 2*d*x] + 2232*a^5*b*d*x*Cosh[4*c + 2*d*x] + 5688*a^4*b^2
*d*x*Cosh[4*c + 2*d*x] + 7272*a^3*b^3*d*x*Cosh[4*c + 2*d*x] + 4608*a^2*b^4*
d*x*Cosh[4*c + 2*d*x] + 1152*a*b^5*d*x*Cosh[4*c + 2*d*x] + 144*a^6*d*x*Cosh
[2*c + 4*d*x] + 720*a^5*b*d*x*Cosh[2*c + 4*d*x] + 1296*a^4*b^2*d*x*Cosh[2*c
 + 4*d*x] + 1008*a^3*b^3*d*x*Cosh[2*c + 4*d*x] + 288*a^2*b^4*d*x*Cosh[2*c +
 4*d*x] + 144*a^6*d*x*Cosh[6*c + 4*d*x] + 720*a^5*b*d*x*Cosh[6*c + 4*d*x] +
 1296*a^4*b^2*d*x*Cosh[6*c + 4*d*x] + 1008*a^3*b^3*d*x*Cosh[6*c + 4*d*x] +
288*a^2*b^4*d*x*Cosh[6*c + 4*d*x] + 24*a^6*d*x*Cosh[4*c + 6*d*x] + 72*a^5*b
*d*x*Cosh[4*c + 6*d*x] + 72*a^4*b^2*d*x*Cosh[4*c + 6*d*x] + 24*a^3*b^3*d*x*
Cosh[4*c + 6*d*x] + 24*a^6*d*x*Cosh[8*c + 6*d*x] + 72*a^5*b*d*x*Cosh[8*c +
6*d*x] + 72*a^4*b^2*d*x*Cosh[8*c + 6*d*x] + 24*a^3*b^3*d*x*Cosh[8*c + 6*d*x
] + 870*a^5*b*Sinh[2*c] + 4292*a^4*b^2*Sinh[2*c] + 8792*a^3*b^3*Sinh[2*c] +
 9936*a^2*b^4*Sinh[2*c] + 5824*a*b^5*Sinh[2*c] + 1408*b^6*Sinh[2*c] - 870*a
^5*b*Sinh[2*d*x] - 3792*a^4*b^2*Sinh[2*d*x] - 6432*a^3*b^3*Sinh[2*d*x] - 46
08*a^2*b^4*Sinh[2*d*x] - 1248*a*b^5*Sinh[2*d*x] + 435*a^5*b*Sinh[4*c + 2*d*
x] + 2124*a^4*b^2*Sinh[4*c + 2*d*x] + 3972*a^3*b^3*Sinh[4*c + 2*d*x] + 3072
*a^2*b^4*Sinh[4*c + 2*d*x] + 864*a*b^5*Sinh[4*c + 2*d*x] - 435*a^5*b*Sinh[2
*c + 4*d*x] - 1374*a^4*b^2*Sinh[2*c + 4*d*x] - 1248*a^3*b^3*Sinh[2*c + 4*d*
x] - 384*a^2*b^4*Sinh[2*c + 4*d*x] + 87*a^5*b*Sinh[6*c + 4*d*x] + 366*a^4*b
^2*Sinh[6*c + 4*d*x] + 408*a^3*b^3*Sinh[6*c + 4*d*x] + 144*a^2*b^4*Sinh[6*c
 + 4*d*x] - 87*a^5*b*Sinh[4*c + 6*d*x] - 116*a^4*b^2*Sinh[4*c + 6*d*x] - 44
*a^3*b^3*Sinh[4*c + 6*d*x]))/(3072*a^4*(a + b)^3*d*(a + b*Sech[c + d*x]^2)^
4)
```

Maple [B] time = 0.119, size = 2892, normalized size = 14.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(d*x+c)^2)^4,x)
```

```
[Out] -2/d*b^5/a^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*
x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^2+2*a*b+b^2)*tanh(1/
2*d*x+1/2*c)^7-145/4/d*b*a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4
+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^2+2*a*
b+b^2)*tanh(1/2*d*x+1/2*c)^7-1/2/d*b^4/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(
1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^3
/(a+b)/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-37/4/d*b^3/a/(tanh(1/2*d*x+1/2
*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/
```

```
2*c)^2*b+a+b)^3/(a+b)/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-145/8/d*b/(tanh
(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tan
h(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9-145/8/d*b
/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a
-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3-35/
16/d*b^(3/2)/a^2/(a^3+3*a^2*b+3*a*b^2+b^3)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(
1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-37/4/d*b^3/a/(t
anh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*
tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^3+3*a^2*b+3*a*b^2+b^3)*tanh(1/2*d*x+1/2*c
)^5+11/4/d*b^3/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(
1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)*tanh(1/
2*d*x+1/2*c)^9-1/d*b^3/a^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4
+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)*tanh(1/2*
d*x+1/2*c)^11-35/32/d*b^(1/2)/a/(a^3+3*a^2*b+3*a*b^2+b^3)/(a+b)^(1/2)*ln((a
+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+
1/d/a^4*ln(tanh(1/2*d*x+1/2*c)+1)-1/d/a^4*ln(tanh(1/2*d*x+1/2*c)-1)-37/d*b^
2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*
a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^3+3*a^2*b+3*a*b^2+b^3)*tanh(1/2*d*x+1
/2*c)^5-1/d*b^3/a^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh
(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)*tanh(1/2*d*x+1/2
*c)+35/16/d*b^(3/2)/a^2/(a^3+3*a^2*b+3*a*b^2+b^3)/(a+b)^(1/2)*ln(-(a+b)^(1/
2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2))+7/4/d*b
^(5/2)/a^3/(a^3+3*a^2*b+3*a*b^2+b^3)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d
*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2))+1/2/d*b^(7/2)/a^4/(a
^3+3*a^2*b+3*a*b^2+b^3)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2
*tanh(1/2*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2))-145/4/d*b*a/(tanh(1/2*d*x+1/2*c)^
4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)
^2*b+a+b)^3/(a^3+3*a^2*b+3*a*b^2+b^3)*tanh(1/2*d*x+1/2*c)^5-29/8/d*b/a/(tan
h(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*ta
nh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)*tanh(1/2*d*x+1/2*c)^11-2/d*b^5/a^3/(tanh
(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tan
h(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^3+3*a^2*b+3*a*b^2+b^3)*tanh(1/2*d*x+1/2*c)^5
+3/d*b^4/a^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*
x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)*tanh(1/2*d*x+
1/2*c)^3-1/2/d*b^(7/2)/a^4/(a^3+3*a^2*b+3*a*b^2+b^3)/(a+b)^(1/2)*ln((a+b)^(
1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-29/8/
d*b/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c
)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)*tanh(1/2*d*x+1/2*c)-13/4/d*b^2
/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)
^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)*tanh(1/2*d*x+1/2*c)^11+35/32/d*
b^(1/2)/a/(a^3+3*a^2*b+3*a*b^2+b^3)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*
x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2))-37/d*b^2/(tanh(1/2*d*
x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d
*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-281/24/d*b
^2/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)
^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3
+11/4/d*b^3/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2
*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)*tanh(1/2*d
*x+1/2*c)^3-1/2/d*b^4/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+
2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^3+3*a^2*b+3*a
*b^2+b^3)*tanh(1/2*d*x+1/2*c)^5+3/d*b^4/a^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh
(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^
3/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9-281/24/d*b^2/a/(tanh(1/2*d*x+1/2*c)
^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c
)^2*b+a+b)^3/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9-13/4/d*b^2/a^2/(tanh(1/2
*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/
2*d*x+1/2*c)^2*b+a+b)^3/(a+b)*tanh(1/2*d*x+1/2*c)-7/4/d*b^(5/2)/a^3/(a^3+3*
a^2*b+3*a*b^2+b^3)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(
1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)**2)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18109, size = 811, normalized size = 3.92

$$-\frac{\left(35\,a^3b+70\,a^2b^2+56\,ab^3+16\,b^4\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{16\,\left(a^7d+3\,a^6bd+3\,a^5b^2d+a^4b^3d\right)\sqrt{-ab-b^2}}+\frac{87\,a^5be^{(10\,dx+10\,c)}+366\,a^4b^2e^{(10\,dx+10\,c)}+408\,a^3b^3e^{(10\,c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] -1/16*(35*a^3*b + 70*a^2*b^2 + 56*a*b^3 + 16*b^4)*arctan(1/2*(a*e^(2*d*x +
2*c) + a + 2*b)/sqrt(-a*b - b^2))/((a^7*d + 3*a^6*b*d + 3*a^5*b^2*d + a^4*b
^3*d)*sqrt(-a*b - b^2)) + 1/24*(87*a^5*b*e^(10*d*x + 10*c) + 366*a^4*b^2*e^
(10*d*x + 10*c) + 408*a^3*b^3*e^(10*d*x + 10*c) + 144*a^2*b^4*e^(10*d*x + 1
0*c) + 435*a^5*b*e^(8*d*x + 8*c) + 2124*a^4*b^2*e^(8*d*x + 8*c) + 3972*a^3*
b^3*e^(8*d*x + 8*c) + 3072*a^2*b^4*e^(8*d*x + 8*c) + 864*a*b^5*e^(8*d*x + 8
*c) + 870*a^5*b*e^(6*d*x + 6*c) + 4292*a^4*b^2*e^(6*d*x + 6*c) + 8792*a^3*b
^3*e^(6*d*x + 6*c) + 9936*a^2*b^4*e^(6*d*x + 6*c) + 5824*a*b^5*e^(6*d*x + 6
```

```
*c) + 1408*b^6*e^(6*d*x + 6*c) + 870*a^5*b*e^(4*d*x + 4*c) + 3792*a^4*b^2*e
^(4*d*x + 4*c) + 6432*a^3*b^3*e^(4*d*x + 4*c) + 4608*a^2*b^4*e^(4*d*x + 4*c
) + 1248*a*b^5*e^(4*d*x + 4*c) + 435*a^5*b*e^(2*d*x + 2*c) + 1374*a^4*b^2*e
^(2*d*x + 2*c) + 1248*a^3*b^3*e^(2*d*x + 2*c) + 384*a^2*b^4*e^(2*d*x + 2*c)
 + 87*a^5*b + 116*a^4*b^2 + 44*a^3*b^3)/((a^7*d + 3*a^6*b*d + 3*a^5*b^2*d +
 a^4*b^3*d)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c)
+ a)^3) + (d*x + c)/(a^4*d)
```

3.170 $\int \left(1-\text{sech}^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=29

$$\sqrt{\tanh^2(x)}\coth(x)\log(\cosh(x))-\frac{1}{2}\tanh^2(x)^{3/2}\coth(x)$$

```
[Out] Coth[x]*Log[Cosh[x]]*Sqrt[Tanh[x]^2] - (Coth[x]*(Tanh[x]^2)^(3/2))/2
```

Rubi [A] time = 0.0310718, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4121, 3658, 3473, 3475}

$$\sqrt{\tanh^2(x)}\coth(x)\log(\cosh(x))-\frac{1}{2}\tanh^2(x)^{3/2}\coth(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sech[x]^2)^(3/2),x]
```

```
[Out] Coth[x]*Log[Cosh[x]]*Sqrt[Tanh[x]^2] - (Coth[x]*(Tanh[x]^2)^(3/2))/2
```

Rule 4121

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(1-\operatorname{sech}^2(x)\right)^{3/2} dx = \int \tanh^2(x)^{3/2} dx$$
$$= \left(\coth(x)\sqrt{\tanh^2(x)}\right) \int \tanh^3(x)\, dx$$
$$= -\frac{1}{2}\coth(x)\tanh^2(x)^{3/2} + \left(\coth(x)\sqrt{\tanh^2(x)}\right) \int \tanh(x)\, dx$$
$$= \coth(x)\log(\cosh(x))\sqrt{\tanh^2(x)} - \frac{1}{2}\coth(x)\tanh^2(x)^{3/2}$$

Mathematica [A] time = 0.0225163, size = 25, normalized size = 0.86

$$\frac{1}{2}\sqrt{\tanh^2(x)}(\operatorname{csch}(x)\operatorname{sech}(x) + 2\coth(x)\log(\cosh(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sech[x]^2)^(3/2),x]
```

```
[Out] ((2*Coth[x]*Log[Cosh[x]] + Csch[x]*Sech[x])*Sqrt[Tanh[x]^2])/2
```

Maple [B] time = 0.108, size = 120, normalized size = 4.1

$$-\frac{\left(e^{2x}+1\right)x}{e^{2x}-1}\sqrt{\frac{\left(e^{2x}-1\right)^2}{\left(e^{2x}+1\right)^2}} + 2\frac{e^{2x}}{\left(e^{2x}-1\right)\left(e^{2x}+1\right)}\sqrt{\frac{\left(e^{2x}-1\right)^2}{\left(e^{2x}+1\right)^2}} + \frac{\left(e^{2x}+1\right)\ln\left(e^{2x}+1\right)}{e^{2x}-1}\sqrt{\frac{\left(e^{2x}-1\right)^2}{\left(e^{2x}+1\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sech(x)^2)^(3/2),x)
```

```
[Out] -1/(exp(2*x)-1)*(exp(2*x)+1)*((exp(2*x)-1)^2/(exp(2*x)+1)^2)^(1/2)*x+2/(exp
(2*x)-1)/(exp(2*x)+1)*((exp(2*x)-1)^2/(exp(2*x)+1)^2)^(1/2)*exp(2*x)+1/(exp
(2*x)-1)*(exp(2*x)+1)*((exp(2*x)-1)^2/(exp(2*x)+1)^2)^(1/2)*ln(exp(2*x)+1)
```

Maxima [A] time = 1.64972, size = 45, normalized size = 1.55

$$-x - \frac{2\,e^{(-2\,x)}}{2\,e^{(-2\,x)} + e^{(-4\,x)} + 1} - \log\left(e^{(-2\,x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -x - 2*e^(-2*x)/(2*e^(-2*x) + e^(-4*x) + 1) - log(e^(-2*x) + 1)
```

Fricas [B] time = 1.95346, size = 620, normalized size = 21.38

$$-\frac{x\cosh(x)^4 + 4\,x\cosh(x)\sinh(x)^3 + x\sinh(x)^4 + 2\,(x-1)\cosh(x)^2 + 2\left(3\,x\cosh(x)^2 + x - 1\right)\sinh(x)^2 - \left(\cosh(x)^4 \right.}{\cosh(x)^4 + 4\,\cosh(x)\sinh(x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -(x*cosh(x)^4 + 4*x*cosh(x)*sinh(x)^3 + x*sinh(x)^4 + 2*(x - 1)*cosh(x)^2 +
 2*(3*x*cosh(x)^2 + x - 1)*sinh(x)^2 - (cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + s
inh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cos
h(x))*sinh(x) + 1)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 4*(x*cosh(x)^3 + (x
 - 1)*cosh(x))*sinh(x) + x)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 +
2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(
x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(1 - \operatorname{sech}^2(x)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sech(x)**2)**(3/2),x)
```

```
[Out] Integral((1 - sech(x)**2)**(3/2), x)
```

Giac [B] time = 1.12704, size = 97, normalized size = 3.34

$$-x\operatorname{sgn}\left(e^{(4x)}-1\right)+\log\left(e^{(2x)}+1\right)\operatorname{sgn}\left(e^{(4x)}-1\right)-\frac{3\,e^{(4x)}\operatorname{sgn}\left(e^{(4x)}-1\right)+2\,e^{(2x)}\operatorname{sgn}\left(e^{(4x)}-1\right)+3\operatorname{sgn}\left(e^{(4x)}-1\right)}{2\left(e^{(2x)}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -x*sgn(e^(4*x) - 1) + log(e^(2*x) + 1)*sgn(e^(4*x) - 1) - 1/2*(3*e^(4*x)*sg
n(e^(4*x) - 1) + 2*e^(2*x)*sgn(e^(4*x) - 1) + 3*sgn(e^(4*x) - 1))/(e^(2*x)
+ 1)^2
```

3.171 $\int \sqrt{1-\operatorname{sech}^2(x)}\,dx$

Optimal. Leaf size=14

$$\sqrt{\tanh^2(x)}\coth(x)\log(\cosh(x))$$

```
[Out] Coth[x]*Log[Cosh[x]]*Sqrt[Tanh[x]^2]
```

Rubi [A] time = 0.0218037, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4121, 3658, 3475}

$$\sqrt{\tanh^2(x)}\coth(x)\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Sech[x]^2],x]
```

```
[Out] Coth[x]*Log[Cosh[x]]*Sqrt[Tanh[x]^2]
```

Rule 4121

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1-\operatorname{sech}^2(x)}\,dx &= \int \sqrt{\tanh^2(x)}\,dx\\ &= \left(\coth(x)\sqrt{\tanh^2(x)}\right)\int \tanh(x)\,dx\\ &= \coth(x)\log(\cosh(x))\sqrt{\tanh^2(x)}\end{aligned}$$

Mathematica [A] time = 0.0058679, size = 14, normalized size = 1.

$$\sqrt{\tanh^2(x)}\coth(x)\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Sech[x]^2],x]
```

```
[Out] Coth[x]*Log[Cosh[x]]*Sqrt[Tanh[x]^2]
```

Maple [B] time = 0.116, size = 79, normalized size = 5.6

$$-\frac{\left(e^{2x}+1\right)x}{e^{2x}-1}\sqrt{\frac{\left(e^{2x}-1\right)^{2}}{\left(e^{2x}+1\right)^{2}}}+\frac{\left(e^{2x}+1\right)\ln\left(e^{2x}+1\right)}{e^{2x}-1}\sqrt{\frac{\left(e^{2x}-1\right)^{2}}{\left(e^{2x}+1\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sech(x)^2)^(1/2),x)
```

```
[Out] -1/(exp(2*x)-1)*(exp(2*x)+1)*((exp(2*x)-1)^2/(exp(2*x)+1)^2)^(1/2)*x+1/(exp
(2*x)-1)*(exp(2*x)+1)*((exp(2*x)-1)^2/(exp(2*x)+1)^2)^(1/2)*ln(exp(2*x)+1)
```

Maxima [A] time = 1.67558, size = 18, normalized size = 1.29

$$-x-\log\left(e^{(-2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -x - log(e^(-2*x) + 1)
```

Fricas [A] time = 1.87648, size = 55, normalized size = 3.93

$$-x+\log\left(\frac{2\cosh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -x + log(2*cosh(x)/(cosh(x) - sinh(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{1-\operatorname{sech}^{2}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sech(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(1 - sech(x)**2), x)
```

Giac [B] time = 1.1261, size = 35, normalized size = 2.5

$$-x\operatorname{sgn}\left(e^{(4x)}-1\right)+\log\left(e^{(2x)}+1\right)\operatorname{sgn}\left(e^{(4x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -x*sgn(e^(4*x) - 1) + log(e^(2*x) + 1)*sgn(e^(4*x) - 1)
```

3.172 $\int \frac{1}{\sqrt{1-\mathbf{sech}^2(x)}} \, dx$

Optimal. Leaf size=14

$$\frac{\tanh(x)\log(\sinh(x))}{\sqrt{\tanh^2(x)}}$$

```
[Out] (Log[Sinh[x]]*Tanh[x])/Sqrt[Tanh[x]^2]
```

Rubi [A] time = 0.0243822, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4121, 3658, 3475}

$$\frac{\tanh(x)\log(\sinh(x))}{\sqrt{\tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 - Sech[x]^2],x]
```

```
[Out] (Log[Sinh[x]]*Tanh[x])/Sqrt[Tanh[x]^2]
```

Rule 4121

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-\operatorname{sech}^2(x)}} \, dx &= \int \frac{1}{\sqrt{\tanh^2(x)}} \, dx\\ &= \frac{\tanh(x) \int \coth(x) \, dx}{\sqrt{\tanh^2(x)}}\\ &= \frac{\log(\sinh(x))\tanh(x)}{\sqrt{\tanh^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.0107964, size = 14, normalized size = 1.

$$\frac{\tanh(x)\log(\sinh(x))}{\sqrt{\tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - Sech[x]^2],x]
```

```
[Out] (Log[Sinh[x]]*Tanh[x])/Sqrt[Tanh[x]^2]
```

Maple [B] time = 0.099, size = 79, normalized size = 5.6

$$-\frac{\left(e^{2x}-1\right)x}{e^{2x}+1}\frac{1}{\sqrt{\frac{\left(e^{2x}-1\right)^2}{\left(e^{2x}+1\right)^2}}}+\frac{\left(e^{2x}-1\right)\ln\left(e^{2x}-1\right)}{e^{2x}+1}\frac{1}{\sqrt{\frac{\left(e^{2x}-1\right)^2}{\left(e^{2x}+1\right)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sech(x)^2)^(1/2),x)
```

```
[Out] -1/((exp(2*x)-1)^2/(exp(2*x)+1)^2)^(1/2)/(exp(2*x)+1)*(exp(2*x)-1)*x+1/((ex
p(2*x)-1)^2/(exp(2*x)+1)^2)^(1/2)/(exp(2*x)+1)*(exp(2*x)-1)*ln(exp(2*x)-1)
```

Maxima [A] time = 1.68178, size = 30, normalized size = 2.14

$$-x-\log\left(e^{(-x)}+1\right)-\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -x - log(e^(-x) + 1) - log(e^(-x) - 1)
```

Fricas [A] time = 1.85823, size = 55, normalized size = 3.93

$$-x+\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -x + log(2*sinh(x)/(cosh(x) - sinh(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{1-\operatorname{sech}^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sech(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(1 - sech(x)**2), x)
```

Giac [B] time = 1.13699, size = 42, normalized size = 3.

$$-\frac{x}{\operatorname{sgn}\left(e^{(4x)}-1\right)}+\frac{\log\left(\left|e^{(2x)}-1\right|\right)}{\operatorname{sgn}\left(e^{(4x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -x/sgn(e^(4*x) - 1) + log(abs(e^(2*x) - 1))/sgn(e^(4*x) - 1)
```

3.173 $\int \left(-1+\operatorname{sech}^2(x)\right)^{3/2} dx$

Optimal. Leaf size=34

$$\frac{1}{2}\tanh(x)\sqrt{-\tanh^2(x)}-\sqrt{-\tanh^2(x)}\coth(x)\log(\cosh(x))$$

```
[Out] -(Coth[x]*Log[Cosh[x]]*Sqrt[-Tanh[x]^2]) + (Tanh[x]*Sqrt[-Tanh[x]^2])/2
```

Rubi [A] time = 0.0288396, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4121, 3658, 3473, 3475}

$$\frac{1}{2}\tanh(x)\sqrt{-\tanh^2(x)}-\sqrt{-\tanh^2(x)}\coth(x)\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Sech[x]^2)^(3/2),x]
```

```
[Out] -(Coth[x]*Log[Cosh[x]]*Sqrt[-Tanh[x]^2]) + (Tanh[x]*Sqrt[-Tanh[x]^2])/2
```

Rule 4121

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(-1+\operatorname{sech}^2(x)\right)^{3/2} dx = \int \left(-\tanh^2(x)\right)^{3/2} dx$$
$$= -\left(\left(\coth(x)\sqrt{-\tanh^2(x)}\right)\int \tanh^3(x)\,dx\right)$$
$$= \frac{1}{2}\tanh(x)\sqrt{-\tanh^2(x)} - \left(\coth(x)\sqrt{-\tanh^2(x)}\right)\int \tanh(x)\,dx$$
$$= -\coth(x)\log(\cosh(x))\sqrt{-\tanh^2(x)} + \frac{1}{2}\tanh(x)\sqrt{-\tanh^2(x)}$$

Mathematica [A] time = 0.0154117, size = 27, normalized size = 0.79

$$-\frac{1}{2}\sqrt{-\tanh^2(x)}(\operatorname{csch}(x)\operatorname{sech}(x)+2\coth(x)\log(\cosh(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Sech[x]^2)^(3/2),x]
```

```
[Out] -((2*Coth[x]*Log[Cosh[x]] + Csch[x]*Sech[x])*Sqrt[-Tanh[x]^2])/2
```

Maple [B] time = 0.102, size = 123, normalized size = 3.6

$$\frac{\left(e^{2x}+1\right)x}{e^{2x}-1}\sqrt{-\frac{\left(e^{2x}-1\right)^2}{\left(e^{2x}+1\right)^2}}-2\frac{e^{2x}}{\left(e^{2x}-1\right)\left(e^{2x}+1\right)}\sqrt{-\frac{\left(e^{2x}-1\right)^2}{\left(e^{2x}+1\right)^2}}-\frac{\left(e^{2x}+1\right)\ln\left(e^{2x}+1\right)}{e^{2x}-1}\sqrt{-\frac{\left(e^{2x}-1\right)^2}{\left(e^{2x}+1\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+sech(x)^2)^(3/2),x)
```

```
[Out] 1/(exp(2*x)-1)*(exp(2*x)+1)*(-(exp(2*x)-1)^2/(exp(2*x)+1)^2)^(1/2)*x-2/(exp
(2*x)-1)/(exp(2*x)+1)*(-(exp(2*x)-1)^2/(exp(2*x)+1)^2)^(1/2)*exp(2*x)-1/(ex
p(2*x)-1)*(exp(2*x)+1)*(-(exp(2*x)-1)^2/(exp(2*x)+1)^2)^(1/2)*ln(exp(2*x)+1
)
```

Maxima [C] time = 1.66048, size = 45, normalized size = 1.32

$$i\,x+\frac{2i\,e^{(-2\,x)}}{2\,e^{(-2\,x)}+e^{(-4\,x)}+1}+i\,\log\left(e^{(-2\,x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] I*x + 2*I*e^(-2*x)/(2*e^(-2*x) + e^(-4*x) + 1) + I*log(e^(-2*x) + 1)
```

Fricas [A] time = 2.05415, size = 4, normalized size = 0.12

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\operatorname{sech}^2(x) - 1\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sech(x)**2)**(3/2),x)
```

```
[Out] Integral((sech(x)**2 - 1)**(3/2), x)
```

Giac [C] time = 1.13726, size = 112, normalized size = 3.29

$$-ix\operatorname{sgn}\left(-e^{(4x)}+1\right)+i\log\left(e^{(2x)}+1\right)\operatorname{sgn}\left(-e^{(4x)}+1\right)-\frac{i\left(3e^{(4x)}\operatorname{sgn}\left(-e^{(4x)}+1\right)+2e^{(2x)}\operatorname{sgn}\left(-e^{(4x)}+1\right)+3\operatorname{sgn}\left(-e^{(4x)}+1\right)\right)}{2\left(e^{(2x)}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -I*x*sgn(-e^(4*x) + 1) + I*log(e^(2*x) + 1)*sgn(-e^(4*x) + 1) - 1/2*I*(3*e^
(4*x)*sgn(-e^(4*x) + 1) + 2*e^(2*x)*sgn(-e^(4*x) + 1) + 3*sgn(-e^(4*x) + 1)
)/(e^(2*x) + 1)^2
```

3.174 $\int \sqrt{-1+\operatorname{sech}^2(x)}\,dx$

Optimal. Leaf size=16

$$\sqrt{-\tanh^2(x)}\coth(x)\log(\cosh(x))$$

```
[Out] Coth[x]*Log[Cosh[x]]*Sqrt[-Tanh[x]^2]
```

Rubi [A] time = 0.0195109, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4121, 3658, 3475}

$$\sqrt{-\tanh^2(x)}\coth(x)\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + Sech[x]^2],x]
```

```
[Out] Coth[x]*Log[Cosh[x]]*Sqrt[-Tanh[x]^2]
```

Rule 4121

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{-1+\operatorname{sech}^2(x)}\,dx &= \int \sqrt{-\tanh^2(x)}\,dx\\ &= \left(\coth(x)\sqrt{-\tanh^2(x)}\right)\int \tanh(x)\,dx\\ &= \coth(x)\log(\cosh(x))\sqrt{-\tanh^2(x)}\end{aligned}$$

Mathematica [A] time = 0.0064218, size = 16, normalized size = 1.

$$\sqrt{-\tanh^2(x)}\coth(x)\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + Sech[x]^2],x]
```

```
[Out] Coth[x]*Log[Cosh[x]]*Sqrt[-Tanh[x]^2]
```

Maple [B] time = 0.122, size = 81, normalized size = 5.1

$$-\frac{\left(e^{2x}+1\right)x}{e^{2x}-1}\sqrt{-\frac{\left(e^{2x}-1\right)^{2}}{\left(e^{2x}+1\right)^{2}}}+\frac{\left(e^{2x}+1\right)\ln\left(e^{2x}+1\right)}{e^{2x}-1}\sqrt{-\frac{\left(e^{2x}-1\right)^{2}}{\left(e^{2x}+1\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+sech(x)^2)^(1/2),x)
```

```
[Out] -1/(exp(2*x)-1)*(exp(2*x)+1)*(-(exp(2*x)-1)^2/(exp(2*x)+1)^2)^(1/2)*x+1/(ex
p(2*x)-1)*(exp(2*x)+1)*(-(exp(2*x)-1)^2/(exp(2*x)+1)^2)^(1/2)*ln(exp(2*x)+1
)
```

Maxima [C] time = 1.70793, size = 18, normalized size = 1.12

$$-ix-i\log\left(e^{(-2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -I*x - I*log(e^(-2*x) + 1)
```

Fricas [A] time = 1.97561, size = 4, normalized size = 0.25

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\operatorname{sech}^{2}(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sech(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(sech(x)**2 - 1), x)
```

Giac [C] time = 1.12261, size = 42, normalized size = 2.62

$$i\,x\mathrm{sgn}\left(-e^{(4\,x)}+1\right)-i\,\log\left(e^{(2\,x)}+1\right)\mathrm{sgn}\left(-e^{(4\,x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] I*x*sgn(-e^(4*x) + 1) - I*log(e^(2*x) + 1)*sgn(-e^(4*x) + 1)
```

3.175 $\int \frac{1}{\sqrt{-1+\mathbf{sech}^2(x)}} \, dx$

Optimal. Leaf size=16

$$\frac{\tanh(x)\log(\sinh(x))}{\sqrt{-\tanh^2(x)}}$$

```
[Out] (Log[Sinh[x]]*Tanh[x])/Sqrt[-Tanh[x]^2]
```

Rubi [A] time = 0.0219229, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4121, 3658, 3475}

$$\frac{\tanh(x)\log(\sinh(x))}{\sqrt{-\tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-1 + Sech[x]^2],x]
```

```
[Out] (Log[Sinh[x]]*Tanh[x])/Sqrt[-Tanh[x]^2]
```

Rule 4121

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-1+\operatorname{sech}^2(x)}} \, dx &= \int \frac{1}{\sqrt{-\tanh^2(x)}} \, dx \\
&= \frac{\tanh(x) \int \coth(x) \, dx}{\sqrt{-\tanh^2(x)}} \\
&= \frac{\log(\sinh(x))\tanh(x)}{\sqrt{-\tanh^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.0086223, size = 16, normalized size = 1.

$$\frac{\tanh(x)\log(\sinh(x))}{\sqrt{-\tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 + Sech[x]^2],x]
```

```
[Out] (Log[Sinh[x]]*Tanh[x])/Sqrt[-Tanh[x]^2]
```

Maple [B] time = 0.101, size = 81, normalized size = 5.1

$$-\frac{(e^{2x}-1)x}{e^{2x}+1}\frac{1}{\sqrt{-\frac{(e^{2x}-1)^2}{(e^{2x}+1)^2}}}+\frac{(e^{2x}-1)\ln(e^{2x}-1)}{e^{2x}+1}\frac{1}{\sqrt{-\frac{(e^{2x}-1)^2}{(e^{2x}+1)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+sech(x)^2)^(1/2),x)
```

```
[Out] -1/(-(exp(2*x)-1)^2/(exp(2*x)+1)^2)^(1/2)/(exp(2*x)+1)*(exp(2*x)-1)*x+1/(-(
exp(2*x)-1)^2/(exp(2*x)+1)^2)^(1/2)/(exp(2*x)+1)*(exp(2*x)-1)*ln(exp(2*x)-1
)
```

Maxima [C] time = 1.70369, size = 30, normalized size = 1.88

$$ix + i\log\left(e^{(-x)}+1\right) + i\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] I*x + I*log(e^(-x) + 1) + I*log(e^(-x) - 1)
```

Fricas [A] time = 1.96996, size = 4, normalized size = 0.25

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{\operatorname{sech}^2(x)-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+sech(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(sech(x)**2 - 1), x)
```

Giac [C] time = 1.14028, size = 50, normalized size = 3.12

$$-\frac{ix}{\operatorname{sgn}\left(-e^{(4x)}+1\right)}+\frac{i\log\left(-ie^{(2x)}+i\right)}{\operatorname{sgn}\left(-e^{(4x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -I*x/sgn(-e^(4*x) + 1) + I*log(-I*e^(2*x) + I)/sgn(-e^(4*x) + 1)
```

3.176 $\int \sqrt{a+b\,\mathbf{sech}^2(x)}\tanh^5(x)\,dx$

Optimal. Leaf size=83

$$-\frac{\left(a+b\,\mathrm{sech}^2(x)\right)^{5/2}}{5b^2}+\frac{(a+2b)\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}{3b^2}-\sqrt{a+b\,\mathrm{sech}^2(x)}+\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)$$

```
[Out] Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - Sqrt[a + b*Sech[x]^2] + ((
a + 2*b)*(a + b*Sech[x]^2)^(3/2))/(3*b^2) - (a + b*Sech[x]^2)^(5/2)/(5*b^2)
```

Rubi [A] time = 0.146731, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {4139, 446, 88, 50, 63, 208}

$$-\frac{\left(a+b\,\mathrm{sech}^2(x)\right)^{5/2}}{5b^2}+\frac{(a+2b)\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}{3b^2}-\sqrt{a+b\,\mathrm{sech}^2(x)}+\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sech[x]^2]*Tanh[x]^5,x]
```

```
[Out] Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - Sqrt[a + b*Sech[x]^2] + ((
a + 2*b)*(a + b*Sech[x]^2)^(3/2))/(3*b^2) - (a + b*Sech[x]^2)^(5/2)/(5*b^2)
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{a+b\text{sech}^2(x)}\tanh^5(x)\,dx &= -\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2\sqrt{a+bx^2}}{x}\,dx, x, \text{sech}(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(-1+x)^2\sqrt{a+bx}}{x}\,dx, x, \text{sech}^2(x)\right)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int\left(\frac{(-a-2b)\sqrt{a+bx}}{b}+\frac{\sqrt{a+bx}}{x}+\frac{(a+bx)^{3/2}}{b}\right)dx, x, \text{sech}^2(x)\right)\right)\\
&= \frac{(a+2b)\left(a+b\text{sech}^2(x)\right)^{3/2}}{3b^2}-\frac{\left(a+b\text{sech}^2(x)\right)^{5/2}}{5b^2}-\frac{1}{2}\text{Subst}\left(\int\frac{\sqrt{a+bx}}{x}\,dx, x, \text{sech}^2(
\\
&= -\sqrt{a+b\text{sech}^2(x)}+\frac{(a+2b)\left(a+b\text{sech}^2(x)\right)^{3/2}}{3b^2}-\frac{\left(a+b\text{sech}^2(x)\right)^{5/2}}{5b^2}-\frac{1}{2}a\,\text{Subst}\Bigg(\int
\\
&= -\sqrt{a+b\text{sech}^2(x)}+\frac{(a+2b)\left(a+b\text{sech}^2(x)\right)^{3/2}}{3b^2}-\frac{\left(a+b\text{sech}^2(x)\right)^{5/2}}{5b^2}-\frac{a\,\text{Subst}\left(\int \frac{}{-\frac{a}{b}
\\
&= \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)-\sqrt{a+b\text{sech}^2(x)}+\frac{(a+2b)\left(a+b\text{sech}^2(x)\right)^{3/2}}{3b^2}-\frac{(a+
\end{aligned}
$$

Mathematica [A] time = 0.707993, size = 114, normalized size = 1.37

$$
\frac{1}{15}\cosh(x)\sqrt{a+b\text{sech}^2(x)}\left(\left(\frac{2a^2}{b^2}+\frac{10a}{b}-15\right)\text{sech}(x)+\left(10-\frac{a}{b}\right)\text{sech}^3(x)+\frac{15\sqrt{2}\sqrt{a}\log\left(\sqrt{a\cosh(2x)+a+2b}+\sqrt{2}\right.}{\sqrt{a\cosh(2x)+a+2b}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sech[x]^2]*Tanh[x]^5,x]
```

```
[Out] (Cosh[x]*Sqrt[a + b*Sech[x]^2]*((15*Sqrt[2]*Sqrt[a]*Log[Sqrt[2]*Sqrt[a]*Cos
h[x] + Sqrt[a + 2*b + a*Cosh[2*x]]])/Sqrt[a + 2*b + a*Cosh[2*x]] + (-15 + (
2*a^2)/b^2 + (10*a)/b)*Sech[x] + (10 - a/b)*Sech[x]^3 - 3*Sech[x]^5))/15
```

Maple [F] time = 0.156, size = 0, normalized size = 0.

$$
\int \sqrt{a+b\left(\text{sech}\left(x\right)\right)^2}\left(\tanh\left(x\right)\right)^5\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)^(1/2)*tanh(x)^5,x)
```

```
[Out] int((a+b*sech(x)^2)^(1/2)*tanh(x)^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\mathrm{sech}\,(x)^2 + a}\tanh\,(x)^5\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a)*tanh(x)^5, x)
```

Fricas [B] time = 5.88927, size = 12714, normalized size = 153.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^5,x, algorithm="fricas")
```

```
[Out] [1/60*(15*(b^2*cosh(x)^10 + 10*b^2*cosh(x)*sinh(x)^9 + b^2*sinh(x)^10 + 5*b
^2*cosh(x)^8 + 5*(9*b^2*cosh(x)^2 + b^2)*sinh(x)^8 + 10*b^2*cosh(x)^6 + 40*
(3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^7 + 10*(21*b^2*cosh(x)^4 + 14*b^2*c
osh(x)^2 + b^2)*sinh(x)^6 + 10*b^2*cosh(x)^4 + 4*(63*b^2*cosh(x)^5 + 70*b^2
*cosh(x)^3 + 15*b^2*cosh(x))*sinh(x)^5 + 10*(21*b^2*cosh(x)^6 + 35*b^2*cosh
(x)^4 + 15*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 5*b^2*cosh(x)^2 + 40*(3*b^2*cos
h(x)^7 + 7*b^2*cosh(x)^5 + 5*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^3 + 5*(9*
b^2*cosh(x)^8 + 28*b^2*cosh(x)^6 + 30*b^2*cosh(x)^4 + 12*b^2*cosh(x)^2 + b^
2)*sinh(x)^2 + b^2 + 10*(b^2*cosh(x)^9 + 4*b^2*cosh(x)^7 + 6*b^2*cosh(x)^5
+ 4*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a)*log(((a^3 + 2*a^2*b + a*b
^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*
b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2
*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh
(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*
b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (7
0*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^
3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b +
a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3
+ 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x
)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b
^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh
(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b
+ b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b +
 b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*s
inh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(
x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)
^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*
(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2
 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2
```

```
*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*
b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 1
4*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)
^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3
+ 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 15*(b^2*cosh
(x)^10 + 10*b^2*cosh(x)*sinh(x)^9 + b^2*sinh(x)^10 + 5*b^2*cosh(x)^8 + 5*(9
*b^2*cosh(x)^2 + b^2)*sinh(x)^8 + 10*b^2*cosh(x)^6 + 40*(3*b^2*cosh(x)^3 +
b^2*cosh(x))*sinh(x)^7 + 10*(21*b^2*cosh(x)^4 + 14*b^2*cosh(x)^2 + b^2)*sin
h(x)^6 + 10*b^2*cosh(x)^4 + 4*(63*b^2*cosh(x)^5 + 70*b^2*cosh(x)^3 + 15*b^2
*cosh(x))*sinh(x)^5 + 10*(21*b^2*cosh(x)^6 + 35*b^2*cosh(x)^4 + 15*b^2*cosh
(x)^2 + b^2)*sinh(x)^4 + 5*b^2*cosh(x)^2 + 40*(3*b^2*cosh(x)^7 + 7*b^2*cosh
(x)^5 + 5*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^3 + 5*(9*b^2*cosh(x)^8 + 28*
b^2*cosh(x)^6 + 30*b^2*cosh(x)^4 + 12*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2
+ 10*(b^2*cosh(x)^9 + 4*b^2*cosh(x)^7 + 6*b^2*cosh(x)^5 + 4*b^2*cosh(x)^3 +
 b^2*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 +
a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*
sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*co
sh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2)) + 4*sqrt(2)*((2*a^2 + 10*a*b - 15*b^2)*cosh(x)^8 + 8*(2*a^2 + 10*a*b -
15*b^2)*cosh(x)*sinh(x)^7 + (2*a^2 + 10*a*b - 15*b^2)*sinh(x)^8 + 4*(2*a^2
+ 9*a*b - 5*b^2)*cosh(x)^6 + 4*(7*(2*a^2 + 10*a*b - 15*b^2)*cosh(x)^2 + 2*a
^2 + 9*a*b - 5*b^2)*sinh(x)^6 + 8*(7*(2*a^2 + 10*a*b - 15*b^2)*cosh(x)^3 +
3*(2*a^2 + 9*a*b - 5*b^2)*cosh(x))*sinh(x)^5 + 2*(6*a^2 + 26*a*b - 29*b^2)*
cosh(x)^4 + 2*(35*(2*a^2 + 10*a*b - 15*b^2)*cosh(x)^4 + 30*(2*a^2 + 9*a*b -
 5*b^2)*cosh(x)^2 + 6*a^2 + 26*a*b - 29*b^2)*sinh(x)^4 + 8*(7*(2*a^2 + 10*a
*b - 15*b^2)*cosh(x)^5 + 10*(2*a^2 + 9*a*b - 5*b^2)*cosh(x)^3 + (6*a^2 + 26
*a*b - 29*b^2)*cosh(x))*sinh(x)^3 + 4*(2*a^2 + 9*a*b - 5*b^2)*cosh(x)^2 + 4
*(7*(2*a^2 + 10*a*b - 15*b^2)*cosh(x)^6 + 15*(2*a^2 + 9*a*b - 5*b^2)*cosh(x
)^4 + 3*(6*a^2 + 26*a*b - 29*b^2)*cosh(x)^2 + 2*a^2 + 9*a*b - 5*b^2)*sinh(x
)^2 + 2*a^2 + 10*a*b - 15*b^2 + 8*((2*a^2 + 10*a*b - 15*b^2)*cosh(x)^7 + 3*
(2*a^2 + 9*a*b - 5*b^2)*cosh(x)^5 + (6*a^2 + 26*a*b - 29*b^2)*cosh(x)^3 + (
2*a^2 + 9*a*b - 5*b^2)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b^2*cosh(x)^10 + 10
*b^2*cosh(x)*sinh(x)^9 + b^2*sinh(x)^10 + 5*b^2*cosh(x)^8 + 5*(9*b^2*cosh(x
)^2 + b^2)*sinh(x)^8 + 10*b^2*cosh(x)^6 + 40*(3*b^2*cosh(x)^3 + b^2*cosh(x)
)*sinh(x)^7 + 10*(21*b^2*cosh(x)^4 + 14*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 10
*b^2*cosh(x)^4 + 4*(63*b^2*cosh(x)^5 + 70*b^2*cosh(x)^3 + 15*b^2*cosh(x))*s
inh(x)^5 + 10*(21*b^2*cosh(x)^6 + 35*b^2*cosh(x)^4 + 15*b^2*cosh(x)^2 + b^2
)*sinh(x)^4 + 5*b^2*cosh(x)^2 + 40*(3*b^2*cosh(x)^7 + 7*b^2*cosh(x)^5 + 5*b
^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^3 + 5*(9*b^2*cosh(x)^8 + 28*b^2*cosh(x)
^6 + 30*b^2*cosh(x)^4 + 12*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 10*(b^2*c
osh(x)^9 + 4*b^2*cosh(x)^7 + 6*b^2*cosh(x)^5 + 4*b^2*cosh(x)^3 + b^2*cosh(x
))*sinh(x)), -1/30*(15*(b^2*cosh(x)^10 + 10*b^2*cosh(x)*sinh(x)^9 + b^2*sin
h(x)^10 + 5*b^2*cosh(x)^8 + 5*(9*b^2*cosh(x)^2 + b^2)*sinh(x)^8 + 10*b^2*co
sh(x)^6 + 40*(3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^7 + 10*(21*b^2*cosh(x)
^4 + 14*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 10*b^2*cosh(x)^4 + 4*(63*b^2*cosh(
x)^5 + 70*b^2*cosh(x)^3 + 15*b^2*cosh(x))*sinh(x)^5 + 10*(21*b^2*cosh(x)^6
+ 35*b^2*cosh(x)^4 + 15*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 5*b^2*cosh(x)^2 +
40*(3*b^2*cosh(x)^7 + 7*b^2*cosh(x)^5 + 5*b^2*cosh(x)^3 + b^2*cosh(x))*sinh
(x)^3 + 5*(9*b^2*cosh(x)^8 + 28*b^2*cosh(x)^6 + 30*b^2*cosh(x)^4 + 12*b^2*c
osh(x)^2 + b^2)*sinh(x)^2 + b^2 + 10*(b^2*cosh(x)^9 + 4*b^2*cosh(x)^7 + 6*b
^2*cosh(x)^5 + 4*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt
(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)
*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)
*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(
x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*c
osh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2
*a^2 + 3*a*b)*cosh(x))*sinh(x))) + 15*(b^2*cosh(x)^10 + 10*b^2*cosh(x)*sinh
```

```
(x)^9 + b^2*sinh(x)^10 + 5*b^2*cosh(x)^8 + 5*(9*b^2*cosh(x)^2 + b^2)*sinh(x
)^8 + 10*b^2*cosh(x)^6 + 40*(3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^7 + 10*
(21*b^2*cosh(x)^4 + 14*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 10*b^2*cosh(x)^4 +
4*(63*b^2*cosh(x)^5 + 70*b^2*cosh(x)^3 + 15*b^2*cosh(x))*sinh(x)^5 + 10*(21
*b^2*cosh(x)^6 + 35*b^2*cosh(x)^4 + 15*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 5*b
^2*cosh(x)^2 + 40*(3*b^2*cosh(x)^7 + 7*b^2*cosh(x)^5 + 5*b^2*cosh(x)^3 + b^
2*cosh(x))*sinh(x)^3 + 5*(9*b^2*cosh(x)^8 + 28*b^2*cosh(x)^6 + 30*b^2*cosh(
x)^4 + 12*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 10*(b^2*cosh(x)^9 + 4*b^2*
cosh(x)^7 + 6*b^2*cosh(x)^5 + 4*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(
-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)
*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
+ sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2
*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a
 + 2*b)*cosh(x))*sinh(x) + a)) - 2*sqrt(2)*((2*a^2 + 10*a*b - 15*b^2)*cosh(
x)^8 + 8*(2*a^2 + 10*a*b - 15*b^2)*cosh(x)*sinh(x)^7 + (2*a^2 + 10*a*b - 15
*b^2)*sinh(x)^8 + 4*(2*a^2 + 9*a*b - 5*b^2)*cosh(x)^6 + 4*(7*(2*a^2 + 10*a*
b - 15*b^2)*cosh(x)^2 + 2*a^2 + 9*a*b - 5*b^2)*sinh(x)^6 + 8*(7*(2*a^2 + 10
*a*b - 15*b^2)*cosh(x)^3 + 3*(2*a^2 + 9*a*b - 5*b^2)*cosh(x))*sinh(x)^5 + 2
*(6*a^2 + 26*a*b - 29*b^2)*cosh(x)^4 + 2*(35*(2*a^2 + 10*a*b - 15*b^2)*cosh
(x)^4 + 30*(2*a^2 + 9*a*b - 5*b^2)*cosh(x)^2 + 6*a^2 + 26*a*b - 29*b^2)*sin
h(x)^4 + 8*(7*(2*a^2 + 10*a*b - 15*b^2)*cosh(x)^5 + 10*(2*a^2 + 9*a*b - 5*b
^2)*cosh(x)^3 + (6*a^2 + 26*a*b - 29*b^2)*cosh(x))*sinh(x)^3 + 4*(2*a^2 + 9
*a*b - 5*b^2)*cosh(x)^2 + 4*(7*(2*a^2 + 10*a*b - 15*b^2)*cosh(x)^6 + 15*(2*
a^2 + 9*a*b - 5*b^2)*cosh(x)^4 + 3*(6*a^2 + 26*a*b - 29*b^2)*cosh(x)^2 + 2*
a^2 + 9*a*b - 5*b^2)*sinh(x)^2 + 2*a^2 + 10*a*b - 15*b^2 + 8*((2*a^2 + 10*a
*b - 15*b^2)*cosh(x)^7 + 3*(2*a^2 + 9*a*b - 5*b^2)*cosh(x)^5 + (6*a^2 + 26*
a*b - 29*b^2)*cosh(x)^3 + (2*a^2 + 9*a*b - 5*b^2)*cosh(x))*sinh(x))*sqrt((a
*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x
)^2)))/(b^2*cosh(x)^10 + 10*b^2*cosh(x)*sinh(x)^9 + b^2*sinh(x)^10 + 5*b^2*
cosh(x)^8 + 5*(9*b^2*cosh(x)^2 + b^2)*sinh(x)^8 + 10*b^2*cosh(x)^6 + 40*(3*
b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^7 + 10*(21*b^2*cosh(x)^4 + 14*b^2*cosh
(x)^2 + b^2)*sinh(x)^6 + 10*b^2*cosh(x)^4 + 4*(63*b^2*cosh(x)^5 + 70*b^2*co
sh(x)^3 + 15*b^2*cosh(x))*sinh(x)^5 + 10*(21*b^2*cosh(x)^6 + 35*b^2*cosh(x)
^4 + 15*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 5*b^2*cosh(x)^2 + 40*(3*b^2*cosh(x
)^7 + 7*b^2*cosh(x)^5 + 5*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^3 + 5*(9*b^2
*cosh(x)^8 + 28*b^2*cosh(x)^6 + 30*b^2*cosh(x)^4 + 12*b^2*cosh(x)^2 + b^2)*
sinh(x)^2 + b^2 + 10*(b^2*cosh(x)^9 + 4*b^2*cosh(x)^7 + 6*b^2*cosh(x)^5 + 4
*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\operatorname{sech}^2(x)}\tanh^5(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)**2)**(1/2)*tanh(x)**5,x)
```

```
[Out] Integral(sqrt(a + b*sech(x)**2)*tanh(x)**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\operatorname{sech}(x)^2+a}\tanh(x)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a)*tanh(x)^5, x)
```

3.177 $\int \sqrt{a+b\,\mathbf{sech}^2(x)}\tanh^4(x)\,dx$

Optimal. Leaf size=125

$$-\frac{\left(a^2+6ab-3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{8b^{3/2}}-\frac{1}{4}\tanh^3(x)\sqrt{a-b\tanh^2(x)+b}+\frac{(a-3b)\tanh(x)\sqrt{a-b\tanh^2(x)+b}}{8b}$$

```
[Out] -((a^2 + 6*a*b - 3*b^2)*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]
)/(8*b^(3/2)) + Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]
] + ((a - 3*b)*Tanh[x]*Sqrt[a + b - b*Tanh[x]^2])/(8*b) - (Tanh[x]^3*Sqrt[a
 + b - b*Tanh[x]^2])/4
```

Rubi [A] time = 0.312771, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {4141, 1975, 478, 582, 523, 217, 203, 377, 206}

$$-\frac{\left(a^2+6ab-3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{8b^{3/2}}-\frac{1}{4}\tanh^3(x)\sqrt{a-b\tanh^2(x)+b}+\frac{(a-3b)\tanh(x)\sqrt{a-b\tanh^2(x)+b}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sech[x]^2]*Tanh[x]^4,x]
```

```
[Out] -((a^2 + 6*a*b - 3*b^2)*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]
)/(8*b^(3/2)) + Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]
] + ((a - 3*b)*Tanh[x]*Sqrt[a + b - b*Tanh[x]^2])/(8*b) - (Tanh[x]^3*Sqrt[a
 + b - b*Tanh[x]^2])/4
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
```

```
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+b\operatorname{sech}^2(x)}\tanh^4(x)\,dx = \text{Subst}\left(\int \frac{x^4\sqrt{a+b\left(1-x^2\right)}}{1-x^2}\,dx, x, \tanh(x)\right)$$

$$= \text{Subst}\left(\int \frac{x^4\sqrt{a+b-bx^2}}{1-x^2}\,dx, x, \tanh(x)\right)$$

$$= -\frac{1}{4}\tanh^3(x)\sqrt{a+b-b\tanh^2(x)} + \frac{1}{4}\text{Subst}\left(\int \frac{x^2\left(3(a+b)+(a-3b)x^2\right)}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx, x, \text{ta}\right.$$

$$= \frac{(a-3b)\tanh(x)\sqrt{a+b-b\tanh^2(x)}}{8b} - \frac{1}{4}\tanh^3(x)\sqrt{a+b-b\tanh^2(x)} - \frac{\text{Subst}\left(\right.}{}$$

$$= \frac{(a-3b)\tanh(x)\sqrt{a+b-b\tanh^2(x)}}{8b} - \frac{1}{4}\tanh^3(x)\sqrt{a+b-b\tanh^2(x)} + a\,\text{Subst}$$

$$= \frac{(a-3b)\tanh(x)\sqrt{a+b-b\tanh^2(x)}}{8b} - \frac{1}{4}\tanh^3(x)\sqrt{a+b-b\tanh^2(x)} + a\,\text{Subst}$$

$$= -\frac{\left(a^2+6ab-3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{8b^{3/2}} + \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right) + \frac{(}{}$$

Mathematica [A] time = 0.439764, size = 192, normalized size = 1.54

$$-\frac{\cosh(x)\sqrt{a+b\operatorname{sech}^2(x)}\left(\sqrt{2}\left(a^2+6ab-3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sinh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right) - 8\sqrt{2}\sqrt{a}b^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right) - 2b^{3/2}\right.}{8b^{3/2}\sqrt{a\cosh(2x)+a+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sech[x]^2]*Tanh[x]^4,x]
```

```
[Out] -(Cosh[x]*Sqrt[a + b*Sech[x]^2]*(Sqrt[2]*(a^2 + 6*a*b - 3*b^2)*ArcTan[(Sqrt
[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]] - 8*Sqrt[2]*Sqrt[a]*b^(3/
2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]] - (a - 5*
b)*Sqrt[b]*Sqrt[a + 2*b + a*Cosh[2*x]]*Sech[x]*Tanh[x] - 2*b^(3/2)*Sqrt[a +
 2*b + a*Cosh[2*x]]*Sech[x]^3*Tanh[x]))/(8*b^(3/2)*Sqrt[a + 2*b + a*Cosh[2*
x]])
```

Maple [F] time = 0.125, size = 0, normalized size = 0.

$$\int \sqrt{a+b\left(\operatorname{sech}(x)\right)^2}\left(\tanh(x)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)^(1/2)*tanh(x)^4,x)
```

```
[Out] int((a+b*sech(x)^2)^(1/2)*tanh(x)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\mathrm{sech}\,(x)^2 + a}\tanh\,(x)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a)*tanh(x)^4, x)
```

Fricas [B] time = 5.80809, size = 24571, normalized size = 196.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^4,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*c
osh(x)^6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2
*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x
)^2 + 3*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh
(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 +
9*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5
 + 3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8
*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*
(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a
*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*
a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sin
h(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*
b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14
*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*
a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh
(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)
*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*
cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 -
 a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x
))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x
)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5
+ (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(
cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sin
h(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a^2
 + 6*a*b - 3*b^2)*cosh(x)^8 + 8*(a^2 + 6*a*b - 3*b^2)*cosh(x)*sinh(x)^7 + (
a^2 + 6*a*b - 3*b^2)*sinh(x)^8 + 4*(a^2 + 6*a*b - 3*b^2)*cosh(x)^6 + 4*(7*(
a^2 + 6*a*b - 3*b^2)*cosh(x)^2 + a^2 + 6*a*b - 3*b^2)*sinh(x)^6 + 8*(7*(a^2
 + 6*a*b - 3*b^2)*cosh(x)^3 + 3*(a^2 + 6*a*b - 3*b^2)*cosh(x))*sinh(x)^5 +
6*(a^2 + 6*a*b - 3*b^2)*cosh(x)^4 + 2*(35*(a^2 + 6*a*b - 3*b^2)*cosh(x)^4 +
 30*(a^2 + 6*a*b - 3*b^2)*cosh(x)^2 + 3*a^2 + 18*a*b - 9*b^2)*sinh(x)^4 + 8
*(7*(a^2 + 6*a*b - 3*b^2)*cosh(x)^5 + 10*(a^2 + 6*a*b - 3*b^2)*cosh(x)^3 +
3*(a^2 + 6*a*b - 3*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + 6*a*b - 3*b^2)*cosh(x
)^2 + 4*(7*(a^2 + 6*a*b - 3*b^2)*cosh(x)^6 + 15*(a^2 + 6*a*b - 3*b^2)*cosh(
x)^4 + 9*(a^2 + 6*a*b - 3*b^2)*cosh(x)^2 + a^2 + 6*a*b - 3*b^2)*sinh(x)^2 +
 a^2 + 6*a*b - 3*b^2 + 8*((a^2 + 6*a*b - 3*b^2)*cosh(x)^7 + 3*(a^2 + 6*a*b
- 3*b^2)*cosh(x)^5 + 3*(a^2 + 6*a*b - 3*b^2)*cosh(x)^3 + (a^2 + 6*a*b - 3*b
^2)*cosh(x))*sinh(x))*sqrt(-b)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*
sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x
```

```
)^2 + a + 3*b)*sinh(x)^2 + 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(
x)^2 - 1)*sqrt(-b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))
*sinh(x) + a - b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(
x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) +
 4*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)
^6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(
x)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 +
3*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3
+ 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2*
cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 + 3*b
^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x
)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)
*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a
)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
 + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2
+ 2*cosh(x)*sinh(x) + sinh(x)^2)) + 2*sqrt(2)*((a*b - 5*b^2)*cosh(x)^6 + 6*
(a*b - 5*b^2)*cosh(x)*sinh(x)^5 + (a*b - 5*b^2)*sinh(x)^6 + (a*b + 3*b^2)*c
osh(x)^4 + (15*(a*b - 5*b^2)*cosh(x)^2 + a*b + 3*b^2)*sinh(x)^4 + 4*(5*(a*b
 - 5*b^2)*cosh(x)^3 + (a*b + 3*b^2)*cosh(x))*sinh(x)^3 - (a*b + 3*b^2)*cosh
(x)^2 + (15*(a*b - 5*b^2)*cosh(x)^4 + 6*(a*b + 3*b^2)*cosh(x)^2 - a*b - 3*b
^2)*sinh(x)^2 - a*b + 5*b^2 + 2*(3*(a*b - 5*b^2)*cosh(x)^5 + 2*(a*b + 3*b^2
)*cosh(x)^3 - (a*b + 3*b^2)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)
^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b^2*cosh(x)^8
+ 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh
(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 + 3*b^2*cosh(
x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 + 3*b^2)*sinh(x)^4 +
 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*s
inh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2*cosh(x)^2 + b^2)*s
inh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 + 3*b^2*cosh(x)^3 + b^2
*cosh(x))*sinh(x)), -1/8*(((a^2 + 6*a*b - 3*b^2)*cosh(x)^8 + 8*(a^2 + 6*a*b
 - 3*b^2)*cosh(x)*sinh(x)^7 + (a^2 + 6*a*b - 3*b^2)*sinh(x)^8 + 4*(a^2 + 6*
a*b - 3*b^2)*cosh(x)^6 + 4*(7*(a^2 + 6*a*b - 3*b^2)*cosh(x)^2 + a^2 + 6*a*b
 - 3*b^2)*sinh(x)^6 + 8*(7*(a^2 + 6*a*b - 3*b^2)*cosh(x)^3 + 3*(a^2 + 6*a*b
 - 3*b^2)*cosh(x))*sinh(x)^5 + 6*(a^2 + 6*a*b - 3*b^2)*cosh(x)^4 + 2*(35*(a
^2 + 6*a*b - 3*b^2)*cosh(x)^4 + 30*(a^2 + 6*a*b - 3*b^2)*cosh(x)^2 + 3*a^2
+ 18*a*b - 9*b^2)*sinh(x)^4 + 8*(7*(a^2 + 6*a*b - 3*b^2)*cosh(x)^5 + 10*(a^
2 + 6*a*b - 3*b^2)*cosh(x)^3 + 3*(a^2 + 6*a*b - 3*b^2)*cosh(x))*sinh(x)^3 +
 4*(a^2 + 6*a*b - 3*b^2)*cosh(x)^2 + 4*(7*(a^2 + 6*a*b - 3*b^2)*cosh(x)^6 +
 15*(a^2 + 6*a*b - 3*b^2)*cosh(x)^4 + 9*(a^2 + 6*a*b - 3*b^2)*cosh(x)^2 + a
^2 + 6*a*b - 3*b^2)*sinh(x)^2 + a^2 + 6*a*b - 3*b^2 + 8*((a^2 + 6*a*b - 3*b
^2)*cosh(x)^7 + 3*(a^2 + 6*a*b - 3*b^2)*cosh(x)^5 + 3*(a^2 + 6*a*b - 3*b^2)
*cosh(x)^3 + (a^2 + 6*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(b)*arctan(sqrt(2)
*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2
+ a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*co
sh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(
3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*si
nh(x) + a)) - 2*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 +
4*b^2*cosh(x)^6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8
*(7*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2
*cosh(x)^2 + 3*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b
^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(
x)^4 + 9*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*co
sh(x)^5 + 3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x
)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)
^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3
 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4
 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)
^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 +
```

```
 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2
+ 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a
^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b
^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2
 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 +
4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh
(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))
*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2
*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cos
h(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sin
h(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x
)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6))
- 2*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x
)^6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh
(x)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 +
 3*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3
 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2
*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 + 3*
b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(
x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b
)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(
a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2
 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - sqrt(2)*((a*b - 5*b^2)*cosh(x)^6 + 6*(
a*b - 5*b^2)*cosh(x)*sinh(x)^5 + (a*b - 5*b^2)*sinh(x)^6 + (a*b + 3*b^2)*co
sh(x)^4 + (15*(a*b - 5*b^2)*cosh(x)^2 + a*b + 3*b^2)*sinh(x)^4 + 4*(5*(a*b
- 5*b^2)*cosh(x)^3 + (a*b + 3*b^2)*cosh(x))*sinh(x)^3 - (a*b + 3*b^2)*cosh(
x)^2 + (15*(a*b - 5*b^2)*cosh(x)^4 + 6*(a*b + 3*b^2)*cosh(x)^2 - a*b - 3*b^
2)*sinh(x)^2 - a*b + 5*b^2 + 2*(3*(a*b - 5*b^2)*cosh(x)^5 + 2*(a*b + 3*b^2)
*cosh(x)^3 - (a*b + 3*b^2)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^
2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b^2*cosh(x)^8 +
 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh(
x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 + 3*b^2*cosh(x
))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 + 3*b^2)*sinh(x)^4 +
4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*si
nh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2*cosh(x)^2 + b^2)*si
nh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 + 3*b^2*cosh(x)^3 + b^2*
cosh(x))*sinh(x)), -1/16*(8*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*
sinh(x)^8 + 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*c
osh(x)^4 + 8*(7*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x
)^4 + 30*b^2*cosh(x)^2 + 3*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh
(x)^5 + 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 +
15*b^2*cosh(x)^4 + 9*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^
7 + 3*b^2*cosh(x)^5 + 3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(-a)*arct
an(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*s
qrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
sinh(x)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2
 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(
2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) + 8*(b^2*cosh(x)^8 + 8*b
^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh(x)^2
 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 + 3*b^2*cosh(x))*s
inh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 + 3*b^2)*sinh(x)^4 + 4*b^
2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x
)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2*cosh(x)^2 + b^2)*sinh(x
)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 + 3*b^2*cosh(x)^3 + b^2*cosh
(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh
(x)^2 + 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*s
inh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 +
```

```
4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - ((a^2 + 6*a*b - 3*b^2)*
cosh(x)^8 + 8*(a^2 + 6*a*b - 3*b^2)*cosh(x)*sinh(x)^7 + (a^2 + 6*a*b - 3*b^
2)*sinh(x)^8 + 4*(a^2 + 6*a*b - 3*b^2)*cosh(x)^6 + 4*(7*(a^2 + 6*a*b - 3*b^
2)*cosh(x)^2 + a^2 + 6*a*b - 3*b^2)*sinh(x)^6 + 8*(7*(a^2 + 6*a*b - 3*b^2)*
cosh(x)^3 + 3*(a^2 + 6*a*b - 3*b^2)*cosh(x))*sinh(x)^5 + 6*(a^2 + 6*a*b - 3
*b^2)*cosh(x)^4 + 2*(35*(a^2 + 6*a*b - 3*b^2)*cosh(x)^4 + 30*(a^2 + 6*a*b -
 3*b^2)*cosh(x)^2 + 3*a^2 + 18*a*b - 9*b^2)*sinh(x)^4 + 8*(7*(a^2 + 6*a*b -
 3*b^2)*cosh(x)^5 + 10*(a^2 + 6*a*b - 3*b^2)*cosh(x)^3 + 3*(a^2 + 6*a*b - 3
*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + 6*a*b - 3*b^2)*cosh(x)^2 + 4*(7*(a^2 +
6*a*b - 3*b^2)*cosh(x)^6 + 15*(a^2 + 6*a*b - 3*b^2)*cosh(x)^4 + 9*(a^2 + 6*
a*b - 3*b^2)*cosh(x)^2 + a^2 + 6*a*b - 3*b^2)*sinh(x)^2 + a^2 + 6*a*b - 3*b
^2 + 8*((a^2 + 6*a*b - 3*b^2)*cosh(x)^7 + 3*(a^2 + 6*a*b - 3*b^2)*cosh(x)^5
 + 3*(a^2 + 6*a*b - 3*b^2)*cosh(x)^3 + (a^2 + 6*a*b - 3*b^2)*cosh(x))*sinh(
x))*sqrt(-b)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b
)*sinh(x)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sin
h(x)^2 + 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)
*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
 + sinh(x)^2)) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b)/
(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^
2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) - 2*sqrt(2)*((a*b -
 5*b^2)*cosh(x)^6 + 6*(a*b - 5*b^2)*cosh(x)*sinh(x)^5 + (a*b - 5*b^2)*sinh(
x)^6 + (a*b + 3*b^2)*cosh(x)^4 + (15*(a*b - 5*b^2)*cosh(x)^2 + a*b + 3*b^2)
*sinh(x)^4 + 4*(5*(a*b - 5*b^2)*cosh(x)^3 + (a*b + 3*b^2)*cosh(x))*sinh(x)^
3 - (a*b + 3*b^2)*cosh(x)^2 + (15*(a*b - 5*b^2)*cosh(x)^4 + 6*(a*b + 3*b^2)
*cosh(x)^2 - a*b - 3*b^2)*sinh(x)^2 - a*b + 5*b^2 + 2*(3*(a*b - 5*b^2)*cosh
(x)^5 + 2*(a*b + 3*b^2)*cosh(x)^3 - (a*b + 3*b^2)*cosh(x))*sinh(x))*sqrt((a
*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x
)^2)))/(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cos
h(x)^6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*c
osh(x)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^
2 + 3*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x
)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*
b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 +
 3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)), -1/8*(4*(b^2*cosh(x)^8 + 8*b^2*co
sh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh(x)^2 + b^
2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x
)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 + 3*b^2)*sinh(x)^4 + 4*b^2*cos
h(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 +
 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2*cosh(x)^2 + b^2)*sinh(x)^2 +
 b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 + 3*b^2*cosh(x)^3 + b^2*cosh(x))*
sinh(x))*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sin
h(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2
 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3
 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)
*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) +
4*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)^
6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(x
)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 + 3
*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3 +
 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2*c
osh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 + 3*b^
2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*
cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
 a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*c
osh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 +
 a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + (
(a^2 + 6*a*b - 3*b^2)*cosh(x)^8 + 8*(a^2 + 6*a*b - 3*b^2)*cosh(x)*sinh(x)^7
 + (a^2 + 6*a*b - 3*b^2)*sinh(x)^8 + 4*(a^2 + 6*a*b - 3*b^2)*cosh(x)^6 + 4*
(7*(a^2 + 6*a*b - 3*b^2)*cosh(x)^2 + a^2 + 6*a*b - 3*b^2)*sinh(x)^6 + 8*(7*
```

```
(a^2 + 6*a*b - 3*b^2)*cosh(x)^3 + 3*(a^2 + 6*a*b - 3*b^2)*cosh(x))*sinh(x)^
5 + 6*(a^2 + 6*a*b - 3*b^2)*cosh(x)^4 + 2*(35*(a^2 + 6*a*b - 3*b^2)*cosh(x)
^4 + 30*(a^2 + 6*a*b - 3*b^2)*cosh(x)^2 + 3*a^2 + 18*a*b - 9*b^2)*sinh(x)^4
 + 8*(7*(a^2 + 6*a*b - 3*b^2)*cosh(x)^5 + 10*(a^2 + 6*a*b - 3*b^2)*cosh(x)^
3 + 3*(a^2 + 6*a*b - 3*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + 6*a*b - 3*b^2)*co
sh(x)^2 + 4*(7*(a^2 + 6*a*b - 3*b^2)*cosh(x)^6 + 15*(a^2 + 6*a*b - 3*b^2)*c
osh(x)^4 + 9*(a^2 + 6*a*b - 3*b^2)*cosh(x)^2 + a^2 + 6*a*b - 3*b^2)*sinh(x)
^2 + a^2 + 6*a*b - 3*b^2 + 8*((a^2 + 6*a*b - 3*b^2)*cosh(x)^7 + 3*(a^2 + 6*
a*b - 3*b^2)*cosh(x)^5 + 3*(a^2 + 6*a*b - 3*b^2)*cosh(x)^3 + (a^2 + 6*a*b -
 3*b^2)*cosh(x))*sinh(x))*sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sin
h(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(c
osh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(
x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*si
nh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - sqrt(2)*((a*b
 - 5*b^2)*cosh(x)^6 + 6*(a*b - 5*b^2)*cosh(x)*sinh(x)^5 + (a*b - 5*b^2)*sin
h(x)^6 + (a*b + 3*b^2)*cosh(x)^4 + (15*(a*b - 5*b^2)*cosh(x)^2 + a*b + 3*b^
2)*sinh(x)^4 + 4*(5*(a*b - 5*b^2)*cosh(x)^3 + (a*b + 3*b^2)*cosh(x))*sinh(x
)^3 - (a*b + 3*b^2)*cosh(x)^2 + (15*(a*b - 5*b^2)*cosh(x)^4 + 6*(a*b + 3*b^
2)*cosh(x)^2 - a*b - 3*b^2)*sinh(x)^2 - a*b + 5*b^2 + 2*(3*(a*b - 5*b^2)*co
sh(x)^5 + 2*(a*b + 3*b^2)*cosh(x)^3 - (a*b + 3*b^2)*cosh(x))*sinh(x))*sqrt(
(a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh
(x)^2)))/(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*c
osh(x)^6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2
*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x
)^2 + 3*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh
(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 +
9*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5
 + 3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\,\mathrm{sech}^2(x)}\tanh^4(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)**2)**(1/2)*tanh(x)**4,x)
```

```
[Out] Integral(sqrt(a + b*sech(x)**2)*tanh(x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\mathrm{sech}(x)^2 + a}\tanh(x)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a)*tanh(x)^4, x)
```

3.178 $\int \sqrt{a+b\,\mathbf{sech}^2(x)}\tanh^3(x)\,dx$

Optimal. Leaf size=59

$$\frac{\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}{3b}-\sqrt{a+b\,\mathrm{sech}^2(x)}+\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)$$

```
[Out] Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - Sqrt[a + b*Sech[x]^2] + (a
 + b*Sech[x]^2)^(3/2)/(3*b)
```

Rubi [A] time = 0.106641, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {4139, 446, 80, 50, 63, 208}

$$\frac{\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}{3b}-\sqrt{a+b\,\mathrm{sech}^2(x)}+\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sech[x]^2]*Tanh[x]^3,x]
```

```
[Out] Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - Sqrt[a + b*Sech[x]^2] + (a
 + b*Sech[x]^2)^(3/2)/(3*b)
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{a+b\operatorname{sech}^2(x)}\tanh^3(x)\,dx &= \operatorname{Subst}\left(\int \frac{\left(-1+x^2\right)\sqrt{a+bx^2}}{x}\,dx, x, \operatorname{sech}(x)\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(-1+x)\sqrt{a+bx}}{x}\,dx, x, \operatorname{sech}^2(x)\right)\\
&= \frac{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}{3b} - \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \operatorname{sech}^2(x)\right)\\
&= -\sqrt{a+b\operatorname{sech}^2(x)} + \frac{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}{3b} - \frac{1}{2}a\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \operatorname{sech}^2(x)\right)\\
&= -\sqrt{a+b\operatorname{sech}^2(x)} + \frac{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}{3b} - \frac{a\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{sech}^2(x)}\right)}{b}\\
&= \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right) - \sqrt{a+b\operatorname{sech}^2(x)} + \frac{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}{3b}
\end{aligned}
$$

Mathematica [A] time = 0.373467, size = 90, normalized size = 1.53

$$
\frac{1}{3}\cosh(x)\sqrt{a+b\operatorname{sech}^2(x)}\left(\left(\frac{a}{b}-3\right)\operatorname{sech}(x)+\frac{3\sqrt{2}\sqrt{a}\log\left(\sqrt{a\cosh(2x)+a+2b}+\sqrt{2}\sqrt{a}\cosh(x)\right)}{\sqrt{a\cosh(2x)+a+2b}}+\operatorname{sech}^3(x)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sech[x]^2]*Tanh[x]^3,x]
```

```
[Out] (Cosh[x]*Sqrt[a + b*Sech[x]^2]*((3*Sqrt[2]*Sqrt[a]*Log[Sqrt[2]*Sqrt[a]*Cosh
[x] + Sqrt[a + 2*b + a*Cosh[2*x]]])/Sqrt[a + 2*b + a*Cosh[2*x]] + (-3 + a/b
)*Sech[x] + Sech[x]^3))/3
```

Maple [F] time = 0.112, size = 0, normalized size = 0.

$$
\int \sqrt{a+b\left(\operatorname{sech}(x)\right)^2}\left(\tanh(x)\right)^3\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)^(1/2)*tanh(x)^3,x)
```

```
[Out] int((a+b*sech(x)^2)^(1/2)*tanh(x)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\mathrm{sech}\,(x)^2 + a}\tanh\,(x)^3\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a)*tanh(x)^3, x)
```

Fricas [B] time = 3.26579, size = 7024, normalized size = 119.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^3,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sinh(x)^6 + 3*b*cosh(x)^4
 + 3*(5*b*cosh(x)^2 + b)*sinh(x)^4 + 4*(5*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x
)^3 + 3*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh(x)^2 + b)*sinh(x)^2 + 6*(
b*cosh(x)^5 + 2*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(a)*log(((a^3 + 2
*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (
a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh
(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cos
h(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*
a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*co
sh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^
2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3
+ 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)
^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a
^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a
^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9
*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(
a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a
^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*
a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b
+ b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b +
b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^
2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x
)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(
x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 +
2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5
+ (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(
x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^
3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) +
3*(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sinh(x)^6 + 3*b*cosh(x)^4 + 3*(5
```

```
*b*cosh(x)^2 + b)*sinh(x)^4 + 4*(5*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^3 + 3
*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh(x)^2 + b)*sinh(x)^2 + 6*(b*cosh(
x)^5 + 2*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(a)*log(-(a*cosh(x)^4 +
4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)
*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a
)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
 + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*co
sh(x)*sinh(x) + sinh(x)^2)) + 4*sqrt(2)*((a - 3*b)*cosh(x)^4 + 4*(a - 3*b)*
cosh(x)*sinh(x)^3 + (a - 3*b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a - 3
*b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a - 3*b)*cosh(x)^3 + (a - b)*cosh(x)
)*sinh(x) + a - 3*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2
- 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 + b
*sinh(x)^6 + 3*b*cosh(x)^4 + 3*(5*b*cosh(x)^2 + b)*sinh(x)^4 + 4*(5*b*cosh(
x)^3 + 3*b*cosh(x))*sinh(x)^3 + 3*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh
(x)^2 + b)*sinh(x)^2 + 6*(b*cosh(x)^5 + 2*b*cosh(x)^3 + b*cosh(x))*sinh(x)
+ b), -1/6*(3*(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sinh(x)^6 + 3*b*cosh
(x)^4 + 3*(5*b*cosh(x)^2 + b)*sinh(x)^4 + 4*(5*b*cosh(x)^3 + 3*b*cosh(x))*s
inh(x)^3 + 3*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh(x)^2 + b)*sinh(x)^2
+ 6*(b*cosh(x)^5 + 2*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(-a)*arctan(
sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2
+ a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*s
inh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*
b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3
+ (2*a^2 + 3*a*b)*cosh(x))*sinh(x))) + 3*(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)
^5 + b*sinh(x)^6 + 3*b*cosh(x)^4 + 3*(5*b*cosh(x)^2 + b)*sinh(x)^4 + 4*(5*b
*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^3 + 3*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 + 6*
b*cosh(x)^2 + b)*sinh(x)^2 + 6*(b*cosh(x)^5 + 2*b*cosh(x)^3 + b*cosh(x))*si
nh(x) + b)*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*
cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh
(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(
a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - 2*sqrt(2)*((a - 3*b)*cosh(
x)^4 + 4*(a - 3*b)*cosh(x)*sinh(x)^3 + (a - 3*b)*sinh(x)^4 + 2*(a - b)*cosh
(x)^2 + 2*(3*(a - 3*b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a - 3*b)*cosh(x)^
3 + (a - b)*cosh(x))*sinh(x) + a - 3*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a
 + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^6 + 6*b*co
sh(x)*sinh(x)^5 + b*sinh(x)^6 + 3*b*cosh(x)^4 + 3*(5*b*cosh(x)^2 + b)*sinh(
x)^4 + 4*(5*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^3 + 3*b*cosh(x)^2 + 3*(5*b*c
osh(x)^4 + 6*b*cosh(x)^2 + b)*sinh(x)^2 + 6*(b*cosh(x)^5 + 2*b*cosh(x)^3 +
b*cosh(x))*sinh(x) + b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \operatorname{sech}^2(x)} \tanh^3(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)**2)**(1/2)*tanh(x)**3,x)
```

```
[Out] Integral(sqrt(a + b*sech(x)**2)*tanh(x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\mathrm{sech}\,(x)^2 + a}\tanh\,(x)^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a)*tanh(x)^3, x)
```

3.179 $\int \sqrt{a+b\,\mathrm{sech}^2(x)}\tanh^2(x)\,dx$

Optimal. Leaf size=87

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)-\frac{1}{2}\tanh(x)\sqrt{a-b\tanh^2(x)+b}-\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{2\sqrt{b}}$$

```
[Out] -((a - b)*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]])/(2*Sqrt[b])
+ Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]] - (Tanh[x]*S
qrt[a + b - b*Tanh[x]^2])/2
```

Rubi [A] time = 0.218031, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {4141, 1975, 478, 523, 217, 203, 377, 206}

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)-\frac{1}{2}\tanh(x)\sqrt{a-b\tanh^2(x)+b}-\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sech[x]^2]*Tanh[x]^2,x]
```

```
[Out] -((a - b)*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]])/(2*Sqrt[b])
+ Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]] - (Tanh[x]*S
qrt[a + b - b*Tanh[x]^2])/2
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a+b\operatorname{sech}^2(x)}\tanh^2(x)\,dx &= \text{Subst}\left(\int \frac{x^2\sqrt{a+b\left(1-x^2\right)}}{1-x^2}\,dx, x, \tanh(x)\right)\\
&= \text{Subst}\left(\int \frac{x^2\sqrt{a+b-bx^2}}{1-x^2}\,dx, x, \tanh(x)\right)\\
&= -\frac{1}{2}\tanh(x)\sqrt{a+b-b\tanh^2(x)}+\frac{1}{2}\text{Subst}\left(\int \frac{a+b+(a-b)x^2}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x\right.\\
&= -\frac{1}{2}\tanh(x)\sqrt{a+b-b\tanh^2(x)}+a\,\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x\right.\\
&= -\frac{1}{2}\tanh(x)\sqrt{a+b-b\tanh^2(x)}+a\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right.\\
&= -\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{2\sqrt{b}}+\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)-\frac{1}{2}\tanh(x)\sqrt{}
\end{aligned}$$

Mathematica [A] time = 0.265298, size = 150, normalized size = 1.72

$$-\frac{\cosh(x)\sqrt{a+b\operatorname{sech}^2(x)}\left(\sqrt{2}(a-b)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sinh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)-2\sqrt{2}\sqrt{a}\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)+\sqrt{b}\tanh(x)\operatorname{sec}\right.}{2\sqrt{b}\sqrt{a\cosh(2x)+a+2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sech[x]^2]*Tanh[x]^2,x]
```

```
[Out] -(Cosh[x]*Sqrt[a + b*Sech[x]^2]*(Sqrt[2]*(a - b)*ArcTan[(Sqrt[2]*Sqrt[b]*Si
nh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]] - 2*Sqrt[2]*Sqrt[a]*Sqrt[b]*ArcTanh[(Sq
rt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]] + Sqrt[b]*Sqrt[a + 2*b
+ a*Cosh[2*x]]*Sech[x]*Tanh[x]))/(2*Sqrt[b]*Sqrt[a + 2*b + a*Cosh[2*x]])
```

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int \sqrt{a + b\,(\operatorname{sech}(x))^2}\,(\tanh(x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)^(1/2)*tanh(x)^2,x)
```

```
[Out] int((a+b*sech(x)^2)^(1/2)*tanh(x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\operatorname{sech}(x)^2 + a}\,\tanh(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a)*tanh(x)^2, x)
```

Fricas [B] time = 3.54754, size = 12953, normalized size = 148.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^2,x, algorithm="fricas")
```

```
[Out] [1/4*((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 +
2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*
sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8
- 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^
6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a
^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 -
 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2
- b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(
a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)
^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqr
t(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(
x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cos
h(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh
```

```
(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3
 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2
+ a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x
)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^
3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x
)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)
*sinh(x)^5 + sinh(x)^6)) + ((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3
 + (a - b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a - b
)*sinh(x)^2 + 4*((a - b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a - b)*sqrt
(-b)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x
)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 +
 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a
*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x
)^2)) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)
^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*co
sh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + (b*cosh(x)^4 + 4*b*cosh(x
)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2
 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(a)*log(-(a*cosh(x)^4 + 4*a
*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 +
 a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*
sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cos
h(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 2*sqrt(2)*(b*cosh(x)^2 + 2*b*cos
h(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^4 + 4*b*cosh(x)*si
nh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4
*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b), -1/4*(2*((a - b)*cosh(x)^4 + 4*(a
- b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a
- b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a - b)*cosh(x)^3 + (a - b)*cosh(x))
*sinh(x) + a - b)*sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + s
inh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2
 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a
*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2
+ 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - (b*cosh(x)^4 + 4*b*co
sh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(
x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(a)*log((a*b^2*cosh(x)^
8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6
 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 -
 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 +
 (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2
)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4
*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 +
2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3
 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2
*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 -
 b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*
a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x
)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*s
inh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(
x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(
x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^
3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) -
(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b
*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(a
)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(
x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)
*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b
)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*c
```

```
osh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 2*sqrt(
2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt((a*cosh(x)^2
 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*c
osh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cos
h(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b), -1/4*(2*(
b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*
cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(-a
)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt
(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh
(x) + sinh(x)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4
 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2
 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) + 2*(b*cosh(x)^4 +
 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b
)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(-a)*arctan(sqrt
(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt((a*cosh(x
)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(
a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 +
 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x)
)*sinh(x) + a)) - ((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b
)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a - b)*sinh(x)
^2 + 4*((a - b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a - b)*sqrt(-b)*log(
-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(
a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 + 2*sqrt(2
)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a*cosh(x)^
2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4
*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)^4 + 4*co
sh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 +
 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + 2*sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(
x)*sinh(x) + b*sinh(x)^2 - b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(c
osh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^4 + 4*b*cosh(x)*sinh
(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(
b*cosh(x)^3 + b*cosh(x))*sinh(x) + b), -1/2*((b*cosh(x)^4 + 4*b*cosh(x)*sin
h(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*
(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2
 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*si
nh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(x
)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (
6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2
+ 3*a*b)*cosh(x))*sinh(x))) + (b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh
(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 +
b*cosh(x))*sinh(x) + b)*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh
(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(c
osh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(
x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*si
nh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + ((a - b)*cosh
(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(a - b)*cosh(x)
^2 + 2*(3*(a - b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a - b)*cosh(x)^3 + (a
- b)*cosh(x))*sinh(x) + a - b)*sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x
)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*
b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*
sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*
b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + sqrt(2)*
(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt((a*cosh(x)^2 + a
*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh
(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x
)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\operatorname{sech}^2(x)}\tanh^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)**2)**(1/2)*tanh(x)**2,x)

[Out] Integral(sqrt(a + b*sech(x)**2)*tanh(x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\operatorname{sech}(x)^2 + a}\tanh(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^2,x, algorithm="giac")

[Out] integrate(sqrt(b*sech(x)^2 + a)*tanh(x)^2, x)
```

3.180 $\int \sqrt{a+b\,\mathbf{sech}^2(x)}\tanh(x)\,dx$

Optimal. Leaf size=40

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)-\sqrt{a+b\text{sech}^2(x)}$$

```
[Out] Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - Sqrt[a + b*Sech[x]^2]
```

Rubi [A] time = 0.0630451, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4139, 266, 50, 63, 208}

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)-\sqrt{a+b\text{sech}^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sech[x]^2]*Tanh[x],x]
```

```
[Out] Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - Sqrt[a + b*Sech[x]^2]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a+b\mathrm{sech}^2(x)}\tanh(x)\,dx &= -\mathrm{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x}\,dx,x,\mathrm{sech}(x)\right)\\
&= -\left(\frac{1}{2}\mathrm{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,\mathrm{sech}^2(x)\right)\right)\\
&= -\sqrt{a+b\mathrm{sech}^2(x)}-\frac{1}{2}a\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,\mathrm{sech}^2(x)\right)\\
&= -\sqrt{a+b\mathrm{sech}^2(x)}-\frac{a\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\mathrm{sech}^2(x)}\right)}{b}\\
&= \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\mathrm{sech}^2(x)}}{\sqrt{a}}\right)-\sqrt{a+b\mathrm{sech}^2(x)}
\end{aligned}$$

Mathematica [B] time = 0.256738, size = 90, normalized size = 2.25

$$-\frac{\sqrt{a+b\mathrm{sech}^2(x)}\left(-\sqrt{2}\sqrt{a}\sqrt{b}\cosh(x)\sqrt{\frac{a\cosh(2x)+a+2b}{b}}\sinh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{b}}\right)+a\cosh(2x)+a+2b\right)}{a\cosh(2x)+a+2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sech[x]^2]*Tanh[x],x]
```

```
[Out] -(((a + 2*b + a*Cosh[2*x] - Sqrt[2]*Sqrt[a]*Sqrt[b]*ArcSinh[(Sqrt[a]*Cosh[x
])/Sqrt[b]]*Cosh[x]*Sqrt[(a + 2*b + a*Cosh[2*x])/b])*Sqrt[a + b*Sech[x]^2])
/(a + 2*b + a*Cosh[2*x]))
```

Maple [A] time = 0.034, size = 43, normalized size = 1.1

$$-\sqrt{a+b\left(\mathrm{sech}\left(x\right)\right)^{2}}+\sqrt{a}\ln\left(\frac{1}{\mathrm{sech}\left(x\right)}\left(2\,a+2\,\sqrt{a}\sqrt{a+b\left(\mathrm{sech}\left(x\right)\right)^{2}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)^(1/2)*tanh(x),x)
```

```
[Out] -(a+b*sech(x)^2)^(1/2)+a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sech(x)^2)^(1/2))/sec
h(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\mathrm{sech}\left(x\right)^{2}+a}\tanh\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a)*tanh(x), x)
```

Fricas [B] time = 2.28792, size = 4658, normalized size = 116.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x),x, algorithm="fricas")
```

```
[Out] [1/4*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*log(((a^3 + 2
*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (
a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh
(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cos
h(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*
a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*co
sh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^
2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3
+ 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)
^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a
^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a
^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9
*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(
a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a
^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*
a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b
+ b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b +
b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^
2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x
)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(
x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 +
2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5
+ (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(
x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^
3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) +
(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*log(-(a*cosh(x)^4 +
 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b
)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(
a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*c
osh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(
x)*sinh(x) + sinh(x)^2 + 1), -1/2*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2 + 1)*sqrt(-a)*arctan(sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh
(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a +
 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 +
 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*
cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(
2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(x))*sinh(x))) + (cosh(x)^2 +
 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*
cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
 a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*c
osh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 +
```

```
 a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + 2
*sqrt(2)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*
sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\,\mathrm{sech}^2\,(x)}\,\tanh\,(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)**2)**(1/2)*tanh(x),x)
```

```
[Out] Integral(sqrt(a + b*sech(x)**2)*tanh(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\mathrm{sech}\,(x)^2 + a}\,\tanh\,(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a)*tanh(x), x)
```

3.181 $\int \sqrt{a + b\,\mathbf{sech}^2(x)}\,dx$

Optimal. Leaf size=59

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a - b\tanh^2(x) + b}}\right) + \sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a - b\tanh^2(x) + b}}\right)$$

```
[Out] Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]] + Sqrt[a]*ArcTa
nh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]
```

Rubi [A] time = 0.0454513, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4128, 402, 217, 203, 377, 206}

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a - b\tanh^2(x) + b}}\right) + \sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a - b\tanh^2(x) + b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]] + Sqrt[a]*ArcTa
nh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a+b\text{sech}^2(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+b-bx^2}}{1-x^2}\,dx,x,\tanh(x)\right)\\
&= a\,\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}}\,dx,x,\tanh(x)\right)+b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b-bx^2}}\,dx,x,\tanh(x)\right)\\
&= a\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)+b\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)\\
&= \sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)+\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)
\end{aligned}$$

Mathematica [B] time = 0.679204, size = 134, normalized size = 2.27

$$\frac{\sqrt{2}\cosh(x)\sqrt{a+b\text{sech}^2(x)}\left(\sqrt{a}\sqrt{a+b}\sinh^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a+b}}\right)\sqrt{\frac{a\cosh(2x)+a+2b}{a+b}}+\sqrt{b}\sqrt{a\cosh(2x)+a+2b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sinh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)\right)}{a\cosh(2x)+a+2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] (Sqrt[2]*Cosh[x]*(Sqrt[b]*ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a
*Cosh[2*x]]]*Sqrt[a + 2*b + a*Cosh[2*x]] + Sqrt[a]*Sqrt[a + b]*ArcSinh[(Sqr
t[a]*Sinh[x])/Sqrt[a + b]]*Sqrt[(a + 2*b + a*Cosh[2*x])/(a + b)])*Sqrt[a +
b*Sech[x]^2])/(a + 2*b + a*Cosh[2*x])
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int \sqrt{a+b\left(\text{sech}\left(x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int((a+b*sech(x)^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\text{sech}\left(x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a), x)
```

Fricas [B] time = 2.77008, size = 8756, normalized size = 148.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(
x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sin
h(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3
+ 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*
b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a
*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3
+ 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*co
sh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2
+ sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*
cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^
2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2
*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh
(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(
x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*c
osh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3
+ (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*c
osh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*co
sh(x)*sinh(x)^5 + sinh(x)^6)) + 1/2*sqrt(-b)*log(-((a - b)*cosh(x)^4 + 4*(a
 - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*
(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*s
inh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)
/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a - b)*cosh(x)^3 + (a +
 3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^
4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*s
inh(x) + 1)) + 1/4*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*si
nh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(
2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^
2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4
*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x
) + sinh(x)^2)), sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + si
nh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2
- 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*
sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 +
 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + 1/4*sqrt(a)*log((a*b^2
*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*
cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*co
sh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*co
sh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*
cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 +
 (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cos
h(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b
 + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^
6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*co
```

```
sh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 -
(a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*
b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*c
osh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x
)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b
^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(
x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20
*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(
x)^6)) + 1/4*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^
4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*
sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*co
sh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + si
nh(x)^2)), -1/2*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x)
+ b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cos
h(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sin
h(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 -
 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(
x))) - 1/2*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2 + 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*
cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh
(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(
a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + 1/2*sqrt(-b)*log(-((a - b)
*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(a + 3*b)*
cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x
)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a*cosh(x)^2 + a*sin
h(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a - b)
*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)^4 + 4*cosh(x)*sin
h(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(
x)^3 + cosh(x))*sinh(x) + 1)), -1/2*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 +
2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(
x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(x)^4
 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a
*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3
*a*b)*cosh(x))*sinh(x))) - 1/2*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(
x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a +
2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x
)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a +
2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + sqrt(b
)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sq
rt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + s
inh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)
*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a +
2*b)*cosh(x))*sinh(x) + a))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\,\mathrm{sech}^2\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sech(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \operatorname{sech}(x)^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a), x)
```

3.182 $\int \coth(x)\sqrt{a+b\,\mathbf{sech}^2(x)}\,dx$

Optimal. Leaf size=56

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)-\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a+b}}\right)$$

```
[Out] Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - Sqrt[a + b]*ArcTanh[Sqrt[a
 + b*Sech[x]^2]/Sqrt[a + b]]
```

Rubi [A] time = 0.107893, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4139, 446, 83, 63, 208}

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)-\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a+b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]*Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - Sqrt[a + b]*ArcTanh[Sqrt[a
 + b*Sech[x]^2]/Sqrt[a + b]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \coth(x)\sqrt{a+b\text{sech}^2(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x\left(-1+x^2\right)}\,dx, x, \text{sech}(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{(-1+x)x}\,dx, x, \text{sech}^2(x)\right)\\
&= -\left(\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \text{sech}^2(x)\right)\right)+\frac{1}{2}(a+b)\,\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+bx}}\,dx,\right.\\
&= -\frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{sech}^2(x)}\right)}{b}+\frac{(a+b)\,\text{Subst}\left(\int \frac{1}{-1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{sec}}\right.}{b}\\
&= \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)-\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)
\end{aligned}$$

Mathematica [A] time = 0.120651, size = 111, normalized size = 1.98

$$\frac{\sqrt{2}\cosh(x)\sqrt{a+b\text{sech}^2(x)}\left(\sqrt{a}\log\left(\sqrt{a\cosh(2x)+a+2b}+\sqrt{2}\sqrt{a}\cosh(x)\right)-\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{2a+2b}\cosh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)\right)}{\sqrt{a\cosh(2x)+a+2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]*Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] (Sqrt[2]*Cosh[x]*(-(Sqrt[a + b]*ArcTanh[(Sqrt[2*a + 2*b]*Cosh[x])/Sqrt[a +
2*b + a*Cosh[2*x]]]) + Sqrt[a]*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b +
 a*Cosh[2*x]]])*Sqrt[a + b*Sech[x]^2])/Sqrt[a + 2*b + a*Cosh[2*x]]
```

Maple [F] time = 0.116, size = 0, normalized size = 0.

$$\int \coth(x)\sqrt{a+b\left(\text{sech}(x)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)*(a+b*sech(x)^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\text{sech}(x)^2+a}\coth(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a)*coth(x), x)
```

Fricas [B] time = 2.91413, size = 10376, normalized size = 185.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*
b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a
^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(
a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)
*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^
3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 +
 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^
2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*
b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x
)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*c
osh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b
 + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*
a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a
*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b +
 b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*c
osh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(
x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2
 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*
(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*s
qrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b +
 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3
 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)
^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*
sinh(x)^5 + sinh(x)^6)) + 1/2*sqrt(a + b)*log(((2*a + b)*cosh(x)^4 + 4*(2*a
 + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a + 3*b)*cosh(x)^2 + 2
*(3*(2*a + b)*cosh(x)^2 + 2*a + 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*c
osh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2
 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cos
h(x)^3 + (2*a + 3*b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sin
h(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(
x)^3 - cosh(x))*sinh(x) + 1)) + 1/4*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)
*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2
+ sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*c
osh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(
x) + sinh(x)^2)), sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x
) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sin
h(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*
sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + 1/4*sqrt(a)
*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)
*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b
```

^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*
b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 +
3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b
+ 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*
a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4
+ 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2
+ b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 +
2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 1
5*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3
+ 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*c
osh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*si
nh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x
)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3
*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(
a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4
*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b
+ b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(
x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))
+ 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b
^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*
cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2
+ 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 +
sinh(x)^6)) + 1/4*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sin
h(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(cosh(x)
^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(
x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)
^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)),
-1/2*sqrt(-a)*arctan(sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x)
+ (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*
b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*
(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cos
h(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(
a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(x))*sinh(x))) - 1/2*sqrt(-a)*ar
ctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*sqrt(
(a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh
(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*co
sh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b
)*cosh(x))*sinh(x) + a)) + 1/2*sqrt(a + b)*log(((2*a + b)*cosh(x)^4 + 4*(2*
a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a + 3*b)*cosh(x)^2 +
2*(3*(2*a + b)*cosh(x)^2 + 2*a + 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*
cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^
2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*co
sh(x)^3 + (2*a + 3*b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*si
nh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh
(x)^3 - cosh(x))*sinh(x) + 1)), -1/2*sqrt(-a)*arctan(sqrt(2)*((a + b)*cosh(
x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*
cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)
*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 +
3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh
(x))*sinh(x))) - 1/2*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x)
+ sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^
3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(
x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + sqrt(-a - b)*arc
tan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sq
rt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + s
inh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)
cosh(x)^2 + 2(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a +

```
2*b)*cosh(x))*sinh(x) + a))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\operatorname{sech}^2(x)}\coth(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sech(x)**2)*coth(x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.183 $\int \coth^2(x)\sqrt{a+b\mathbf{sech}^2(x)}\,dx$

Optimal. Leaf size=48

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)-\coth(x)\sqrt{a-b\tanh^2(x)+b}$$

```
[Out] Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]] - Coth[x]*Sqrt
[a + b - b*Tanh[x]^2]
```

Rubi [A] time = 0.177677, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {4141, 1975, 475, 12, 377, 206}

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)-\coth(x)\sqrt{a-b\tanh^2(x)+b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2*Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]] - Coth[x]*Sqrt
[a + b - b*Tanh[x]^2]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \coth^2(x)\sqrt{a+b\text{sech}^2(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+b\left(1-x^2\right)}}{x^2\left(1-x^2\right)}\,dx,x,\tanh(x)\right)\\
&= \text{Subst}\left(\int \frac{\sqrt{a+b-bx^2}}{x^2\left(1-x^2\right)}\,dx,x,\tanh(x)\right)\\
&= -\coth(x)\sqrt{a+b-b\tanh^2(x)}+\text{Subst}\left(\int \frac{a}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx,x,\tanh(x)\right)\\
&= -\coth(x)\sqrt{a+b-b\tanh^2(x)}+a\,\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx,x,\tanh(x)\right)\\
&= -\coth(x)\sqrt{a+b-b\tanh^2(x)}+a\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)\\
&= \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)-\coth(x)\sqrt{a+b-b\tanh^2(x)}
\end{aligned}$$

Mathematica [A] time = 0.525326, size = 75, normalized size = 1.56

$$\sqrt{a+b\text{sech}^2(x)}\left(\frac{\sqrt{2}\sqrt{a}\cosh(x)\sinh^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b}\sqrt{\frac{a\cosh(2x)+a+2b}{a+b}}}-\coth(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2*Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] ((Sqrt[2]*Sqrt[a]*ArcSinh[(Sqrt[a]*Sinh[x])/Sqrt[a + b]]*Cosh[x])/(Sqrt[a +
 b]*Sqrt[(a + 2*b + a*Cosh[2*x])/(a + b)]) - Coth[x])*Sqrt[a + b*Sech[x]^2]
```

Maple [F] time = 0.119, size = 0, normalized size = 0.

$$\int (\coth(x))^2\sqrt{a+b(\text{sech}(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2*(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^2*(a+b*sech(x)^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \operatorname{sech}(x)^2 + a} \coth(x)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a)*coth(x)^2, x)
```

Fricas [B] time = 2.52916, size = 3892, normalized size = 81.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*log((a*b^2*co
sh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cos
h(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(
x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(
x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cos
h(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a
^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x
)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b +
3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 +
 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(
x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^
2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*
sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh
(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2
 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)
*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))
*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*co
sh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^
6)) + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*log(-(a*cosh(
x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*c
osh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh
(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x)
 + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*sqrt((a*cosh
(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))
)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1), -1/2*((cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*co
sh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
 a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(x)^4 + 4*a
*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cos
h(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*
cosh(x))*sinh(x))) + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-
a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*
sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
```

```
sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*
b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a
+ 2*b)*cosh(x))*sinh(x) + a)) + 2*sqrt(2)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
 a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh
(x)*sinh(x) + sinh(x)^2 - 1)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\operatorname{sech}^2(x)}\coth^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2*(a+b*sech(x)**2)**(1/2),x)

[Out] Integral(sqrt(a + b*sech(x)**2)*coth(x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\operatorname{sech}(x)^2 + a}\coth(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(a+b*sech(x)^2)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*sech(x)^2 + a)*coth(x)^2, x)
```

3.184 $\int \coth^3(x)\sqrt{a+b\operatorname{sech}^2(x)}\,dx$

Optimal. Leaf size=83

$$-\frac{1}{2}\coth^2(x)\sqrt{a+b\operatorname{sech}^2(x)}+\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)-\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a+b}}\right)}{2\sqrt{a+b}}$$

```
[Out] Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - ((2*a + b)*ArcTanh[Sqrt[a
+ b*Sech[x]^2]/Sqrt[a + b]])/(2*Sqrt[a + b]) - (Coth[x]^2*Sqrt[a + b*Sech[x
]^2])/2
```

Rubi [A] time = 0.143286, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {4139, 446, 99, 156, 63, 208}

$$-\frac{1}{2}\coth^2(x)\sqrt{a+b\operatorname{sech}^2(x)}+\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)-\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a+b}}\right)}{2\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3*Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - ((2*a + b)*ArcTanh[Sqrt[a
+ b*Sech[x]^2]/Sqrt[a + b]])/(2*Sqrt[a + b]) - (Coth[x]^2*Sqrt[a + b*Sech[x
]^2])/2
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \coth^3(x)\sqrt{a+b\operatorname{sech}^2(x)}\,dx &= -\operatorname{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x\left(-1+x^2\right)^2}\,dx, x, \operatorname{sech}(x)\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{(-1+x)^2x}\,dx, x, \operatorname{sech}^2(x)\right)\right)\\
&= -\frac{1}{2}\coth^2(x)\sqrt{a+b\operatorname{sech}^2(x)} - \frac{1}{2}\operatorname{Subst}\left(\int \frac{-a-\frac{bx}{2}}{(-1+x)x\sqrt{a+bx}}\,dx, x, \operatorname{sech}^2(x)\right)\\
&= -\frac{1}{2}\coth^2(x)\sqrt{a+b\operatorname{sech}^2(x)} - \frac{1}{2}a\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \operatorname{sech}^2(x)\right) - \frac{1}{4}(-2a-\\
&= -\frac{1}{2}\coth^2(x)\sqrt{a+b\operatorname{sech}^2(x)} - \frac{a\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{sech}^2(x)}\right)}{b} - \frac{(-2a-}{}\\
&= \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right) - \frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a+b}}\right)}{2\sqrt{a+b}} - \frac{1}{2}\coth^2(x)\sqrt{a+b}
\end{aligned}$$

Mathematica [A] time = 0.514248, size = 156, normalized size = 1.88

$$-\frac{\sqrt{a+b\operatorname{sech}^2(x)}\left(\sqrt{2}(2a+b)\cosh(x)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a+b}\cosh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)+\sqrt{a+b}\left(\coth^2(x)\sqrt{a\cosh(2x)+a+2b}-2\sqrt{2}\sqrt{a}\,\mathrm{c}\right.\right.}{2\sqrt{a+b}\sqrt{a\cosh(2x)+a+2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3*Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] -((Sqrt[2]*(2*a + b)*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Cosh[x])/Sqrt[a + 2*b + a
*Cosh[2*x]]]*Cosh[x] + Sqrt[a + b]*(Sqrt[a + 2*b + a*Cosh[2*x]]*Coth[x]^2 -
 2*Sqrt[2]*Sqrt[a]*Cosh[x]*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*C
osh[2*x]]]))*Sqrt[a + b*Sech[x]^2])/(2*Sqrt[a + b]*Sqrt[a + 2*b + a*Cosh[2*
```

```
x]])
```

Maple [F] time = 0.122, size = 0, normalized size = 0.

$$\int (\coth(x))^3 \sqrt{a + b(\operatorname{sech}(x))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3*(a+b*sech(x)^2)^(1/2),x)

[Out] int(coth(x)^3*(a+b*sech(x)^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\operatorname{sech}(x)^2 + a}\coth(x)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3*(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*sech(x)^2 + a)*coth(x)^3, x)
```

Fricas [B] time = 4.2813, size = 15598, normalized size = 187.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3*(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")

[Out] [1/4*(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4
- 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - a - b)*sinh(x)^2 + 4*((a +
 b)*cosh(x)^3 - (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(a)*log(((a^3 + 2*a^2
*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3
+ 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^
6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)
^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*
b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x
)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 +
30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*
a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 +
 (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b
)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b
 + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b
^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2
+ 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 +
 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b
+ b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^
2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)
*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 +
```

```
a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3
+ (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2
 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a^
2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (6
*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))/
(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*si
nh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((2*
a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 - 2*
(2*a + b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 - 2*a - b)*sinh(x)^2 + 4*((2
*a + b)*cosh(x)^3 - (2*a + b)*cosh(x))*sinh(x) + 2*a + b)*sqrt(a + b)*log((
(2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 +
 2*(2*a + 3*b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a + 3*b)*sinh(x)^2
- 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqr
t((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + si
nh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a + 3*b)*cosh(x))*sinh(x) + 2*a + b
)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x
)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)) + ((a + b)*cosh(x
)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*(a + b)*cosh(x)^2
 + 2*(3*(a + b)*cosh(x)^2 - a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 - (a +
b)*cosh(x))*sinh(x) + a + b)*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x
)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(
2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^
2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4
*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + si
nh(x)^2)) - 2*sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a +
 b)*sinh(x)^2 + a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)
*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)
^2 - a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 - (a + b)*cosh(x))*sinh(x) + a
 + b), 1/4*(2*((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a +
 b)*sinh(x)^4 - 2*(2*a + b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 - 2*a - b)
*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3 - (2*a + b)*cosh(x))*sinh(x) + 2*a + b)
*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1
)*sqrt(-a - b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*co
sh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x
)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*
cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + ((a + b)*cosh(x)^4 + 4*(a +
b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*(a + b)*cosh(x)^2 + 2*(3*(a +
b)*cosh(x)^2 - a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 - (a + b)*cosh(x))*s
inh(x) + a + b)*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2
*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(
2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 +
 b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2
*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x
)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*
cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^
3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a
^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh
(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b
 + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^
3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)
*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 +
 (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^
2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a
*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4
*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2
)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cos
h(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x)
)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)
```

```
*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3
+ 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)
^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) +
 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 +
 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)
*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)
^2 - a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 - (a + b)*cosh(x))*sinh(x) + a
 + b)*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b
*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(
x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2
*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(
x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 2*sqrt(2)*(
(a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)*
sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
 sinh(x)^2)))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*si
nh(x)^4 - 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - a - b)*sinh(x)^2 +
 4*((a + b)*cosh(x)^3 - (a + b)*cosh(x))*sinh(x) + a + b), -1/4*(2*((a + b)
*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*(a + b)*co
sh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3
 - (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(-a)*arctan(sqrt(2)*((a + b)*cosh(x
)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*c
osh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*
sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 +
3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(
x))*sinh(x))) + 2*((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b
)*sinh(x)^4 - 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - a - b)*sinh(x)
^2 + 4*((a + b)*cosh(x)^3 - (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(-a)*arct
an(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*sqrt((a
*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x
)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh
(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*
cosh(x))*sinh(x) + a)) - ((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)
^3 + (2*a + b)*sinh(x)^4 - 2*(2*a + b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2
 - 2*a - b)*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3 - (2*a + b)*cosh(x))*sinh(x)
 + 2*a + b)*sqrt(a + b)*log(((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh
(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a + 3*b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh
(x)^2 + 2*a + 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + s
inh(x)^2 + 1)*sqrt(a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(
x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a + 3*
b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4
 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*si
nh(x) + 1)) + 2*sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a
 + b)*sinh(x)^2 + a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x
)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(
x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(
x)^2 - a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 - (a + b)*cosh(x))*sinh(x) +
 a + b), -1/2*(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*s
inh(x)^4 - 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - a - b)*sinh(x)^2
+ 4*((a + b)*cosh(x)^3 - (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(-a)*arctan(
sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2
+ a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*s
inh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*
b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3
+ (2*a^2 + 3*a*b)*cosh(x))*sinh(x))) + ((a + b)*cosh(x)^4 + 4*(a + b)*cosh(
x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(
x)^2 - a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 - (a + b)*cosh(x))*sinh(x) +
 a + b)*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2
```

```
- 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)
^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*c
osh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - ((2*a + b)*cosh(x)^4 + 4*(2*a
 + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 - 2*(2*a + b)*cosh(x)^2 + 2*(
3*(2*a + b)*cosh(x)^2 - 2*a - b)*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3 - (2*a
+ b)*cosh(x))*sinh(x) + 2*a + b)*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2
*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt((a*cosh(x)^2 + a*sinh(x
)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 +
4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x
)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)
) + sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x
)^2 + a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh
(x)*sinh(x) + sinh(x)^2)))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3
 + (a + b)*sinh(x)^4 - 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - a - b
)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 - (a + b)*cosh(x))*sinh(x) + a + b)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3*(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\mathrm{sech}\,(x)^2 + a}\coth\,(x)^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3*(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a)*coth(x)^3, x)
```

3.185 $\int \coth^4(x)\sqrt{a+b\,\mathbf{sech}^2(x)}\,dx$

Optimal. Leaf size=84

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)-\frac{1}{3}\coth^3(x)\sqrt{a-b\tanh^2(x)+b}-\frac{(3a+2b)\coth(x)\sqrt{a-b\tanh^2(x)+b}}{3(a+b)}$$

```
[Out] Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]] - ((3*a + 2*b)
*Coth[x]*Sqrt[a + b - b*Tanh[x]^2])/(3*(a + b)) - (Coth[x]^3*Sqrt[a + b - b
*Tanh[x]^2])/3
```

Rubi [A] time = 0.248464, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {4141, 1975, 475, 583, 12, 377, 206}

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)-\frac{1}{3}\coth^3(x)\sqrt{a-b\tanh^2(x)+b}-\frac{(3a+2b)\coth(x)\sqrt{a-b\tanh^2(x)+b}}{3(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4*Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]] - ((3*a + 2*b)
*Coth[x]*Sqrt[a + b - b*Tanh[x]^2])/(3*(a + b)) - (Coth[x]^3*Sqrt[a + b - b
*Tanh[x]^2])/3
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \coth^4(x)\sqrt{a+b\text{sech}^2(x)}\,dx = \text{Subst}\left(\int \frac{\sqrt{a+b\left(1-x^2\right)}}{x^4\left(1-x^2\right)}\,dx, x, \tanh(x)\right)$$
$$= \text{Subst}\left(\int \frac{\sqrt{a+b-bx^2}}{x^4\left(1-x^2\right)}\,dx, x, \tanh(x)\right)$$
$$= -\frac{1}{3}\coth^3(x)\sqrt{a+b-b\tanh^2(x)} + \frac{1}{3}\text{Subst}\left(\int \frac{3a+2b-2bx^2}{x^2\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx, x, \tan$$
$$= -\frac{(3a+2b)\coth(x)\sqrt{a+b-b\tanh^2(x)}}{3(a+b)} - \frac{1}{3}\coth^3(x)\sqrt{a+b-b\tanh^2(x)} - \frac{\text{Subst}}{}$$
$$= -\frac{(3a+2b)\coth(x)\sqrt{a+b-b\tanh^2(x)}}{3(a+b)} - \frac{1}{3}\coth^3(x)\sqrt{a+b-b\tanh^2(x)} + a\,\text{Subs}$$
$$= -\frac{(3a+2b)\coth(x)\sqrt{a+b-b\tanh^2(x)}}{3(a+b)} - \frac{1}{3}\coth^3(x)\sqrt{a+b-b\tanh^2(x)} + a\,\text{Subs}$$
$$= \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right) - \frac{(3a+2b)\coth(x)\sqrt{a+b-b\tanh^2(x)}}{3(a+b)} - \frac{1}{3}\text{cot}$$

Mathematica [A] time = 0.587006, size = 149, normalized size = 1.77

$$\frac{\sqrt{2}\cosh(x)\sqrt{a\sinh^2(x)+a+b}\sqrt{a+b\text{sech}^2(x)}\left(3\sqrt{a}(a+b)\sinh^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a+b}}\right) - \sqrt{a+b}\text{csch}(x)\sqrt{\frac{a\sinh^2(x)+a+b}{a+b}}\left((a+\right.\right.}{3(a+b)^{3/2}\sqrt{\frac{a\sinh^2(x)+a+b}{a+b}}\sqrt{a\cosh(2x)+a+2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4*Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] (Sqrt[2]*Cosh[x]*Sqrt[a + b*Sech[x]^2]*Sqrt[a + b + a*Sinh[x]^2]*(3*Sqrt[a]
*(a + b)*ArcSinh[(Sqrt[a]*Sinh[x])/Sqrt[a + b]] - Sqrt[a + b]*Csch[x]*(4*a
+ 3*b + (a + b)*Csch[x]^2)*Sqrt[(a + b + a*Sinh[x]^2)/(a + b)]))/(3*(a + b)
^(3/2)*Sqrt[a + 2*b + a*Cosh[2*x]]*Sqrt[(a + b + a*Sinh[x]^2)/(a + b)])
```

Maple [F] time = 0.126, size = 0, normalized size = 0.

$$\int (\coth (x))^4 \sqrt{a + b (\operatorname{sech} (x))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4*(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^4*(a+b*sech(x)^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \operatorname{sech} (x)^2 + a} \coth (x)^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4*(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a)*coth(x)^4, x)
```

Fricas [B] time = 3.42388, size = 7036, normalized size = 83.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4*(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x)^5 + (a + b)*sinh(x)
^6 - 3*(a + b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 - a - b)*sinh(x)^4 + 4*(5
*(a + b)*cosh(x)^3 - 3*(a + b)*cosh(x))*sinh(x)^3 + 3*(a + b)*cosh(x)^2 + 3
*(5*(a + b)*cosh(x)^4 - 6*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 6*((a + b)
*cosh(x)^5 - 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) - a - b)*sqrt(a
)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a
*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*
(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b +
 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a
*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)
*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 +
3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a
^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(
```

```
b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 +
 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*
sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2
- a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^
2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a +
2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 -
3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*
a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*si
nh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(
x)^5 + sinh(x)^6)) + 3*((a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x)^5 + (
a + b)*sinh(x)^6 - 3*(a + b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 - a - b)*si
nh(x)^4 + 4*(5*(a + b)*cosh(x)^3 - 3*(a + b)*cosh(x))*sinh(x)^3 + 3*(a + b)
*cosh(x)^2 + 3*(5*(a + b)*cosh(x)^4 - 6*(a + b)*cosh(x)^2 + a + b)*sinh(x)^
2 + 6*((a + b)*cosh(x)^5 - 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) -
 a - b)*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2
*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)
^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(
x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)
^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2)) - 4*sqrt(2)*((4*a + 3*b)*cosh(x)^4 + 4*(4*a + 3*b)*cosh(x)*sinh(x)^3 +
 (4*a + 3*b)*sinh(x)^4 - 2*(2*a + b)*cosh(x)^2 + 2*(3*(4*a + 3*b)*cosh(x)^2
 - 2*a - b)*sinh(x)^2 + 4*((4*a + 3*b)*cosh(x)^3 - (2*a + b)*cosh(x))*sinh(
x) + 4*a + 3*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)))/((a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x
)^5 + (a + b)*sinh(x)^6 - 3*(a + b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 - a
- b)*sinh(x)^4 + 4*(5*(a + b)*cosh(x)^3 - 3*(a + b)*cosh(x))*sinh(x)^3 + 3*
(a + b)*cosh(x)^2 + 3*(5*(a + b)*cosh(x)^4 - 6*(a + b)*cosh(x)^2 + a + b)*s
inh(x)^2 + 6*((a + b)*cosh(x)^5 - 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*si
nh(x) - a - b), -1/6*(3*((a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x)^5 +
(a + b)*sinh(x)^6 - 3*(a + b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 - a - b)*s
inh(x)^4 + 4*(5*(a + b)*cosh(x)^3 - 3*(a + b)*cosh(x))*sinh(x)^3 + 3*(a + b
)*cosh(x)^2 + 3*(5*(a + b)*cosh(x)^4 - 6*(a + b)*cosh(x)^2 + a + b)*sinh(x)
^2 + 6*((a + b)*cosh(x)^5 - 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x)
- a - b)*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sin
h(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2
- 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3
+ a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)
*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) +
3*((a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x)^5 + (a + b)*sinh(x)^6 - 3*
(a + b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 - a - b)*sinh(x)^4 + 4*(5*(a + b
)*cosh(x)^3 - 3*(a + b)*cosh(x))*sinh(x)^3 + 3*(a + b)*cosh(x)^2 + 3*(5*(a
+ b)*cosh(x)^4 - 6*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 6*((a + b)*cosh(x
)^5 - 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) - a - b)*sqrt(-a)*arct
an(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt((a
*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x
)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh
(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*
cosh(x))*sinh(x) + a)) + 2*sqrt(2)*((4*a + 3*b)*cosh(x)^4 + 4*(4*a + 3*b)*c
osh(x)*sinh(x)^3 + (4*a + 3*b)*sinh(x)^4 - 2*(2*a + b)*cosh(x)^2 + 2*(3*(4*
a + 3*b)*cosh(x)^2 - 2*a - b)*sinh(x)^2 + 4*((4*a + 3*b)*cosh(x)^3 - (2*a +
 b)*cosh(x))*sinh(x) + 4*a + 3*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b
)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a + b)*cosh(x)^6 + 6*(a +
 b)*cosh(x)*sinh(x)^5 + (a + b)*sinh(x)^6 - 3*(a + b)*cosh(x)^4 + 3*(5*(a +
 b)*cosh(x)^2 - a - b)*sinh(x)^4 + 4*(5*(a + b)*cosh(x)^3 - 3*(a + b)*cosh(
x))*sinh(x)^3 + 3*(a + b)*cosh(x)^2 + 3*(5*(a + b)*cosh(x)^4 - 6*(a + b)*co
sh(x)^2 + a + b)*sinh(x)^2 + 6*((a + b)*cosh(x)^5 - 2*(a + b)*cosh(x)^3 + (
a + b)*cosh(x))*sinh(x) - a - b)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4*(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\mathrm{sech}\,(x)^2 + a}\coth\,(x)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4*(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a)*coth(x)^4, x)
```

3.186 $\int \coth^5(x)\sqrt{a+b\mathbf{sech}^2(x)}\,dx$

Optimal. Leaf size=125

$$-\frac{(8a^2+12ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{8(a+b)^{3/2}}-\frac{1}{4}\coth^4(x)\sqrt{a+b\text{sech}^2(x)}-\frac{(4a+3b)\coth^2(x)\sqrt{a+b\text{sech}^2(x)}}{8(a+b)}+\sqrt{a}\,\text{t}$$

```
[Out] Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - ((8*a^2 + 12*a*b + 3*b^2)*
ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a + b]])/(8*(a + b)^(3/2)) - ((4*a + 3*b
)*Coth[x]^2*Sqrt[a + b*Sech[x]^2])/(8*(a + b)) - (Coth[x]^4*Sqrt[a + b*Sech
[x]^2])/4
```

Rubi [A] time = 0.220621, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {4139, 446, 99, 151, 156, 63, 208}

$$-\frac{(8a^2+12ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{8(a+b)^{3/2}}-\frac{1}{4}\coth^4(x)\sqrt{a+b\text{sech}^2(x)}-\frac{(4a+3b)\coth^2(x)\sqrt{a+b\text{sech}^2(x)}}{8(a+b)}+\sqrt{a}\,\text{t}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^5*Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - ((8*a^2 + 12*a*b + 3*b^2)*
ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a + b]])/(8*(a + b)^(3/2)) - ((4*a + 3*b
)*Coth[x]^2*Sqrt[a + b*Sech[x]^2])/(8*(a + b)) - (Coth[x]^4*Sqrt[a + b*Sech
[x]^2])/4
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
```

```
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \coth^5(x)\sqrt{a+b\,\mathrm{sech}^2(x)}\,dx = \mathrm{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x\left(-1+x^2\right)^3}\,dx, x, \mathrm{sech}(x)\right)$$
$$= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{\sqrt{a+bx}}{(-1+x)^3x}\,dx, x, \mathrm{sech}^2(x)\right)$$
$$= -\frac{1}{4}\coth^4(x)\sqrt{a+b\,\mathrm{sech}^2(x)} + \frac{1}{4}\,\mathrm{Subst}\left(\int \frac{-2a-\frac{3bx}{2}}{(-1+x)^2x\sqrt{a+bx}}\,dx, x, \mathrm{sech}^2(x)\right)$$
$$= -\frac{(4a+3b)\coth^2(x)\sqrt{a+b\,\mathrm{sech}^2(x)}}{8(a+b)} - \frac{1}{4}\coth^4(x)\sqrt{a+b\,\mathrm{sech}^2(x)} - \frac{\mathrm{Subst}\left(\int \frac{-2a(\ldots}{(\ldots}\right.}{\ldots}$$
$$= -\frac{(4a+3b)\coth^2(x)\sqrt{a+b\,\mathrm{sech}^2(x)}}{8(a+b)} - \frac{1}{4}\coth^4(x)\sqrt{a+b\,\mathrm{sech}^2(x)} - \frac{1}{2}a\,\mathrm{Subst}\left(\int \ldots\right.$$
$$= -\frac{(4a+3b)\coth^2(x)\sqrt{a+b\,\mathrm{sech}^2(x)}}{8(a+b)} - \frac{1}{4}\coth^4(x)\sqrt{a+b\,\mathrm{sech}^2(x)} - \frac{a\,\mathrm{Subst}\left(\int \frac{\ldots}{-\frac{a}{b}\ldots}\right.}{\ldots}$$
$$= \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right) - \frac{\left(8a^2+12ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\mathrm{sech}^2(x)}}{\sqrt{a+b}}\right)}{8(a+b)^{3/2}} - \frac{(4a+3\ldots}{\ldots}$$

Mathematica [A] time = 0.909891, size = 191, normalized size = 1.53

$$-\frac{\cosh(x)\sqrt{a+b\,\mathrm{sech}^2(x)}\left(\sqrt{2}\left(8a^2+12ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a+b}\cosh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)+\sqrt{a+b}\left(\frac{1}{2}\coth(x)\mathrm{csch}^3(x)\sqrt{a\cosh(2x)}\ldots\right.\right.}{8(a+b)^{3/2}\sqrt{a\cosh(2x)\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^5*Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] -(Cosh[x]*(Sqrt[2]*(8*a^2 + 12*a*b + 3*b^2)*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Co
sh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]] + Sqrt[a + b]*((Sqrt[a + 2*b + a*Cosh[2
*x]]*(-2*a - b + (6*a + 5*b)*Cosh[2*x])*Coth[x]*Csch[x]^3)/2 - 8*Sqrt[2]*Sq
rt[a]*(a + b)*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*Cosh[2*x]]]))*
Sqrt[a + b*Sech[x]^2])/(8*(a + b)^(3/2)*Sqrt[a + 2*b + a*Cosh[2*x]])
```

Maple [F] time = 0.128, size = 0, normalized size = 0.

$$\int (\coth(x))^5\sqrt{a+b(\mathrm{sech}(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5*(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^5*(a+b*sech(x)^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\mathrm{sech}\,(x)^2 + a}\coth\,(x)^5\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5*(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a)*coth(x)^5, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5*(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**5*(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\mathrm{sech}\,(x)^2 + a}\coth\,(x)^5\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5*(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a)*coth(x)^5, x)
```

3.187 $\int \left(a+b\operatorname{sech}^2(x)\right)^{3/2}\tanh^3(x)\,dx$

Optimal. Leaf size=76

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)+\frac{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}{5b}-\frac{1}{3}\left(a+b\operatorname{sech}^2(x)\right)^{3/2}-a\sqrt{a+b\operatorname{sech}^2(x)}$$

```
[Out] a^(3/2)*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - a*Sqrt[a + b*Sech[x]^2] -
(a + b*Sech[x]^2)^(3/2)/3 + (a + b*Sech[x]^2)^(5/2)/(5*b)
```

Rubi [A] time = 0.127669, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {4139, 446, 80, 50, 63, 208}

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)+\frac{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}{5b}-\frac{1}{3}\left(a+b\operatorname{sech}^2(x)\right)^{3/2}-a\sqrt{a+b\operatorname{sech}^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[x]^2)^(3/2)*Tanh[x]^3,x]
```

```
[Out] a^(3/2)*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - a*Sqrt[a + b*Sech[x]^2] -
(a + b*Sech[x]^2)^(3/2)/3 + (a + b*Sech[x]^2)^(5/2)/(5*b)
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \left(a+b\operatorname{sech}^2(x)\right)^{3/2}\tanh^3(x)\,dx = \text{Subst}\left(\int \frac{\left(-1+x^2\right)\left(a+bx^2\right)^{3/2}}{x}\,dx,x,\operatorname{sech}(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{(-1+x)(a+bx)^{3/2}}{x}\,dx,x,\operatorname{sech}^2(x)\right)$$

$$= \frac{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}{5b} - \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx,x,\operatorname{sech}^2(x)\right)$$

$$= -\frac{1}{3}\left(a+b\operatorname{sech}^2(x)\right)^{3/2} + \frac{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}{5b} - \frac{1}{2}a\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,\operatorname{sech}^2(x\right.$$

$$= -a\sqrt{a+b\operatorname{sech}^2(x)} - \frac{1}{3}\left(a+b\operatorname{sech}^2(x)\right)^{3/2} + \frac{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}{5b} - \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{}{x\ldots}\right.$$

$$= -a\sqrt{a+b\operatorname{sech}^2(x)} - \frac{1}{3}\left(a+b\operatorname{sech}^2(x)\right)^{3/2} + \frac{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}{5b} - \frac{a^2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\ldots}\right.}{}$$

$$= a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right) - a\sqrt{a+b\operatorname{sech}^2(x)} - \frac{1}{3}\left(a+b\operatorname{sech}^2(x)\right)^{3/2} + \frac{(a+bs\ldots}{}$$

Mathematica [A] time = 0.971459, size = 129, normalized size = 1.7

$$\cosh^3(x)\left(a+b\operatorname{sech}^2(x)\right)^{3/2}\left(\frac{2\sqrt{2}a^{3/2}\log\left(\sqrt{a\cosh(2x)+a+2b}+\sqrt{2}\sqrt{a}\cosh(x)\right)}{(a\cosh(2x)+a+2b)^{3/2}} + \frac{2\left(b(6a-5b)\operatorname{sech}^3(x)+a(3a-20b\ldots\right.}{15b(a\cosh(2x)+\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[x]^2)^(3/2)*Tanh[x]^3,x]
```

```
[Out] Cosh[x]^3*(a + b*Sech[x]^2)^(3/2)*((2*Sqrt[2]*a^(3/2)*Log[Sqrt[2]*Sqrt[a]*C
osh[x] + Sqrt[a + 2*b + a*Cosh[2*x]]])/(a + 2*b + a*Cosh[2*x])^(3/2) + (2*(
a*(3*a - 20*b)*Sech[x] + (6*a - 5*b)*b*Sech[x]^3 + 3*b^2*Sech[x]^5))/(15*b*
(a + 2*b + a*Cosh[2*x])))
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int \left(a + b\,(\operatorname{sech}(x))^2\right)^{\frac{3}{2}} (\tanh(x))^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)^(3/2)*tanh(x)^3,x)
```

```
[Out] int((a+b*sech(x)^2)^(3/2)*tanh(x)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\operatorname{sech}(x)^2 + a\right)^{\frac{3}{2}} \tanh(x)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sech(x)^2 + a)^(3/2)*tanh(x)^3, x)
```

Fricas [B] time = 6.0479, size = 12290, normalized size = 161.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x)^3,x, algorithm="fricas")
```

```
[Out] [1/60*(15*(a*b*cosh(x)^10 + 10*a*b*cosh(x)*sinh(x)^9 + a*b*sinh(x)^10 + 5*a
*b*cosh(x)^8 + 5*(9*a*b*cosh(x)^2 + a*b)*sinh(x)^8 + 10*a*b*cosh(x)^6 + 40*
(3*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x)^7 + 10*(21*a*b*cosh(x)^4 + 14*a*b*c
osh(x)^2 + a*b)*sinh(x)^6 + 10*a*b*cosh(x)^4 + 4*(63*a*b*cosh(x)^5 + 70*a*b
*cosh(x)^3 + 15*a*b*cosh(x))*sinh(x)^5 + 10*(21*a*b*cosh(x)^6 + 35*a*b*cosh
(x)^4 + 15*a*b*cosh(x)^2 + a*b)*sinh(x)^4 + 5*a*b*cosh(x)^2 + 40*(3*a*b*cos
h(x)^7 + 7*a*b*cosh(x)^5 + 5*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x)^3 + 5*(9*
a*b*cosh(x)^8 + 28*a*b*cosh(x)^6 + 30*a*b*cosh(x)^4 + 12*a*b*cosh(x)^2 + a*
b)*sinh(x)^2 + a*b + 10*(a*b*cosh(x)^9 + 4*a*b*cosh(x)^7 + 6*a*b*cosh(x)^5
+ 4*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x))*sqrt(a)*log(((a^3 + 2*a^2*b + a*b
^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*
b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2
*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh
(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*
b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (7
0*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^
3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b +
a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3
+ 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x
)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b
^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh
(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b
+ b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b +
 b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*s
```

```
inh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(
x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)
^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*
(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2
 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2
*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*
b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 1
4*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)
^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3
+ 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 15*(a*b*cosh
(x)^10 + 10*a*b*cosh(x)*sinh(x)^9 + a*b*sinh(x)^10 + 5*a*b*cosh(x)^8 + 5*(9
*a*b*cosh(x)^2 + a*b)*sinh(x)^8 + 10*a*b*cosh(x)^6 + 40*(3*a*b*cosh(x)^3 +
a*b*cosh(x))*sinh(x)^7 + 10*(21*a*b*cosh(x)^4 + 14*a*b*cosh(x)^2 + a*b)*sin
h(x)^6 + 10*a*b*cosh(x)^4 + 4*(63*a*b*cosh(x)^5 + 70*a*b*cosh(x)^3 + 15*a*b
*cosh(x))*sinh(x)^5 + 10*(21*a*b*cosh(x)^6 + 35*a*b*cosh(x)^4 + 15*a*b*cosh
(x)^2 + a*b)*sinh(x)^4 + 5*a*b*cosh(x)^2 + 40*(3*a*b*cosh(x)^7 + 7*a*b*cosh
(x)^5 + 5*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x)^3 + 5*(9*a*b*cosh(x)^8 + 28*
a*b*cosh(x)^6 + 30*a*b*cosh(x)^4 + 12*a*b*cosh(x)^2 + a*b)*sinh(x)^2 + a*b
+ 10*(a*b*cosh(x)^9 + 4*a*b*cosh(x)^7 + 6*a*b*cosh(x)^5 + 4*a*b*cosh(x)^3 +
 a*b*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 +
a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*
sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*co
sh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2)) + 4*sqrt(2)*((3*a^2 - 20*a*b)*cosh(x)^8 + 8*(3*a^2 - 20*a*b)*cosh(x)*si
nh(x)^7 + (3*a^2 - 20*a*b)*sinh(x)^8 + 4*(3*a^2 - 14*a*b - 5*b^2)*cosh(x)^6
 + 4*(7*(3*a^2 - 20*a*b)*cosh(x)^2 + 3*a^2 - 14*a*b - 5*b^2)*sinh(x)^6 + 8*
(7*(3*a^2 - 20*a*b)*cosh(x)^3 + 3*(3*a^2 - 14*a*b - 5*b^2)*cosh(x))*sinh(x)
^5 + 2*(9*a^2 - 36*a*b + 4*b^2)*cosh(x)^4 + 2*(35*(3*a^2 - 20*a*b)*cosh(x)^
4 + 30*(3*a^2 - 14*a*b - 5*b^2)*cosh(x)^2 + 9*a^2 - 36*a*b + 4*b^2)*sinh(x)
^4 + 8*(7*(3*a^2 - 20*a*b)*cosh(x)^5 + 10*(3*a^2 - 14*a*b - 5*b^2)*cosh(x)^
3 + (9*a^2 - 36*a*b + 4*b^2)*cosh(x))*sinh(x)^3 + 4*(3*a^2 - 14*a*b - 5*b^2
)*cosh(x)^2 + 4*(7*(3*a^2 - 20*a*b)*cosh(x)^6 + 15*(3*a^2 - 14*a*b - 5*b^2)
*cosh(x)^4 + 3*(9*a^2 - 36*a*b + 4*b^2)*cosh(x)^2 + 3*a^2 - 14*a*b - 5*b^2)
*sinh(x)^2 + 3*a^2 - 20*a*b + 8*((3*a^2 - 20*a*b)*cosh(x)^7 + 3*(3*a^2 - 14
*a*b - 5*b^2)*cosh(x)^5 + (9*a^2 - 36*a*b + 4*b^2)*cosh(x)^3 + (3*a^2 - 14*
a*b - 5*b^2)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(
cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^10 + 10*b*cosh(x)*s
inh(x)^9 + b*sinh(x)^10 + 5*b*cosh(x)^8 + 5*(9*b*cosh(x)^2 + b)*sinh(x)^8 +
 40*(3*b*cosh(x)^3 + b*cosh(x))*sinh(x)^7 + 10*b*cosh(x)^6 + 10*(21*b*cosh(
x)^4 + 14*b*cosh(x)^2 + b)*sinh(x)^6 + 4*(63*b*cosh(x)^5 + 70*b*cosh(x)^3 +
 15*b*cosh(x))*sinh(x)^5 + 10*b*cosh(x)^4 + 10*(21*b*cosh(x)^6 + 35*b*cosh(
x)^4 + 15*b*cosh(x)^2 + b)*sinh(x)^4 + 40*(3*b*cosh(x)^7 + 7*b*cosh(x)^5 +
5*b*cosh(x)^3 + b*cosh(x))*sinh(x)^3 + 5*b*cosh(x)^2 + 5*(9*b*cosh(x)^8 + 2
8*b*cosh(x)^6 + 30*b*cosh(x)^4 + 12*b*cosh(x)^2 + b)*sinh(x)^2 + 10*(b*cosh
(x)^9 + 4*b*cosh(x)^7 + 6*b*cosh(x)^5 + 4*b*cosh(x)^3 + b*cosh(x))*sinh(x)
+ b), -1/30*(15*(a*b*cosh(x)^10 + 10*a*b*cosh(x)*sinh(x)^9 + a*b*sinh(x)^10
 + 5*a*b*cosh(x)^8 + 5*(9*a*b*cosh(x)^2 + a*b)*sinh(x)^8 + 10*a*b*cosh(x)^6
 + 40*(3*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x)^7 + 10*(21*a*b*cosh(x)^4 + 14
*a*b*cosh(x)^2 + a*b)*sinh(x)^6 + 10*a*b*cosh(x)^4 + 4*(63*a*b*cosh(x)^5 +
70*a*b*cosh(x)^3 + 15*a*b*cosh(x))*sinh(x)^5 + 10*(21*a*b*cosh(x)^6 + 35*a*
b*cosh(x)^4 + 15*a*b*cosh(x)^2 + a*b)*sinh(x)^4 + 5*a*b*cosh(x)^2 + 40*(3*a
*b*cosh(x)^7 + 7*a*b*cosh(x)^5 + 5*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x)^3 +
 5*(9*a*b*cosh(x)^8 + 28*a*b*cosh(x)^6 + 30*a*b*cosh(x)^4 + 12*a*b*cosh(x)^
2 + a*b)*sinh(x)^2 + a*b + 10*(a*b*cosh(x)^9 + 4*a*b*cosh(x)^7 + 6*a*b*cosh
(x)^5 + 4*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*((a
 + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(-
a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 +
```

```
(a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^
2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 +
3*a*b)*cosh(x))*sinh(x))) + 15*(a*b*cosh(x)^10 + 10*a*b*cosh(x)*sinh(x)^9 +
 a*b*sinh(x)^10 + 5*a*b*cosh(x)^8 + 5*(9*a*b*cosh(x)^2 + a*b)*sinh(x)^8 + 1
0*a*b*cosh(x)^6 + 40*(3*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x)^7 + 10*(21*a*b
*cosh(x)^4 + 14*a*b*cosh(x)^2 + a*b)*sinh(x)^6 + 10*a*b*cosh(x)^4 + 4*(63*a
*b*cosh(x)^5 + 70*a*b*cosh(x)^3 + 15*a*b*cosh(x))*sinh(x)^5 + 10*(21*a*b*co
sh(x)^6 + 35*a*b*cosh(x)^4 + 15*a*b*cosh(x)^2 + a*b)*sinh(x)^4 + 5*a*b*cosh
(x)^2 + 40*(3*a*b*cosh(x)^7 + 7*a*b*cosh(x)^5 + 5*a*b*cosh(x)^3 + a*b*cosh(
x))*sinh(x)^3 + 5*(9*a*b*cosh(x)^8 + 28*a*b*cosh(x)^6 + 30*a*b*cosh(x)^4 +
12*a*b*cosh(x)^2 + a*b)*sinh(x)^2 + a*b + 10*(a*b*cosh(x)^9 + 4*a*b*cosh(x)
^7 + 6*a*b*cosh(x)^5 + 4*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x))*sqrt(-a)*arc
tan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*sqrt((
a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(
x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cos
h(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)
*cosh(x))*sinh(x) + a)) - 2*sqrt(2)*((3*a^2 - 20*a*b)*cosh(x)^8 + 8*(3*a^2
- 20*a*b)*cosh(x)*sinh(x)^7 + (3*a^2 - 20*a*b)*sinh(x)^8 + 4*(3*a^2 - 14*a*
b - 5*b^2)*cosh(x)^6 + 4*(7*(3*a^2 - 20*a*b)*cosh(x)^2 + 3*a^2 - 14*a*b - 5
*b^2)*sinh(x)^6 + 8*(7*(3*a^2 - 20*a*b)*cosh(x)^3 + 3*(3*a^2 - 14*a*b - 5*b
^2)*cosh(x))*sinh(x)^5 + 2*(9*a^2 - 36*a*b + 4*b^2)*cosh(x)^4 + 2*(35*(3*a^
2 - 20*a*b)*cosh(x)^4 + 30*(3*a^2 - 14*a*b - 5*b^2)*cosh(x)^2 + 9*a^2 - 36*
a*b + 4*b^2)*sinh(x)^4 + 8*(7*(3*a^2 - 20*a*b)*cosh(x)^5 + 10*(3*a^2 - 14*a
*b - 5*b^2)*cosh(x)^3 + (9*a^2 - 36*a*b + 4*b^2)*cosh(x))*sinh(x)^3 + 4*(3*
a^2 - 14*a*b - 5*b^2)*cosh(x)^2 + 4*(7*(3*a^2 - 20*a*b)*cosh(x)^6 + 15*(3*a
^2 - 14*a*b - 5*b^2)*cosh(x)^4 + 3*(9*a^2 - 36*a*b + 4*b^2)*cosh(x)^2 + 3*a
^2 - 14*a*b - 5*b^2)*sinh(x)^2 + 3*a^2 - 20*a*b + 8*((3*a^2 - 20*a*b)*cosh(
x)^7 + 3*(3*a^2 - 14*a*b - 5*b^2)*cosh(x)^5 + (9*a^2 - 36*a*b + 4*b^2)*cosh
(x)^3 + (3*a^2 - 14*a*b - 5*b^2)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*si
nh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)
^10 + 10*b*cosh(x)*sinh(x)^9 + b*sinh(x)^10 + 5*b*cosh(x)^8 + 5*(9*b*cosh(x
)^2 + b)*sinh(x)^8 + 40*(3*b*cosh(x)^3 + b*cosh(x))*sinh(x)^7 + 10*b*cosh(x
)^6 + 10*(21*b*cosh(x)^4 + 14*b*cosh(x)^2 + b)*sinh(x)^6 + 4*(63*b*cosh(x)^
5 + 70*b*cosh(x)^3 + 15*b*cosh(x))*sinh(x)^5 + 10*b*cosh(x)^4 + 10*(21*b*co
sh(x)^6 + 35*b*cosh(x)^4 + 15*b*cosh(x)^2 + b)*sinh(x)^4 + 40*(3*b*cosh(x)^
7 + 7*b*cosh(x)^5 + 5*b*cosh(x)^3 + b*cosh(x))*sinh(x)^3 + 5*b*cosh(x)^2 +
5*(9*b*cosh(x)^8 + 28*b*cosh(x)^6 + 30*b*cosh(x)^4 + 12*b*cosh(x)^2 + b)*si
nh(x)^2 + 10*(b*cosh(x)^9 + 4*b*cosh(x)^7 + 6*b*cosh(x)^5 + 4*b*cosh(x)^3 +
 b*cosh(x))*sinh(x) + b)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)**2)**(3/2)*tanh(x)**3,x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\,\mathrm{sech}\left(x\right)^2+a\right)^{\frac{3}{2}}\tanh\left(x\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x)^3,x, algorithm="giac")
```

```
[Out] integrate((b*sech(x)^2 + a)^(3/2)*tanh(x)^3, x)
```

3.188 $\int \left(a+b\,\mathbf{sech}^2(x)\right)^{3/2}\tanh^2(x)\,dx$

Optimal. Leaf size=125

$$-\frac{\left(3a^2-6ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{8\sqrt{b}}+a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)+\frac{1}{4}b\tanh^3(x)\sqrt{a-b\tanh^2(x)+b}-$$

```
[Out] -((3*a^2 - 6*a*b - b^2)*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]
)/(8*Sqrt[b]) + a^(3/2)*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]
] - ((5*a + b)*Tanh[x]*Sqrt[a + b - b*Tanh[x]^2])/8 + (b*Tanh[x]^3*Sqrt[a +
 b - b*Tanh[x]^2])/4
```

Rubi [A] time = 0.360966, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {4141, 1975, 477, 582, 523, 217, 203, 377, 206}

$$-\frac{\left(3a^2-6ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{8\sqrt{b}}+a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)+\frac{1}{4}b\tanh^3(x)\sqrt{a-b\tanh^2(x)+b}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[x]^2)^(3/2)*Tanh[x]^2,x]
```


```
[Out] -((3*a^2 - 6*a*b - b^2)*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]
)/(8*Sqrt[b]) + a^(3/2)*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]
] - ((5*a + b)*Tanh[x]*Sqrt[a + b - b*Tanh[x]^2])/8 + (b*Tanh[x]^3*Sqrt[a +
 b - b*Tanh[x]^2])/4
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
```

```
, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\text{sech}^2(x)\right)^{3/2}\tanh^2(x)\,dx = \text{Subst}\left(\int \frac{x^2\left(a+b\left(1-x^2\right)\right)^{3/2}}{1-x^2}dx, x, \tanh(x)\right)$$

$$= \text{Subst}\left(\int \frac{x^2\left(a+b-bx^2\right)^{3/2}}{1-x^2}dx, x, \tanh(x)\right)$$

$$= \frac{1}{4}b\tanh^3(x)\sqrt{a+b-b\tanh^2(x)} - \frac{1}{4}\text{Subst}\left(\int \frac{x^2\left(-(a+b)(4a+b)+b(5a+b)\right.}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\right.$$

S

$$= -\frac{1}{8}(5a+b)\tanh(x)\sqrt{a+b-b\tanh^2(x)} + \frac{1}{4}b\tanh^3(x)\sqrt{a+b-b\tanh^2(x)} + -$$

$$= -\frac{1}{8}(5a+b)\tanh(x)\sqrt{a+b-b\tanh^2(x)} + \frac{1}{4}b\tanh^3(x)\sqrt{a+b-b\tanh^2(x)} + a^2$$

$$= -\frac{1}{8}(5a+b)\tanh(x)\sqrt{a+b-b\tanh^2(x)} + \frac{1}{4}b\tanh^3(x)\sqrt{a+b-b\tanh^2(x)} + a^2$$

$$= -\frac{\left(3a^2-6ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{8\sqrt{b}} + a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right) -$$

Mathematica [A] time = 0.808369, size = 197, normalized size = 1.58

$$-\frac{\cosh^3(x)\left(a+b\text{sech}^2(x)\right)^{3/2}\left(\sqrt{2}\left(3a^2-6ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sinh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right) - 8\sqrt{2}a^{3/2}\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right) + 2\right.}{4\sqrt{b}(a\cosh(2x)+a+2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[x]^2)^(3/2)*Tanh[x]^2,x]
```

```
[Out] -(Cosh[x]^3*(a + b*Sech[x]^2)^(3/2)*(Sqrt[2]*(3*a^2 - 6*a*b - b^2)*ArcTan[(
Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]] - 8*Sqrt[2]*a^(3/2)*S
qrt[b]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]] + (5*
a - b)*Sqrt[b]*Sqrt[a + 2*b + a*Cosh[2*x]]*Sech[x]*Tanh[x] + 2*b^(3/2)*Sqrt
[a + 2*b + a*Cosh[2*x]]*Sech[x]^3*Tanh[x]))/(4*Sqrt[b]*(a + 2*b + a*Cosh[2*
x])^(3/2))
```

Maple [F] time = 0.09, size = 0, normalized size = 0.

$$\int \left(a+b\left(\text{sech}\left(x\right)\right)^2\right)^{\frac{3}{2}}\left(\tanh\left(x\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)^(3/2)*tanh(x)^2,x)
```

```
[Out] int((a+b*sech(x)^2)^(3/2)*tanh(x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{sech}(x)^2 + a\right)^{\frac{3}{2}} \tanh(x)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sech(x)^2 + a)^(3/2)*tanh(x)^2, x)
```

Fricas [B] time = 5.82784, size = 23956, normalized size = 191.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x)^2,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(a*b*cosh(x)^8 + 8*a*b*cosh(x)*sinh(x)^7 + a*b*sinh(x)^8 + 4*a*b*c
osh(x)^6 + 4*(7*a*b*cosh(x)^2 + a*b)*sinh(x)^6 + 6*a*b*cosh(x)^4 + 8*(7*a*b
*cosh(x)^3 + 3*a*b*cosh(x))*sinh(x)^5 + 2*(35*a*b*cosh(x)^4 + 30*a*b*cosh(x
)^2 + 3*a*b)*sinh(x)^4 + 4*a*b*cosh(x)^2 + 8*(7*a*b*cosh(x)^5 + 10*a*b*cosh
(x)^3 + 3*a*b*cosh(x))*sinh(x)^3 + 4*(7*a*b*cosh(x)^6 + 15*a*b*cosh(x)^4 +
9*a*b*cosh(x)^2 + a*b)*sinh(x)^2 + a*b + 8*(a*b*cosh(x)^7 + 3*a*b*cosh(x)^5
 + 3*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8
*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*
(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a
*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*
a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sin
h(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*
b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14
*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*
a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh
(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)
*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*
cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 -
 a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x
))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x
)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5
+ (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(
cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sin
h(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((3*a
^2 - 6*a*b - b^2)*cosh(x)^8 + 8*(3*a^2 - 6*a*b - b^2)*cosh(x)*sinh(x)^7 + (
3*a^2 - 6*a*b - b^2)*sinh(x)^8 + 4*(3*a^2 - 6*a*b - b^2)*cosh(x)^6 + 4*(7*(
3*a^2 - 6*a*b - b^2)*cosh(x)^2 + 3*a^2 - 6*a*b - b^2)*sinh(x)^6 + 8*(7*(3*a
^2 - 6*a*b - b^2)*cosh(x)^3 + 3*(3*a^2 - 6*a*b - b^2)*cosh(x))*sinh(x)^5 +
6*(3*a^2 - 6*a*b - b^2)*cosh(x)^4 + 2*(35*(3*a^2 - 6*a*b - b^2)*cosh(x)^4 +
 30*(3*a^2 - 6*a*b - b^2)*cosh(x)^2 + 9*a^2 - 18*a*b - 3*b^2)*sinh(x)^4 + 8
*(7*(3*a^2 - 6*a*b - b^2)*cosh(x)^5 + 10*(3*a^2 - 6*a*b - b^2)*cosh(x)^3 +
3*(3*a^2 - 6*a*b - b^2)*cosh(x))*sinh(x)^3 + 4*(3*a^2 - 6*a*b - b^2)*cosh(x
)^2 + 4*(7*(3*a^2 - 6*a*b - b^2)*cosh(x)^6 + 15*(3*a^2 - 6*a*b - b^2)*cosh(
x)^4 + 9*(3*a^2 - 6*a*b - b^2)*cosh(x)^2 + 3*a^2 - 6*a*b - b^2)*sinh(x)^2 +
 3*a^2 - 6*a*b - b^2 + 8*((3*a^2 - 6*a*b - b^2)*cosh(x)^7 + 3*(3*a^2 - 6*a*
b - b^2)*cosh(x)^5 + 3*(3*a^2 - 6*a*b - b^2)*cosh(x)^3 + (3*a^2 - 6*a*b - b
^2)*cosh(x))*sinh(x))*sqrt(-b)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*
sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x
```

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)^2 + a + 3*b)*sinh(x)^2 + 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(
x)^2 - 1)*sqrt(-b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))
*sinh(x) + a - b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(
x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) +
 4*(a*b*cosh(x)^8 + 8*a*b*cosh(x)*sinh(x)^7 + a*b*sinh(x)^8 + 4*a*b*cosh(x)
^6 + 4*(7*a*b*cosh(x)^2 + a*b)*sinh(x)^6 + 6*a*b*cosh(x)^4 + 8*(7*a*b*cosh(
x)^3 + 3*a*b*cosh(x))*sinh(x)^5 + 2*(35*a*b*cosh(x)^4 + 30*a*b*cosh(x)^2 +
3*a*b)*sinh(x)^4 + 4*a*b*cosh(x)^2 + 8*(7*a*b*cosh(x)^5 + 10*a*b*cosh(x)^3
+ 3*a*b*cosh(x))*sinh(x)^3 + 4*(7*a*b*cosh(x)^6 + 15*a*b*cosh(x)^4 + 9*a*b*
cosh(x)^2 + a*b)*sinh(x)^2 + a*b + 8*(a*b*cosh(x)^7 + 3*a*b*cosh(x)^5 + 3*a
*b*cosh(x)^3 + a*b*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x
)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)
*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a
)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
 + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2
+ 2*cosh(x)*sinh(x) + sinh(x)^2)) - 2*sqrt(2)*((5*a*b - b^2)*cosh(x)^6 + 6*
(5*a*b - b^2)*cosh(x)*sinh(x)^5 + (5*a*b - b^2)*sinh(x)^6 + (5*a*b + 7*b^2)
*cosh(x)^4 + (15*(5*a*b - b^2)*cosh(x)^2 + 5*a*b + 7*b^2)*sinh(x)^4 + 4*(5*
(5*a*b - b^2)*cosh(x)^3 + (5*a*b + 7*b^2)*cosh(x))*sinh(x)^3 - (5*a*b + 7*b
^2)*cosh(x)^2 + (15*(5*a*b - b^2)*cosh(x)^4 + 6*(5*a*b + 7*b^2)*cosh(x)^2 -
 5*a*b - 7*b^2)*sinh(x)^2 - 5*a*b + b^2 + 2*(3*(5*a*b - b^2)*cosh(x)^5 + 2*
(5*a*b + 7*b^2)*cosh(x)^3 - (5*a*b + 7*b^2)*cosh(x))*sinh(x))*sqrt((a*cosh(
x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))
/(b*cosh(x)^8 + 8*b*cosh(x)*sinh(x)^7 + b*sinh(x)^8 + 4*b*cosh(x)^6 + 4*(7*
b*cosh(x)^2 + b)*sinh(x)^6 + 8*(7*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^5 + 6*
b*cosh(x)^4 + 2*(35*b*cosh(x)^4 + 30*b*cosh(x)^2 + 3*b)*sinh(x)^4 + 8*(7*b*
cosh(x)^5 + 10*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^3 + 4*b*cosh(x)^2 + 4*(7*
b*cosh(x)^6 + 15*b*cosh(x)^4 + 9*b*cosh(x)^2 + b)*sinh(x)^2 + 8*(b*cosh(x)^
7 + 3*b*cosh(x)^5 + 3*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b), -1/8*(((3*a^2
- 6*a*b - b^2)*cosh(x)^8 + 8*(3*a^2 - 6*a*b - b^2)*cosh(x)*sinh(x)^7 + (3*a
^2 - 6*a*b - b^2)*sinh(x)^8 + 4*(3*a^2 - 6*a*b - b^2)*cosh(x)^6 + 4*(7*(3*a
^2 - 6*a*b - b^2)*cosh(x)^2 + 3*a^2 - 6*a*b - b^2)*sinh(x)^6 + 8*(7*(3*a^2
- 6*a*b - b^2)*cosh(x)^3 + 3*(3*a^2 - 6*a*b - b^2)*cosh(x))*sinh(x)^5 + 6*(
3*a^2 - 6*a*b - b^2)*cosh(x)^4 + 2*(35*(3*a^2 - 6*a*b - b^2)*cosh(x)^4 + 30
*(3*a^2 - 6*a*b - b^2)*cosh(x)^2 + 9*a^2 - 18*a*b - 3*b^2)*sinh(x)^4 + 8*(7
*(3*a^2 - 6*a*b - b^2)*cosh(x)^5 + 10*(3*a^2 - 6*a*b - b^2)*cosh(x)^3 + 3*(
3*a^2 - 6*a*b - b^2)*cosh(x))*sinh(x)^3 + 4*(3*a^2 - 6*a*b - b^2)*cosh(x)^2
 + 4*(7*(3*a^2 - 6*a*b - b^2)*cosh(x)^6 + 15*(3*a^2 - 6*a*b - b^2)*cosh(x)^
4 + 9*(3*a^2 - 6*a*b - b^2)*cosh(x)^2 + 3*a^2 - 6*a*b - b^2)*sinh(x)^2 + 3*
a^2 - 6*a*b - b^2 + 8*((3*a^2 - 6*a*b - b^2)*cosh(x)^7 + 3*(3*a^2 - 6*a*b -
 b^2)*cosh(x)^5 + 3*(3*a^2 - 6*a*b - b^2)*cosh(x)^3 + (3*a^2 - 6*a*b - b^2)
*cosh(x))*sinh(x))*sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
sinh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 +
a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2
 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - 2*(a*b*cosh(x)^8 + 8
*a*b*cosh(x)*sinh(x)^7 + a*b*sinh(x)^8 + 4*a*b*cosh(x)^6 + 4*(7*a*b*cosh(x)
^2 + a*b)*sinh(x)^6 + 6*a*b*cosh(x)^4 + 8*(7*a*b*cosh(x)^3 + 3*a*b*cosh(x))
*sinh(x)^5 + 2*(35*a*b*cosh(x)^4 + 30*a*b*cosh(x)^2 + 3*a*b)*sinh(x)^4 + 4*
a*b*cosh(x)^2 + 8*(7*a*b*cosh(x)^5 + 10*a*b*cosh(x)^3 + 3*a*b*cosh(x))*sinh
(x)^3 + 4*(7*a*b*cosh(x)^6 + 15*a*b*cosh(x)^4 + 9*a*b*cosh(x)^2 + a*b)*sinh
(x)^2 + a*b + 8*(a*b*cosh(x)^7 + 3*a*b*cosh(x)^5 + 3*a*b*cosh(x)^3 + a*b*co
sh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 +
a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2
 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x
)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a
^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x
)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(
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x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^
2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)
*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)
^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(
x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)
^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 -
 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^
2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4
*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)
*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sin
h(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(
x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) - 2*(a*b*cosh(x)^8 + 8*a*b*cosh(x)
*sinh(x)^7 + a*b*sinh(x)^8 + 4*a*b*cosh(x)^6 + 4*(7*a*b*cosh(x)^2 + a*b)*si
nh(x)^6 + 6*a*b*cosh(x)^4 + 8*(7*a*b*cosh(x)^3 + 3*a*b*cosh(x))*sinh(x)^5 +
 2*(35*a*b*cosh(x)^4 + 30*a*b*cosh(x)^2 + 3*a*b)*sinh(x)^4 + 4*a*b*cosh(x)^
2 + 8*(7*a*b*cosh(x)^5 + 10*a*b*cosh(x)^3 + 3*a*b*cosh(x))*sinh(x)^3 + 4*(7
*a*b*cosh(x)^6 + 15*a*b*cosh(x)^4 + 9*a*b*cosh(x)^2 + a*b)*sinh(x)^2 + a*b
+ 8*(a*b*cosh(x)^7 + 3*a*b*cosh(x)^5 + 3*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(
x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a
+ b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 +
 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2
 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 +
 (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2))
 + sqrt(2)*((5*a*b - b^2)*cosh(x)^6 + 6*(5*a*b - b^2)*cosh(x)*sinh(x)^5 + (
5*a*b - b^2)*sinh(x)^6 + (5*a*b + 7*b^2)*cosh(x)^4 + (15*(5*a*b - b^2)*cosh
(x)^2 + 5*a*b + 7*b^2)*sinh(x)^4 + 4*(5*(5*a*b - b^2)*cosh(x)^3 + (5*a*b +
7*b^2)*cosh(x))*sinh(x)^3 - (5*a*b + 7*b^2)*cosh(x)^2 + (15*(5*a*b - b^2)*c
osh(x)^4 + 6*(5*a*b + 7*b^2)*cosh(x)^2 - 5*a*b - 7*b^2)*sinh(x)^2 - 5*a*b +
 b^2 + 2*(3*(5*a*b - b^2)*cosh(x)^5 + 2*(5*a*b + 7*b^2)*cosh(x)^3 - (5*a*b
+ 7*b^2)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^8 + 8*b*cosh(x)*sinh(x)
^7 + b*sinh(x)^8 + 4*b*cosh(x)^6 + 4*(7*b*cosh(x)^2 + b)*sinh(x)^6 + 8*(7*b
*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^5 + 6*b*cosh(x)^4 + 2*(35*b*cosh(x)^4 + 3
0*b*cosh(x)^2 + 3*b)*sinh(x)^4 + 8*(7*b*cosh(x)^5 + 10*b*cosh(x)^3 + 3*b*co
sh(x))*sinh(x)^3 + 4*b*cosh(x)^2 + 4*(7*b*cosh(x)^6 + 15*b*cosh(x)^4 + 9*b*
cosh(x)^2 + b)*sinh(x)^2 + 8*(b*cosh(x)^7 + 3*b*cosh(x)^5 + 3*b*cosh(x)^3 +
 b*cosh(x))*sinh(x) + b), -1/16*(8*(a*b*cosh(x)^8 + 8*a*b*cosh(x)*sinh(x)^7
 + a*b*sinh(x)^8 + 4*a*b*cosh(x)^6 + 4*(7*a*b*cosh(x)^2 + a*b)*sinh(x)^6 +
6*a*b*cosh(x)^4 + 8*(7*a*b*cosh(x)^3 + 3*a*b*cosh(x))*sinh(x)^5 + 2*(35*a*b
*cosh(x)^4 + 30*a*b*cosh(x)^2 + 3*a*b)*sinh(x)^4 + 4*a*b*cosh(x)^2 + 8*(7*a
*b*cosh(x)^5 + 10*a*b*cosh(x)^3 + 3*a*b*cosh(x))*sinh(x)^3 + 4*(7*a*b*cosh(
x)^6 + 15*a*b*cosh(x)^4 + 9*a*b*cosh(x)^2 + a*b)*sinh(x)^2 + a*b + 8*(a*b*c
osh(x)^7 + 3*a*b*cosh(x)^5 + 3*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x))*sqrt(-
a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqr
t(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sin
h(x) + sinh(x)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4
 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^
2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) + 8*(a*b*cosh(x)^
8 + 8*a*b*cosh(x)*sinh(x)^7 + a*b*sinh(x)^8 + 4*a*b*cosh(x)^6 + 4*(7*a*b*co
sh(x)^2 + a*b)*sinh(x)^6 + 6*a*b*cosh(x)^4 + 8*(7*a*b*cosh(x)^3 + 3*a*b*cos
h(x))*sinh(x)^5 + 2*(35*a*b*cosh(x)^4 + 30*a*b*cosh(x)^2 + 3*a*b)*sinh(x)^4
 + 4*a*b*cosh(x)^2 + 8*(7*a*b*cosh(x)^5 + 10*a*b*cosh(x)^3 + 3*a*b*cosh(x))
*sinh(x)^3 + 4*(7*a*b*cosh(x)^6 + 15*a*b*cosh(x)^4 + 9*a*b*cosh(x)^2 + a*b)
*sinh(x)^2 + a*b + 8*(a*b*cosh(x)^7 + 3*a*b*cosh(x)^5 + 3*a*b*cosh(x)^3 + a
*b*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cosh(x)^2 + a
*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(
x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)) - ((3*a^2 - 6*a*b - b^2)*cos
h(x)^8 + 8*(3*a^2 - 6*a*b - b^2)*cosh(x)*sinh(x)^7 + (3*a^2 - 6*a*b - b^2)*
sinh(x)^8 + 4*(3*a^2 - 6*a*b - b^2)*cosh(x)^6 + 4*(7*(3*a^2 - 6*a*b - b^2)*
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cosh(x)^2 + 3*a^2 - 6*a*b - b^2)*sinh(x)^6 + 8*(7*(3*a^2 - 6*a*b - b^2)*cos
h(x)^3 + 3*(3*a^2 - 6*a*b - b^2)*cosh(x))*sinh(x)^5 + 6*(3*a^2 - 6*a*b - b^
2)*cosh(x)^4 + 2*(35*(3*a^2 - 6*a*b - b^2)*cosh(x)^4 + 30*(3*a^2 - 6*a*b -
b^2)*cosh(x)^2 + 9*a^2 - 18*a*b - 3*b^2)*sinh(x)^4 + 8*(7*(3*a^2 - 6*a*b -
b^2)*cosh(x)^5 + 10*(3*a^2 - 6*a*b - b^2)*cosh(x)^3 + 3*(3*a^2 - 6*a*b - b^
2)*cosh(x))*sinh(x)^3 + 4*(3*a^2 - 6*a*b - b^2)*cosh(x)^2 + 4*(7*(3*a^2 - 6
*a*b - b^2)*cosh(x)^6 + 15*(3*a^2 - 6*a*b - b^2)*cosh(x)^4 + 9*(3*a^2 - 6*a
*b - b^2)*cosh(x)^2 + 3*a^2 - 6*a*b - b^2)*sinh(x)^2 + 3*a^2 - 6*a*b - b^2
+ 8*((3*a^2 - 6*a*b - b^2)*cosh(x)^7 + 3*(3*a^2 - 6*a*b - b^2)*cosh(x)^5 +
3*(3*a^2 - 6*a*b - b^2)*cosh(x)^3 + (3*a^2 - 6*a*b - b^2)*cosh(x))*sinh(x))
*sqrt(-b)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*s
inh(x)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x
)^2 + 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sq
rt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + s
inh(x)^2)) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b)/(co
sh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 +
 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + 2*sqrt(2)*((5*a*b -
b^2)*cosh(x)^6 + 6*(5*a*b - b^2)*cosh(x)*sinh(x)^5 + (5*a*b - b^2)*sinh(x)^
6 + (5*a*b + 7*b^2)*cosh(x)^4 + (15*(5*a*b - b^2)*cosh(x)^2 + 5*a*b + 7*b^2
)*sinh(x)^4 + 4*(5*(5*a*b - b^2)*cosh(x)^3 + (5*a*b + 7*b^2)*cosh(x))*sinh(
x)^3 - (5*a*b + 7*b^2)*cosh(x)^2 + (15*(5*a*b - b^2)*cosh(x)^4 + 6*(5*a*b +
 7*b^2)*cosh(x)^2 - 5*a*b - 7*b^2)*sinh(x)^2 - 5*a*b + b^2 + 2*(3*(5*a*b -
b^2)*cosh(x)^5 + 2*(5*a*b + 7*b^2)*cosh(x)^3 - (5*a*b + 7*b^2)*cosh(x))*sin
h(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sin
h(x) + sinh(x)^2)))/(b*cosh(x)^8 + 8*b*cosh(x)*sinh(x)^7 + b*sinh(x)^8 + 4*
b*cosh(x)^6 + 4*(7*b*cosh(x)^2 + b)*sinh(x)^6 + 8*(7*b*cosh(x)^3 + 3*b*cosh
(x))*sinh(x)^5 + 6*b*cosh(x)^4 + 2*(35*b*cosh(x)^4 + 30*b*cosh(x)^2 + 3*b)*
sinh(x)^4 + 8*(7*b*cosh(x)^5 + 10*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^3 + 4*
b*cosh(x)^2 + 4*(7*b*cosh(x)^6 + 15*b*cosh(x)^4 + 9*b*cosh(x)^2 + b)*sinh(x
)^2 + 8*(b*cosh(x)^7 + 3*b*cosh(x)^5 + 3*b*cosh(x)^3 + b*cosh(x))*sinh(x) +
 b), -1/8*(4*(a*b*cosh(x)^8 + 8*a*b*cosh(x)*sinh(x)^7 + a*b*sinh(x)^8 + 4*a
*b*cosh(x)^6 + 4*(7*a*b*cosh(x)^2 + a*b)*sinh(x)^6 + 6*a*b*cosh(x)^4 + 8*(7
*a*b*cosh(x)^3 + 3*a*b*cosh(x))*sinh(x)^5 + 2*(35*a*b*cosh(x)^4 + 30*a*b*co
sh(x)^2 + 3*a*b)*sinh(x)^4 + 4*a*b*cosh(x)^2 + 8*(7*a*b*cosh(x)^5 + 10*a*b*
cosh(x)^3 + 3*a*b*cosh(x))*sinh(x)^3 + 4*(7*a*b*cosh(x)^6 + 15*a*b*cosh(x)^
4 + 9*a*b*cosh(x)^2 + a*b)*sinh(x)^2 + a*b + 8*(a*b*cosh(x)^7 + 3*a*b*cosh(
x)^5 + 3*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(b*c
osh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^
2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*
b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(
x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3
 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) + ((3*a^2 - 6*a*b - b^2)*cosh(x)^8 + 8*
(3*a^2 - 6*a*b - b^2)*cosh(x)*sinh(x)^7 + (3*a^2 - 6*a*b - b^2)*sinh(x)^8 +
 4*(3*a^2 - 6*a*b - b^2)*cosh(x)^6 + 4*(7*(3*a^2 - 6*a*b - b^2)*cosh(x)^2 +
 3*a^2 - 6*a*b - b^2)*sinh(x)^6 + 8*(7*(3*a^2 - 6*a*b - b^2)*cosh(x)^3 + 3*
(3*a^2 - 6*a*b - b^2)*cosh(x))*sinh(x)^5 + 6*(3*a^2 - 6*a*b - b^2)*cosh(x)^
4 + 2*(35*(3*a^2 - 6*a*b - b^2)*cosh(x)^4 + 30*(3*a^2 - 6*a*b - b^2)*cosh(x
)^2 + 9*a^2 - 18*a*b - 3*b^2)*sinh(x)^4 + 8*(7*(3*a^2 - 6*a*b - b^2)*cosh(x
)^5 + 10*(3*a^2 - 6*a*b - b^2)*cosh(x)^3 + 3*(3*a^2 - 6*a*b - b^2)*cosh(x))
*sinh(x)^3 + 4*(3*a^2 - 6*a*b - b^2)*cosh(x)^2 + 4*(7*(3*a^2 - 6*a*b - b^2)
*cosh(x)^6 + 15*(3*a^2 - 6*a*b - b^2)*cosh(x)^4 + 9*(3*a^2 - 6*a*b - b^2)*c
osh(x)^2 + 3*a^2 - 6*a*b - b^2)*sinh(x)^2 + 3*a^2 - 6*a*b - b^2 + 8*((3*a^2
 - 6*a*b - b^2)*cosh(x)^7 + 3*(3*a^2 - 6*a*b - b^2)*cosh(x)^5 + 3*(3*a^2 -
6*a*b - b^2)*cosh(x)^3 + (3*a^2 - 6*a*b - b^2)*cosh(x))*sinh(x))*sqrt(b)*ar
ctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt((
a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(
x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cos
h(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)
*cosh(x))*sinh(x) + a)) + 4*(a*b*cosh(x)^8 + 8*a*b*cosh(x)*sinh(x)^7 + a*b*
```

```
sinh(x)^8 + 4*a*b*cosh(x)^6 + 4*(7*a*b*cosh(x)^2 + a*b)*sinh(x)^6 + 6*a*b*c
osh(x)^4 + 8*(7*a*b*cosh(x)^3 + 3*a*b*cosh(x))*sinh(x)^5 + 2*(35*a*b*cosh(x
)^4 + 30*a*b*cosh(x)^2 + 3*a*b)*sinh(x)^4 + 4*a*b*cosh(x)^2 + 8*(7*a*b*cosh
(x)^5 + 10*a*b*cosh(x)^3 + 3*a*b*cosh(x))*sinh(x)^3 + 4*(7*a*b*cosh(x)^6 +
15*a*b*cosh(x)^4 + 9*a*b*cosh(x)^2 + a*b)*sinh(x)^2 + a*b + 8*(a*b*cosh(x)^
7 + 3*a*b*cosh(x)^5 + 3*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x))*sqrt(-a)*arct
an(sqrt(2)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sin
h(x)^2 + a)) + sqrt(2)*((5*a*b - b^2)*cosh(x)^6 + 6*(5*a*b - b^2)*cosh(x)*s
inh(x)^5 + (5*a*b - b^2)*sinh(x)^6 + (5*a*b + 7*b^2)*cosh(x)^4 + (15*(5*a*b
 - b^2)*cosh(x)^2 + 5*a*b + 7*b^2)*sinh(x)^4 + 4*(5*(5*a*b - b^2)*cosh(x)^3
 + (5*a*b + 7*b^2)*cosh(x))*sinh(x)^3 - (5*a*b + 7*b^2)*cosh(x)^2 + (15*(5*
a*b - b^2)*cosh(x)^4 + 6*(5*a*b + 7*b^2)*cosh(x)^2 - 5*a*b - 7*b^2)*sinh(x)
^2 - 5*a*b + b^2 + 2*(3*(5*a*b - b^2)*cosh(x)^5 + 2*(5*a*b + 7*b^2)*cosh(x)
^3 - (5*a*b + 7*b^2)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a
+ 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^8 + 8*b*cos
h(x)*sinh(x)^7 + b*sinh(x)^8 + 4*b*cosh(x)^6 + 4*(7*b*cosh(x)^2 + b)*sinh(x
)^6 + 8*(7*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^5 + 6*b*cosh(x)^4 + 2*(35*b*c
osh(x)^4 + 30*b*cosh(x)^2 + 3*b)*sinh(x)^4 + 8*(7*b*cosh(x)^5 + 10*b*cosh(x
)^3 + 3*b*cosh(x))*sinh(x)^3 + 4*b*cosh(x)^2 + 4*(7*b*cosh(x)^6 + 15*b*cosh
(x)^4 + 9*b*cosh(x)^2 + b)*sinh(x)^2 + 8*(b*cosh(x)^7 + 3*b*cosh(x)^5 + 3*b
*cosh(x)^3 + b*cosh(x))*sinh(x) + b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\operatorname{sech}^2(x)\right)^{\frac{3}{2}} \tanh^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)**2)**(3/2)*tanh(x)**2,x)
```

```
[Out] Integral((a + b*sech(x)**2)**(3/2)*tanh(x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\operatorname{sech}(x)^2 + a\right)^{\frac{3}{2}} \tanh(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x)^2,x, algorithm="giac")
```

```
[Out] integrate((b*sech(x)^2 + a)^(3/2)*tanh(x)^2, x)
```

3.189 $\int \left(a+b\,\mathrm{sech}^2(x)\right)^{3/2} \tanh(x)\,dx$

Optimal. Leaf size=57

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)-a\sqrt{a+b\,\mathrm{sech}^2(x)}-\frac{1}{3}\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}$$

```
[Out] a^(3/2)*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - a*Sqrt[a + b*Sech[x]^2] -
(a + b*Sech[x]^2)^(3/2)/3
```

Rubi [A] time = 0.084093, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4139, 266, 50, 63, 208}

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)-a\sqrt{a+b\,\mathrm{sech}^2(x)}-\frac{1}{3}\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[x]^2)^(3/2)*Tanh[x],x]
```

```
[Out] a^(3/2)*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - a*Sqrt[a + b*Sech[x]^2] -
(a + b*Sech[x]^2)^(3/2)/3
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+b\text{sech}^2(x)\right)^{3/2}\tanh(x)\,dx &= -\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x}\,dx, x, \text{sech}(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \text{sech}^2(x)\right)\right)\\
&= -\frac{1}{3}\left(a+b\text{sech}^2(x)\right)^{3/2}-\frac{1}{2}a\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \text{sech}^2(x)\right)\\
&= -a\sqrt{a+b\text{sech}^2(x)}-\frac{1}{3}\left(a+b\text{sech}^2(x)\right)^{3/2}-\frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \text{sech}^2(x)\right)\\
&= -a\sqrt{a+b\text{sech}^2(x)}-\frac{1}{3}\left(a+b\text{sech}^2(x)\right)^{3/2}-\frac{a^2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{sech}^2(x)}\right)}{b}\\
&= a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)-a\sqrt{a+b\text{sech}^2(x)}-\frac{1}{3}\left(a+b\text{sech}^2(x)\right)^{3/2}
\end{aligned}
$$

Mathematica [C] time = 0.113562, size = 65, normalized size = 1.14

$$
-\frac{2b\left(a+b\text{sech}^2(x)\right)^{3/2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{a\cosh^2(x)}{b}\right)}{3\sqrt{\frac{a\cosh^2(x)}{b}+1}(a\cosh(2x)+a+2b)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[x]^2)^(3/2)*Tanh[x],x]
```

```
[Out] (-2*b*Hypergeometric2F1[-3/2, -3/2, -1/2, -((a*Cosh[x]^2)/b)]*(a + b*Sech[x
]^2)^(3/2))/(3*Sqrt[1 + (a*Cosh[x]^2)/b]*(a + 2*b + a*Cosh[2*x]))
```

Maple [A] time = 0.014, size = 56, normalized size = 1.

$$
-\frac{1}{3}\left(a+b\left(\text{sech}\left(x\right)\right)^2\right)^{\frac{3}{2}}+\ln\left(\frac{1}{\text{sech}\left(x\right)}\left(2\,a+2\,\sqrt{a}\sqrt{a+b\left(\text{sech}\left(x\right)\right)^2}\right)\right)a^{\frac{3}{2}}-a\sqrt{a+b\left(\text{sech}\left(x\right)\right)^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)^(3/2)*tanh(x),x)
```

```
[Out] -1/3*(a+b*sech(x)^2)^(3/2)+ln((2*a+2*a^(1/2)*(a+b*sech(x)^2)^(1/2))/sech(x)
)*a^(3/2)-a*(a+b*sech(x)^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\operatorname{sech}(x)^2+a\right)^{\frac{3}{2}}\tanh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(x)^2 + a)^(3/2)*tanh(x), x)
```

Fricas [B] time = 3.28383, size = 6842, normalized size = 120.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 + 3*a*cosh(x)^4
 + 3*(5*a*cosh(x)^2 + a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x
)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 + 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*(
a*cosh(x)^5 + 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*sqrt(a)*log(((a^3 + 2
*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (
a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh
(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cos
h(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*
a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*co
sh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^
2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3
+ 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)
^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a
^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a
^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9
*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(
a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a
^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*
a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b
+ b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b +
b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^
2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x
)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(
x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 +
2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5
+ (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(
x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^
3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) +
3*(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 + 3*a*cosh(x)^4 + 3*(5
*a*cosh(x)^2 + a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^3 + 3
*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 + 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*(a*cosh(
x)^5 + 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*sqrt(a)*log(-(a*cosh(x)^4 +
4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)
*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a
)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
 + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*co
```

```
sh(x)*sinh(x) + sinh(x)^2)) - 16*sqrt(2)*(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)
^3 + a*sinh(x)^4 + (2*a + b)*cosh(x)^2 + (6*a*cosh(x)^2 + 2*a + b)*sinh(x)^
2 + 2*(2*a*cosh(x)^3 + (2*a + b)*cosh(x))*sinh(x) + a)*sqrt((a*cosh(x)^2 +
a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(
x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 1)*sinh(x)^4 + 3*
cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 + 6*cosh
(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(x)^3 + cosh(x))*
sinh(x) + 1), -1/6*(3*(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 +
3*a*cosh(x)^4 + 3*(5*a*cosh(x)^2 + a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 + 3*a*co
sh(x))*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 + 6*a*cosh(x)^2 + a)*si
nh(x)^2 + 6*(a*cosh(x)^5 + 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*sqrt(-a)
*arctan(sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*si
nh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2
 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*c
osh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(
a^2 + a*b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*co
sh(x)^3 + (2*a^2 + 3*a*b)*cosh(x))*sinh(x))) + 3*(a*cosh(x)^6 + 6*a*cosh(x)
*sinh(x)^5 + a*sinh(x)^6 + 3*a*cosh(x)^4 + 3*(5*a*cosh(x)^2 + a)*sinh(x)^4
+ 4*(5*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x
)^4 + 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*(a*cosh(x)^5 + 2*a*cosh(x)^3 + a*cos
h(x))*sinh(x) + a)*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x
)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3
+ a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)
^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + 8*sqrt(2)*(a*cosh(
x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + (2*a + b)*cosh(x)^2 + (6*a*cos
h(x)^2 + 2*a + b)*sinh(x)^2 + 2*(2*a*cosh(x)^3 + (2*a + b)*cosh(x))*sinh(x)
 + a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sin
h(x) + sinh(x)^2)))/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cos
h(x)^2 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3
 + 3*(5*cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5
 + 2*cosh(x)^3 + cosh(x))*sinh(x) + 1)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\operatorname{sech}^2(x)\right)^{\frac{3}{2}}\tanh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)**2)**(3/2)*tanh(x),x)
```

```
[Out] Integral((a + b*sech(x)**2)**(3/2)*tanh(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\operatorname{sech}(x)^2 + a\right)^{\frac{3}{2}}\tanh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x),x, algorithm="giac")
```

```
[Out] integrate((b*sech(x)^2 + a)^(3/2)*tanh(x), x)
```

3.190 $\int \left(a+b\,\mathbf{sech}^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=88

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)+\frac{1}{2}b\tanh(x)\sqrt{a-b\tanh^2(x)+b}+\frac{1}{2}\sqrt{b}(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)$$

```
[Out] (Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]])/2 +
 a^(3/2)*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]] + (b*Tanh[x]*
Sqrt[a + b - b*Tanh[x]^2])/2
```

Rubi [A] time = 0.0912239, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {4128, 416, 523, 217, 203, 377, 206}

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)+\frac{1}{2}b\tanh(x)\sqrt{a-b\tanh^2(x)+b}+\frac{1}{2}\sqrt{b}(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]])/2 +
 a^(3/2)*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]] + (b*Tanh[x]*
Sqrt[a + b - b*Tanh[x]^2])/2
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\operatorname{sech}^2(x)\right)^{3/2}\,dx = \operatorname{Subst}\left(\int \frac{\left(a+b-bx^2\right)^{3/2}}{1-x^2}\,dx, x, \tanh(x)\right)$$
$$= \frac{1}{2}b\tanh(x)\sqrt{a+b-b\tanh^2(x)} - \frac{1}{2}\operatorname{Subst}\left(\int \frac{-(a+b)(2a+b)+b(3a+b)x^2}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right.$$
$$= \frac{1}{2}b\tanh(x)\sqrt{a+b-b\tanh^2(x)} + a^2\operatorname{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right) + \frac{1}{2}(b($$
$$= \frac{1}{2}b\tanh(x)\sqrt{a+b-b\tanh^2(x)} + a^2\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right) + \frac{1}{2}(b(3$$
$$= \frac{1}{2}\sqrt{b}(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right) + a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right) + \frac{1}{2}b\tanh($$

Mathematica [A] time = 0.259941, size = 152, normalized size = 1.73

$$\frac{\operatorname{sech}(x)\left(a\cosh^2(x)+b\right)\sqrt{a+b\operatorname{sech}^2(x)}\left(2\sqrt{2}a^{3/2}\cosh^2(x)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)+b\sinh(x)\sqrt{a\cosh(2x)+a+2b}+\right.}{(a\cosh(2x)+a+2b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] ((b + a*Cosh[x]^2)*Sech[x]*Sqrt[a + b*Sech[x]^2]*(Sqrt[2]*Sqrt[b]*(3*a + b)
*ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]]*Cosh[x]^2 +
2*Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2
*x]]]*Cosh[x]^2 + b*Sqrt[a + 2*b + a*Cosh[2*x]]*Sinh[x]))/(a + 2*b + a*Cosh
[2*x])^(3/2)
```

Maple [F] time = 0.082, size = 0, normalized size = 0.

$$\int \left(a+b\left(\operatorname{sech}(x)\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)^(3/2),x)
```

```
[Out] int((a+b*sech(x)^2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{sech}(x)^2 + a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(x)^2 + a)^(3/2), x)
```

Fricas [B] time = 3.59063, size = 12542, normalized size = 142.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*a*cosh(x)^2 +
2*(3*a*cosh(x)^2 + a)*sinh(x)^2 + 4*(a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*
sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8
- 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^
6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a
^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 -
 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2
- b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(
a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)
^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqr
t(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(
x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cos
h(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh
(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3
 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2
+ a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x
)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^
3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x
)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)
*sinh(x)^5 + sinh(x)^6)) + ((3*a + b)*cosh(x)^4 + 4*(3*a + b)*cosh(x)*sinh(
x)^3 + (3*a + b)*sinh(x)^4 + 2*(3*a + b)*cosh(x)^2 + 2*(3*(3*a + b)*cosh(x)
^2 + 3*a + b)*sinh(x)^2 + 4*((3*a + b)*cosh(x)^3 + (3*a + b)*cosh(x))*sinh(
x) + 3*a + b)*sqrt(-b)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^
3 + (a - b)*sinh(x)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a
+ 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1
)*sqrt(-b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x
)*sinh(x) + sinh(x)^2)) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x)
 + a - b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1
)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + (a*cosh
```

```
(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*a*cosh(x
)^2 + a)*sinh(x)^2 + 4*(a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*sqrt(a)*log(-
(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 +
2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x
) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))
*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 2*sqrt(2)*(b*c
osh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt((a*cosh(x)^2 + a*sin
h(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^4
+ 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(
x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1), 1/4*(2*((3*a + b)*cosh(x)^4 +
4*(3*a + b)*cosh(x)*sinh(x)^3 + (3*a + b)*sinh(x)^4 + 2*(3*a + b)*cosh(x)^2
 + 2*(3*(3*a + b)*cosh(x)^2 + 3*a + b)*sinh(x)^2 + 4*((3*a + b)*cosh(x)^3 +
 (3*a + b)*cosh(x))*sinh(x) + 3*a + b)*sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 +
2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2
+ a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*
cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2
+ a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) +
(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*a
*cosh(x)^2 + a)*sinh(x)^2 + 4*(a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*sqrt(a
)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a
*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*
(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b +
 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a
*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)
*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 +
3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a
^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(
b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 +
 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*
sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2
- a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^
2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a +
2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 -
3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*
a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*si
nh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(
x)^5 + sinh(x)^6)) + (a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2
*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a)*sinh(x)^2 + 4*(a*cosh(x)^3 + a*cosh(x)
)*sinh(x) + a)*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x
)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(
cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 +
a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*
cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
sinh(x)^2)) + 2*sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 -
b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2)))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x
)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1), -1
/4*(2*(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*a*cosh(x)^2 +
2*(3*a*cosh(x)^2 + a)*sinh(x)^2 + 4*(a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*
sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 +
a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(
x)*sinh(x) + sinh(x)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sin
h(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^
2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) + 2*(a*cosh
(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*a*cosh(x
)^2 + a)*sinh(x)^2 + 4*(a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*sqrt(-a)*arct
an(sqrt(2)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sin
```

```
h(x)^2 + a)) - ((3*a + b)*cosh(x)^4 + 4*(3*a + b)*cosh(x)*sinh(x)^3 + (3*a
+ b)*sinh(x)^4 + 2*(3*a + b)*cosh(x)^2 + 2*(3*(3*a + b)*cosh(x)^2 + 3*a + b
)*sinh(x)^2 + 4*((3*a + b)*cosh(x)^3 + (3*a + b)*cosh(x))*sinh(x) + 3*a + b
)*sqrt(-b)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*
sinh(x)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(
x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*s
qrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
sinh(x)^2)) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b)/(c
osh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2
+ 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) - 2*sqrt(2)*(b*cosh(x
)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt((a*cosh(x)^2 + a*sinh(x)^
2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^4 + 4*c
osh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2
+ 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1), -1/2*((a*cosh(x)^4 + 4*a*cosh(x)*si
nh(x)^3 + a*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a)*sinh(x)^2 + 4
*(a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^
2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*s
inh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(
x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 +
(6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2
 + 3*a*b)*cosh(x))*sinh(x))) - ((3*a + b)*cosh(x)^4 + 4*(3*a + b)*cosh(x)*s
inh(x)^3 + (3*a + b)*sinh(x)^4 + 2*(3*a + b)*cosh(x)^2 + 2*(3*(3*a + b)*cos
h(x)^2 + 3*a + b)*sinh(x)^2 + 4*((3*a + b)*cosh(x)^3 + (3*a + b)*cosh(x))*s
inh(x) + 3*a + b)*sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + s
inh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2
 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a
*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2
+ 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + (a*cosh(x)^4 + 4*a*co
sh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a)*sinh(
x)^2 + 4*(a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*sqrt(-a)*arctan(sqrt(2)*sqr
t(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sin
h(x) + sinh(x)^2))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)) -
 sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt((a*cosh
(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))
)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x
)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\operatorname{sech}^2(x)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sech(x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\operatorname{sech}(x)^2 + a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sech(x)^2 + a)^(3/2), x)
```

3.191 $\int \coth(x) \left(a + b\,\mathbf{sech}^2(x)\right)^{3/2} dx$

Optimal. Leaf size=70

$$a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+b\,\text{sech}^2(x)}}{\sqrt{a}}\right) + b\sqrt{a+b\,\text{sech}^2(x)} - (a+b)^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+b\,\text{sech}^2(x)}}{\sqrt{a+b}}\right)$$

```
[Out] a^(3/2)*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - (a + b)^(3/2)*ArcTanh[Sqrt
[a + b*Sech[x]^2]/Sqrt[a + b]] + b*Sqrt[a + b*Sech[x]^2]
```

Rubi [A] time = 0.131956, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4139, 446, 84, 156, 63, 208}

$$a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+b\,\text{sech}^2(x)}}{\sqrt{a}}\right) + b\sqrt{a+b\,\text{sech}^2(x)} - (a+b)^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+b\,\text{sech}^2(x)}}{\sqrt{a+b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]*(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] a^(3/2)*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - (a + b)^(3/2)*ArcTanh[Sqrt
[a + b*Sech[x]^2]/Sqrt[a + b]] + b*Sqrt[a + b*Sech[x]^2]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \coth(x)\left(a+b\mathrm{sech}^2(x)\right)^{3/2}dx = \text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x\left(-1+x^2\right)}dx, x, \mathrm{sech}(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{(-1+x)x}dx, x, \mathrm{sech}^2(x)\right)$$

$$= b\sqrt{a+b\mathrm{sech}^2(x)} + \frac{1}{2}\text{Subst}\left(\int \frac{a^2+b(2a+b)x}{(-1+x)x\sqrt{a+bx}}dx, x, \mathrm{sech}^2(x)\right)$$

$$= b\sqrt{a+b\mathrm{sech}^2(x)} - \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}dx, x, \mathrm{sech}^2(x)\right) + \frac{1}{2}(a+b)^2\,\text{Subst}\left(\int\right.$$

$$= b\sqrt{a+b\mathrm{sech}^2(x)} - \frac{a^2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}dx, x, \sqrt{a+b\mathrm{sech}^2(x)}\right)}{b} + \frac{(a+b)^2\,\text{Subst}\left(\int\right.}{}$$

$$= a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\mathrm{sech}^2(x)}}{\sqrt{a}}\right) - (a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\mathrm{sech}^2(x)}}{\sqrt{a+b}}\right) + b\sqrt{a+b\mathrm{sech}}$$

Mathematica [B] time = 0.490602, size = 159, normalized size = 2.27

$$-\frac{2\left(a\cosh^2(x)+b\right)\sqrt{a+b\mathrm{sech}^2(x)}\left(\sqrt{2}(a+b)^2\cosh(x)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a+b}\cosh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)-\sqrt{a+b}\left(\sqrt{2}a^{3/2}\cosh(x)\log\left(\sqrt{a\cos}\right.\right.\right.}{\sqrt{a+b}(a\cosh(2x)+a+2b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]*(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] (-2*(b + a*Cosh[x]^2)*(Sqrt[2]*(a + b)^2*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Cosh[
x])/Sqrt[a + 2*b + a*Cosh[2*x]]]*Cosh[x] - Sqrt[a + b]*(b*Sqrt[a + 2*b + a*
Cosh[2*x]] + Sqrt[2]*a^(3/2)*Cosh[x]*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a +
 2*b + a*Cosh[2*x]]]))*Sqrt[a + b*Sech[x]^2])/(Sqrt[a + b]*(a + 2*b + a*Cos
h[2*x])^(3/2))
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int \coth(x)\left(a+b(\mathrm{sech}(x))^2\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a+b*sech(x)^2)^(3/2),x)
```

```
[Out] int(coth(x)*(a+b*sech(x)^2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\,\mathrm{sech}\left(x\right)^2+a\right)^{\frac{3}{2}}\coth\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(x)^2 + a)^(3/2)*coth(x), x)
```

Fricas [B] time = 3.58444, size = 12166, normalized size = 173.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)*sqrt(a)*log(((a
^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)
^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3
)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^
2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^
3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b
^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b +
9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14
*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*c
osh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3
 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3
 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2
*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6
 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6
+ 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^
2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 +
2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*
a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*si
nh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*
cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a
*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(
a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh
(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))
*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*co
sh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^
6)) + 2*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2
+ a + b)*sqrt(a + b)*log(((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)
^3 + (2*a + b)*sinh(x)^4 + 2*(2*a + 3*b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)
^2 + 2*a + 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh
(x)^2 + 1)*sqrt(a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a + 3*b)*
```

```
cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 +
2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(
x) + 1)) + (a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)*sqrt(a)*lo
g(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(
3*a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sin
h(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)
/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*sqrt(2)*b*sqrt((a*cosh(x)
^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(
cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1), 1/4*(4*((a + b)*cosh(x)^2 +
 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)*sqrt(-a - b)*arctan
(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt(
(a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh
(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*co
sh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b
)*cosh(x))*sinh(x) + a)) + (a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2
 + a)*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a
*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*
a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*
(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2
)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a
^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4
+ 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)
^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2
*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(
x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*
cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*
b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2
*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*
a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b
+ b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*
cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh
(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^
2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6
*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*
sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
 sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b
+ 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^
3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x
)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)
*sinh(x)^5 + sinh(x)^6)) + (a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2
 + a)*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b
*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(
x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2
*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(
x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*sqrt(2)*b
*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
+ sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1), -1/2*((a*co
sh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)*sqrt(-a)*arctan(sqrt(2)*((
a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(
-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(
x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 +
 (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)
^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 +
 3*a*b)*cosh(x))*sinh(x))) + (a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)
^2 + a)*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2
- 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)
^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*c
```

```
osh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - ((a + b)*cosh(x)^2 + 2*(a + b
)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)*sqrt(a + b)*log(((2*a + b)*c
osh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a + 3
*b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a + 3*b)*sinh(x)^2 - 2*sqrt(2)
*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt((a*cosh(x
)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) +
 4*((2*a + b)*cosh(x)^3 + (2*a + 3*b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^
4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cos
h(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)) - 2*sqrt(2)*b*sqrt((a*cosh(x
)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/
(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1), -1/2*((a*cosh(x)^2 + 2*a*c
osh(x)*sinh(x) + a*sinh(x)^2 + a)*sqrt(-a)*arctan(sqrt(2)*((a + b)*cosh(x)^
2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cos
h(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)
)/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*si
nh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + 3*
a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(x)
)*sinh(x))) + (a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)*sqrt(-a
)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*s
qrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b
)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a +
 2*b)*cosh(x))*sinh(x) + a)) - 2*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sin
h(x) + (a + b)*sinh(x)^2 + a + b)*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 +
2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt((a*cosh(x)^2 + a*sinh(
x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 +
 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(
x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a
)) - 2*sqrt(2)*b*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*
cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 +
 1)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sech(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\operatorname{sech}(x)^2 + a\right)^{\frac{3}{2}} \coth(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sech(x)^2 + a)^(3/2)*coth(x), x)
```

3.192 $\int \coth^2(x)\left(a+b\mathbf{sech}^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=81

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)-b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)-(a+b)\coth(x)\sqrt{a-b\tanh^2(x)+b}$$

```
[Out] -(b^(3/2)*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]) + a^(3/2)*Ar
cTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]] - (a + b)*Coth[x]*Sqrt[a
 + b - b*Tanh[x]^2]
```

Rubi [A] time = 0.232478, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {4141, 1975, 474, 523, 217, 203, 377, 206}

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)-b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)-(a+b)\coth(x)\sqrt{a-b\tanh^2(x)+b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2*(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] -(b^(3/2)*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]) + a^(3/2)*Ar
cTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]] - (a + b)*Coth[x]*Sqrt[a
 + b - b*Tanh[x]^2]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \coth^2(x)\left(a+b\operatorname{sech}^2(x)\right)^{3/2}dx &= \text{Subst}\left(\int \frac{\left(a+b\left(1-x^2\right)\right)^{3/2}}{x^2\left(1-x^2\right)}\,dx,x,\tanh(x)\right)\\
&= \text{Subst}\left(\int \frac{\left(a+b-bx^2\right)^{3/2}}{x^2\left(1-x^2\right)}\,dx,x,\tanh(x)\right)\\
&= -(a+b)\coth(x)\sqrt{a+b-b\tanh^2(x)}+\text{Subst}\left(\int \frac{a^2-b^2+b^2x^2}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx,x,\text{ta}\right.\\
&= -(a+b)\coth(x)\sqrt{a+b-b\tanh^2(x)}+a^2\,\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx,x,\right.\\
&= -(a+b)\coth(x)\sqrt{a+b-b\tanh^2(x)}+a^2\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{\tanh(x}{\sqrt{a+b-b\,\text{ta}}}\right.\\
&= -b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)+a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)-(a+
\end{aligned}
$$

Mathematica [A] time = 0.3267, size = 144, normalized size = 1.78

$$
-\frac{2\left(a\cosh^2(x)+b\right)\sqrt{a+b\operatorname{sech}^2(x)}\left(-\sqrt{2}a^{3/2}\cosh(x)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)+\sqrt{2}b^{3/2}\cosh(x)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sinh(x}{\sqrt{a\cosh(2x)+a}}\right.\right.}{(a\cosh(2x)+a+2b)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2*(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] (-2*(b + a*Cosh[x]^2)*(Sqrt[2]*b^(3/2)*ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqr
t[a + 2*b + a*Cosh[2*x]]]*Cosh[x] - Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a
]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]]*Cosh[x] + (a + b)*Sqrt[a + 2*b + a*
Cosh[2*x]]*Coth[x])*Sqrt[a + b*Sech[x]^2])/(a + 2*b + a*Cosh[2*x])^(3/2)
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int (\coth (x))^2 \left(a + b (\operatorname{sech} (x))^2\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2*(a+b*sech(x)^2)^(3/2),x)
```

```
[Out] int(coth(x)^2*(a+b*sech(x)^2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{sech} (x)^2 + a\right)^{\frac{3}{2}} \coth (x)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(x)^2 + a)^(3/2)*coth(x)^2, x)
```

Fricas [B] time = 3.45242, size = 10029, normalized size = 123.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(a)*log((a*
b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^
3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2
*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)
*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^
3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^
3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*
cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^
2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(
x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2
*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3
 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4
*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b
)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cos
h(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2
```

- b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*co
sh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 +
 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + si
nh(x)^6)) + 2*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt(-b
)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4
 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 + 2*
sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a*co
sh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2
)) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)^4
+ 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(
x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + (a*cosh(x)^2 + 2*a*cosh(x)*s
inh(x) + a*sinh(x)^2 - a)*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3
 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2
+ sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*c
osh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)
*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*(a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2
+ a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2 - 1), -1/4*(4*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) +
 b*sinh(x)^2 - b)*sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + s
inh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2
 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a
sinh(x)^4 + 2(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2
+ 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - (a*cosh(x)^2 + 2*a*co
sh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cos
h(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*
cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3
)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh
(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4
*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^
2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cos
h(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*
a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x
)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4
 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2
+ (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(
3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)
*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
+ sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4
*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6
+ 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 1
5*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) - (a*cosh(x)^2 +
2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh
(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a +
b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt
(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(
x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^
2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*sqrt(2)*(a + b)*sqrt((a*cosh(x)^2 +
 a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh
(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1), -1/2*((a*cosh(x)^2 + 2*a*cosh(x
)*sinh(x) + a*sinh(x)^2 - a)*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cos
h(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(x)^4 + 4*a*
b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh
(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*c
osh(x))*sinh(x))) + (a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*s
qrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^2 + 2*a*cosh(x)*sin
h(x) + a*sinh(x)^2 + a)) - (b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2

```
 - b)*sqrt(-b)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a -
 b)*sinh(x)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*s
inh(x)^2 + 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-
b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2)) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b
)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x
)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + 2*sqrt(2)*(a +
b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1), -1/2*((a*
cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(-a)*arctan(sqrt(2)*
(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh
(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))
/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*c
osh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(
x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) + 2*(b*cosh(x)^2 + 2*b*cosh(x)*sinh
(x) + b*sinh(x)^2 - b)*sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x
) + sinh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^
3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(
x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + (a*cosh(x)^2 + 2
*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt
((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sin
h(x)^2))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)) + 2*sqrt(2)
*(a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*
sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2*(a+b*sech(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\,\mathrm{sech}\,(x)^2 + a\right)^{\frac{3}{2}} \coth\,(x)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sech(x)^2 + a)^(3/2)*coth(x)^2, x)
```

3.193 $\int \left(a+b\,\textbf{sech}^2(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=170

$$\frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a-b\tanh^2(c+dx)+b}}\right)}{8d}+\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a-b\tanh^2(c+dx)+b}}\right)}{d}+\frac{b\tanh(c+dx)\left(a-b\tanh^2(c\right.}{4d}$$

```
[Out] (Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a +
b - b*Tanh[c + d*x]^2]])/(8*d) + (a^(5/2)*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/S
qrt[a + b - b*Tanh[c + d*x]^2]])/d + (b*(7*a + 3*b)*Tanh[c + d*x]*Sqrt[a +
b - b*Tanh[c + d*x]^2])/(8*d) + (b*Tanh[c + d*x]*(a + b - b*Tanh[c + d*x]^2
)^(3/2))/(4*d)
```

Rubi [A] time = 0.194354, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4128, 416, 528, 523, 217, 203, 377, 206}

$$\frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a-b\tanh^2(c+dx)+b}}\right)}{8d}+\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a-b\tanh^2(c+dx)+b}}\right)}{d}+\frac{b\tanh(c+dx)\left(a-b\tanh^2(c\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^(5/2),x]
```

```
[Out] (Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a +
b - b*Tanh[c + d*x]^2]])/(8*d) + (a^(5/2)*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/S
qrt[a + b - b*Tanh[c + d*x]^2]])/d + (b*(7*a + 3*b)*Tanh[c + d*x]*Sqrt[a +
b - b*Tanh[c + d*x]^2])/(8*d) + (b*Tanh[c + d*x]*(a + b - b*Tanh[c + d*x]^2
)^(3/2))/(4*d)
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^{5/2}dx = \frac{\text{Subst}\left(\int \frac{(a+b-bx^2)^{5/2}}{1-x^2}dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\tanh(c+dx)\left(a+b-b\tanh^2(c+dx)\right)^{3/2}}{4d} - \frac{\text{Subst}\left(\int \frac{\sqrt{a+b-bx^2}\left((a+b)(b-4(a+b))+b(7a+3b)x^2\right)}{1-x^2}\right.}{4d}$$

$$= \frac{b(7a+3b)\tanh(c+dx)\sqrt{a+b-b\tanh^2(c+dx)}}{8d} + \frac{b\tanh(c+dx)\left(a+b-b\tanh^2(c-\right.}{4d}$$

$$= \frac{b(7a+3b)\tanh(c+dx)\sqrt{a+b-b\tanh^2(c+dx)}}{8d} + \frac{b\tanh(c+dx)\left(a+b-b\tanh^2(c-\right.}{4d}$$

$$= \frac{b(7a+3b)\tanh(c+dx)\sqrt{a+b-b\tanh^2(c+dx)}}{8d} + \frac{b\tanh(c+dx)\left(a+b-b\tanh^2(c-\right.}{4d}$$

$$= \frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b-b\tanh^2(c+dx)}}\right)}{8d} + \frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a+b-b\tanh^2(c+dx)}}\right)}{d} + \cdots$$

Mathematica [A] time = 9.0174, size = 280, normalized size = 1.65

$$\frac{\cosh^5(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^{5/2}\left(\sqrt{b}\left(15a^2+10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a\sinh^2(c+dx)+a+b}}\right)+8a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a\sinh^2(c+dx)}}\right.\right.}{\sqrt{2}d(a\cosh(2c+2dx)+a+2b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^(5/2),x]
```

```
[Out] ((Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a +
 b + a*Sinh[c + d*x]^2]] + 8*a^(5/2)*ArcTanh[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a
 + b + a*Sinh[c + d*x]^2]])*Cosh[c + d*x]^5*(a + b*Sech[c + d*x]^2)^(5/2))/
(Sqrt[2]*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^(5/2)) + (Cosh[c + d*x]^5*(a + b
*Sech[c + d*x]^2)^(5/2)*((b^2*Sech[c]*Sech[c + d*x]^4*Sinh[d*x])/d + (3*Sec
h[c]*Sech[c + d*x]^2*(3*a*b*Sinh[d*x] + b^2*Sinh[d*x]))/(2*d) + (3*b*(3*a +
 b)*Sech[c + d*x]*Tanh[c])/(2*d) + (b^2*Sech[c + d*x]^3*Tanh[c])/d))/(a + 2
*b + a*Cosh[2*c + 2*d*x])^2
```

Maple [F] time = 0.193, size = 0, normalized size = 0.

$$\int \left(a+b\left(\operatorname{sech}\left(dx+c\right)\right)^{2}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^(5/2),x)
```

```
[Out] int((a+b*sech(d*x+c)^2)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\operatorname{sech}\left(dx+c\right)^{2}+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(d*x + c)^2 + a)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\,\mathrm{sech}\,(dx+c)^2+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*x + c)^2 + a)^(5/2), x)
```

3.194 $\int \frac{\tanh^5(x)}{\sqrt{a+b\,\mathrm{sech}^2(x)}}\,dx$

Optimal. Leaf size=66

$$-\frac{\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}{3b^2}+\frac{(a+2b)\sqrt{a+b\,\mathrm{sech}^2(x)}}{b^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/Sqrt[a] + ((a + 2*b)*Sqrt[a + b*Sech
[x]^2])/b^2 - (a + b*Sech[x]^2)^(3/2)/(3*b^2)
```

Rubi [A] time = 0.13142, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {4139, 446, 88, 63, 208}

$$-\frac{\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}{3b^2}+\frac{(a+2b)\sqrt{a+b\,\mathrm{sech}^2(x)}}{b^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^5/Sqrt[a + b*Sech[x]^2],x]
```


```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/Sqrt[a] + ((a + 2*b)*Sqrt[a + b*Sech
[x]^2])/b^2 - (a + b*Sech[x]^2)^(3/2)/(3*b^2)
```


Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^5(x)}{\sqrt{a+b\operatorname{sech}^2(x)}}\,dx &= -\operatorname{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x\sqrt{a+bx^2}}\,dx,x,\operatorname{sech}(x)\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{(-1+x)^2}{x\sqrt{a+bx}}\,dx,x,\operatorname{sech}^2(x)\right)\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{-a-2b}{b\sqrt{a+bx}}+\frac{1}{x\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b}\right)dx,x,\operatorname{sech}^2(x)\right)\right)\\
&= \frac{(a+2b)\sqrt{a+b\operatorname{sech}^2(x)}}{b^2}-\frac{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}{3b^2}-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,\operatorname{sech}^2(x)\right)\\
&= \frac{(a+2b)\sqrt{a+b\operatorname{sech}^2(x)}}{b^2}-\frac{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}{3b^2}-\frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\operatorname{sech}^2(x)}\right)}{b}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{(a+2b)\sqrt{a+b\operatorname{sech}^2(x)}}{b^2}-\frac{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}{3b^2}
\end{aligned}
$$

Mathematica [A] time = 0.491037, size = 109, normalized size = 1.65

$$
\frac{\operatorname{sech}(x)\left(\frac{\operatorname{sech}(x)(a\cosh(2x)+a+2b)\left(2a-b\operatorname{sech}^2(x)+6b\right)}{3b^2}+\frac{\sqrt{2}\sqrt{a\cosh(2x)+a+2b}\log\left(\sqrt{a\cosh(2x)+a+2b}+\sqrt{2}\sqrt{a}\cosh(x)\right)}{\sqrt{a}}\right)}{2\sqrt{a+b\operatorname{sech}^2(x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] (Sech[x]*((Sqrt[2]*Sqrt[a + 2*b + a*Cosh[2*x]]*Log[Sqrt[2]*Sqrt[a]*Cosh[x]
+ Sqrt[a + 2*b + a*Cosh[2*x]]])/Sqrt[a] + ((a + 2*b + a*Cosh[2*x])*Sech[x]*
(2*a + 6*b - b*Sech[x]^2))/(3*b^2)))/(2*Sqrt[a + b*Sech[x]^2])
```

Maple [F] time = 0.125, size = 0, normalized size = 0.

$$
\int (\tanh(x))^5 \frac{1}{\sqrt{a+b(\operatorname{sech}(x))^2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(tanh(x)^5/(a+b*sech(x)^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^5}{\sqrt{b\,\mathrm{sech}(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^5/sqrt(b*sech(x)^2 + a), x)
```

Fricas [B] time = 3.49577, size = 7483, normalized size = 113.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 + 3*b^2*c
osh(x)^4 + 3*(5*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 3*b^2*cosh(x)^2 + 4*(5*b^2
*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 3*(5*b^2*cosh(x)^4 + 6*b^2*cosh(x)^
2 + b^2)*sinh(x)^2 + b^2 + 6*(b^2*cosh(x)^5 + 2*b^2*cosh(x)^3 + b^2*cosh(x)
)*sinh(x))*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*
b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3
 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3
+ 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b +
a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 +
 (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(
x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*co
sh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 +
5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*
sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*
b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3
*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^
2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2
 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2
*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b +
b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)
*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cos
h(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^
5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqr
t(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh
(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a
^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 +
(2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*c
osh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*co
sh(x)*sinh(x)^5 + sinh(x)^6)) + 3*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5
+ b^2*sinh(x)^6 + 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 3
*b^2*cosh(x)^2 + 4*(5*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 3*(5*b^2*c
osh(x)^4 + 6*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 6*(b^2*cosh(x)^5 + 2*b^
2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)
```

```
*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2
+ sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*c
osh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(
x) + sinh(x)^2)) + 8*sqrt(2)*((a^2 + 3*a*b)*cosh(x)^4 + 4*(a^2 + 3*a*b)*cos
h(x)*sinh(x)^3 + (a^2 + 3*a*b)*sinh(x)^4 + 2*(a^2 + 2*a*b)*cosh(x)^2 + 2*(3
*(a^2 + 3*a*b)*cosh(x)^2 + a^2 + 2*a*b)*sinh(x)^2 + a^2 + 3*a*b + 4*((a^2 +
 3*a*b)*cosh(x)^3 + (a^2 + 2*a*b)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*s
inh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*b^2*co
sh(x)^6 + 6*a*b^2*cosh(x)*sinh(x)^5 + a*b^2*sinh(x)^6 + 3*a*b^2*cosh(x)^4 +
 3*a*b^2*cosh(x)^2 + 3*(5*a*b^2*cosh(x)^2 + a*b^2)*sinh(x)^4 + 4*(5*a*b^2*c
osh(x)^3 + 3*a*b^2*cosh(x))*sinh(x)^3 + a*b^2 + 3*(5*a*b^2*cosh(x)^4 + 6*a*
b^2*cosh(x)^2 + a*b^2)*sinh(x)^2 + 6*(a*b^2*cosh(x)^5 + 2*a*b^2*cosh(x)^3 +
 a*b^2*cosh(x))*sinh(x)), -1/6*(3*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5
+ b^2*sinh(x)^6 + 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 3
*b^2*cosh(x)^2 + 4*(5*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 3*(5*b^2*c
osh(x)^4 + 6*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 6*(b^2*cosh(x)^5 + 2*b^
2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*((a + b)*cosh(x
)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*c
osh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*
sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 +
3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(
x))*sinh(x))) + 3*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6
+ 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 3*b^2*cosh(x)^2 +
 4*(5*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 3*(5*b^2*cosh(x)^4 + 6*b^2
*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 6*(b^2*cosh(x)^5 + 2*b^2*cosh(x)^3 + b^
2*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x)
+ sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(
x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3
 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x
)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - 4*sqrt(2)*((a^2 +
 3*a*b)*cosh(x)^4 + 4*(a^2 + 3*a*b)*cosh(x)*sinh(x)^3 + (a^2 + 3*a*b)*sinh(
x)^4 + 2*(a^2 + 2*a*b)*cosh(x)^2 + 2*(3*(a^2 + 3*a*b)*cosh(x)^2 + a^2 + 2*a
*b)*sinh(x)^2 + a^2 + 3*a*b + 4*((a^2 + 3*a*b)*cosh(x)^3 + (a^2 + 2*a*b)*co
sh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)))/(a*b^2*cosh(x)^6 + 6*a*b^2*cosh(x)*sinh(x)^5
+ a*b^2*sinh(x)^6 + 3*a*b^2*cosh(x)^4 + 3*a*b^2*cosh(x)^2 + 3*(5*a*b^2*cosh
(x)^2 + a*b^2)*sinh(x)^4 + 4*(5*a*b^2*cosh(x)^3 + 3*a*b^2*cosh(x))*sinh(x)^
3 + a*b^2 + 3*(5*a*b^2*cosh(x)^4 + 6*a*b^2*cosh(x)^2 + a*b^2)*sinh(x)^2 + 6
*(a*b^2*cosh(x)^5 + 2*a*b^2*cosh(x)^3 + a*b^2*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\,\mathrm{sech}^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(tanh(x)**5/sqrt(a + b*sech(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^5}{\sqrt{b\operatorname{sech}(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(x)^5/sqrt(b*sech(x)^2 + a), x)
```

3.195 $\int \frac{\tanh^4(x)}{\sqrt{a+b\mathbf{sech}^2(x)}}\,dx$

Optimal. Leaf size=90

$$-\frac{(a+3b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{2b^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}}+\frac{\tanh(x)\sqrt{a-b\tanh^2(x)+b}}{2b}$$

```
[Out] -((a + 3*b)*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]])/(2*b^(3/2)
) + ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/Sqrt[a] + (Tanh[x]
*Sqrt[a + b - b*Tanh[x]^2])/(2*b)
```

Rubi [A] time = 0.228319, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {4141, 1975, 479, 523, 217, 203, 377, 206}

$$-\frac{(a+3b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{2b^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}}+\frac{\tanh(x)\sqrt{a-b\tanh^2(x)+b}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] -((a + 3*b)*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]])/(2*b^(3/2)
) + ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/Sqrt[a] + (Tanh[x]
*Sqrt[a + b - b*Tanh[x]^2])/(2*b)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)\sqrt{a+b\left(1-x^2\right)}}\,dx, x, \tanh(x)\right)$$

$$= \text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)$$

$$= \frac{\tanh(x)\sqrt{a+b-b\tanh^2(x)}}{2b} - \frac{\text{Subst}\left(\int \frac{a+b+(-a-3b)x^2}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)}{2b}$$

$$= \frac{\tanh(x)\sqrt{a+b-b\tanh^2(x)}}{2b} - \frac{(a+3b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)}{2b} + \text{Subst}\left(\int \frac{}{(1-}\right.$$

$$= \frac{\tanh(x)\sqrt{a+b-b\tanh^2(x)}}{2b} - \frac{(a+3b)\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{2b} + \text{Subst}\left(\int \frac{}{1}\right.$$

$$= -\frac{(a+3b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{2b^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{\sqrt{a}} + \frac{\tanh(x)\sqrt{a+b-b\tanh^2(x)}}{2b}$$

Mathematica [A] time = 0.494854, size = 169, normalized size = 1.88

$$\frac{\text{sech}(x)\left(2\sqrt{2}b^{3/2}\sqrt{a\cosh(2x)+a+2b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)+\sqrt{a}\left(\sqrt{b}\tanh(x)\text{sech}(x)(a\cosh(2x)+a+2b)-\sqrt{}\right.\right.}{4\sqrt{a}b^{3/2}\sqrt{a+b\text{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] (Sech[x]*(2*Sqrt[2]*b^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b
+ a*Cosh[2*x]]]*Sqrt[a + 2*b + a*Cosh[2*x]] + Sqrt[a]*(-(Sqrt[2]*(a + 3*b)*
ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]]*Sqrt[a + 2*b
+ a*Cosh[2*x]]) + Sqrt[b]*(a + 2*b + a*Cosh[2*x])*Sech[x]*Tanh[x])))/(4*Sqr
t[a]*b^(3/2)*Sqrt[a + b*Sech[x]^2])
```

Maple [F] time = 0.125, size = 0, normalized size = 0.

$$\int (\tanh (x))^{4} \frac{1}{\sqrt{a + b (\mathrm{sech} (x))^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(tanh(x)^4/(a+b*sech(x)^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh (x)^{4}}{\sqrt{b\, \mathrm{sech} (x)^{2} + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^4/sqrt(b*sech(x)^2 + a), x)
```

Fricas [B] time = 3.84137, size = 13245, normalized size = 147.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*b^2*cosh
(x)^2 + 2*(3*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + b^2*
cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7
+ a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b
^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh
(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4
*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh
(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sin
h(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*
b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^
```

```
2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(
x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cos
h(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(
x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5
 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x
)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) +
 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^
2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*s
inh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sin
h(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) - ((a^2 + 3*a*b)*cosh(x)^4 + 4*(
a^2 + 3*a*b)*cosh(x)*sinh(x)^3 + (a^2 + 3*a*b)*sinh(x)^4 + 2*(a^2 + 3*a*b)*
cosh(x)^2 + 2*(3*(a^2 + 3*a*b)*cosh(x)^2 + a^2 + 3*a*b)*sinh(x)^2 + a^2 + 3
*a*b + 4*((a^2 + 3*a*b)*cosh(x)^3 + (a^2 + 3*a*b)*cosh(x))*sinh(x))*sqrt(-b
)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4
 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 - 2*
sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a*co
sh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2
)) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)^4
+ 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(
x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + (b^2*cosh(x)^4 + 4*b^2*cosh(
x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*b^2*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + b^2)*
sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a)*log(-(a*
cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(
3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*si
nh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 2*sqrt(2)*(a*b*co
sh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2 - a*b)*sqrt((a*cosh(x)^2 +
a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*b^2
*cosh(x)^4 + 4*a*b^2*cosh(x)*sinh(x)^3 + a*b^2*sinh(x)^4 + 2*a*b^2*cosh(x)^
2 + a*b^2 + 2*(3*a*b^2*cosh(x)^2 + a*b^2)*sinh(x)^2 + 4*(a*b^2*cosh(x)^3 +
a*b^2*cosh(x))*sinh(x)), -1/4*(2*((a^2 + 3*a*b)*cosh(x)^4 + 4*(a^2 + 3*a*b)
*cosh(x)*sinh(x)^3 + (a^2 + 3*a*b)*sinh(x)^4 + 2*(a^2 + 3*a*b)*cosh(x)^2 +
2*(3*(a^2 + 3*a*b)*cosh(x)^2 + a^2 + 3*a*b)*sinh(x)^2 + a^2 + 3*a*b + 4*((a
^2 + 3*a*b)*cosh(x)^3 + (a^2 + 3*a*b)*cosh(x))*sinh(x))*sqrt(b)*arctan(sqrt
(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)
^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a
*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 +
2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))
*sinh(x) + a)) - (b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 +
 2*b^2*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(
x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)
*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh
(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*co
sh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^
4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14
*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*c
osh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)
^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^
2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5
+ b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4
*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (1
5*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^
2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqr
t((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + si
nh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2
*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*
cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*co
sh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) - (b^2*cosh(x)^4 + 4*
```

```
b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*b^2*cosh(x)^2 + 2*(3*b^2*cosh(x)^
2 + b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a)
*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x
)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*
sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)
/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*co
sh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 2*sqrt(2
)*(a*b*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2 - a*b)*sqrt((a*cos
h(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)
))/(a*b^2*cosh(x)^4 + 4*a*b^2*cosh(x)*sinh(x)^3 + a*b^2*sinh(x)^4 + 2*a*b^2
*cosh(x)^2 + a*b^2 + 2*(3*a*b^2*cosh(x)^2 + a*b^2)*sinh(x)^2 + 4*(a*b^2*cos
h(x)^3 + a*b^2*cosh(x))*sinh(x)), -1/4*(2*(b^2*cosh(x)^4 + 4*b^2*cosh(x)*si
nh(x)^3 + b^2*sinh(x)^4 + 2*b^2*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + b^2)*sinh(
x)^2 + b^2 + 4*(b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(
2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*c
osh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b
)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*co
sh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) + 2*(b^2*cosh(x)^4 + 4*b^2*cosh(
x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*b^2*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + b^2)*
sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(-a)*arctan(
sqrt(2)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*
cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x
)^2 + a)) + ((a^2 + 3*a*b)*cosh(x)^4 + 4*(a^2 + 3*a*b)*cosh(x)*sinh(x)^3 +
(a^2 + 3*a*b)*sinh(x)^4 + 2*(a^2 + 3*a*b)*cosh(x)^2 + 2*(3*(a^2 + 3*a*b)*co
sh(x)^2 + a^2 + 3*a*b)*sinh(x)^2 + a^2 + 3*a*b + 4*((a^2 + 3*a*b)*cosh(x)^3
 + (a^2 + 3*a*b)*cosh(x))*sinh(x))*sqrt(-b)*log(-((a - b)*cosh(x)^4 + 4*(a
- b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(
a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*si
nh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a - b)*cosh(x)^3 + (a +
3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4
 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*si
nh(x) + 1)) - 2*sqrt(2)*(a*b*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x
)^2 - a*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x
)*sinh(x) + sinh(x)^2)))/(a*b^2*cosh(x)^4 + 4*a*b^2*cosh(x)*sinh(x)^3 + a*b
^2*sinh(x)^4 + 2*a*b^2*cosh(x)^2 + a*b^2 + 2*(3*a*b^2*cosh(x)^2 + a*b^2)*si
nh(x)^2 + 4*(a*b^2*cosh(x)^3 + a*b^2*cosh(x))*sinh(x)), -1/2*((b^2*cosh(x)^
4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*b^2*cosh(x)^2 + 2*(3*b^2*co
sh(x)^2 + b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*s
qrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a
)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x
)*sinh(x) + sinh(x)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh
(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2
 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) + ((a^2 + 3*
a*b)*cosh(x)^4 + 4*(a^2 + 3*a*b)*cosh(x)*sinh(x)^3 + (a^2 + 3*a*b)*sinh(x)^
4 + 2*(a^2 + 3*a*b)*cosh(x)^2 + 2*(3*(a^2 + 3*a*b)*cosh(x)^2 + a^2 + 3*a*b)
*sinh(x)^2 + a^2 + 3*a*b + 4*((a^2 + 3*a*b)*cosh(x)^3 + (a^2 + 3*a*b)*cosh(
x))*sinh(x))*sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x
)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*
cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh
(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(
a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + (b^2*cosh(x)^4 + 4*b^2*cos
h(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*b^2*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + b^2
)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(-a)*arcta
n(sqrt(2)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh
(x)^2 + a)) - sqrt(2)*(a*b*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^
2 - a*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*
```

```
sinh(x) + sinh(x)^2)))/(a*b^2*cosh(x)^4 + 4*a*b^2*cosh(x)*sinh(x)^3 + a*b^2
*sinh(x)^4 + 2*a*b^2*cosh(x)^2 + a*b^2 + 2*(3*a*b^2*cosh(x)^2 + a*b^2)*sinh
(x)^2 + 4*(a*b^2*cosh(x)^3 + a*b^2*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\operatorname{sech}^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(tanh(x)**4/sqrt(a + b*sech(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^4}{\sqrt{b\operatorname{sech}(x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(x)^4/sqrt(b*sech(x)^2 + a), x)
```

3.196 $\int \frac{\tanh^3(x)}{\sqrt{a+b\,\mathbf{sech}^2(x)}}\,dx$

Optimal. Leaf size=42

$$\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/Sqrt[a] + Sqrt[a + b*Sech[x]^2]/b
```

Rubi [A] time = 0.0929296, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {4139, 446, 80, 63, 208}

$$\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/Sqrt[a] + Sqrt[a + b*Sech[x]^2]/b
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^3(x)}{\sqrt{a+b\operatorname{sech}^2(x)}}\,dx &= \text{Subst}\left(\int \frac{-1+x^2}{x\sqrt{a+bx^2}}\,dx, x, \operatorname{sech}(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{-1+x}{x\sqrt{a+bx}}\,dx, x, \operatorname{sech}^2(x)\right)\\
&= \frac{\sqrt{a+b\operatorname{sech}^2(x)}}{b} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \operatorname{sech}^2(x)\right)\\
&= \frac{\sqrt{a+b\operatorname{sech}^2(x)}}{b} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{sech}^2(x)}\right)}{b}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{\sqrt{a+b\operatorname{sech}^2(x)}}{b}
\end{aligned}$$

Mathematica [B] time = 0.211837, size = 105, normalized size = 2.5

$$\frac{\operatorname{sech}^2(x)(a\cosh(2x)+a+2b)}{2b\sqrt{a+b\operatorname{sech}^2(x)}} + \frac{\operatorname{sech}(x)\sqrt{a\cosh(2x)+a+2b}\log\left(\sqrt{a\cosh(2x)+a+2b}+\sqrt{2}\sqrt{a}\cosh(x)\right)}{\sqrt{2}\sqrt{a}\sqrt{a+b\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] (Sqrt[a + 2*b + a*Cosh[2*x]]*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a
*Cosh[2*x]]]*Sech[x])/(Sqrt[2]*Sqrt[a]*Sqrt[a + b*Sech[x]^2]) + ((a + 2*b +
 a*Cosh[2*x])*Sech[x]^2)/(2*b*Sqrt[a + b*Sech[x]^2])
```

Maple [F] time = 0.121, size = 0, normalized size = 0.

$$\int (\tanh(x))^3 \frac{1}{\sqrt{a+b(\operatorname{sech}(x))^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(tanh(x)^3/(a+b*sech(x)^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^3}{\sqrt{b\,\mathrm{sech}(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^3/sqrt(b*sech(x)^2 + a), x)
```

Fricas [B] time = 2.61953, size = 4733, normalized size = 112.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*sqrt(a)*log(((a
^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)
^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3
)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^
2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^
3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b
^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b +
9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14
*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*c
osh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3
 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3
 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2
*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6
 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6
+ 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^
2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 +
2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*
a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*si
nh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*
cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a
*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(
a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh
(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))
*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*co
sh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^
6)) + (b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*sqrt(a)*log(-(a
*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*c
osh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*co
sh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cos
h(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*sqrt(2)*a*sqrt((a*cosh(x)^2 +
a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*b*c
osh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2 + a*b), -1/2*((b*cosh(x)^2
 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*sqrt(-a)*arctan(sqrt(2)*((a + b)*
cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqr
t((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + si
nh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 +
```

```
 a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*
a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)
*cosh(x))*sinh(x))) + (b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)
*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sq
rt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*si
nh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*
(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^
3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - 2*sqrt(2)*a*sqrt((a*cosh(x)^2 + a*si
nh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*b*cosh(
x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2 + a*b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^3(x)}{\sqrt{a + b\,\mathrm{sech}^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(tanh(x)**3/sqrt(a + b*sech(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^3}{\sqrt{b\,\mathrm{sech}(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(x)^3/sqrt(b*sech(x)^2 + a), x)
```

3.197 $\int \frac{\tanh^2(x)}{\sqrt{a+b\,\mathrm{sech}^2(x)}}\,dx$

Optimal. Leaf size=60

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{b}}$$

```
[Out] -(ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/Sqrt[b]) + ArcTanh[(S
qrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/Sqrt[a]
```

Rubi [A] time = 0.192646, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {4141, 1975, 483, 217, 203, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] -(ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/Sqrt[b]) + ArcTanh[(S
qrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/Sqrt[a]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^2(x)}{\sqrt{a+b\operatorname{sech}^2(x)}}\,dx &= \text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)\sqrt{a+b\left(1-x^2\right)}}\,dx,x,\tanh(x)\right)\\
&= \text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx,x,\tanh(x)\right)\\
&= -\text{Subst}\left(\int \frac{1}{\sqrt{a+b-bx^2}}\,dx,x,\tanh(x)\right)+\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx,x,\tanh(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)-\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.168394, size = 107, normalized size = 1.78

$$\frac{\operatorname{sech}(x)\sqrt{a\cosh(2x)+a+2b}\left(\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)}{\sqrt{a}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sinh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)}{\sqrt{b}}\right)}{\sqrt{2}\sqrt{a+b\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] ((-(ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]]/Sqrt[b])
+ ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]]/Sqrt[a])*S
qrt[a + 2*b + a*Cosh[2*x]]*Sech[x])/(Sqrt[2]*Sqrt[a + b*Sech[x]^2])
```

Maple [F] time = 0.115, size = 0, normalized size = 0.

$$\int (\tanh(x))^2 \frac{1}{\sqrt{a + b(\operatorname{sech}(x))^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(tanh(x)^2/(a+b*sech(x)^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^2}{\sqrt{b\operatorname{sech}(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^2/sqrt(b*sech(x)^2 + a), x)
```

Fricas [B] time = 3.04135, size = 8377, normalized size = 139.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*b*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*si
nh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*
sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a
^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9
*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10
*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a
^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)
*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)
^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b
^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3
*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*
b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*c
osh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*si
nh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^
2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)
^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 1
5*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6
*cosh(x)*sinh(x)^5 + sinh(x)^6)) - 2*a*sqrt(-b)*log(-((a - b)*cosh(x)^4 + 4
*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*
(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x
)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2
*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a - b)*cosh(x)^3 + (
a + 3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(
```

```
x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x)
)*sinh(x) + 1)) + sqrt(a)*b*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*s
inh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt
(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)
^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) +
4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(
x) + sinh(x)^2)))/(a*b), -1/4*(4*a*sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 + 2*co
sh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a
+ 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh
(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a
+ 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - sqrt
(a)*b*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 -
2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6
+ 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2
*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 3
0*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 -
b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^
3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4
 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(
2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)
^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(
x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x
)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 -
 (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^
7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3
+ 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^
4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*s
inh(x)^5 + sinh(x)^6)) - sqrt(a)*b*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^
3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2
 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*
cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x
)*sinh(x) + sinh(x)^2)))/(a*b), -1/2*(sqrt(-a)*b*arctan(sqrt(2)*(b*cosh(x)^
2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*s
inh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(
x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 +
(6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2
 + 3*a*b)*cosh(x))*sinh(x))) + sqrt(-a)*b*arctan(sqrt(2)*sqrt(-a)*sqrt((a*c
osh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)) + a*sqrt(-b)*log
(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2*
(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 - 2*sqrt(
2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a*cosh(x)
^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) +
4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)^4 + 4*c
osh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2
+ 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)))/(a*b), -1/2*(sqrt(-a)*b*arctan(sqr
t(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a
*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x
)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a
*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*
cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) + 2*a*sqrt(b)*arctan(sqrt(2)*(
cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 +
a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh
(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*
a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh
(x) + a)) + sqrt(-a)*b*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x
)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^2 +
```

```
2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)))/(a*b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2(x)}{\sqrt{a + b\,\mathrm{sech}^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(tanh(x)**2/sqrt(a + b*sech(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^2}{\sqrt{b\,\mathrm{sech}(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(x)^2/sqrt(b*sech(x)^2 + a), x)
```

3.198 $\int \frac{\tanh(x)}{\sqrt{a+b\mathbf{sech}^2(x)}}\,dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/Sqrt[a]
```

Rubi [A] time = 0.0532532, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4139, 266, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/Sqrt[a + b*Sech[x]^2],x]

[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/Sqrt[a]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tanh(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx = -\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx, x, \text{sech}(x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \text{sech}^2(x)\right)\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{sech}^2(x)}\right)}{b}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [B] time = 0.0809718, size = 70, normalized size = 2.8

$$\frac{\text{sech}(x)\sqrt{a\cosh(2x)+a+2b}\log\left(\sqrt{a\cosh(2x)+a+2b}+\sqrt{2}\sqrt{a}\cosh(x)\right)}{\sqrt{2}\sqrt{a}\sqrt{a+b\text{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] (Sqrt[a + 2*b + a*Cosh[2*x]]*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a
*Cosh[2*x]]]*Sech[x])/(Sqrt[2]*Sqrt[a]*Sqrt[a + b*Sech[x]^2])
```

Maple [A] time = 0.035, size = 30, normalized size = 1.2

$$\ln\left(\frac{1}{\text{sech}(x)}\left(2a+2\sqrt{a}\sqrt{a+b(\text{sech}(x))^2}\right)\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] 1/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sech(x)^2)^(1/2))/sech(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{\sqrt{b\,\text{sech}(x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)/sqrt(b*sech(x)^2 + a), x)
```

Fricas [B] time = 2.17882, size = 4004, normalized size = 160.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a
*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*
a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*
(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2
)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a
^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4
+ 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)
^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2
*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(
x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*
cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*
b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2
*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*
a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b
+ b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*
cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh
(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^
2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6
*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*
sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
 sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b
+ 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^
3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x
)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)
*sinh(x)^5 + sinh(x)^6)) + sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^
3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)
*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2
+ a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(
a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh
(x)^2)))/a, -1/2*(sqrt(-a)*arctan(sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*co
sh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(
x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*c
osh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2
 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 +
 a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(x))*sinh(x))) + sq
rt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(
-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(
x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a
+ 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 +
 (a + 2*b)*cosh(x))*sinh(x) + a)))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{\sqrt{a + b\operatorname{sech}^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(tanh(x)/sqrt(a + b*sech(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{\sqrt{b\,\mathrm{sech}(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(x)/sqrt(b*sech(x)^2 + a), x)
```

3.199 $\int \frac{1}{\sqrt{a+b\mathbf{sech}^2(x)}}\,dx$

Optimal. Leaf size=29

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}}$$

```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/Sqrt[a]
```

Rubi [A] time = 0.0254476, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4128, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/Sqrt[a]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)$$
$$= \text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{\sqrt{a}}$$

Mathematica [B] time = 0.0401439, size = 62, normalized size = 2.14

$$\frac{\text{sech}(x)\sqrt{a\cosh(2x)+a+2b}\tanh^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a\sinh^2(x)+a+b}}\right)}{\sqrt{2}\sqrt{a}\sqrt{a+b\text{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] (ArcTanh[(Sqrt[a]*Sinh[x])/Sqrt[a + b + a*Sinh[x]^2]]*Sqrt[a + 2*b + a*Cosh
[2*x]]*Sech[x])/(Sqrt[2]*Sqrt[a]*Sqrt[a + b*Sech[x]^2])
```

Maple [F] time = 0.149, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b(\text{sech}(x))^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(1/(a+b*sech(x)^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\,\text{sech}(x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sech(x)^2 + a), x)
```

Fricas [B] time = 2.16516, size = 3023, normalized size = 104.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh
(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*si
nh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3
 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a
*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(
a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3
 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*c
osh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2
 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2
*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b
^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^
2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cos
h(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh
(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*
cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3
 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*
cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*c
osh(x)*sinh(x)^5 + sinh(x)^6)) + sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*si
nh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sin
h(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sq
rt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + s
inh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*
cosh(x)*sinh(x) + sinh(x)^2)))/a, -1/2*(sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^
2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*s
inh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(
x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 +
(6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2
 + 3*a*b)*cosh(x))*sinh(x))) + sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos
h(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)
)/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b\,\mathrm{sech}^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sech(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\,\mathrm{sech}(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sech(x)^2 + a), x)
```

3.200 $\int \frac{\coth(x)}{\sqrt{a+b\,\mathrm{sech}^2(x)}}\,dx$

Optimal. Leaf size=56

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/Sqrt[a] - ArcTanh[Sqrt[a + b*Sech[x]
^2]/Sqrt[a + b]]/Sqrt[a + b]
```

Rubi [A] time = 0.102984, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4139, 446, 86, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/Sqrt[a] - ArcTanh[Sqrt[a + b*Sech[x]
^2]/Sqrt[a + b]]/Sqrt[a + b]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth(x)}{\sqrt{a+b\operatorname{sech}^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{x\left(-1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \operatorname{sech}(x)\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(-1+x)x\sqrt{a+bx}}\,dx, x, \operatorname{sech}^2(x)\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+bx}}\,dx, x, \operatorname{sech}^2(x)\right) - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \operatorname{sech}^2(x)\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{-1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{sech}^2(x)}\right)}{b} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{sech}^2(x)}\right)}{b}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}
\end{aligned}$$

Mathematica [B] time = 0.258608, size = 124, normalized size = 2.21

$$\frac{\operatorname{sech}(x)\sqrt{a\cosh(2x)+a+2b}\left(\sqrt{a+b}\log\left(\sqrt{a\cosh(2x)+a+2b}+\sqrt{2}\sqrt{a}\cosh(x)\right)-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a+b}\cosh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)\right)}{\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{a+b\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] (Sqrt[a + 2*b + a*Cosh[2*x]]*(-(Sqrt[a]*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Cosh[x
])/Sqrt[a + 2*b + a*Cosh[2*x]]]) + Sqrt[a + b]*Log[Sqrt[2]*Sqrt[a]*Cosh[x]
+ Sqrt[a + 2*b + a*Cosh[2*x]]])*Sech[x])/(Sqrt[2]*Sqrt[a]*Sqrt[a + b]*Sqrt[
a + b*Sech[x]^2])
```

Maple [F] time = 0.12, size = 0, normalized size = 0.

$$\int \coth(x)\frac{1}{\sqrt{a+b\left(\operatorname{sech}(x)\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)/(a+b*sech(x)^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\sqrt{b\,\mathrm{sech}(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)/sqrt(b*sech(x)^2 + a), x)
```

Fricas [B] time = 3.02827, size = 10525, normalized size = 187.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a + b)*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a
^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*
a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b
^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b
 + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^
5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*co
sh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)
*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3
 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x
))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b +
 a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3
+ 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(
(a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (
a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2
+ 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b
 + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a
*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*
cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(
x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*
sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*s
inh(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 +
5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3
 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 1
5*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6
*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 2*sqrt(a + b)*a*log(((2*a + b)*cosh(x)^4
 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a + 3*b)*cosh
(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a + 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x
)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt((a*cosh(x)^2 + a*
sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a
 + b)*cosh(x)^3 + (2*a + 3*b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*co
sh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 +
 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)) + (a + b)*sqrt(a)*log(-(a*cosh(x)^4
+ 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 +
b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt
(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(
x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*
```

```
cosh(x)*sinh(x) + sinh(x)^2)))/(a^2 + a*b), 1/4*(4*a*sqrt(-a - b)*arctan(sq
rt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt((a*
cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(
x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*c
osh(x))*sinh(x) + a)) + (a + b)*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x
)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)
*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a
^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*
(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)
*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2
*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*
b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cos
h(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b
 + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(1
4*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*
cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sin
h(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cos
h(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh
(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 +
 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x
)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a
^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 +
2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*
cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(
x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(
x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b +
9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cos
h(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(
x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + (a + b)*sqrt(a)*log(-(
a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*
cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^2 + a*b), -1/2*(sqrt(-a)*(a +
 b)*arctan(sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)
*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x
)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b
)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (
6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)
*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(x))*sinh(x))) + sqrt(-a)*(a + b)*arctan(s
qrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cos
h(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)
)/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^
2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh
(x))*sinh(x) + a)) - sqrt(a + b)*a*log(((2*a + b)*cosh(x)^4 + 4*(2*a + b)*c
osh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a + 3*b)*cosh(x)^2 + 2*(3*(2*
a + b)*cosh(x)^2 + 2*a + 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*
sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a +
2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3
+ (2*a + 3*b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3
+ sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 -
cosh(x))*sinh(x) + 1)))/(a^2 + a*b), -1/2*(sqrt(-a)*(a + b)*arctan(sqrt(2)*
((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqr
t(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sin
h(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3
 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(
x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2
 + 3*a*b)*cosh(x))*sinh(x))) + sqrt(-a)*(a + b)*arctan(sqrt(2)*(cosh(x)^2 +
```

```
 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^
2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*
a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^
2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a))
- 2*a*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2 + 1)*sqrt(-a - b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*s
inh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 +
4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)))/(a^2 + a*b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\sqrt{a+b\operatorname{sech}^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(coth(x)/sqrt(a + b*sech(x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.201 $\int \frac{\coth^2(x)}{\sqrt{a+b\,\mathbf{sech}^2(x)}}\,dx$

Optimal. Leaf size=53

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}}-\frac{\coth(x)\sqrt{a-b\tanh^2(x)+b}}{a+b}$$

```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/Sqrt[a] - (Coth[x]*Sqr
t[a + b - b*Tanh[x]^2])/(a + b)
```

Rubi [A] time = 0.188142, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {4141, 1975, 480, 12, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}}-\frac{\coth(x)\sqrt{a-b\tanh^2(x)+b}}{a+b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/Sqrt[a] - (Coth[x]*Sqr
t[a + b - b*Tanh[x]^2])/(a + b)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^2(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{x^2\left(1-x^2\right)\sqrt{a+b\left(1-x^2\right)}}\,dx, x, \tanh(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{x^2\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)\\
&= -\frac{\coth(x)\sqrt{a+b-b\tanh^2(x)}}{a+b}+\frac{\text{Subst}\left(\int \frac{a+b}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)}{a+b}\\
&= -\frac{\coth(x)\sqrt{a+b-b\tanh^2(x)}}{a+b}+\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)\\
&= -\frac{\coth(x)\sqrt{a+b-b\tanh^2(x)}}{a+b}+\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{\sqrt{a}}-\frac{\coth(x)\sqrt{a+b-b\tanh^2(x)}}{a+b}
\end{aligned}$$

Mathematica [A] time = 0.113547, size = 94, normalized size = 1.77

$$\frac{\text{sech}(x)\sqrt{a\cosh(2x)+a+2b}\left((a+b)\tanh^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a\sinh^2(x)+a+b}}\right)-\sqrt{a}\text{csch}(x)\sqrt{a\sinh^2(x)+a+b}\right)}{\sqrt{2}\sqrt{a}(a+b)\sqrt{a+b\text{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] (Sqrt[a + 2*b + a*Cosh[2*x]]*Sech[x]*((a + b)*ArcTanh[(Sqrt[a]*Sinh[x])/Sqr
t[a + b + a*Sinh[x]^2]] - Sqrt[a]*Csch[x]*Sqrt[a + b + a*Sinh[x]^2]))/(Sqrt
[2]*Sqrt[a]*(a + b)*Sqrt[a + b*Sech[x]^2])
```

Maple [F] time = 0.124, size = 0, normalized size = 0.

$$\int (\coth (x))^{2} \frac{1}{\sqrt{a+b (\operatorname{sech} (x))^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^2/(a+b*sech(x)^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth (x)^{2}}{\sqrt{b \operatorname{sech} (x)^{2} + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)^2/sqrt(b*sech(x)^2 + a), x)
```

Fricas [B] time = 2.48388, size = 3951, normalized size = 74.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 -
a - b)*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sin
h(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*s
inh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^
3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*
a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*
(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^
3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*
cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^
2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^
2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*
b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b
^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*co
sh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sin
h(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2
*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^
3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15
*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*
cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*si
nh(x) + (a + b)*sinh(x)^2 - a - b)*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*
sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*s
inh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*
sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
```

```
 sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 +
2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*a*sqrt((a*cosh(x)^2 + a*sinh(x)
^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^2 + a*b)*co
sh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a^2 + a*b)*sinh(x)^2 - a^2 - a*b
), -1/2*(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2
 - a - b)*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*si
nh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2
 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3
 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b
)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) +
 ((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a - b
)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*
b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^2 + 2*a*cosh(x)*
sinh(x) + a*sinh(x)^2 + a)) + 2*sqrt(2)*a*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
 a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^2 + a*b)*cosh(x
)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a^2 + a*b)*sinh(x)^2 - a^2 - a*b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(x)}{\sqrt{a + b\operatorname{sech}^2(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(coth(x)**2/sqrt(a + b*sech(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)^2}{\sqrt{b\operatorname{sech}(x)^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(coth(x)^2/sqrt(b*sech(x)^2 + a), x)
```

3.202 $\int \frac{\coth^3(x)}{\sqrt{a+b\,\mathbf{sech}^2(x)}}\,dx$

Optimal. Leaf size=90

$$-\frac{\coth^2(x)\sqrt{a+b\operatorname{sech}^2(x)}}{2(a+b)}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a+b}}\right)}{2(a+b)^{3/2}}$$

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/Sqrt[a] - ((2*a + 3*b)*ArcTanh[Sqrt[
a + b*Sech[x]^2]/Sqrt[a + b]])/(2*(a + b)^(3/2)) - (Coth[x]^2*Sqrt[a + b*Se
ch[x]^2])/(2*(a + b))
```

Rubi [A] time = 0.160626, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {4139, 446, 103, 156, 63, 208}

$$-\frac{\coth^2(x)\sqrt{a+b\operatorname{sech}^2(x)}}{2(a+b)}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a+b}}\right)}{2(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/Sqrt[a] - ((2*a + 3*b)*ArcTanh[Sqrt[
a + b*Sech[x]^2]/Sqrt[a + b]])/(2*(a + b)^(3/2)) - (Coth[x]^2*Sqrt[a + b*Se
ch[x]^2])/(2*(a + b))
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^3(x)}{\sqrt{a+b\,\text{sech}^2(x)}}\,dx &= -\text{Subst}\left(\int \frac{1}{x\left(-1+x^2\right)^2\sqrt{a+bx^2}}\,dx, x, \text{sech}(x)\right)\\
&= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(-1+x)^2 x\sqrt{a+bx}}\,dx, x, \text{sech}^2(x)\right)\right)\\
&= -\frac{\coth^2(x)\sqrt{a+b\,\text{sech}^2(x)}}{2(a+b)} + \frac{\text{Subst}\left(\int \frac{a+b+\frac{bx}{2}}{(-1+x)x\sqrt{a+bx}}\,dx, x, \text{sech}^2(x)\right)}{2(a+b)}\\
&= -\frac{\coth^2(x)\sqrt{a+b\,\text{sech}^2(x)}}{2(a+b)} - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \text{sech}^2(x)\right) + \frac{(2a+3b)\,\text{Subst}\left(\int \cdots\right.}{4\cdots}\\
&= -\frac{\coth^2(x)\sqrt{a+b\,\text{sech}^2(x)}}{2(a+b)} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\,\text{sech}^2(x)}\right)}{b} + \frac{(2a+3b)\,\text{Subst}\left(\int \cdots\right.}{\cdots}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\text{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\,\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{2(a+b)^{3/2}} - \frac{\coth^2(x)\sqrt{a+b\,\text{sech}^2(x)}}{2(a+b)}
\end{aligned}
$$

Mathematica [A] time = 0.826402, size = 159, normalized size = 1.77

$$
\frac{\frac{\sqrt{2}\,\text{sech}(x)\sqrt{a\cosh(2x)+a+2b}\left(2(a+b)^{3/2}\log\left(\sqrt{a\cosh(2x)+a+2b}+\sqrt{2}\sqrt{a}\cosh(x)\right)-\sqrt{a}(2a+3b)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a+b}\cosh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)\right)}{\sqrt{a}\sqrt{a+b}} - \text{csch}^2(x)(a\cosh(2x)+\cdots}{4(a+b)\sqrt{a+b\,\text{sech}^2(x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] (-((a + 2*b + a*Cosh[2*x])*Csch[x]^2) + (Sqrt[2]*Sqrt[a + 2*b + a*Cosh[2*x]
]*(-(Sqrt[a]*(2*a + 3*b)*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Cosh[x])/Sqrt[a + 2*b
 + a*Cosh[2*x]]]) + 2*(a + b)^(3/2)*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a +
```

```
2*b + a*Cosh[2*x]]])*Sech[x])/(Sqrt[a]*Sqrt[a + b]))/(4*(a + b)*Sqrt[a + b*
Sech[x]^2])
```

Maple [F] time = 0.13, size = 0, normalized size = 0.

$$\int (\coth(x))^3 \frac{1}{\sqrt{a + b(\operatorname{sech}(x))^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^3/(a+b*sech(x)^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)^3}{\sqrt{b\operatorname{sech}(x)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)^3/sqrt(b*sech(x)^2 + a), x)
```

Fricas [B] time = 4.51437, size = 17920, normalized size = 199.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a^2 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x
)^3 + (a^2 + 2*a*b + b^2)*sinh(x)^4 - 2*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 2*(
3*(a^2 + 2*a*b + b^2)*cosh(x)^2 - a^2 - 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*
b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(x)^3 - (a^2 + 2*a*b + b^2)*cosh(x))*s
inh(x))*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b +
 a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 +
5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 1
4*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b
^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6
*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^
4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(
x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a
^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sin
h(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2
)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^
2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 +
 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 +
2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*
```

```
b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2
)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*co
sh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x
)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 +
 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a
)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
 + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*
b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*
a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh
(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(
x)*sinh(x)^5 + sinh(x)^6)) + ((2*a^2 + 3*a*b)*cosh(x)^4 + 4*(2*a^2 + 3*a*b)
*cosh(x)*sinh(x)^3 + (2*a^2 + 3*a*b)*sinh(x)^4 - 2*(2*a^2 + 3*a*b)*cosh(x)^
2 + 2*(3*(2*a^2 + 3*a*b)*cosh(x)^2 - 2*a^2 - 3*a*b)*sinh(x)^2 + 2*a^2 + 3*a
*b + 4*((2*a^2 + 3*a*b)*cosh(x)^3 - (2*a^2 + 3*a*b)*cosh(x))*sinh(x))*sqrt(
a + b)*log(((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)
*sinh(x)^4 + 2*(2*a + 3*b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a + 3*b
)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqr
t(a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*
sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a + 3*b)*cosh(x))*sinh(
x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2
 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)) + ((a
^2 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^3 + (a^
2 + 2*a*b + b^2)*sinh(x)^4 - 2*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 2*(3*(a^2 +
2*a*b + b^2)*cosh(x)^2 - a^2 - 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 +
 4*((a^2 + 2*a*b + b^2)*cosh(x)^3 - (a^2 + 2*a*b + b^2)*cosh(x))*sinh(x))*s
qrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x
)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh
(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(co
sh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sin
h(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 2*sqrt(2)*((a^2 +
a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a^2 + a*b)*sinh(x)^2 + a^
2 + a*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*
sinh(x) + sinh(x)^2)))/((a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 4*(a^3 + 2*a^2*
b + a*b^2)*cosh(x)*sinh(x)^3 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^4 + a^3 + 2*
a^2*b + a*b^2 - 2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2 - 2*(a^3 + 2*a^2*b + a*
b^2 - 3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 2*a^2*b +
a*b^2)*cosh(x)^3 - (a^3 + 2*a^2*b + a*b^2)*cosh(x))*sinh(x)), 1/4*(2*((2*a^
2 + 3*a*b)*cosh(x)^4 + 4*(2*a^2 + 3*a*b)*cosh(x)*sinh(x)^3 + (2*a^2 + 3*a*b
)*sinh(x)^4 - 2*(2*a^2 + 3*a*b)*cosh(x)^2 + 2*(3*(2*a^2 + 3*a*b)*cosh(x)^2
 - 2*a^2 - 3*a*b)*sinh(x)^2 + 2*a^2 + 3*a*b + 4*((2*a^2 + 3*a*b)*cosh(x)^3 -
 (2*a^2 + 3*a*b)*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 +
 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt((a*cosh(x)^2 + a*sinh
(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4
+ 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh
(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) +
a)) + ((a^2 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x
)^3 + (a^2 + 2*a*b + b^2)*sinh(x)^4 - 2*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 2*(
3*(a^2 + 2*a*b + b^2)*cosh(x)^2 - a^2 - 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*
b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(x)^3 - (a^2 + 2*a*b + b^2)*cosh(x))*s
inh(x))*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b +
 a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 +
5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 1
4*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b
^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6
*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^
4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(
x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a
^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sin
h(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2
```

)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^
2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 +
 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 +
2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*
b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2
)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*co
sh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x
)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 +
 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a
)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
 + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*
b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*
a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh
(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(
x)*sinh(x)^5 + sinh(x)^6)) + ((a^2 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*
b + b^2)*cosh(x)*sinh(x)^3 + (a^2 + 2*a*b + b^2)*sinh(x)^4 - 2*(a^2 + 2*a*b
 + b^2)*cosh(x)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^2 - a^2 - 2*a*b - b^2)
*sinh(x)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(x)^3 - (a^2 +
2*a*b + b^2)*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh
(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sqr
t(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x
)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) +
 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
sinh(x)^2)) - 2*sqrt(2)*((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh
(x) + (a^2 + a*b)*sinh(x)^2 + a^2 + a*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^3 + 2*a^2*b + a*
b^2)*cosh(x)^4 + 4*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^3 + (a^3 + 2*a^2
*b + a*b^2)*sinh(x)^4 + a^3 + 2*a^2*b + a*b^2 - 2*(a^3 + 2*a^2*b + a*b^2)*c
osh(x)^2 - 2*(a^3 + 2*a^2*b + a*b^2 - 3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*
sinh(x)^2 + 4*((a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 - (a^3 + 2*a^2*b + a*b^2)*
cosh(x))*sinh(x)), -1/4*(2*((a^2 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b
+ b^2)*cosh(x)*sinh(x)^3 + (a^2 + 2*a*b + b^2)*sinh(x)^4 - 2*(a^2 + 2*a*b +
 b^2)*cosh(x)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^2 - a^2 - 2*a*b - b^2)*s
inh(x)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(x)^3 - (a^2 + 2*
a*b + b^2)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*((a + b)*cosh(x)^2 + 2
*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^
2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a
^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)
^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + 3*a*b)*
sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(x))*sin
h(x))) + 2*((a^2 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(x)*s
inh(x)^3 + (a^2 + 2*a*b + b^2)*sinh(x)^4 - 2*(a^2 + 2*a*b + b^2)*cosh(x)^2
+ 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^2 - a^2 - 2*a*b - b^2)*sinh(x)^2 + a^2 +
 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(x)^3 - (a^2 + 2*a*b + b^2)*cosh(
x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(
x)^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*si
nh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4
*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - ((2*a^2 + 3*a*b)*cosh(x)
^4 + 4*(2*a^2 + 3*a*b)*cosh(x)*sinh(x)^3 + (2*a^2 + 3*a*b)*sinh(x)^4 - 2*(2
*a^2 + 3*a*b)*cosh(x)^2 + 2*(3*(2*a^2 + 3*a*b)*cosh(x)^2 - 2*a^2 - 3*a*b)*s
inh(x)^2 + 2*a^2 + 3*a*b + 4*((2*a^2 + 3*a*b)*cosh(x)^3 - (2*a^2 + 3*a*b)*c
osh(x))*sinh(x))*sqrt(a + b)*log(((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)
*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a + 3*b)*cosh(x)^2 + 2*(3*(2*a + b)
*cosh(x)^2 + 2*a + 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x
) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(
cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a
 + 3*b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh
(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x

```
))*sinh(x) + 1)) + 2*sqrt(2)*((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)
*sinh(x) + (a^2 + a*b)*sinh(x)^2 + a^2 + a*b)*sqrt((a*cosh(x)^2 + a*sinh(x)
^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^3 + 2*a^2*b
 + a*b^2)*cosh(x)^4 + 4*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^3 + (a^3 +
2*a^2*b + a*b^2)*sinh(x)^4 + a^3 + 2*a^2*b + a*b^2 - 2*(a^3 + 2*a^2*b + a*b
^2)*cosh(x)^2 - 2*(a^3 + 2*a^2*b + a*b^2 - 3*(a^3 + 2*a^2*b + a*b^2)*cosh(x
)^2)*sinh(x)^2 + 4*((a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 - (a^3 + 2*a^2*b + a*
b^2)*cosh(x))*sinh(x)), -1/2*(((a^2 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a
*b + b^2)*cosh(x)*sinh(x)^3 + (a^2 + 2*a*b + b^2)*sinh(x)^4 - 2*(a^2 + 2*a*
b + b^2)*cosh(x)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^2 - a^2 - 2*a*b - b^2
)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(x)^3 - (a^2 +
 2*a*b + b^2)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*((a + b)*cosh(x)^2
+ 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(
x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/
((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh
(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + 3*a*
b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(x))*
sinh(x))) + ((a^2 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(x)*
sinh(x)^3 + (a^2 + 2*a*b + b^2)*sinh(x)^4 - 2*(a^2 + 2*a*b + b^2)*cosh(x)^2
 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^2 - a^2 - 2*a*b - b^2)*sinh(x)^2 + a^2
+ 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(x)^3 - (a^2 + 2*a*b + b^2)*cosh
(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh
(x)^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*s
inh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 +
4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - ((2*a^2 + 3*a*b)*cosh(x
)^4 + 4*(2*a^2 + 3*a*b)*cosh(x)*sinh(x)^3 + (2*a^2 + 3*a*b)*sinh(x)^4 - 2*(
2*a^2 + 3*a*b)*cosh(x)^2 + 2*(3*(2*a^2 + 3*a*b)*cosh(x)^2 - 2*a^2 - 3*a*b)*
sinh(x)^2 + 2*a^2 + 3*a*b + 4*((2*a^2 + 3*a*b)*cosh(x)^3 - (2*a^2 + 3*a*b)*
cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x
) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sin
h(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*
sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + sqrt(2)*((a
^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a^2 + a*b)*sinh(x)^2
 + a^2 + a*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2)))/((a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 4*(a^3 + 2
*a^2*b + a*b^2)*cosh(x)*sinh(x)^3 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^4 + a^3
 + 2*a^2*b + a*b^2 - 2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2 - 2*(a^3 + 2*a^2*b
 + a*b^2 - 3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 2*a^2
*b + a*b^2)*cosh(x)^3 - (a^3 + 2*a^2*b + a*b^2)*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3(x)}{\sqrt{a + b\operatorname{sech}^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(coth(x)**3/sqrt(a + b*sech(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)^3}{\sqrt{b\,\mathrm{sech}(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(coth(x)^3/sqrt(b*sech(x)^2 + a), x)
```

3.203 $\int \frac{\tanh^5(x)}{\left(a+b\,\mathbf{sech}^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=68

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{(a+b)^2}{ab^2\sqrt{a+b\text{sech}^2(x)}}-\frac{\sqrt{a+b\text{sech}^2(x)}}{b^2}$$

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/a^(3/2) - (a + b)^2/(a*b^2*Sqrt[a +
b*Sech[x]^2]) - Sqrt[a + b*Sech[x]^2]/b^2
```

Rubi [A] time = 0.147392, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {4139, 446, 87, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{(a+b)^2}{ab^2\sqrt{a+b\text{sech}^2(x)}}-\frac{\sqrt{a+b\text{sech}^2(x)}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^5/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/a^(3/2) - (a + b)^2/(a*b^2*Sqrt[a +
b*Sech[x]^2]) - Sqrt[a + b*Sech[x]^2]/b^2
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^5(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}\,dx &= -\operatorname{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x\left(a+bx^2\right)^{3/2}}\,dx, x, \operatorname{sech}(x)\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{(-1+x)^2}{x(a+bx)^{3/2}}\,dx, x, \operatorname{sech}^2(x)\right)\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{(a+b)^2}{ab(a+bx)^{3/2}}+\frac{1}{b\sqrt{a+bx}}+\frac{1}{ax\sqrt{a+bx}}\right)dx, x, \operatorname{sech}^2(x)\right)\right)\\
&= -\frac{(a+b)^2}{ab^2\sqrt{a+b\operatorname{sech}^2(x)}}-\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{b^2}-\frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \operatorname{sech}^2(x)\right)}{2a}\\
&= -\frac{(a+b)^2}{ab^2\sqrt{a+b\operatorname{sech}^2(x)}}-\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{b^2}-\frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{sech}^2(x)}\right)}{ab}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{(a+b)^2}{ab^2\sqrt{a+b\operatorname{sech}^2(x)}}-\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{b^2}
\end{aligned}$$

Mathematica [A] time = 0.577225, size = 129, normalized size = 1.9

$$\frac{\operatorname{sech}^3(x)\left(\frac{\sqrt{2}(a\cosh(2x)+a+2b)^{3/2}\log\left(\sqrt{a\cosh(2x)+a+2b}+\sqrt{2}\sqrt{a}\cosh(x)\right)}{a^{3/2}}-\frac{\operatorname{sech}(x)(a\cosh(2x)+a+2b)\left(\left(2a^2+2ab+b^2\right)\cosh(2x)+2a^2+4ab+b^2\right)}{ab^2}\right)}{4\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] (Sech[x]^3*((Sqrt[2]*(a + 2*b + a*Cosh[2*x])^(3/2)*Log[Sqrt[2]*Sqrt[a]*Cosh
[x] + Sqrt[a + 2*b + a*Cosh[2*x]]])/a^(3/2) - ((a + 2*b + a*Cosh[2*x])*(2*a
^2 + 4*a*b + b^2 + (2*a^2 + 2*a*b + b^2)*Cosh[2*x])*Sech[x])/(a*b^2)))/(4*(
a + b*Sech[x]^2)^(3/2))
```

Maple [F] time = 0.104, size = 0, normalized size = 0.

$$\int (\tanh(x))^5\left(a+b(\operatorname{sech}(x))^2\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a+b*sech(x)^2)^(3/2),x)
```

```
[Out] int(tanh(x)^5/(a+b*sech(x)^2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^5}{\left(b\,\mathrm{sech}(x)^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^5/(b*sech(x)^2 + a)^(3/2), x)
```

Fricas [B] time = 4.18523, size = 8805, normalized size = 129.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*b^2*cosh(x)^6 + 6*a*b^2*cosh(x)*sinh(x)^5 + a*b^2*sinh(x)^6 + (3*a
*b^2 + 4*b^3)*cosh(x)^4 + (15*a*b^2*cosh(x)^2 + 3*a*b^2 + 4*b^3)*sinh(x)^4
+ 4*(5*a*b^2*cosh(x)^3 + (3*a*b^2 + 4*b^3)*cosh(x))*sinh(x)^3 + a*b^2 + (3*
a*b^2 + 4*b^3)*cosh(x)^2 + (15*a*b^2*cosh(x)^4 + 3*a*b^2 + 4*b^3 + 6*(3*a*b
^2 + 4*b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*a*b^2*cosh(x)^5 + 2*(3*a*b^2 + 4*b^
3)*cosh(x)^3 + (3*a*b^2 + 4*b^3)*cosh(x))*sinh(x))*sqrt(a)*log(((a^3 + 2*a^
2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3
 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)
^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x
)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2
*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(
x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 +
 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2
*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3
+ (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*
b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*
b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*
b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2
 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2
+ 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b
 + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b
^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2
)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 +
 a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3
 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^
2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a
^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (
6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))
/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*s
inh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + (a*
```

```
b^2*cosh(x)^6 + 6*a*b^2*cosh(x)*sinh(x)^5 + a*b^2*sinh(x)^6 + (3*a*b^2 + 4*
b^3)*cosh(x)^4 + (15*a*b^2*cosh(x)^2 + 3*a*b^2 + 4*b^3)*sinh(x)^4 + 4*(5*a*
b^2*cosh(x)^3 + (3*a*b^2 + 4*b^3)*cosh(x))*sinh(x)^3 + a*b^2 + (3*a*b^2 + 4
*b^3)*cosh(x)^2 + (15*a*b^2*cosh(x)^4 + 3*a*b^2 + 4*b^3 + 6*(3*a*b^2 + 4*b^
3)*cosh(x)^2)*sinh(x)^2 + 2*(3*a*b^2*cosh(x)^5 + 2*(3*a*b^2 + 4*b^3)*cosh(x
)^3 + (3*a*b^2 + 4*b^3)*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*c
osh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh
(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqr
t((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + si
nh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)
*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*((2*a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 4
*(2*a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^3 + (2*a^3 + 2*a^2*b + a*b^2)*si
nh(x)^4 + 2*a^3 + 2*a^2*b + a*b^2 + 2*(2*a^3 + 4*a^2*b + a*b^2)*cosh(x)^2 +
 2*(2*a^3 + 4*a^2*b + a*b^2 + 3*(2*a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x
)^2 + 4*((2*a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + (2*a^3 + 4*a^2*b + a*b^2)*co
sh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)))/(a^3*b^2*cosh(x)^6 + 6*a^3*b^2*cosh(x)*sinh(x
)^5 + a^3*b^2*sinh(x)^6 + a^3*b^2 + (3*a^3*b^2 + 4*a^2*b^3)*cosh(x)^4 + (15
*a^3*b^2*cosh(x)^2 + 3*a^3*b^2 + 4*a^2*b^3)*sinh(x)^4 + 4*(5*a^3*b^2*cosh(x
)^3 + (3*a^3*b^2 + 4*a^2*b^3)*cosh(x))*sinh(x)^3 + (3*a^3*b^2 + 4*a^2*b^3)*
cosh(x)^2 + (15*a^3*b^2*cosh(x)^4 + 3*a^3*b^2 + 4*a^2*b^3 + 6*(3*a^3*b^2 +
4*a^2*b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*a^3*b^2*cosh(x)^5 + 2*(3*a^3*b^2 + 4
*a^2*b^3)*cosh(x)^3 + (3*a^3*b^2 + 4*a^2*b^3)*cosh(x))*sinh(x)), -1/2*((a*b
^2*cosh(x)^6 + 6*a*b^2*cosh(x)*sinh(x)^5 + a*b^2*sinh(x)^6 + (3*a*b^2 + 4*b
^3)*cosh(x)^4 + (15*a*b^2*cosh(x)^2 + 3*a*b^2 + 4*b^3)*sinh(x)^4 + 4*(5*a*b
^2*cosh(x)^3 + (3*a*b^2 + 4*b^3)*cosh(x))*sinh(x)^3 + a*b^2 + (3*a*b^2 + 4*
b^3)*cosh(x)^2 + (15*a*b^2*cosh(x)^4 + 3*a*b^2 + 4*b^3 + 6*(3*a*b^2 + 4*b^3
)*cosh(x)^2)*sinh(x)^2 + 2*(3*a*b^2*cosh(x)^5 + 2*(3*a*b^2 + 4*b^3)*cosh(x)
^3 + (3*a*b^2 + 4*b^3)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*((a + b)*c
osh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqrt
((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sin
h(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 +
a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a
^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*
cosh(x))*sinh(x))) + (a*b^2*cosh(x)^6 + 6*a*b^2*cosh(x)*sinh(x)^5 + a*b^2*s
inh(x)^6 + (3*a*b^2 + 4*b^3)*cosh(x)^4 + (15*a*b^2*cosh(x)^2 + 3*a*b^2 + 4*
b^3)*sinh(x)^4 + 4*(5*a*b^2*cosh(x)^3 + (3*a*b^2 + 4*b^3)*cosh(x))*sinh(x)^
3 + a*b^2 + (3*a*b^2 + 4*b^3)*cosh(x)^2 + (15*a*b^2*cosh(x)^4 + 3*a*b^2 + 4
*b^3 + 6*(3*a*b^2 + 4*b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*a*b^2*cosh(x)^5 + 2*
(3*a*b^2 + 4*b^3)*cosh(x)^3 + (3*a*b^2 + 4*b^3)*cosh(x))*sinh(x))*sqrt(-a)*
arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*sqr
t((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + si
nh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*
cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2
*b)*cosh(x))*sinh(x) + a)) + 2*sqrt(2)*((2*a^3 + 2*a^2*b + a*b^2)*cosh(x)^4
 + 4*(2*a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^3 + (2*a^3 + 2*a^2*b + a*b^2
)*sinh(x)^4 + 2*a^3 + 2*a^2*b + a*b^2 + 2*(2*a^3 + 4*a^2*b + a*b^2)*cosh(x)
^2 + 2*(2*a^3 + 4*a^2*b + a*b^2 + 3*(2*a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*si
nh(x)^2 + 4*((2*a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + (2*a^3 + 4*a^2*b + a*b^2
)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^3*b^2*cosh(x)^6 + 6*a^3*b^2*cosh(x)*si
nh(x)^5 + a^3*b^2*sinh(x)^6 + a^3*b^2 + (3*a^3*b^2 + 4*a^2*b^3)*cosh(x)^4 +
 (15*a^3*b^2*cosh(x)^2 + 3*a^3*b^2 + 4*a^2*b^3)*sinh(x)^4 + 4*(5*a^3*b^2*co
sh(x)^3 + (3*a^3*b^2 + 4*a^2*b^3)*cosh(x))*sinh(x)^3 + (3*a^3*b^2 + 4*a^2*b
^3)*cosh(x)^2 + (15*a^3*b^2*cosh(x)^4 + 3*a^3*b^2 + 4*a^2*b^3 + 6*(3*a^3*b^
2 + 4*a^2*b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*a^3*b^2*cosh(x)^5 + 2*(3*a^3*b^2
 + 4*a^2*b^3)*cosh(x)^3 + (3*a^3*b^2 + 4*a^2*b^3)*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^5(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/(a+b*sech(x)**2)**(3/2),x)
```

```
[Out] Integral(tanh(x)**5/(a + b*sech(x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^5}{\left(b\,\mathrm{sech}(x)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(x)^5/(b*sech(x)^2 + a)^(3/2), x)
```

3.204 $\int \frac{\tanh^4(x)}{\left(a+b\,\mathbf{sech}^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=86

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{3/2}}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{b^{3/2}}-\frac{(a+b)\tanh(x)}{ab\sqrt{a-b\tanh^2(x)+b}}$$

```
[Out] ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/b^(3/2) + ArcTanh[(Sqrt
[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(3/2) - ((a + b)*Tanh[x])/(a*b*Sq
rt[a + b - b*Tanh[x]^2])
```

Rubi [A] time = 0.252348, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {4141, 1975, 470, 523, 217, 203, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{3/2}}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{b^{3/2}}-\frac{(a+b)\tanh(x)}{ab\sqrt{a-b\tanh^2(x)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/b^(3/2) + ArcTanh[(Sqrt
[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(3/2) - ((a + b)*Tanh[x])/(a*b*Sq
rt[a + b - b*Tanh[x]^2])
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
```

```
p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^4(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}\,dx &= \operatorname{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)\left(a+b\left(1-x^2\right)\right)^{3/2}}\,dx, x, \tanh(x)\right)\\
&= \operatorname{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)\left(a+b-bx^2\right)^{3/2}}\,dx, x, \tanh(x)\right)\\
&= -\frac{(a+b)\tanh(x)}{ab\sqrt{a+b-b\tanh^2(x)}}+\frac{\operatorname{Subst}\left(\int \frac{a+b-ax^2}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)}{ab}\\
&= -\frac{(a+b)\tanh(x)}{ab\sqrt{a+b-b\tanh^2(x)}}+\frac{\operatorname{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)}{a}+\frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+b-bx^2}}\right.}{b}\\
&= -\frac{(a+b)\tanh(x)}{ab\sqrt{a+b-b\tanh^2(x)}}+\frac{\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a}+\frac{\operatorname{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x,\right.}{b}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{b^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{3/2}}-\frac{(a+b)\tanh(x)}{ab\sqrt{a+b-b\tanh^2(x)}}
\end{aligned}
$$

Mathematica [A] time = 0.381261, size = 169, normalized size = 1.97

$$-\frac{\operatorname{sech}^3(x)(a\cosh(2x)+a+2b)\left(\sqrt{a}\left(2\sqrt{b}(a+b)\sinh(x)-\sqrt{2}a\sqrt{a\cosh(2x)+a+2b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sinh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)\right)-\sqrt{2}b^{3/2}\cdots}{4a^{3/2}b^{3/2}\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] -((a + 2*b + a*Cosh[2*x])*Sech[x]^3*(-(Sqrt[2]*b^(3/2)*ArcTanh[(Sqrt[2]*Sqr
t[a]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]]*Sqrt[a + 2*b + a*Cosh[2*x]]) + S
qrt[a]*(-(Sqrt[2]*a*ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[
2*x]]]*Sqrt[a + 2*b + a*Cosh[2*x]]) + 2*Sqrt[b]*(a + b)*Sinh[x])))/(4*a^(3/
2)*b^(3/2)*(a + b*Sech[x]^2)^(3/2))
```

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int (\tanh(x))^4 \left(a+b\,(\operatorname{sech}(x))^2\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*sech(x)^2)^(3/2),x)
```

```
[Out] int(tanh(x)^4/(a+b*sech(x)^2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^4}{\left(b\operatorname{sech}(x)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^4/(b*sech(x)^2 + a)^(3/2), x)
```

Fricas [B] time = 4.313, size = 14456, normalized size = 168.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*b^2*cosh(x)^4 + 4*a*b^2*cosh(x)*sinh(x)^3 + a*b^2*sinh(x)^4 + a*b^
2 + 2*(a*b^2 + 2*b^3)*cosh(x)^2 + 2*(3*a*b^2*cosh(x)^2 + a*b^2 + 2*b^3)*sin
h(x)^2 + 4*(a*b^2*cosh(x)^3 + (a*b^2 + 2*b^3)*cosh(x))*sinh(x))*sqrt(a)*log
((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2
```

```
- b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a
*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*
b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2
- b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh
(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2
*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 +
3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*c
osh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5
*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(
x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2
 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4
*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*
b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b
)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)
^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5
+ sinh(x)^6)) - 2*(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4
+ a^3 + 2*(a^3 + 2*a^2*b)*cosh(x)^2 + 2*(3*a^3*cosh(x)^2 + a^3 + 2*a^2*b)*s
inh(x)^2 + 4*(a^3*cosh(x)^3 + (a^3 + 2*a^2*b)*cosh(x))*sinh(x))*sqrt(-b)*lo
g(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2
*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 + 2*sqrt
(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a*cosh(x
)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) +
 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)^4 + 4*
cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2
 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + (a*b^2*cosh(x)^4 + 4*a*b^2*cosh(
x)*sinh(x)^3 + a*b^2*sinh(x)^4 + a*b^2 + 2*(a*b^2 + 2*b^3)*cosh(x)^2 + 2*(3
*a*b^2*cosh(x)^2 + a*b^2 + 2*b^3)*sinh(x)^2 + 4*(a*b^2*cosh(x)^3 + (a*b^2 +
 2*b^3)*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3
 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2
+ sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*c
osh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)
*sinh(x) + sinh(x)^2)) + 4*sqrt(2)*(a^2*b + a*b^2 - (a^2*b + a*b^2)*cosh(x)
^2 - 2*(a^2*b + a*b^2)*cosh(x)*sinh(x) - (a^2*b + a*b^2)*sinh(x)^2)*sqrt((a
*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x
)^2)))/(a^3*b^2*cosh(x)^4 + 4*a^3*b^2*cosh(x)*sinh(x)^3 + a^3*b^2*sinh(x)^4
 + a^3*b^2 + 2*(a^3*b^2 + 2*a^2*b^3)*cosh(x)^2 + 2*(3*a^3*b^2*cosh(x)^2 + a
^3*b^2 + 2*a^2*b^3)*sinh(x)^2 + 4*(a^3*b^2*cosh(x)^3 + (a^3*b^2 + 2*a^2*b^3
)*cosh(x))*sinh(x)), 1/4*(4*(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*
sinh(x)^4 + a^3 + 2*(a^3 + 2*a^2*b)*cosh(x)^2 + 2*(3*a^3*cosh(x)^2 + a^3 +
2*a^2*b)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + (a^3 + 2*a^2*b)*cosh(x))*sinh(x))*s
qrt(b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(
b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a +
 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 +
(a + 2*b)*cosh(x))*sinh(x) + a)) + (a*b^2*cosh(x)^4 + 4*a*b^2*cosh(x)*sinh(
x)^3 + a*b^2*sinh(x)^4 + a*b^2 + 2*(a*b^2 + 2*b^3)*cosh(x)^2 + 2*(3*a*b^2*c
osh(x)^2 + a*b^2 + 2*b^3)*sinh(x)^2 + 4*(a*b^2*cosh(x)^3 + (a*b^2 + 2*b^3)*
cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7
+ a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b
^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh
(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4
*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh
(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sin
h(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*
b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^
2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(
x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cos
```

```
h(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(
x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5
 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x
)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) +
 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^
2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*s
inh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sin
h(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + (a*b^2*cosh(x)^4 + 4*a*b^2*cos
h(x)*sinh(x)^3 + a*b^2*sinh(x)^4 + a*b^2 + 2*(a*b^2 + 2*b^3)*cosh(x)^2 + 2*
(3*a*b^2*cosh(x)^2 + a*b^2 + 2*b^3)*sinh(x)^2 + 4*(a*b^2*cosh(x)^3 + (a*b^2
 + 2*b^3)*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)
^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^
2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a
*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x
)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(
x)*sinh(x) + sinh(x)^2)) + 4*sqrt(2)*(a^2*b + a*b^2 - (a^2*b + a*b^2)*cosh(
x)^2 - 2*(a^2*b + a*b^2)*cosh(x)*sinh(x) - (a^2*b + a*b^2)*sinh(x)^2)*sqrt(
(a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh
(x)^2)))/(a^3*b^2*cosh(x)^4 + 4*a^3*b^2*cosh(x)*sinh(x)^3 + a^3*b^2*sinh(x)
^4 + a^3*b^2 + 2*(a^3*b^2 + 2*a^2*b^3)*cosh(x)^2 + 2*(3*a^3*b^2*cosh(x)^2 +
 a^3*b^2 + 2*a^2*b^3)*sinh(x)^2 + 4*(a^3*b^2*cosh(x)^3 + (a^3*b^2 + 2*a^2*b
^3)*cosh(x))*sinh(x)), -1/2*((a*b^2*cosh(x)^4 + 4*a*b^2*cosh(x)*sinh(x)^3 +
 a*b^2*sinh(x)^4 + a*b^2 + 2*(a*b^2 + 2*b^3)*cosh(x)^2 + 2*(3*a*b^2*cosh(x)
^2 + a*b^2 + 2*b^3)*sinh(x)^2 + 4*(a*b^2*cosh(x)^3 + (a*b^2 + 2*b^3)*cosh(x
))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*
sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)
^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a
*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x)))
 + (a*b^2*cosh(x)^4 + 4*a*b^2*cosh(x)*sinh(x)^3 + a*b^2*sinh(x)^4 + a*b^2 +
 2*(a*b^2 + 2*b^3)*cosh(x)^2 + 2*(3*a*b^2*cosh(x)^2 + a*b^2 + 2*b^3)*sinh(x
)^2 + 4*(a*b^2*cosh(x)^3 + (a*b^2 + 2*b^3)*cosh(x))*sinh(x))*sqrt(-a)*arcta
n(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt((a*
cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(
x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*c
osh(x))*sinh(x) + a)) + (a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh
(x)^4 + a^3 + 2*(a^3 + 2*a^2*b)*cosh(x)^2 + 2*(3*a^3*cosh(x)^2 + a^3 + 2*a^
2*b)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + (a^3 + 2*a^2*b)*cosh(x))*sinh(x))*sqrt(
-b)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)
^4 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 +
2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a*
cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
^2)) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)^
4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cos
h(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) - 2*sqrt(2)*(a^2*b + a*b^2 -
 (a^2*b + a*b^2)*cosh(x)^2 - 2*(a^2*b + a*b^2)*cosh(x)*sinh(x) - (a^2*b + a
*b^2)*sinh(x)^2)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*
cosh(x)*sinh(x) + sinh(x)^2)))/(a^3*b^2*cosh(x)^4 + 4*a^3*b^2*cosh(x)*sinh(
x)^3 + a^3*b^2*sinh(x)^4 + a^3*b^2 + 2*(a^3*b^2 + 2*a^2*b^3)*cosh(x)^2 + 2*
(3*a^3*b^2*cosh(x)^2 + a^3*b^2 + 2*a^2*b^3)*sinh(x)^2 + 4*(a^3*b^2*cosh(x)^
3 + (a^3*b^2 + 2*a^2*b^3)*cosh(x))*sinh(x)), -1/2*((a*b^2*cosh(x)^4 + 4*a*b
^2*cosh(x)*sinh(x)^3 + a*b^2*sinh(x)^4 + a*b^2 + 2*(a*b^2 + 2*b^3)*cosh(x)^
2 + 2*(3*a*b^2*cosh(x)^2 + a*b^2 + 2*b^3)*sinh(x)^2 + 4*(a*b^2*cosh(x)^3 +
(a*b^2 + 2*b^3)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*
b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)
^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(x)^4 +
 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b
*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a
```

```
*b)*cosh(x))*sinh(x))) + (a*b^2*cosh(x)^4 + 4*a*b^2*cosh(x)*sinh(x)^3 + a*b
^2*sinh(x)^4 + a*b^2 + 2*(a*b^2 + 2*b^3)*cosh(x)^2 + 2*(3*a*b^2*cosh(x)^2 +
 a*b^2 + 2*b^3)*sinh(x)^2 + 4*(a*b^2*cosh(x)^3 + (a*b^2 + 2*b^3)*cosh(x))*s
inh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2
+ 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)
^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*c
osh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - 2*(a^3*cosh(x)^4 + 4*a^3*cosh
(x)*sinh(x)^3 + a^3*sinh(x)^4 + a^3 + 2*(a^3 + 2*a^2*b)*cosh(x)^2 + 2*(3*a^
3*cosh(x)^2 + a^3 + 2*a^2*b)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + (a^3 + 2*a^2*b)
*cosh(x))*sinh(x))*sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
sinh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 +
a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2
 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - 2*sqrt(2)*(a^2*b + a
*b^2 - (a^2*b + a*b^2)*cosh(x)^2 - 2*(a^2*b + a*b^2)*cosh(x)*sinh(x) - (a^2
*b + a*b^2)*sinh(x)^2)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^3*b^2*cosh(x)^4 + 4*a^3*b^2*cosh(x)
*sinh(x)^3 + a^3*b^2*sinh(x)^4 + a^3*b^2 + 2*(a^3*b^2 + 2*a^2*b^3)*cosh(x)^
2 + 2*(3*a^3*b^2*cosh(x)^2 + a^3*b^2 + 2*a^2*b^3)*sinh(x)^2 + 4*(a^3*b^2*co
sh(x)^3 + (a^3*b^2 + 2*a^2*b^3)*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^4(x)}{\left(a + b\operatorname{sech}^2(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+b*sech(x)**2)**(3/2),x)
```

```
[Out] Integral(tanh(x)**4/(a + b*sech(x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^4}{\left(b\operatorname{sech}(x)^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(x)^4/(b*sech(x)^2 + a)^(3/2), x)
```

3.205 $\int \frac{\tanh^3(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=49

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{a+b}{ab\sqrt{a+b\,\mathrm{sech}^2(x)}}$$

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/a^(3/2) - (a + b)/(a*b*Sqrt[a + b*Se
ch[x]^2])
```

Rubi [A] time = 0.114608, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {4139, 446, 78, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{a+b}{ab\sqrt{a+b\,\mathrm{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/a^(3/2) - (a + b)/(a*b*Sqrt[a + b*Se
ch[x]^2])
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^3(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}\,dx &= \text{Subst}\left(\int \frac{-1+x^2}{x\left(a+bx^2\right)^{3/2}}\,dx, x, \operatorname{sech}(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{-1+x}{x(a+bx)^{3/2}}\,dx, x, \operatorname{sech}^2(x)\right)\\
&= -\frac{a+b}{ab\sqrt{a+b\operatorname{sech}^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \operatorname{sech}^2(x)\right)}{2a}\\
&= -\frac{a+b}{ab\sqrt{a+b\operatorname{sech}^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{sech}^2(x)}\right)}{ab}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{a+b}{ab\sqrt{a+b\operatorname{sech}^2(x)}}
\end{aligned}$$

Mathematica [B] time = 0.229031, size = 103, normalized size = 2.1

$$\frac{\operatorname{sech}^3(x)\left(\sqrt{2}(a\cosh(2x)+a+2b)^{3/2}\log\left(\sqrt{a\cosh(2x)+a+2b}+\sqrt{2}\sqrt{a}\cosh(x)\right)-\frac{2\sqrt{a}(a+b)\cosh(x)(a\cosh(2x)+a+2b)}{b}\right)}{4a^{3/2}\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] (((-2*Sqrt[a]*(a + b)*Cosh[x]*(a + 2*b + a*Cosh[2*x]))/b + Sqrt[2]*(a + 2*b
 + a*Cosh[2*x])^(3/2)*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*Cosh[2
*x]]])*Sech[x]^3)/(4*a^(3/2)*(a + b*Sech[x]^2)^(3/2))
```

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int (\tanh(x))^3 \left(a+b(\operatorname{sech}(x))^2\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*sech(x)^2)^(3/2),x)
```

[Out] int(tanh(x)^3/(a+b*sech(x)^2)^(3/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^3}{\left(b\,\mathrm{sech}(x)^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^3/(b*sech(x)^2 + a)^(3/2), x)
```

Fricas [B] time = 2.74828, size = 6137, normalized size = 125.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 + 2*(a*b + 2
*b^2)*cosh(x)^2 + 2*(3*a*b*cosh(x)^2 + a*b + 2*b^2)*sinh(x)^2 + a*b + 4*(a*
b*cosh(x)^3 + (a*b + 2*b^2)*cosh(x))*sinh(x))*sqrt(a)*log(((a^3 + 2*a^2*b +
 a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*
a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 +
2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*
sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b +
4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4
+ (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(
2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*
b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*
a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*co
sh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4
*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*
cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*
a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a
*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^
2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*c
osh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cos
h(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2
+ 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3
*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a
 + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b
+ a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3
 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cos
h(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x
)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + (a*b*cos
h(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 + 2*(a*b + 2*b^2)*cosh(x)^
2 + 2*(3*a*b*cosh(x)^2 + a*b + 2*b^2)*sinh(x)^2 + a*b + 4*(a*b*cosh(x)^3 +
(a*b + 2*b^2)*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sin
h(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sq
rt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(
x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))
```

```
+ 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2)) - 4*sqrt(2)*((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sin
h(x) + (a^2 + a*b)*sinh(x)^2 + a^2 + a*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
 a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^3*b*cosh(x)^4 +
4*a^3*b*cosh(x)*sinh(x)^3 + a^3*b*sinh(x)^4 + a^3*b + 2*(a^3*b + 2*a^2*b^2)
*cosh(x)^2 + 2*(3*a^3*b*cosh(x)^2 + a^3*b + 2*a^2*b^2)*sinh(x)^2 + 4*(a^3*b
*cosh(x)^3 + (a^3*b + 2*a^2*b^2)*cosh(x))*sinh(x)), -1/2*((a*b*cosh(x)^4 +
4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 + 2*(a*b + 2*b^2)*cosh(x)^2 + 2*(3*
a*b*cosh(x)^2 + a*b + 2*b^2)*sinh(x)^2 + a*b + 4*(a*b*cosh(x)^3 + (a*b + 2*
b^2)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*((a + b)*cosh(x)^2 + 2*(a +
b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*
sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a
*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (
2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x
)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(x))*sinh(x)))
 + (a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 + 2*(a*b + 2*b^
2)*cosh(x)^2 + 2*(3*a*b*cosh(x)^2 + a*b + 2*b^2)*sinh(x)^2 + a*b + 4*(a*b*c
osh(x)^3 + (a*b + 2*b^2)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x)
^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh
(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4
+ 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh
(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) +
a)) + 2*sqrt(2)*((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a
^2 + a*b)*sinh(x)^2 + a^2 + a*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)
/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^3*b*cosh(x)^4 + 4*a^3*b*c
osh(x)*sinh(x)^3 + a^3*b*sinh(x)^4 + a^3*b + 2*(a^3*b + 2*a^2*b^2)*cosh(x)^
2 + 2*(3*a^3*b*cosh(x)^2 + a^3*b + 2*a^2*b^2)*sinh(x)^2 + 4*(a^3*b*cosh(x)^
3 + (a^3*b + 2*a^2*b^2)*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^3(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*sech(x)**2)**(3/2),x)
```

```
[Out] Integral(tanh(x)**3/(a + b*sech(x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^3}{\left(b\operatorname{sech}(x)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(x)^3/(b*sech(x)^2 + a)^(3/2), x)
```

3.206 $\int \frac{\tanh^2(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=51

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{3/2}}-\frac{\tanh(x)}{a\sqrt{a-b\tanh^2(x)+b}}$$

```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(3/2) - Tanh[x]/(a*S
qrt[a + b - b*Tanh[x]^2])
```

Rubi [A] time = 0.217583, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {4141, 1975, 471, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{3/2}}-\frac{\tanh(x)}{a\sqrt{a-b\tanh^2(x)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(3/2) - Tanh[x]/(a*S
qrt[a + b - b*Tanh[x]^2])
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^2(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}\,dx &= \text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)\left(a+b\left(1-x^2\right)\right)^{3/2}}\,dx, x, \tanh(x)\right)\\
&= \text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)\left(a+b-bx^2\right)^{3/2}}\,dx, x, \tanh(x)\right)\\
&= -\frac{\tanh(x)}{a\sqrt{a+b-b\tanh^2(x)}}+\frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)}{a}\\
&= -\frac{\tanh(x)}{a\sqrt{a+b-b\tanh^2(x)}}+\frac{\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{3/2}}-\frac{\tanh(x)}{a\sqrt{a+b-b\tanh^2(x)}}
\end{aligned}
$$

Mathematica [B] time = 0.713224, size = 128, normalized size = 2.51

$$
\frac{\operatorname{sech}^2(x)(a\cosh(2x)+a+2b)\left(\operatorname{sech}(x)\sinh^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a+b}}\right)(a\cosh(2x)+a+2b)-2\sqrt{a}\sqrt{a+b}\tanh(x)\sqrt{\frac{a\sinh^2(x)+a+b}{a+b}}\right.}{4a^{3/2}\sqrt{a+b}\sqrt{\frac{a\sinh^2(x)+a+b}{a+b}}\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*x])*Sech[x]^2*(ArcSinh[(Sqrt[a]*Sinh[x])/Sqrt[a + b]]*
(a + 2*b + a*Cosh[2*x])*Sech[x] - 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(a + b + a*Sin
h[x]^2)/(a + b)]*Tanh[x]))/(4*a^(3/2)*Sqrt[a + b]*(a + b*Sech[x]^2)^(3/2)*S
qrt[(a + b + a*Sinh[x]^2)/(a + b)])
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$
\int (\tanh(x))^2\left(a+b(\operatorname{sech}(x))^2\right)^{-\frac{3}{2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+b*sech(x)^2)^(3/2),x)
```

```
[Out] int(tanh(x)^2/(a+b*sech(x)^2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^2}{\left(b\,\mathrm{sech}(x)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^2/(b*sech(x)^2 + a)^(3/2), x)
```

Fricas [B] time = 2.6477, size = 5075, normalized size = 99.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh
(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*
cosh(x))*sinh(x) + a)*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x
)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 -
 a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*
sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3
 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*
cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))
*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15
*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh
(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*s
inh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2
*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*c
osh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(
x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*co
sh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2
)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*
a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)
^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2
*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + (a*cosh(x)^4 + 4*a*cosh(x)
*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2
*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)*sqrt(a)*lo
g(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2
 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sin
h(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(c
osh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(
x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*(
a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt((a*cosh(x)^2 + a*
sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^3*cos
h(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + a^3 + 2*(a^3 + 2*a^2*b)*
```

```
cosh(x)^2 + 2*(3*a^3*cosh(x)^2 + a^3 + 2*a^2*b)*sinh(x)^2 + 4*(a^3*cosh(x)^
3 + (a^3 + 2*a^2*b)*cosh(x))*sinh(x)), -1/2*((a*cosh(x)^4 + 4*a*cosh(x)*sin
h(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*
sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)*sqrt(-a)*arcta
n(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sq
rt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + s
inh(x)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2
+ 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2
*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) + (a*cosh(x)^4 + 4*a*cosh
(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a
+ 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)*sqrt(-a
)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*s
qrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b
)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a +
 2*b)*cosh(x))*sinh(x) + a)) + 2*sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x)
 + a*sinh(x)^2 - a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3
+ a^3*sinh(x)^4 + a^3 + 2*(a^3 + 2*a^2*b)*cosh(x)^2 + 2*(3*a^3*cosh(x)^2 +
a^3 + 2*a^2*b)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + (a^3 + 2*a^2*b)*cosh(x))*sinh
(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2(x)}{(a + b\operatorname{sech}^2(x))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*sech(x)**2)**(3/2),x)
```

```
[Out] Integral(tanh(x)**2/(a + b*sech(x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^2}{(b\operatorname{sech}(x)^2 + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(x)^2/(b*sech(x)^2 + a)^(3/2), x)
```

3.207 $\int \frac{\tanh(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=43

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{a\sqrt{a+b\,\mathrm{sech}^2(x)}}$$

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/a^(3/2) - 1/(a*Sqrt[a + b*Sech[x]^2]
)
```

Rubi [A] time = 0.0729075, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4139, 266, 51, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{a\sqrt{a+b\,\mathrm{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + b*Sech[x]^2)^(3/2),x]
```


```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/a^(3/2) - 1/(a*Sqrt[a + b*Sech[x]^2]
)
```


Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}\,dx &= -\operatorname{Subst}\left(\int \frac{1}{x\left(a+bx^2\right)^{3/2}}\,dx, x, \operatorname{sech}(x)\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \operatorname{sech}^2(x)\right)\right)\\
&= -\frac{1}{a\sqrt{a+b\operatorname{sech}^2(x)}} - \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \operatorname{sech}^2(x)\right)}{2a}\\
&= -\frac{1}{a\sqrt{a+b\operatorname{sech}^2(x)}} - \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{sech}^2(x)}\right)}{ab}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{1}{a\sqrt{a+b\operatorname{sech}^2(x)}}
\end{aligned}
$$

Mathematica [B] time = 0.396049, size = 98, normalized size = 2.28

$$
-\frac{\operatorname{sech}^3(x)(a\cosh(2x)+a+2b)\left(2\sqrt{a}\cosh(x)-\sqrt{2}\sqrt{a\cosh(2x)+a+2b}\log\left(\sqrt{a\cosh(2x)+a+2b}+\sqrt{2}\sqrt{a}\cosh(x)\right.\right.}{4a^{3/2}\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] -((a + 2*b + a*Cosh[2*x])*(2*Sqrt[a]*Cosh[x] - Sqrt[2]*Sqrt[a + 2*b + a*Cos
h[2*x]]*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*Cosh[2*x]]])*Sech[x]
^3)/(4*a^(3/2)*(a + b*Sech[x]^2)^(3/2))
```

Maple [A] time = 0.015, size = 46, normalized size = 1.1

$$
-\frac{1}{a}\frac{1}{\sqrt{a+b\left(\operatorname{sech}(x)\right)^2}}+\ln\left(\frac{1}{\operatorname{sech}(x)}\left(2\,a+2\,\sqrt{a}\sqrt{a+b\left(\operatorname{sech}(x)\right)^2}\right)\right)a^{-\frac{3}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*sech(x)^2)^(3/2),x)
```

```
[Out] -1/a/(a+b*sech(x)^2)^(1/2)+1/a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sech(x)^2)^(1/2
))/sech(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{\left(b\,\mathrm{sech}(x)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)/(b*sech(x)^2 + a)^(3/2), x)
```

Fricas [B] time = 2.61665, size = 5840, normalized size = 135.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh
(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*
cosh(x))*sinh(x) + a)*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a
^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8
 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a
*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 +
 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*
sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a
*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^
2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 1
0*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2
)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2
*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4
+ 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + s
qrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(
x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*
(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2
 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a
^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b
 + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^
2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*s
inh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(
2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*c
osh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sin
h(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(
x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + (a*cosh(x)^4 + 4*a*cosh(x)*sinh(
x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*si
nh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)*sqrt(a)*log(-(a*
cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*co
sh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2
 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cos
```

```
h(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh
(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*(a*cosh(x)^2 + 2*a*cosh
(x)*sinh(x) + a*sinh(x)^2 + a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(
cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^3*cosh(x)^4 + 4*a^3*cosh(x)
*sinh(x)^3 + a^3*sinh(x)^4 + a^3 + 2*(a^3 + 2*a^2*b)*cosh(x)^2 + 2*(3*a^3*c
osh(x)^2 + a^3 + 2*a^2*b)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + (a^3 + 2*a^2*b)*co
sh(x))*sinh(x)), -1/2*((a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 +
 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(
x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)*sqrt(-a)*arctan(sqrt(2)*((a + b)*cos
h(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((
a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(
x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*
b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2
 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*co
sh(x))*sinh(x))) + (a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(
a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3
 + (a + 2*b)*cosh(x))*sinh(x) + a)*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*c
osh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*co
sh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 +
a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + 2*
sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)*sqrt((a*cosh(
x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))
/(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + a^3 + 2*(a^3 +
2*a^2*b)*cosh(x)^2 + 2*(3*a^3*cosh(x)^2 + a^3 + 2*a^2*b)*sinh(x)^2 + 4*(a^3
*cosh(x)^3 + (a^3 + 2*a^2*b)*cosh(x))*sinh(x))]
```

Sympy [A] time = 6.61079, size = 44, normalized size = 1.02

$$-\frac{1}{a\sqrt{a+b\operatorname{sech}^2(x)}}-\frac{\operatorname{atan}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{-a}}\right)}{a\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)**2)**(3/2),x)
```

```
[Out] -1/(a*sqrt(a + b*sech(x)**2)) - atan(sqrt(a + b*sech(x)**2)/sqrt(-a))/(a*sq
rt(-a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{\left(b\operatorname{sech}(x)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(x)/(b*sech(x)^2 + a)^(3/2), x)
```

3.208 $\int \frac{1}{\left(a+b\,\mathbf{sech}^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=57

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{3/2}}-\frac{b\tanh(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}$$

```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(3/2) - (b*Tanh[x])/
(a*(a + b)*Sqrt[a + b - b*Tanh[x]^2])
```

Rubi [A] time = 0.0451566, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4128, 382, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{3/2}}-\frac{b\tanh(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[x]^2)^(-3/2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(3/2) - (b*Tanh[x])/
(a*(a + b)*Sqrt[a + b - b*Tanh[x]^2])
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}\,dx = \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\left(a+b-bx^2\right)^{3/2}}\,dx, x, \tanh(x)\right)$$
$$= -\frac{b\tanh(x)}{a(a+b)\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)}{a}$$
$$= -\frac{b\tanh(x)}{a(a+b)\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{3/2}} - \frac{b\tanh(x)}{a(a+b)\sqrt{a+b-b\tanh^2(x)}}$$

Mathematica [A] time = 0.810095, size = 107, normalized size = 1.88

$$\frac{\operatorname{sech}^3(x)(a\cosh(2x)+a+2b)\left((a+b)^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a+b}}\right)\sqrt{\frac{a\cosh(2x)+a+2b}{a+b}}-\sqrt{2}\sqrt{a}b\sinh(x)\right)}{2\sqrt{2}a^{3/2}(a+b)\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[x]^2)^(-3/2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*x])*Sech[x]^3*((a + b)^(3/2)*ArcSinh[(Sqrt[a]*Sinh[x])
/Sqrt[a + b]]*Sqrt[(a + 2*b + a*Cosh[2*x])/(a + b)] - Sqrt[2]*Sqrt[a]*b*Sin
h[x]))/(2*Sqrt[2]*a^(3/2)*(a + b)*(a + b*Sech[x]^2)^(3/2))
```

Maple [F] time = 0.085, size = 0, normalized size = 0.

$$\int \left(a+b\left(\operatorname{sech}(x)\right)^2\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(x)^2)^(3/2),x)
```

```
[Out] int(1/(a+b*sech(x)^2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\operatorname{sech}(x)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(x)^2 + a)^(-3/2), x)
```

Fricas [B] time = 2.81514, size = 5858, normalized size = 102.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b
)*sinh(x)^4 + 2*(a^2 + 3*a*b + 2*b^2)*cosh(x)^2 + 2*(3*(a^2 + a*b)*cosh(x)^
2 + a^2 + 3*a*b + 2*b^2)*sinh(x)^2 + a^2 + a*b + 4*((a^2 + a*b)*cosh(x)^3 +
 (a^2 + 3*a*b + 2*b^2)*cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a
*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(1
4*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b
^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*
b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(
x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b
+ 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a
*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^
2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x
)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*s
inh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*co
sh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a
^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))
*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*
sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 +
(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(co
sh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(
x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a^2 +
 a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 +
 2*(a^2 + 3*a*b + 2*b^2)*cosh(x)^2 + 2*(3*(a^2 + a*b)*cosh(x)^2 + a^2 + 3*a
*b + 2*b^2)*sinh(x)^2 + a^2 + a*b + 4*((a^2 + a*b)*cosh(x)^3 + (a^2 + 3*a*b
 + 2*b^2)*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)
^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^
2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a
*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x
)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(
x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*(a*b*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x)
 + a*b*sinh(x)^2 - a*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^4 + a^3*b)*cosh(x)^4 + 4*(a^4 + a
^3*b)*cosh(x)*sinh(x)^3 + (a^4 + a^3*b)*sinh(x)^4 + a^4 + a^3*b + 2*(a^4 +
3*a^3*b + 2*a^2*b^2)*cosh(x)^2 + 2*(a^4 + 3*a^3*b + 2*a^2*b^2 + 3*(a^4 + a^
3*b)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 + a^3*b)*cosh(x)^3 + (a^4 + 3*a^3*b + 2
*a^2*b^2)*cosh(x))*sinh(x)), -1/2*(((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*c
osh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + 2*(a^2 + 3*a*b + 2*b^2)*cosh(x)^
2 + 2*(3*(a^2 + a*b)*cosh(x)^2 + a^2 + 3*a*b + 2*b^2)*sinh(x)^2 + a^2 + a*b
 + 4*((a^2 + a*b)*cosh(x)^3 + (a^2 + 3*a*b + 2*b^2)*cosh(x))*sinh(x))*sqrt(
-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sq
rt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*si
nh(x) + sinh(x)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^
4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a
```

```
^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) + ((a^2 + a*b)*c
osh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + 2*(a^2
 + 3*a*b + 2*b^2)*cosh(x)^2 + 2*(3*(a^2 + a*b)*cosh(x)^2 + a^2 + 3*a*b + 2*
b^2)*sinh(x)^2 + a^2 + a*b + 4*((a^2 + a*b)*cosh(x)^3 + (a^2 + 3*a*b + 2*b^
2)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x)
 + sinh(x)^2 + 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^
3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(
x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + 2*sqrt(2)*(a*b*c
osh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2 - a*b)*sqrt((a*cosh(x)^2 +
 a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^4
 + a^3*b)*cosh(x)^4 + 4*(a^4 + a^3*b)*cosh(x)*sinh(x)^3 + (a^4 + a^3*b)*sin
h(x)^4 + a^4 + a^3*b + 2*(a^4 + 3*a^3*b + 2*a^2*b^2)*cosh(x)^2 + 2*(a^4 + 3
*a^3*b + 2*a^2*b^2 + 3*(a^4 + a^3*b)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 + a^3*b
)*cosh(x)^3 + (a^4 + 3*a^3*b + 2*a^2*b^2)*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\operatorname{sech}^2(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sech(x)**2)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\operatorname{sech}(x)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sech(x)^2 + a)^(-3/2), x)
```

3.209 $\int \frac{\coth(x)}{\left(a+b\,\mathbf{sech}^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=79

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{b}{a(a+b)\sqrt{a+b\text{sech}^2(x)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}$$

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/a^(3/2) - ArcTanh[Sqrt[a + b*Sech[x]
^2]/Sqrt[a + b]]/(a + b)^(3/2) - b/(a*(a + b)*Sqrt[a + b*Sech[x]^2])
```

Rubi [A] time = 0.140218, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4139, 446, 85, 156, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{b}{a(a+b)\sqrt{a+b\text{sech}^2(x)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/a^(3/2) - ArcTanh[Sqrt[a + b*Sech[x]
^2]/Sqrt[a + b]]/(a + b)^(3/2) - b/(a*(a + b)*Sqrt[a + b*Sech[x]^2])
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}\,dx &= \mathrm{Subst}\left(\int \frac{1}{x\left(-1+x^2\right)\left(a+bx^2\right)^{3/2}}\,dx, x, \mathrm{sech}(x)\right)\\
&= \frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{(-1+x)x(a+bx)^{3/2}}\,dx, x, \mathrm{sech}^2(x)\right)\\
&= -\frac{b}{a(a+b)\sqrt{a+b\,\mathrm{sech}^2(x)}}+\frac{\mathrm{Subst}\left(\int \frac{a+b-bx}{(-1+x)x\sqrt{a+bx}}\,dx, x, \mathrm{sech}^2(x)\right)}{2a(a+b)}\\
&= -\frac{b}{a(a+b)\sqrt{a+b\,\mathrm{sech}^2(x)}}-\frac{\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \mathrm{sech}^2(x)\right)}{2a}+\frac{\mathrm{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+bx}}\,dx,\right.}{2(a+b)}\\
&= -\frac{b}{a(a+b)\sqrt{a+b\,\mathrm{sech}^2(x)}}-\frac{\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\,\mathrm{sech}^2(x)}\right)}{ab}+\frac{\mathrm{Subst}\left(\int \frac{1}{-1-\frac{a}{b}+\frac{x^2}{b}}\right.}{b}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{b}{a(a+b)\sqrt{a+b\,\mathrm{sech}^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.650541, size = 155, normalized size = 1.96

$$\frac{\mathrm{sech}^2(x)\left(\frac{\sqrt{2}\,\mathrm{sech}(x)(a\cosh(2x)+a+2b)^{3/2}\left((a+b)^{3/2}\log\left(\sqrt{a\cosh(2x)+a+2b}+\sqrt{2}\sqrt{a}\cosh(x)\right)-a^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a+b}\cosh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)\right)}{\sqrt{a}\sqrt{a+b}}-2b(a\cosh(2x)+\right.}{4a(a+b)\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] (Sech[x]^2*(-2*b*(a + 2*b + a*Cosh[2*x]) + (Sqrt[2]*(a + 2*b + a*Cosh[2*x])
^(3/2)*(-(a^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Cosh[x])/Sqrt[a + 2*b + a*Co
```

```
sh[2*x]]]) + (a + b)^(3/2)*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*C
osh[2*x]]])*Sech[x])/(Sqrt[a]*Sqrt[a + b])))/(4*a*(a + b)*(a + b*Sech[x]^2)
^(3/2))
```

Maple [F] time = 0.101, size = 0, normalized size = 0.

$$\int \coth(x)\left(a+b(\operatorname{sech}(x))^2\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*sech(x)^2)^(3/2),x)
```

```
[Out] int(coth(x)/(a+b*sech(x)^2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\left(b\operatorname{sech}(x)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)/(b*sech(x)^2 + a)^(3/2), x)
```

Fricas [B] time = 4.7096, size = 18668, normalized size = 236.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 4*(a^3 + 2*a^2*b + a*b^2)*cosh(x
)*sinh(x)^3 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^4 + a^3 + 2*a^2*b + a*b^2 + 2
*(a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*cosh(x)^2 + 2*(a^3 + 4*a^2*b + 5*a*b^2 +
 2*b^3 + 3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 2*a^2*b
 + a*b^2)*cosh(x)^3 + (a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*cosh(x))*sinh(x))*s
qrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*c
osh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b +
 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 +
2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(
x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14
*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3
 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sin
h(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*
a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 +
a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)
^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(
6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b +
```

b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b
^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*
cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)
^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 +
 (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a
^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 +
 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a
*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x
)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b
^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a
^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sin
h(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x
)^5 + sinh(x)^6)) + 2*(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x
)^4 + a^3 + 2*(a^3 + 2*a^2*b)*cosh(x)^2 + 2*(3*a^3*cosh(x)^2 + a^3 + 2*a^2*
b)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + (a^3 + 2*a^2*b)*cosh(x))*sinh(x))*sqrt(a
+ b)*log(((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*s
inh(x)^4 + 2*(2*a + 3*b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a + 3*b)*
sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(
a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*si
nh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a + 3*b)*cosh(x))*sinh(x)
 + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 -
 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)) + ((a^3
 + 2*a^2*b + a*b^2)*cosh(x)^4 + 4*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^3
 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^4 + a^3 + 2*a^2*b + a*b^2 + 2*(a^3 + 4*a
^2*b + 5*a*b^2 + 2*b^3)*cosh(x)^2 + 2*(a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3 + 3*
(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 2*a^2*b + a*b^2)*c
osh(x)^3 + (a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*cosh(x))*sinh(x))*sqrt(a)*log(
-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*
a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(
x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2
*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(
cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*(a^2*b + a*b^2 + (a
^2*b + a*b^2)*cosh(x)^2 + 2*(a^2*b + a*b^2)*cosh(x)*sinh(x) + (a^2*b + a*b^
2)*sinh(x)^2)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2)))/(a^5 + 2*a^4*b + a^3*b^2 + (a^5 + 2*a^4*b + a^3
*b^2)*cosh(x)^4 + 4*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)*sinh(x)^3 + (a^5 + 2*
a^4*b + a^3*b^2)*sinh(x)^4 + 2*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*cosh
(x)^2 + 2*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + 3*(a^5 + 2*a^4*b + a^3*b
^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^3 + (a^5 +
4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*cosh(x))*sinh(x)), 1/4*(4*(a^3*cosh(x)^4 +
 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + a^3 + 2*(a^3 + 2*a^2*b)*cosh(x)^
2 + 2*(3*a^3*cosh(x)^2 + a^3 + 2*a^2*b)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + (a^3
 + 2*a^2*b)*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*co
sh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2
 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a
*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2
 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) +
 ((a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 4*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sin
h(x)^3 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^4 + a^3 + 2*a^2*b + a*b^2 + 2*(a^3
 + 4*a^2*b + 5*a*b^2 + 2*b^3)*cosh(x)^2 + 2*(a^3 + 4*a^2*b + 5*a*b^2 + 2*b^
3 + 3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 2*a^2*b + a*
b^2)*cosh(x)^3 + (a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*cosh(x))*sinh(x))*sqrt(a
)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x
)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*
b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2
*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3
+ 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*
b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14
*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^

```
4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2
 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 +
 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 +
15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3
 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*
cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*s
inh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(
x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 +
3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*
(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 +
4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*
b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh
(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))
 + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 +
b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)
*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^
2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 +
 sinh(x)^6)) + ((a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 4*(a^3 + 2*a^2*b + a*b^
2)*cosh(x)*sinh(x)^3 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^4 + a^3 + 2*a^2*b +
a*b^2 + 2*(a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*cosh(x)^2 + 2*(a^3 + 4*a^2*b +
5*a*b^2 + 2*b^3 + 3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3
 + 2*a^2*b + a*b^2)*cosh(x)^3 + (a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*cosh(x))*s
inh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2
*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a +
 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cos
h(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)
*(a^2*b + a*b^2 + (a^2*b + a*b^2)*cosh(x)^2 + 2*(a^2*b + a*b^2)*cosh(x)*sin
h(x) + (a^2*b + a*b^2)*sinh(x)^2)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b
)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^5 + 2*a^4*b + a^3*b^2 +
(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^4 + 4*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)*s
inh(x)^3 + (a^5 + 2*a^4*b + a^3*b^2)*sinh(x)^4 + 2*(a^5 + 4*a^4*b + 5*a^3*b
^2 + 2*a^2*b^3)*cosh(x)^2 + 2*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + 3*(a
^5 + 2*a^4*b + a^3*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^5 + 2*a^4*b + a^3*b^2)
*cosh(x)^3 + (a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*cosh(x))*sinh(x)), -1/
2*(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 4*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*s
inh(x)^3 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^4 + a^3 + 2*a^2*b + a*b^2 + 2*(a
^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*cosh(x)^2 + 2*(a^3 + 4*a^2*b + 5*a*b^2 + 2*
b^3 + 3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 2*a^2*b +
a*b^2)*cosh(x)^3 + (a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*cosh(x))*sinh(x))*sqrt
(-a)*arctan(sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b
)*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(
x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*
b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 +
(6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b
)*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(x))*sinh(x))) + ((a^3 + 2*a^2*b + a*b^2)
*cosh(x)^4 + 4*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^3 + (a^3 + 2*a^2*b +
 a*b^2)*sinh(x)^4 + a^3 + 2*a^2*b + a*b^2 + 2*(a^3 + 4*a^2*b + 5*a*b^2 + 2*
b^3)*cosh(x)^2 + 2*(a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3 + 3*(a^3 + 2*a^2*b + a*
b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + (a^3 + 4
*a^2*b + 5*a*b^2 + 2*b^3)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x
)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sin
h(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4
 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cos
h(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) +
 a)) - (a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + a^3 + 2*(
a^3 + 2*a^2*b)*cosh(x)^2 + 2*(3*a^3*cosh(x)^2 + a^3 + 2*a^2*b)*sinh(x)^2 +
4*(a^3*cosh(x)^3 + (a^3 + 2*a^2*b)*cosh(x))*sinh(x))*sqrt(a + b)*log(((2*a
+ b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2
```

```
*a + 3*b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a + 3*b)*sinh(x)^2 - 2*s
qrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt((a*
cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a + 3*b)*cosh(x))*sinh(x) + 2*a + b)/(co
sh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 -
 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)) + 2*sqrt(2)*(a^2*b + a
*b^2 + (a^2*b + a*b^2)*cosh(x)^2 + 2*(a^2*b + a*b^2)*cosh(x)*sinh(x) + (a^2
*b + a*b^2)*sinh(x)^2)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^5 + 2*a^4*b + a^3*b^2 + (a^5 + 2*a^
4*b + a^3*b^2)*cosh(x)^4 + 4*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)*sinh(x)^3 +
(a^5 + 2*a^4*b + a^3*b^2)*sinh(x)^4 + 2*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*
b^3)*cosh(x)^2 + 2*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + 3*(a^5 + 2*a^4*
b + a^3*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^3
+ (a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*cosh(x))*sinh(x)), -1/2*(((a^3 +
2*a^2*b + a*b^2)*cosh(x)^4 + 4*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^3 +
(a^3 + 2*a^2*b + a*b^2)*sinh(x)^4 + a^3 + 2*a^2*b + a*b^2 + 2*(a^3 + 4*a^2*
b + 5*a*b^2 + 2*b^3)*cosh(x)^2 + 2*(a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3 + 3*(a^
3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 2*a^2*b + a*b^2)*cosh
(x)^3 + (a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*cosh(x))*sinh(x))*sqrt(-a)*arctan
(sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2
 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*co
sh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*
sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a
*b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3
 + (2*a^2 + 3*a*b)*cosh(x))*sinh(x))) + ((a^3 + 2*a^2*b + a*b^2)*cosh(x)^4
+ 4*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^3 + (a^3 + 2*a^2*b + a*b^2)*sin
h(x)^4 + a^3 + 2*a^2*b + a*b^2 + 2*(a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*cosh(x
)^2 + 2*(a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3 + 3*(a^3 + 2*a^2*b + a*b^2)*cosh(x
)^2)*sinh(x)^2 + 4*((a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + (a^3 + 4*a^2*b + 5*
a*b^2 + 2*b^3)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a
+ 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh
(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a
+ 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - 2*(a
^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + a^3 + 2*(a^3 + 2*a
^2*b)*cosh(x)^2 + 2*(3*a^3*cosh(x)^2 + a^3 + 2*a^2*b)*sinh(x)^2 + 4*(a^3*co
sh(x)^3 + (a^3 + 2*a^2*b)*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt((a*cosh(x)^2
 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*c
osh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*
(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*s
inh(x) + a)) + 2*sqrt(2)*(a^2*b + a*b^2 + (a^2*b + a*b^2)*cosh(x)^2 + 2*(a^
2*b + a*b^2)*cosh(x)*sinh(x) + (a^2*b + a*b^2)*sinh(x)^2)*sqrt((a*cosh(x)^2
 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^
5 + 2*a^4*b + a^3*b^2 + (a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^4 + 4*(a^5 + 2*a^
4*b + a^3*b^2)*cosh(x)*sinh(x)^3 + (a^5 + 2*a^4*b + a^3*b^2)*sinh(x)^4 + 2*
(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*cosh(x)^2 + 2*(a^5 + 4*a^4*b + 5*a^
3*b^2 + 2*a^2*b^3 + 3*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((
a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^3 + (a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3
)*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\left(a + b\operatorname{sech}^2(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)**2)**(3/2),x)
```

```
[Out] Integral(coth(x)/(a + b*sech(x)**2)**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.210 $\int \frac{\coth^2(x)}{\left(a+b\,\mathbf{sech}^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=88

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{3/2}}-\frac{(a-b)\coth(x)\sqrt{a-b\tanh^2(x)+b}}{a(a+b)^2}-\frac{b\coth(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}$$

```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(3/2) - (b*Coth[x])/
(a*(a + b)*Sqrt[a + b - b*Tanh[x]^2]) - ((a - b)*Coth[x]*Sqrt[a + b - b*Tan
h[x]^2])/(a*(a + b)^2)
```

Rubi [A] time = 0.274126, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {4141, 1975, 472, 583, 12, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{3/2}}-\frac{(a-b)\coth(x)\sqrt{a-b\tanh^2(x)+b}}{a(a+b)^2}-\frac{b\coth(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(a + b*Sech[x]^2)^(3/2),x]
```


```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(3/2) - (b*Coth[x])/
(a*(a + b)*Sqrt[a + b - b*Tanh[x]^2]) - ((a - b)*Coth[x]*Sqrt[a + b - b*Tan
h[x]^2])/(a*(a + b)^2)
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^2(x)}{\left(a+b\text{sech}^2(x)\right)^{3/2}}\,dx &= \text{Subst}\left(\int \frac{1}{x^2\left(1-x^2\right)\left(a+b\left(1-x^2\right)\right)^{3/2}}\,dx,x,\tanh(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{x^2\left(1-x^2\right)\left(a+b-bx^2\right)^{3/2}}\,dx,x,\tanh(x)\right)\\
&= -\frac{b\coth(x)}{a(a+b)\sqrt{a+b-b\tanh^2(x)}}-\frac{\text{Subst}\left(\int \frac{-a+b-2bx^2}{x^2\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx,x,\tanh(x)\right)}{a(a+b)}\\
&= -\frac{b\coth(x)}{a(a+b)\sqrt{a+b-b\tanh^2(x)}}-\frac{(a-b)\coth(x)\sqrt{a+b-b\tanh^2(x)}}{a(a+b)^2}+\frac{\text{Subst}\left(\int \frac{(a+b)^2}{\left(1-x^2\right)\sqrt{a+b-}\cdots}\right.}{a(a+\cdots}\\
&= -\frac{b\coth(x)}{a(a+b)\sqrt{a+b-b\tanh^2(x)}}-\frac{(a-b)\coth(x)\sqrt{a+b-b\tanh^2(x)}}{a(a+b)^2}+\frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+b-}\cdots}\right.}{a}\\
&= -\frac{b\coth(x)}{a(a+b)\sqrt{a+b-b\tanh^2(x)}}-\frac{(a-b)\coth(x)\sqrt{a+b-b\tanh^2(x)}}{a(a+b)^2}+\frac{\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\cdots\right.}{a}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{3/2}}-\frac{b\coth(x)}{a(a+b)\sqrt{a+b-b\tanh^2(x)}}-\frac{(a-b)\coth(x)\sqrt{a+b-b\tanh^2(\cdots}}{a(a+b)^2}
\end{aligned}$$

Mathematica [A] time = 0.420422, size = 120, normalized size = 1.36

$$\frac{\operatorname{sech}^3(x)\left(\frac{\sqrt{2}(a\cosh(2x)+a+2b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)}{a^{3/2}}-\frac{(a\cosh(2x)+a+2b)\left(a\operatorname{csch}(x)(a\cosh(2x)+a+2b)+2b^2\sinh(x)\right)}{a(a+b)^2}\right)}{4\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] (Sech[x]^3*((Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cos
h[2*x]]]*(a + 2*b + a*Cosh[2*x])^(3/2))/a^(3/2) - ((a + 2*b + a*Cosh[2*x])*
(a*(a + 2*b + a*Cosh[2*x])*Csch[x] + 2*b^2*Sinh[x]))/(a*(a + b)^2)))/(4*(a
+ b*Sech[x]^2)^(3/2))
```

Maple [F] time = 0.103, size = 0, normalized size = 0.

$$\int (\coth (x))^2 \left(a+b (\operatorname{sech}(x))^2\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*sech(x)^2)^(3/2),x)
```

```
[Out] int(coth(x)^2/(a+b*sech(x)^2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)^2}{\left(b\operatorname{sech}(x)^2+a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)^2/(b*sech(x)^2 + a)^(3/2), x)
```

Fricas [B] time = 4.42849, size = 10016, normalized size = 113.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 6*(a^3 + 2*a^2*b + a*b^2)*cosh(x
)*sinh(x)^5 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^6 + (a^3 + 6*a^2*b + 9*a*b^2
+ 4*b^3)*cosh(x)^4 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3 + 15*(a^3 + 2*a^2*b +
 a*b^2)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + (a^
```

```
3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x))*sinh(x)^3 - a^3 - 2*a^2*b - a*b^2 -
 (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x)^2 + (15*(a^3 + 2*a^2*b + a*b^2)*
cosh(x)^4 - a^3 - 6*a^2*b - 9*a*b^2 - 4*b^3 + 6*(a^3 + 6*a^2*b + 9*a*b^2 +
4*b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 2*(a
^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x)^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^
3)*cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)
^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 -
a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*s
inh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3
+ 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*c
osh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*
sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*
(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(
x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*si
nh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*
cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*co
sh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x
)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cos
h(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)
) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a
*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^
5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*
sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a^3 + 2*a^2*b + a*b^2)*co
sh(x)^6 + 6*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^5 + (a^3 + 2*a^2*b + a*
b^2)*sinh(x)^6 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x)^4 + (a^3 + 6*a^2
*b + 9*a*b^2 + 4*b^3 + 15*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^4 + 4*
(5*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*co
sh(x))*sinh(x)^3 - a^3 - 2*a^2*b - a*b^2 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3
)*cosh(x)^2 + (15*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 - a^3 - 6*a^2*b - 9*a*b
^2 - 4*b^3 + 6*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x)^2)*sinh(x)^2 + 2*(
3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 2*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*c
osh(x)^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x))*sinh(x))*sqrt(a)*log(
-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 +
 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(
x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cos
h(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x)
)*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*((a
^3 + a*b^2)*cosh(x)^4 + 4*(a^3 + a*b^2)*cosh(x)*sinh(x)^3 + (a^3 + a*b^2)*s
inh(x)^4 + a^3 + a*b^2 + 2*(a^3 + 2*a^2*b - a*b^2)*cosh(x)^2 + 2*(a^3 + 2*a
^2*b - a*b^2 + 3*(a^3 + a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + a*b^2)*cosh
(x)^3 + (a^3 + 2*a^2*b - a*b^2)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sin
h(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^5 + 2*a
^4*b + a^3*b^2)*cosh(x)^6 + 6*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)*sinh(x)^5 +
 (a^5 + 2*a^4*b + a^3*b^2)*sinh(x)^6 - a^5 - 2*a^4*b - a^3*b^2 + (a^5 + 6*a
^4*b + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)^4 + (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^
2*b^3 + 15*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^5 + 2*a
^4*b + a^3*b^2)*cosh(x)^3 + (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)
)*sinh(x)^3 - (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)^2 - (a^5 + 6*
a^4*b + 9*a^3*b^2 + 4*a^2*b^3 - 15*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^4 - 6*
(a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a^5 +
2*a^4*b + a^3*b^2)*cosh(x)^5 + 2*(a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*co
sh(x)^3 - (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x))*sinh(x)), -1/2*(
((a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 6*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh
(x)^5 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^6 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^
3)*cosh(x)^4 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3 + 15*(a^3 + 2*a^2*b + a*b^2
)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + (a^3 + 6*
a^2*b + 9*a*b^2 + 4*b^3)*cosh(x))*sinh(x)^3 - a^3 - 2*a^2*b - a*b^2 - (a^3
+ 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x)^2 + (15*(a^3 + 2*a^2*b + a*b^2)*cosh(x
)^4 - a^3 - 6*a^2*b - 9*a*b^2 - 4*b^3 + 6*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)
```

```
*cosh(x)^2)*sinh(x)^2 + 2*(3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 2*(a^3 + 6
*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x)^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cos
h(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) +
 b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh
(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 -
3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x
))) + ((a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 6*(a^3 + 2*a^2*b + a*b^2)*cosh(x
)*sinh(x)^5 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^6 + (a^3 + 6*a^2*b + 9*a*b^2
+ 4*b^3)*cosh(x)^4 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3 + 15*(a^3 + 2*a^2*b +
 a*b^2)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + (a^
3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x))*sinh(x)^3 - a^3 - 2*a^2*b - a*b^2 -
 (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x)^2 + (15*(a^3 + 2*a^2*b + a*b^2)*
cosh(x)^4 - a^3 - 6*a^2*b - 9*a*b^2 - 4*b^3 + 6*(a^3 + 6*a^2*b + 9*a*b^2 +
4*b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 2*(a
^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x)^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^
3)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x)
 + sinh(x)^2 + 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^
3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(
x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + 2*sqrt(2)*((a^3
+ a*b^2)*cosh(x)^4 + 4*(a^3 + a*b^2)*cosh(x)*sinh(x)^3 + (a^3 + a*b^2)*sinh
(x)^4 + a^3 + a*b^2 + 2*(a^3 + 2*a^2*b - a*b^2)*cosh(x)^2 + 2*(a^3 + 2*a^2*
b - a*b^2 + 3*(a^3 + a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + a*b^2)*cosh(x)
^3 + (a^3 + 2*a^2*b - a*b^2)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x
)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^5 + 2*a^4*
b + a^3*b^2)*cosh(x)^6 + 6*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)*sinh(x)^5 + (a
^5 + 2*a^4*b + a^3*b^2)*sinh(x)^6 - a^5 - 2*a^4*b - a^3*b^2 + (a^5 + 6*a^4*
b + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)^4 + (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b
^3 + 15*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^5 + 2*a^4*
b + a^3*b^2)*cosh(x)^3 + (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x))*s
inh(x)^3 - (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)^2 - (a^5 + 6*a^4
*b + 9*a^3*b^2 + 4*a^2*b^3 - 15*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^4 - 6*(a^
5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a^5 + 2*a
^4*b + a^3*b^2)*cosh(x)^5 + 2*(a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*cosh(
x)^3 - (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(x)}{\left(a + b\operatorname{sech}^2(x)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+b*sech(x)**2)**(3/2),x)
```

```
[Out] Integral(coth(x)**2/(a + b*sech(x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)^2}{\left(b\operatorname{sech}(x)^2 + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")

[Out] integrate(coth(x)^2/(b*sech(x)^2 + a)^(3/2), x)
```

3.211 $\int \frac{\tanh^6(x)}{\left(a+b\,\mathbf{sech}^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=118

$$-\frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{b^{5/2}}-\frac{(a+b)\tanh^3(x)}{3ab\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

```
[Out] -(ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/b^(5/2)) + ArcTanh[(S
qrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(5/2) - ((a + b)*Tanh[x]^3)/(3
*a*b*(a + b - b*Tanh[x]^2)^(3/2)) - ((a^(-2) - b^(-2))*Tanh[x])/Sqrt[a + b
- b*Tanh[x]^2]
```

Rubi [A] time = 0.338873, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {4141, 1975, 470, 578, 523, 217, 203, 377, 206}

$$-\frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{b^{5/2}}-\frac{(a+b)\tanh^3(x)}{3ab\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^6/(a + b*Sech[x]^2)^(5/2),x]
```


```
[Out] -(ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/b^(5/2)) + ArcTanh[(S
qrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(5/2) - ((a + b)*Tanh[x]^3)/(3
*a*b*(a + b - b*Tanh[x]^2)^(3/2)) - ((a^(-2) - b^(-2))*Tanh[x])/Sqrt[a + b
- b*Tanh[x]^2]
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tanh^6(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{x^6}{(1-x^2)\left(a+b(1-x^2)\right)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= \text{Subst}\left(\int \frac{x^6}{(1-x^2)\left(a+b-bx^2\right)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= -\frac{(a+b)\tanh^3(x)}{3ab\left(a+b-b\tanh^2(x)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)-3ax^2)}{(1-x^2)(a+b-bx^2)^{3/2}}\,dx, x, \tanh(x)\right)}{3ab}$$

$$= -\frac{(a+b)\tanh^3(x)}{3ab\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}} - \frac{\text{Subst}\left(\int \frac{3(a^2-b^2)-3a^2x^2}{(1-x^2)\sqrt{a+b-bx^2}}\,dx, x, \tanh\right.}{3a^2b^2}$$

$$= -\frac{(a+b)\tanh^3(x)}{3ab\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}}\,dx, x, \tanh\right.}{a^2}$$

$$= -\frac{(a+b)\tanh^3(x)}{3ab\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(}}\right.}{a^2}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{b^{5/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{5/2}} - \frac{(a+b)\tanh^3(x)}{3ab\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{\left(\frac{1}{a^2}\right.}{\sqrt{a+}}$$

Mathematica [A] time = 0.692483, size = 178, normalized size = 1.51

$$\frac{\text{sech}^5(x)\left(\frac{2(a+b)\sinh(x)\left(3a^2+a(3a-4b)\cosh(2x)+4ab-6b^2\right)(a\cosh(2x)+a+2b)}{3a^2b^2} + \frac{\sqrt{2}(a\cosh(2x)+a+2b)^{5/2}\left(b^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)-a^{5/2}\tan^{-1}\right.}{a^{5/2}b^{5/2}}\right.}{8\left(a+b\text{sech}^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^6/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] (Sech[x]^5*((Sqrt[2]*(-(a^(5/2)*ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2
*b + a*Cosh[2*x]]]) + b^(5/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*
b + a*Cosh[2*x]]])*(a + 2*b + a*Cosh[2*x])^(5/2))/(a^(5/2)*b^(5/2)) + (2*(a
 + b)*(a + 2*b + a*Cosh[2*x])*(3*a^2 + 4*a*b - 6*b^2 + a*(3*a - 4*b)*Cosh[2
*x])*Sinh[x])/(3*a^2*b^2)))/(8*(a + b*Sech[x]^2)^(5/2))
```

Maple [F] time = 0.106, size = 0, normalized size = 0.

$$\int (\tanh(x))^6 \left(a+b(\text{sech}(x))^2\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^6/(a+b*sech(x)^2)^(5/2),x)
```

```
[Out] int(tanh(x)^6/(a+b*sech(x)^2)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^6}{\left(b\,\mathrm{sech}(x)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^6/(b*sech(x)^2 + a)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^6(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**6/(a+b*sech(x)**2)**(5/2),x)
```

```
[Out] Integral(tanh(x)**6/(a + b*sech(x)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^6}{\left(b\,\mathrm{sech}(x)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(x)^6/(b*sech(x)^2 + a)^(5/2), x)
```

3.212 $\int \frac{\tanh^5(x)}{\left(a+b\,\mathbf{sech}^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=76

$$-\frac{\frac{1}{a^2}-\frac{1}{b^2}}{\sqrt{a+b\,\mathrm{sech}^2(x)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{(a+b)^2}{3ab^2\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}$$

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/a^(5/2) - (a + b)^2/(3*a*b^2*(a + b*
Sech[x]^2)^(3/2)) - (a^(-2) - b^(-2))/Sqrt[a + b*Sech[x]^2]
```

Rubi [A] time = 0.157898, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {4139, 446, 87, 63, 208}

$$-\frac{\frac{1}{a^2}-\frac{1}{b^2}}{\sqrt{a+b\,\mathrm{sech}^2(x)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{(a+b)^2}{3ab^2\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^5/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/a^(5/2) - (a + b)^2/(3*a*b^2*(a + b*
Sech[x]^2)^(3/2)) - (a^(-2) - b^(-2))/Sqrt[a + b*Sech[x]^2]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^5(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx &= -\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x\left(a+bx^2\right)^{5/2}}\,dx, x, \text{sech}(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(-1+x)^2}{x(a+bx)^{5/2}}\,dx, x, \text{sech}^2(x)\right)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \left(-\frac{(a+b)^2}{ab(a+bx)^{5/2}}+\frac{a^2-b^2}{a^2b(a+bx)^{3/2}}+\frac{1}{a^2x\sqrt{a+bx}}\right)dx, x, \text{sech}^2(x)\right)\right)\\
&= -\frac{(a+b)^2}{3ab^2\left(a+b\text{sech}^2(x)\right)^{3/2}}-\frac{\frac{1}{a^2}-\frac{1}{b^2}}{\sqrt{a+b\text{sech}^2(x)}}-\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \text{sech}^2(x)\right)}{2a^2}\\
&= -\frac{(a+b)^2}{3ab^2\left(a+b\text{sech}^2(x)\right)^{3/2}}-\frac{\frac{1}{a^2}-\frac{1}{b^2}}{\sqrt{a+b\text{sech}^2(x)}}-\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{sech}^2(x)}\right)}{a^2b}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{(a+b)^2}{3ab^2\left(a+b\text{sech}^2(x)\right)^{3/2}}-\frac{\frac{1}{a^2}-\frac{1}{b^2}}{\sqrt{a+b\text{sech}^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.521306, size = 126, normalized size = 1.66

$$\frac{\text{sech}^5(x)\left(\frac{4(a+b)\cosh(x)\left(a^2+a(a-2b)\cosh(2x)+ab-3b^2\right)(a\cosh(2x)+a+2b)}{3a^2b^2}+\frac{\sqrt{2}(a\cosh(2x)+a+2b)^{5/2}\log\left(\sqrt{a\cosh(2x)+a+2b}+\sqrt{2}\sqrt{a}\cosh(x)\right)}{a^{5/2}}\right)}{8\left(a+b\text{sech}^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] (((4*(a + b)*Cosh[x]*(a + 2*b + a*Cosh[2*x])*(a^2 + a*b - 3*b^2 + a*(a - 2*
b)*Cosh[2*x]))/(3*a^2*b^2) + (Sqrt[2]*(a + 2*b + a*Cosh[2*x])^(5/2)*Log[Sqr
t[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*Cosh[2*x]]])/a^(5/2))*Sech[x]^5)/(8
*(a + b*Sech[x]^2)^(5/2))
```

Maple [F] time = 0.108, size = 0, normalized size = 0.

$$\int (\tanh(x))^5\left(a+b(\text{sech}(x))^2\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a+b*sech(x)^2)^(5/2),x)

[Out] int(tanh(x)^5/(a+b*sech(x)^2)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^5}{\left(b\,\mathrm{sech}(x)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")

[Out] integrate(tanh(x)^5/(b*sech(x)^2 + a)^(5/2), x)
```

Fricas [B] time = 4.85522, size = 12878, normalized size = 169.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")

[Out] [1/12*(3*(a^2*b^2*cosh(x)^8 + 8*a^2*b^2*cosh(x)*sinh(x)^7 + a^2*b^2*sinh(x)
^8 + 4*(a^2*b^2 + 2*a*b^3)*cosh(x)^6 + 4*(7*a^2*b^2*cosh(x)^2 + a^2*b^2 + 2
*a*b^3)*sinh(x)^6 + 8*(7*a^2*b^2*cosh(x)^3 + 3*(a^2*b^2 + 2*a*b^3)*cosh(x))
*sinh(x)^5 + 2*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*cosh(x)^4 + 2*(35*a^2*b^2*cosh
(x)^4 + 3*a^2*b^2 + 8*a*b^3 + 8*b^4 + 30*(a^2*b^2 + 2*a*b^3)*cosh(x)^2)*sin
h(x)^4 + a^2*b^2 + 8*(7*a^2*b^2*cosh(x)^5 + 10*(a^2*b^2 + 2*a*b^3)*cosh(x)^
3 + (3*a^2*b^2 + 8*a*b^3 + 8*b^4)*cosh(x))*sinh(x)^3 + 4*(a^2*b^2 + 2*a*b^3
)*cosh(x)^2 + 4*(7*a^2*b^2*cosh(x)^6 + 15*(a^2*b^2 + 2*a*b^3)*cosh(x)^4 + a
^2*b^2 + 2*a*b^3 + 3*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*cosh(x)^2)*sinh(x)^2 + 8
*(a^2*b^2*cosh(x)^7 + 3*(a^2*b^2 + 2*a*b^3)*cosh(x)^5 + (3*a^2*b^2 + 8*a*b^
3 + 8*b^4)*cosh(x)^3 + (a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(x))*sqrt(a)*log(((
a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x
)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^
3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b
^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a
^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*
b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b +
 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(1
4*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*
cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^
3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^
3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^
2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^
6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6
 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a
^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 +
 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2
*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*s
inh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)
*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 +
```

a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*
(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cos
h(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x)
)*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*c
osh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)
^6)) + 3*(a^2*b^2*cosh(x)^8 + 8*a^2*b^2*cosh(x)*sinh(x)^7 + a^2*b^2*sinh(x)
^8 + 4*(a^2*b^2 + 2*a*b^3)*cosh(x)^6 + 4*(7*a^2*b^2*cosh(x)^2 + a^2*b^2 + 2
*a*b^3)*sinh(x)^6 + 8*(7*a^2*b^2*cosh(x)^3 + 3*(a^2*b^2 + 2*a*b^3)*cosh(x))
sinh(x)^5 + 2(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*cosh(x)^4 + 2*(35*a^2*b^2*cosh
(x)^4 + 3*a^2*b^2 + 8*a*b^3 + 8*b^4 + 30*(a^2*b^2 + 2*a*b^3)*cosh(x)^2)*sin
h(x)^4 + a^2*b^2 + 8*(7*a^2*b^2*cosh(x)^5 + 10*(a^2*b^2 + 2*a*b^3)*cosh(x)^
3 + (3*a^2*b^2 + 8*a*b^3 + 8*b^4)*cosh(x))*sinh(x)^3 + 4*(a^2*b^2 + 2*a*b^3
)*cosh(x)^2 + 4*(7*a^2*b^2*cosh(x)^6 + 15*(a^2*b^2 + 2*a*b^3)*cosh(x)^4 + a
^2*b^2 + 2*a*b^3 + 3*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*cosh(x)^2)*sinh(x)^2 + 8
*(a^2*b^2*cosh(x)^7 + 3*(a^2*b^2 + 2*a*b^3)*cosh(x)^5 + (3*a^2*b^2 + 8*a*b^
3 + 8*b^4)*cosh(x)^3 + (a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(x))*sqrt(a)*log(-(
a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*
cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 8*sqrt(2)*((a^4 - a^3*b - 2*a^2
*b^2)*cosh(x)^6 + 6*(a^4 - a^3*b - 2*a^2*b^2)*cosh(x)*sinh(x)^5 + (a^4 - a^
3*b - 2*a^2*b^2)*sinh(x)^6 + 3*(a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3)*cosh(x)^
4 + 3*(a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3 + 5*(a^4 - a^3*b - 2*a^2*b^2)*cosh
(x)^2)*sinh(x)^4 + a^4 - a^3*b - 2*a^2*b^2 + 4*(5*(a^4 - a^3*b - 2*a^2*b^2)
cosh(x)^3 + 3(a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3)*cosh(x))*sinh(x)^3 + 3*(
a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3)*cosh(x)^2 + 3*(5*(a^4 - a^3*b - 2*a^2*b^
2)*cosh(x)^4 + a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3 + 6*(a^4 + a^3*b - 2*a^2*b
^2 - 2*a*b^3)*cosh(x)^2)*sinh(x)^2 + 6*((a^4 - a^3*b - 2*a^2*b^2)*cosh(x)^5
 + 2*(a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3)*cosh(x)^3 + (a^4 + a^3*b - 2*a^2*b
^2 - 2*a*b^3)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^5*b^2*cosh(x)^8 + 8*a^5*b^
2*cosh(x)*sinh(x)^7 + a^5*b^2*sinh(x)^8 + a^5*b^2 + 4*(a^5*b^2 + 2*a^4*b^3)
cosh(x)^6 + 4(7*a^5*b^2*cosh(x)^2 + a^5*b^2 + 2*a^4*b^3)*sinh(x)^6 + 8*(7
*a^5*b^2*cosh(x)^3 + 3*(a^5*b^2 + 2*a^4*b^3)*cosh(x))*sinh(x)^5 + 2*(3*a^5*
b^2 + 8*a^4*b^3 + 8*a^3*b^4)*cosh(x)^4 + 2*(35*a^5*b^2*cosh(x)^4 + 3*a^5*b^
2 + 8*a^4*b^3 + 8*a^3*b^4 + 30*(a^5*b^2 + 2*a^4*b^3)*cosh(x)^2)*sinh(x)^4 +
 8*(7*a^5*b^2*cosh(x)^5 + 10*(a^5*b^2 + 2*a^4*b^3)*cosh(x)^3 + (3*a^5*b^2 +
 8*a^4*b^3 + 8*a^3*b^4)*cosh(x))*sinh(x)^3 + 4*(a^5*b^2 + 2*a^4*b^3)*cosh(x
)^2 + 4*(7*a^5*b^2*cosh(x)^6 + a^5*b^2 + 2*a^4*b^3 + 15*(a^5*b^2 + 2*a^4*b^
3)*cosh(x)^4 + 3*(3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*cosh(x)^2)*sinh(x)^2 +
 8*(a^5*b^2*cosh(x)^7 + 3*(a^5*b^2 + 2*a^4*b^3)*cosh(x)^5 + (3*a^5*b^2 + 8*
a^4*b^3 + 8*a^3*b^4)*cosh(x)^3 + (a^5*b^2 + 2*a^4*b^3)*cosh(x))*sinh(x)), -
1/6*(3*(a^2*b^2*cosh(x)^8 + 8*a^2*b^2*cosh(x)*sinh(x)^7 + a^2*b^2*sinh(x)^8
 + 4*(a^2*b^2 + 2*a*b^3)*cosh(x)^6 + 4*(7*a^2*b^2*cosh(x)^2 + a^2*b^2 + 2*a
*b^3)*sinh(x)^6 + 8*(7*a^2*b^2*cosh(x)^3 + 3*(a^2*b^2 + 2*a*b^3)*cosh(x))*s
inh(x)^5 + 2*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*cosh(x)^4 + 2*(35*a^2*b^2*cosh(x
)^4 + 3*a^2*b^2 + 8*a*b^3 + 8*b^4 + 30*(a^2*b^2 + 2*a*b^3)*cosh(x)^2)*sinh(
x)^4 + a^2*b^2 + 8*(7*a^2*b^2*cosh(x)^5 + 10*(a^2*b^2 + 2*a*b^3)*cosh(x)^3
+ (3*a^2*b^2 + 8*a*b^3 + 8*b^4)*cosh(x))*sinh(x)^3 + 4*(a^2*b^2 + 2*a*b^3)*
cosh(x)^2 + 4*(7*a^2*b^2*cosh(x)^6 + 15*(a^2*b^2 + 2*a*b^3)*cosh(x)^4 + a^2
*b^2 + 2*a*b^3 + 3*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*cosh(x)^2)*sinh(x)^2 + 8*(
a^2*b^2*cosh(x)^7 + 3*(a^2*b^2 + 2*a*b^3)*cosh(x)^5 + (3*a^2*b^2 + 8*a*b^3
+ 8*b^4)*cosh(x)^3 + (a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(x))*sqrt(-a)*arctan(
sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2
+ a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*s
inh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*
b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3

```
+ (2*a^2 + 3*a*b)*cosh(x))*sinh(x))) + 3*(a^2*b^2*cosh(x)^8 + 8*a^2*b^2*cos
h(x)*sinh(x)^7 + a^2*b^2*sinh(x)^8 + 4*(a^2*b^2 + 2*a*b^3)*cosh(x)^6 + 4*(7
*a^2*b^2*cosh(x)^2 + a^2*b^2 + 2*a*b^3)*sinh(x)^6 + 8*(7*a^2*b^2*cosh(x)^3
+ 3*(a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(x)^5 + 2*(3*a^2*b^2 + 8*a*b^3 + 8*b^4
)*cosh(x)^4 + 2*(35*a^2*b^2*cosh(x)^4 + 3*a^2*b^2 + 8*a*b^3 + 8*b^4 + 30*(a
^2*b^2 + 2*a*b^3)*cosh(x)^2)*sinh(x)^4 + a^2*b^2 + 8*(7*a^2*b^2*cosh(x)^5 +
 10*(a^2*b^2 + 2*a*b^3)*cosh(x)^3 + (3*a^2*b^2 + 8*a*b^3 + 8*b^4)*cosh(x))*
sinh(x)^3 + 4*(a^2*b^2 + 2*a*b^3)*cosh(x)^2 + 4*(7*a^2*b^2*cosh(x)^6 + 15*(
a^2*b^2 + 2*a*b^3)*cosh(x)^4 + a^2*b^2 + 2*a*b^3 + 3*(3*a^2*b^2 + 8*a*b^3 +
 8*b^4)*cosh(x)^2)*sinh(x)^2 + 8*(a^2*b^2*cosh(x)^7 + 3*(a^2*b^2 + 2*a*b^3)
*cosh(x)^5 + (3*a^2*b^2 + 8*a*b^3 + 8*b^4)*cosh(x)^3 + (a^2*b^2 + 2*a*b^3)*
cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 +
 a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^
2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - 4*sqrt(2)*((a^4 - a
^3*b - 2*a^2*b^2)*cosh(x)^6 + 6*(a^4 - a^3*b - 2*a^2*b^2)*cosh(x)*sinh(x)^5
 + (a^4 - a^3*b - 2*a^2*b^2)*sinh(x)^6 + 3*(a^4 + a^3*b - 2*a^2*b^2 - 2*a*b
^3)*cosh(x)^4 + 3*(a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3 + 5*(a^4 - a^3*b - 2*a
^2*b^2)*cosh(x)^2)*sinh(x)^4 + a^4 - a^3*b - 2*a^2*b^2 + 4*(5*(a^4 - a^3*b
- 2*a^2*b^2)*cosh(x)^3 + 3*(a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3)*cosh(x))*sin
h(x)^3 + 3*(a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3)*cosh(x)^2 + 3*(5*(a^4 - a^3*
b - 2*a^2*b^2)*cosh(x)^4 + a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3 + 6*(a^4 + a^3
*b - 2*a^2*b^2 - 2*a*b^3)*cosh(x)^2)*sinh(x)^2 + 6*((a^4 - a^3*b - 2*a^2*b^
2)*cosh(x)^5 + 2*(a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3)*cosh(x)^3 + (a^4 + a^3
*b - 2*a^2*b^2 - 2*a*b^3)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2
 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^5*b^2*cosh(x)^
8 + 8*a^5*b^2*cosh(x)*sinh(x)^7 + a^5*b^2*sinh(x)^8 + a^5*b^2 + 4*(a^5*b^2
+ 2*a^4*b^3)*cosh(x)^6 + 4*(7*a^5*b^2*cosh(x)^2 + a^5*b^2 + 2*a^4*b^3)*sinh
(x)^6 + 8*(7*a^5*b^2*cosh(x)^3 + 3*(a^5*b^2 + 2*a^4*b^3)*cosh(x))*sinh(x)^5
 + 2*(3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*cosh(x)^4 + 2*(35*a^5*b^2*cosh(x)^
4 + 3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4 + 30*(a^5*b^2 + 2*a^4*b^3)*cosh(x)^2)
*sinh(x)^4 + 8*(7*a^5*b^2*cosh(x)^5 + 10*(a^5*b^2 + 2*a^4*b^3)*cosh(x)^3 +
(3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*cosh(x))*sinh(x)^3 + 4*(a^5*b^2 + 2*a^4
*b^3)*cosh(x)^2 + 4*(7*a^5*b^2*cosh(x)^6 + a^5*b^2 + 2*a^4*b^3 + 15*(a^5*b^
2 + 2*a^4*b^3)*cosh(x)^4 + 3*(3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*cosh(x)^2)
*sinh(x)^2 + 8*(a^5*b^2*cosh(x)^7 + 3*(a^5*b^2 + 2*a^4*b^3)*cosh(x)^5 + (3*
a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*cosh(x)^3 + (a^5*b^2 + 2*a^4*b^3)*cosh(x))
*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^5(x)}{(a + b\operatorname{sech}^2(x))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/(a+b*sech(x)**2)**(5/2),x)
```

```
[Out] Integral(tanh(x)**5/(a + b*sech(x)**2)**(5/2), x)
```

<u>**Giac**</u> **[F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^5}{\left(b\operatorname{sech}(x)^2 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(x)^5/(b*sech(x)^2 + a)^(5/2), x)
```

3.213 $\int \frac{\tanh^4(x)}{\left(a+b\mathbf{sech}^2(x)\right)^{5/2}} dx$

Optimal. Leaf size=90

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}}+\frac{(a-3b)\tanh(x)}{3a^2b\sqrt{a-b\tanh^2(x)+b}}-\frac{(a+b)\tanh(x)}{3ab\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(5/2) - ((a + b)*Tan
h[x])/(3*a*b*(a + b - b*Tanh[x]^2)^(3/2)) + ((a - 3*b)*Tanh[x])/(3*a^2*b*Sq
rt[a + b - b*Tanh[x]^2])
```

Rubi [A] time = 0.262349, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {4141, 1975, 470, 527, 12, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}}+\frac{(a-3b)\tanh(x)}{3a^2b\sqrt{a-b\tanh^2(x)+b}}-\frac{(a+b)\tanh(x)}{3ab\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(5/2) - ((a + b)*Tan
h[x])/(3*a*b*(a + b - b*Tanh[x]^2)^(3/2)) + ((a - 3*b)*Tanh[x])/(3*a^2*b*Sq
rt[a + b - b*Tanh[x]^2])
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
```

```
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tanh^4(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)\left(a+b\left(1-x^2\right)\right)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= \text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)\left(a+b-bx^2\right)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= -\frac{(a+b)\tanh(x)}{3ab\left(a+b-b\tanh^2(x)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a+b+(-a+2b)x^2}{\left(1-x^2\right)\left(a+b-bx^2\right)^{3/2}}\,dx, x, \tanh(x)\right)}{3ab}$$

$$= -\frac{(a+b)\tanh(x)}{3ab\left(a+b-b\tanh^2(x)\right)^{3/2}} + \frac{(a-3b)\tanh(x)}{3a^2b\sqrt{a+b-b\tanh^2(x)}} - \frac{\text{Subst}\left(\int -\frac{3b(a+b)}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx, x, \text{t}\right.}{3a^2b(a+b)}$$

$$= -\frac{(a+b)\tanh(x)}{3ab\left(a+b-b\tanh^2(x)\right)^{3/2}} + \frac{(a-3b)\tanh(x)}{3a^2b\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx, x, \text{ta}\right.}{a^2}$$

$$= -\frac{(a+b)\tanh(x)}{3ab\left(a+b-b\tanh^2(x)\right)^{3/2}} + \frac{(a-3b)\tanh(x)}{3a^2b\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh}}\right.}{a^2}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{5/2}} - \frac{(a+b)\tanh(x)}{3ab\left(a+b-b\tanh^2(x)\right)^{3/2}} + \frac{(a-3b)\tanh(x)}{3a^2b\sqrt{a+b-b\tanh^2(x)}}$$

Mathematica [B] time = 2.01579, size = 290, normalized size = 3.22

$$\frac{\operatorname{sech}^4(x)\left(\frac{\sqrt{2}\operatorname{csch}(x)\operatorname{sech}(x)\left(-\frac{16\left(a\sinh^2(x)+a+b\right)\left(\frac{a\sinh^2(x)}{a+b}+1\right)\left(\frac{a^2(a+b)\sinh^4(x)}{\left(a\sinh^2(x)+a+b\right)^2}+\frac{3a(a+b)\sinh^2(x)}{a\sinh^2(x)+a+b}-\frac{3\sqrt{a}\sqrt{a+b}\sinh(x)\sinh^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a+b}}\right)}{\sqrt{\frac{a\sinh^2(x)+a+b}{a+b}}}\right)}{a^3}+\frac{12\sinh^4(x)}{a+b}+\frac{2\sin\cdots}{\cdots}\cdots\right)}{\left(a\sinh^2(x)+a+b\right)^{3/2}}\cdots\right)}{384\left(a+\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] (Sech[x]^4*((Sqrt[2]*(a + 2*b + a*Cosh[2*x])^(5/2)*Csch[x]*Sech[x]*(Sinh[x]
^2/(a + b) + (12*Sinh[x]^4)/(a + b) + (2*Sinh[x]^2*(a + b + a*Sinh[x]^2))/(
a + b)^2 - (16*(a + b + a*Sinh[x]^2)*(1 + (a*Sinh[x]^2)/(a + b))*((a^2*(a +
 b)*Sinh[x]^4)/(a + b + a*Sinh[x]^2)^2 + (3*a*(a + b)*Sinh[x]^2)/(a + b + a
*Sinh[x]^2) - (3*Sqrt[a]*Sqrt[a + b]*ArcSinh[(Sqrt[a]*Sinh[x])/Sqrt[a + b]]
*Sinh[x])/Sqrt[(a + b + a*Sinh[x]^2)/(a + b)]))/a^3))/(a + b + a*Sinh[x]^2)
^(3/2) + (8*(a + 2*b + a*Cosh[2*x])*(2*a + 3*b + a*Cosh[2*x])*Tanh[x])/(a +
 b)^2 - (12*(a + 2*b + a*Cosh[2*x])*(b + (3*a + 2*b)*Cosh[2*x])*Tanh[x])/(a
 + b)^2))/(384*(a + b*Sech[x]^2)^(5/2))
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int (\tanh(x))^4\left(a+b(\operatorname{sech}(x))^2\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*sech(x)^2)^(5/2),x)
```

```
[Out] int(tanh(x)^4/(a+b*sech(x)^2)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^4}{\left(b\operatorname{sech}(x)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^4/(b*sech(x)^2 + a)^(5/2), x)
```

Fricas [B] time = 4.46926, size = 9694, normalized size = 107.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2*sinh(x)^8 + 4*(a^2
+ 2*a*b)*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 + a^2 + 2*a*b)*sinh(x)^6 + 8*(7*a^2
*cosh(x)^3 + 3*(a^2 + 2*a*b)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 8*a*b + 8*b^2)
*cosh(x)^4 + 2*(35*a^2*cosh(x)^4 + 30*(a^2 + 2*a*b)*cosh(x)^2 + 3*a^2 + 8*a
*b + 8*b^2)*sinh(x)^4 + 8*(7*a^2*cosh(x)^5 + 10*(a^2 + 2*a*b)*cosh(x)^3 + (
3*a^2 + 8*a*b + 8*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + 2*a*b)*cosh(x)^2 + 4*(
7*a^2*cosh(x)^6 + 15*(a^2 + 2*a*b)*cosh(x)^4 + 3*(3*a^2 + 8*a*b + 8*b^2)*co
sh(x)^2 + a^2 + 2*a*b)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x)^7 + 3*(a^2 + 2*a*b)
*cosh(x)^5 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x)^3 + (a^2 + 2*a*b)*cosh(x))*sin
h(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh
(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*si
nh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3
 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a
*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(
a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3
 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*c
osh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2
 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2
*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b
^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^
2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cos
h(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh
(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*
cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3
 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*
cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*c
osh(x)*sinh(x)^5 + sinh(x)^6)) + 3*(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7
 + a^2*sinh(x)^8 + 4*(a^2 + 2*a*b)*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 + a^2 + 2
*a*b)*sinh(x)^6 + 8*(7*a^2*cosh(x)^3 + 3*(a^2 + 2*a*b)*cosh(x))*sinh(x)^5 +
 2*(3*a^2 + 8*a*b + 8*b^2)*cosh(x)^4 + 2*(35*a^2*cosh(x)^4 + 30*(a^2 + 2*a*
b)*cosh(x)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(x)^4 + 8*(7*a^2*cosh(x)^5 + 10*(
a^2 + 2*a*b)*cosh(x)^3 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x))*sinh(x)^3 + 4*(a^
2 + 2*a*b)*cosh(x)^2 + 4*(7*a^2*cosh(x)^6 + 15*(a^2 + 2*a*b)*cosh(x)^4 + 3*
(3*a^2 + 8*a*b + 8*b^2)*cosh(x)^2 + a^2 + 2*a*b)*sinh(x)^2 + a^2 + 8*(a^2*c
osh(x)^7 + 3*(a^2 + 2*a*b)*cosh(x)^5 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x)^3 +
(a^2 + 2*a*b)*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sin
h(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh
(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqr
t((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + si
nh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*c
osh(x)*sinh(x) + sinh(x)^2)) - 16*sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*si
nh(x)^5 + a^2*sinh(x)^6 + 3*a*b*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a*b)*sinh(
x)^4 - 3*a*b*cosh(x)^2 + 4*(5*a^2*cosh(x)^3 + 3*a*b*cosh(x))*sinh(x)^3 + 3*
(5*a^2*cosh(x)^4 + 6*a*b*cosh(x)^2 - a*b)*sinh(x)^2 - a^2 + 6*(a^2*cosh(x)^
5 + 2*a*b*cosh(x)^3 - a*b*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2
 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^5*cosh(x)^8 +
8*a^5*cosh(x)*sinh(x)^7 + a^5*sinh(x)^8 + 4*(a^5 + 2*a^4*b)*cosh(x)^6 + 4*(
7*a^5*cosh(x)^2 + a^5 + 2*a^4*b)*sinh(x)^6 + 8*(7*a^5*cosh(x)^3 + 3*(a^5 +
2*a^4*b)*cosh(x))*sinh(x)^5 + a^5 + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(x)
^4 + 2*(35*a^5*cosh(x)^4 + 3*a^5 + 8*a^4*b + 8*a^3*b^2 + 30*(a^5 + 2*a^4*b)
```

```
*cosh(x)^2)*sinh(x)^4 + 8*(7*a^5*cosh(x)^5 + 10*(a^5 + 2*a^4*b)*cosh(x)^3 +
 (3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(x))*sinh(x)^3 + 4*(a^5 + 2*a^4*b)*cosh(
x)^2 + 4*(7*a^5*cosh(x)^6 + a^5 + 2*a^4*b + 15*(a^5 + 2*a^4*b)*cosh(x)^4 +
3*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(x)^2)*sinh(x)^2 + 8*(a^5*cosh(x)^7 + 3
*(a^5 + 2*a^4*b)*cosh(x)^5 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(x)^3 + (a^5
 + 2*a^4*b)*cosh(x))*sinh(x)), -1/6*(3*(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(
x)^7 + a^2*sinh(x)^8 + 4*(a^2 + 2*a*b)*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 + a^2
 + 2*a*b)*sinh(x)^6 + 8*(7*a^2*cosh(x)^3 + 3*(a^2 + 2*a*b)*cosh(x))*sinh(x)
^5 + 2*(3*a^2 + 8*a*b + 8*b^2)*cosh(x)^4 + 2*(35*a^2*cosh(x)^4 + 30*(a^2 +
2*a*b)*cosh(x)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(x)^4 + 8*(7*a^2*cosh(x)^5 +
10*(a^2 + 2*a*b)*cosh(x)^3 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x))*sinh(x)^3 + 4
*(a^2 + 2*a*b)*cosh(x)^2 + 4*(7*a^2*cosh(x)^6 + 15*(a^2 + 2*a*b)*cosh(x)^4
+ 3*(3*a^2 + 8*a*b + 8*b^2)*cosh(x)^2 + a^2 + 2*a*b)*sinh(x)^2 + a^2 + 8*(a
^2*cosh(x)^7 + 3*(a^2 + 2*a*b)*cosh(x)^5 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x)^
3 + (a^2 + 2*a*b)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 +
2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(
x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(x)^4
 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a
*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3
*a*b)*cosh(x))*sinh(x))) + 3*(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2
*sinh(x)^8 + 4*(a^2 + 2*a*b)*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 + a^2 + 2*a*b)*
sinh(x)^6 + 8*(7*a^2*cosh(x)^3 + 3*(a^2 + 2*a*b)*cosh(x))*sinh(x)^5 + 2*(3*
a^2 + 8*a*b + 8*b^2)*cosh(x)^4 + 2*(35*a^2*cosh(x)^4 + 30*(a^2 + 2*a*b)*cos
h(x)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(x)^4 + 8*(7*a^2*cosh(x)^5 + 10*(a^2 +
2*a*b)*cosh(x)^3 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + 2*
a*b)*cosh(x)^2 + 4*(7*a^2*cosh(x)^6 + 15*(a^2 + 2*a*b)*cosh(x)^4 + 3*(3*a^2
 + 8*a*b + 8*b^2)*cosh(x)^2 + a^2 + 2*a*b)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x)
^7 + 3*(a^2 + 2*a*b)*cosh(x)^5 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x)^3 + (a^2 +
 2*a*b)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cosh(x)^
2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*
cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)) + 8*sqrt(2)*(a^2*cosh(x
)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a*b*cosh(x)^4 + 3*(5*a^2*
cosh(x)^2 + a*b)*sinh(x)^4 - 3*a*b*cosh(x)^2 + 4*(5*a^2*cosh(x)^3 + 3*a*b*c
osh(x))*sinh(x)^3 + 3*(5*a^2*cosh(x)^4 + 6*a*b*cosh(x)^2 - a*b)*sinh(x)^2 -
 a^2 + 6*(a^2*cosh(x)^5 + 2*a*b*cosh(x)^3 - a*b*cosh(x))*sinh(x))*sqrt((a*c
osh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2)))/(a^5*cosh(x)^8 + 8*a^5*cosh(x)*sinh(x)^7 + a^5*sinh(x)^8 + 4*(a^5 + 2*
a^4*b)*cosh(x)^6 + 4*(7*a^5*cosh(x)^2 + a^5 + 2*a^4*b)*sinh(x)^6 + 8*(7*a^5
*cosh(x)^3 + 3*(a^5 + 2*a^4*b)*cosh(x))*sinh(x)^5 + a^5 + 2*(3*a^5 + 8*a^4*
b + 8*a^3*b^2)*cosh(x)^4 + 2*(35*a^5*cosh(x)^4 + 3*a^5 + 8*a^4*b + 8*a^3*b^
2 + 30*(a^5 + 2*a^4*b)*cosh(x)^2)*sinh(x)^4 + 8*(7*a^5*cosh(x)^5 + 10*(a^5
+ 2*a^4*b)*cosh(x)^3 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(x))*sinh(x)^3 + 4
*(a^5 + 2*a^4*b)*cosh(x)^2 + 4*(7*a^5*cosh(x)^6 + a^5 + 2*a^4*b + 15*(a^5 +
 2*a^4*b)*cosh(x)^4 + 3*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(x)^2)*sinh(x)^2
+ 8*(a^5*cosh(x)^7 + 3*(a^5 + 2*a^4*b)*cosh(x)^5 + (3*a^5 + 8*a^4*b + 8*a^3
*b^2)*cosh(x)^3 + (a^5 + 2*a^4*b)*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^4(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+b*sech(x)**2)**(5/2),x)
```

```
[Out] Integral(tanh(x)**4/(a + b*sech(x)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^4}{\left(b\operatorname{sech}(x)^2 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(x)^4/(b*sech(x)^2 + a)^(5/2), x)
```

3.214 $\int \frac{\tanh^3(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{1}{a^2\sqrt{a+b\operatorname{sech}^2(x)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{a+b}{3ab\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}$$

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/a^(5/2) - (a + b)/(3*a*b*(a + b*Sech
[x]^2)^(3/2)) - 1/(a^2*Sqrt[a + b*Sech[x]^2])
```

Rubi [A] time = 0.124441, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {4139, 446, 78, 51, 63, 208}

$$-\frac{1}{a^2\sqrt{a+b\operatorname{sech}^2(x)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{a+b}{3ab\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(a + b*Sech[x]^2)^(5/2),x]
```


```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/a^(5/2) - (a + b)/(3*a*b*(a + b*Sech
[x]^2)^(3/2)) - 1/(a^2*Sqrt[a + b*Sech[x]^2])
```


Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^3(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}\,dx &= \operatorname{Subst}\left(\int \frac{-1+x^2}{x\left(a+bx^2\right)^{5/2}}\,dx, x, \operatorname{sech}(x)\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{-1+x}{x(a+bx)^{5/2}}\,dx, x, \operatorname{sech}^2(x)\right)\\
&= -\frac{a+b}{3ab\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}-\frac{\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \operatorname{sech}^2(x)\right)}{2a}\\
&= -\frac{a+b}{3ab\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}-\frac{1}{a^2\sqrt{a+b\operatorname{sech}^2(x)}}-\frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \operatorname{sech}^2(x)\right)}{2a^2}\\
&= -\frac{a+b}{3ab\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}-\frac{1}{a^2\sqrt{a+b\operatorname{sech}^2(x)}}-\frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{sech}^2(x)}\right)}{a^2b}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{a+b}{3ab\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}-\frac{1}{a^2\sqrt{a+b\operatorname{sech}^2(x)}}
\end{aligned}
$$

Mathematica [A] time = 0.915025, size = 124, normalized size = 1.82

$$
-\frac{\operatorname{sech}^5(x)(a\cosh(2x)+a+2b)\left(a^{3/2}(a+4b)\cosh(3x)+3\sqrt{a}(a+2b)^2\cosh(x)-3\sqrt{2}b(a\cosh(2x)+a+2b)^{3/2}\log\left(\sqrt{a\text{ c}}\right.\right.}{24a^{5/2}b\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] -((a + 2*b + a*Cosh[2*x])*(3*Sqrt[a]*(a + 2*b)^2*Cosh[x] + a^(3/2)*(a + 4*b
)*Cosh[3*x] - 3*Sqrt[2]*b*(a + 2*b + a*Cosh[2*x])^(3/2)*Log[Sqrt[2]*Sqrt[a]
```

```
*Cosh[x] + Sqrt[a + 2*b + a*Cosh[2*x]]])*Sech[x]^5)/(24*a^(5/2)*b*(a + b*Se
ch[x]^2)^(5/2))
```

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int (\tanh(x))^3 \left(a + b (\mathrm{sech}(x))^2\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*sech(x)^2)^(5/2),x)

[Out] int(tanh(x)^3/(a+b*sech(x)^2)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^3}{\left(b\,\mathrm{sech}(x)^2 + a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")

[Out] integrate(tanh(x)^3/(b*sech(x)^2 + a)^(5/2), x)
```

Fricas [B] time = 4.45586, size = 12041, normalized size = 177.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")

[Out] [1/12*(3*(a^2*b*cosh(x)^8 + 8*a^2*b*cosh(x)*sinh(x)^7 + a^2*b*sinh(x)^8 + 4
*(a^2*b + 2*a*b^2)*cosh(x)^6 + 4*(7*a^2*b*cosh(x)^2 + a^2*b + 2*a*b^2)*sinh
(x)^6 + 8*(7*a^2*b*cosh(x)^3 + 3*(a^2*b + 2*a*b^2)*cosh(x))*sinh(x)^5 + 2*(
3*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x)^4 + 2*(35*a^2*b*cosh(x)^4 + 3*a^2*b + 8*
a*b^2 + 8*b^3 + 30*(a^2*b + 2*a*b^2)*cosh(x)^2)*sinh(x)^4 + 8*(7*a^2*b*cosh
(x)^5 + 10*(a^2*b + 2*a*b^2)*cosh(x)^3 + (3*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x
))*sinh(x)^3 + a^2*b + 4*(a^2*b + 2*a*b^2)*cosh(x)^2 + 4*(7*a^2*b*cosh(x)^6
 + 15*(a^2*b + 2*a*b^2)*cosh(x)^4 + a^2*b + 2*a*b^2 + 3*(3*a^2*b + 8*a*b^2
+ 8*b^3)*cosh(x)^2)*sinh(x)^2 + 8*(a^2*b*cosh(x)^7 + 3*(a^2*b + 2*a*b^2)*co
sh(x)^5 + (3*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x)^3 + (a^2*b + 2*a*b^2)*cosh(x)
)*sinh(x))*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*
b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3
 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3
+ 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b +
a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 +
 (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(
x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*co
sh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 +
```

5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*
sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*
b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3
*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^
2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2
 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2
*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b +
b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)
cosh(x)^2 + (15(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cos
h(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^
5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqr
t(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh
(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a
^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 +
(2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*c
osh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*co
sh(x)*sinh(x)^5 + sinh(x)^6)) + 3*(a^2*b*cosh(x)^8 + 8*a^2*b*cosh(x)*sinh(x
)^7 + a^2*b*sinh(x)^8 + 4*(a^2*b + 2*a*b^2)*cosh(x)^6 + 4*(7*a^2*b*cosh(x)^
2 + a^2*b + 2*a*b^2)*sinh(x)^6 + 8*(7*a^2*b*cosh(x)^3 + 3*(a^2*b + 2*a*b^2)
*cosh(x))*sinh(x)^5 + 2*(3*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x)^4 + 2*(35*a^2*b
*cosh(x)^4 + 3*a^2*b + 8*a*b^2 + 8*b^3 + 30*(a^2*b + 2*a*b^2)*cosh(x)^2)*si
nh(x)^4 + 8*(7*a^2*b*cosh(x)^5 + 10*(a^2*b + 2*a*b^2)*cosh(x)^3 + (3*a^2*b
+ 8*a*b^2 + 8*b^3)*cosh(x))*sinh(x)^3 + a^2*b + 4*(a^2*b + 2*a*b^2)*cosh(x)
^2 + 4*(7*a^2*b*cosh(x)^6 + 15*(a^2*b + 2*a*b^2)*cosh(x)^4 + a^2*b + 2*a*b^
2 + 3*(3*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x)^2)*sinh(x)^2 + 8*(a^2*b*cosh(x)^7
 + 3*(a^2*b + 2*a*b^2)*cosh(x)^5 + (3*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x)^3 +
(a^2*b + 2*a*b^2)*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)
*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2
+ sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*c
osh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(
x) + sinh(x)^2)) - 4*sqrt(2)*((a^3 + 4*a^2*b)*cosh(x)^6 + 6*(a^3 + 4*a^2*b)
*cosh(x)*sinh(x)^5 + (a^3 + 4*a^2*b)*sinh(x)^6 + 3*(a^3 + 4*a^2*b + 4*a*b^2
)*cosh(x)^4 + 3*(a^3 + 4*a^2*b + 4*a*b^2 + 5*(a^3 + 4*a^2*b)*cosh(x)^2)*sin
h(x)^4 + 4*(5*(a^3 + 4*a^2*b)*cosh(x)^3 + 3*(a^3 + 4*a^2*b + 4*a*b^2)*cosh(
x))*sinh(x)^3 + a^3 + 4*a^2*b + 3*(a^3 + 4*a^2*b + 4*a*b^2)*cosh(x)^2 + 3*(
5*(a^3 + 4*a^2*b)*cosh(x)^4 + a^3 + 4*a^2*b + 4*a*b^2 + 6*(a^3 + 4*a^2*b +
4*a*b^2)*cosh(x)^2)*sinh(x)^2 + 6*((a^3 + 4*a^2*b)*cosh(x)^5 + 2*(a^3 + 4*a
^2*b + 4*a*b^2)*cosh(x)^3 + (a^3 + 4*a^2*b + 4*a*b^2)*cosh(x))*sinh(x))*sqr
t((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + si
nh(x)^2)))/(a^5*b*cosh(x)^8 + 8*a^5*b*cosh(x)*sinh(x)^7 + a^5*b*sinh(x)^8 +
 4*(a^5*b + 2*a^4*b^2)*cosh(x)^6 + 4*(7*a^5*b*cosh(x)^2 + a^5*b + 2*a^4*b^2
)*sinh(x)^6 + a^5*b + 8*(7*a^5*b*cosh(x)^3 + 3*(a^5*b + 2*a^4*b^2)*cosh(x))
sinh(x)^5 + 2(3*a^5*b + 8*a^4*b^2 + 8*a^3*b^3)*cosh(x)^4 + 2*(35*a^5*b*co
sh(x)^4 + 3*a^5*b + 8*a^4*b^2 + 8*a^3*b^3 + 30*(a^5*b + 2*a^4*b^2)*cosh(x)^
2)*sinh(x)^4 + 8*(7*a^5*b*cosh(x)^5 + 10*(a^5*b + 2*a^4*b^2)*cosh(x)^3 + (3
*a^5*b + 8*a^4*b^2 + 8*a^3*b^3)*cosh(x))*sinh(x)^3 + 4*(a^5*b + 2*a^4*b^2)*
cosh(x)^2 + 4*(7*a^5*b*cosh(x)^6 + a^5*b + 2*a^4*b^2 + 15*(a^5*b + 2*a^4*b^
2)*cosh(x)^4 + 3*(3*a^5*b + 8*a^4*b^2 + 8*a^3*b^3)*cosh(x)^2)*sinh(x)^2 + 8
*(a^5*b*cosh(x)^7 + 3*(a^5*b + 2*a^4*b^2)*cosh(x)^5 + (3*a^5*b + 8*a^4*b^2
+ 8*a^3*b^3)*cosh(x)^3 + (a^5*b + 2*a^4*b^2)*cosh(x))*sinh(x)), -1/6*(3*(a^
2*b*cosh(x)^8 + 8*a^2*b*cosh(x)*sinh(x)^7 + a^2*b*sinh(x)^8 + 4*(a^2*b + 2*
a*b^2)*cosh(x)^6 + 4*(7*a^2*b*cosh(x)^2 + a^2*b + 2*a*b^2)*sinh(x)^6 + 8*(7
*a^2*b*cosh(x)^3 + 3*(a^2*b + 2*a*b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2*b + 8*
a*b^2 + 8*b^3)*cosh(x)^4 + 2*(35*a^2*b*cosh(x)^4 + 3*a^2*b + 8*a*b^2 + 8*b^
3 + 30*(a^2*b + 2*a*b^2)*cosh(x)^2)*sinh(x)^4 + 8*(7*a^2*b*cosh(x)^5 + 10*(
a^2*b + 2*a*b^2)*cosh(x)^3 + (3*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x))*sinh(x)^3
 + a^2*b + 4*(a^2*b + 2*a*b^2)*cosh(x)^2 + 4*(7*a^2*b*cosh(x)^6 + 15*(a^2*b
 + 2*a*b^2)*cosh(x)^4 + a^2*b + 2*a*b^2 + 3*(3*a^2*b + 8*a*b^2 + 8*b^3)*cos

```
h(x)^2)*sinh(x)^2 + 8*(a^2*b*cosh(x)^7 + 3*(a^2*b + 2*a*b^2)*cosh(x)^5 + (3
*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x)^3 + (a^2*b + 2*a*b^2)*cosh(x))*sinh(x))*s
qrt(-a)*arctan(sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a
+ b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(co
sh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 +
 a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2
 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 +
a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(x))*sinh(x))) + 3*(a^2*b*cosh(x)^8 +
8*a^2*b*cosh(x)*sinh(x)^7 + a^2*b*sinh(x)^8 + 4*(a^2*b + 2*a*b^2)*cosh(x)^6
 + 4*(7*a^2*b*cosh(x)^2 + a^2*b + 2*a*b^2)*sinh(x)^6 + 8*(7*a^2*b*cosh(x)^3
 + 3*(a^2*b + 2*a*b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2*b + 8*a*b^2 + 8*b^3)*c
osh(x)^4 + 2*(35*a^2*b*cosh(x)^4 + 3*a^2*b + 8*a*b^2 + 8*b^3 + 30*(a^2*b +
2*a*b^2)*cosh(x)^2)*sinh(x)^4 + 8*(7*a^2*b*cosh(x)^5 + 10*(a^2*b + 2*a*b^2)
*cosh(x)^3 + (3*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x))*sinh(x)^3 + a^2*b + 4*(a^
2*b + 2*a*b^2)*cosh(x)^2 + 4*(7*a^2*b*cosh(x)^6 + 15*(a^2*b + 2*a*b^2)*cosh
(x)^4 + a^2*b + 2*a*b^2 + 3*(3*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x)^2)*sinh(x)^
2 + 8*(a^2*b*cosh(x)^7 + 3*(a^2*b + 2*a*b^2)*cosh(x)^5 + (3*a^2*b + 8*a*b^2
 + 8*b^3)*cosh(x)^3 + (a^2*b + 2*a*b^2)*cosh(x))*sinh(x))*sqrt(-a)*arctan(s
qrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cos
h(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)
)/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^
2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh
(x))*sinh(x) + a)) + 2*sqrt(2)*((a^3 + 4*a^2*b)*cosh(x)^6 + 6*(a^3 + 4*a^2*
b)*cosh(x)*sinh(x)^5 + (a^3 + 4*a^2*b)*sinh(x)^6 + 3*(a^3 + 4*a^2*b + 4*a*b
^2)*cosh(x)^4 + 3*(a^3 + 4*a^2*b + 4*a*b^2 + 5*(a^3 + 4*a^2*b)*cosh(x)^2)*s
inh(x)^4 + 4*(5*(a^3 + 4*a^2*b)*cosh(x)^3 + 3*(a^3 + 4*a^2*b + 4*a*b^2)*cos
h(x))*sinh(x)^3 + a^3 + 4*a^2*b + 3*(a^3 + 4*a^2*b + 4*a*b^2)*cosh(x)^2 + 3
*(5*(a^3 + 4*a^2*b)*cosh(x)^4 + a^3 + 4*a^2*b + 4*a*b^2 + 6*(a^3 + 4*a^2*b
+ 4*a*b^2)*cosh(x)^2)*sinh(x)^2 + 6*((a^3 + 4*a^2*b)*cosh(x)^5 + 2*(a^3 + 4
*a^2*b + 4*a*b^2)*cosh(x)^3 + (a^3 + 4*a^2*b + 4*a*b^2)*cosh(x))*sinh(x))*s
qrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
sinh(x)^2)))/(a^5*b*cosh(x)^8 + 8*a^5*b*cosh(x)*sinh(x)^7 + a^5*b*sinh(x)^8
 + 4*(a^5*b + 2*a^4*b^2)*cosh(x)^6 + 4*(7*a^5*b*cosh(x)^2 + a^5*b + 2*a^4*b
^2)*sinh(x)^6 + a^5*b + 8*(7*a^5*b*cosh(x)^3 + 3*(a^5*b + 2*a^4*b^2)*cosh(x
))*sinh(x)^5 + 2*(3*a^5*b + 8*a^4*b^2 + 8*a^3*b^3)*cosh(x)^4 + 2*(35*a^5*b*
cosh(x)^4 + 3*a^5*b + 8*a^4*b^2 + 8*a^3*b^3 + 30*(a^5*b + 2*a^4*b^2)*cosh(x
)^2)*sinh(x)^4 + 8*(7*a^5*b*cosh(x)^5 + 10*(a^5*b + 2*a^4*b^2)*cosh(x)^3 +
(3*a^5*b + 8*a^4*b^2 + 8*a^3*b^3)*cosh(x))*sinh(x)^3 + 4*(a^5*b + 2*a^4*b^2
)*cosh(x)^2 + 4*(7*a^5*b*cosh(x)^6 + a^5*b + 2*a^4*b^2 + 15*(a^5*b + 2*a^4*
b^2)*cosh(x)^4 + 3*(3*a^5*b + 8*a^4*b^2 + 8*a^3*b^3)*cosh(x)^2)*sinh(x)^2 +
 8*(a^5*b*cosh(x)^7 + 3*(a^5*b + 2*a^4*b^2)*cosh(x)^5 + (3*a^5*b + 8*a^4*b^
2 + 8*a^3*b^3)*cosh(x)^3 + (a^5*b + 2*a^4*b^2)*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^3(x)}{\left(a + b\operatorname{sech}^2(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*sech(x)**2)**(5/2),x)
```

```
[Out] Integral(tanh(x)**3/(a + b*sech(x)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^3}{\left(b\,\mathrm{sech}(x)^2 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(x)^3/(b*sech(x)^2 + a)^(5/2), x)
```

3.215 $\int \frac{\tanh^2(x)}{\left(a+b\,\mathbf{sech}^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=88

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}}-\frac{(2a+3b)\tanh(x)}{3a^2(a+b)\sqrt{a-b\tanh^2(x)+b}}-\frac{\tanh(x)}{3a\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(5/2) - Tanh[x]/(3*a
*(a + b - b*Tanh[x]^2)^(3/2)) - ((2*a + 3*b)*Tanh[x])/(3*a^2*(a + b)*Sqrt[a
 + b - b*Tanh[x]^2])
```

Rubi [A] time = 0.247904, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {4141, 1975, 471, 527, 12, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}}-\frac{(2a+3b)\tanh(x)}{3a^2(a+b)\sqrt{a-b\tanh^2(x)+b}}-\frac{\tanh(x)}{3a\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(a + b*Sech[x]^2)^(5/2),x]
```


```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(5/2) - Tanh[x]/(3*a
*(a + b - b*Tanh[x]^2)^(3/2)) - ((2*a + 3*b)*Tanh[x])/(3*a^2*(a + b)*Sqrt[a
 + b - b*Tanh[x]^2])
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tanh^2(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}\,dx = \operatorname{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)\left(a+b\left(1-x^2\right)\right)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= \operatorname{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)\left(a+b-bx^2\right)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= -\frac{\tanh(x)}{3a\left(a+b-b\tanh^2(x)\right)^{3/2}} + \frac{\operatorname{Subst}\left(\int \frac{1+2x^2}{\left(1-x^2\right)\left(a+b-bx^2\right)^{3/2}}\,dx, x, \tanh(x)\right)}{3a}$$

$$= -\frac{\tanh(x)}{3a\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{(2a+3b)\tanh(x)}{3a^2(a+b)\sqrt{a+b-b\tanh^2(x)}} - \frac{\operatorname{Subst}\left(\int -\frac{3(a+b)}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx \ldots\right.}{3a^2(a+b)}$$

$$= -\frac{\tanh(x)}{3a\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{(2a+3b)\tanh(x)}{3a^2(a+b)\sqrt{a+b-b\tanh^2(x)}} + \frac{\operatorname{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx, \ldots\right.}{a^2}$$

$$= -\frac{\tanh(x)}{3a\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{(2a+3b)\tanh(x)}{3a^2(a+b)\sqrt{a+b-b\tanh^2(x)}} + \frac{\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tan\ldots}{\sqrt{a+b-b\ldots}}\right.}{a^2}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{5/2}} - \frac{\tanh(x)}{3a\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{(2a+3b)\tanh(x)}{3a^2(a+b)\sqrt{a+b-b\tanh^2(x)}}$$

Mathematica [B] time = 1.15763, size = 290, normalized size = 3.3

$$\frac{\operatorname{sech}^4(x)\left(\frac{\sqrt{2}\operatorname{csch}(x)\operatorname{sech}(x)\left(-\frac{16\left(a\sinh^2(x)+a+b\right)\left(\frac{a\sinh^2(x)}{a+b}+1\right)\left(\frac{a^2(a+b)\sinh^4(x)}{\left(a\sinh^2(x)+a+b\right)^2}+\frac{3a(a+b)\sinh^2(x)}{a\sinh^2(x)+a+b}-\frac{3\sqrt{a}\sqrt{a+b}\sinh(x)\sinh^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a+b}}\right)}{\sqrt{\frac{a\sinh^2(x)+a+b}{a+b}}}\right)}{a^3}+\frac{12\sinh^4(x)}{a+b}+\frac{2\sin\cdots}{\cdots}\right)}{\left(a\sinh^2(x)+a+b\right)^{3/2}}\cdots\right)}{384\left(a+\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] (Sech[x]^4*((Sqrt[2]*(a + 2*b + a*Cosh[2*x])^(5/2)*Csch[x]*Sech[x]*(Sinh[x]
^2/(a + b) + (12*Sinh[x]^4)/(a + b) + (2*Sinh[x]^2*(a + b + a*Sinh[x]^2))/(
a + b)^2 - (16*(a + b + a*Sinh[x]^2)*(1 + (a*Sinh[x]^2)/(a + b))*((a^2*(a +
 b)*Sinh[x]^4)/(a + b + a*Sinh[x]^2)^2 + (3*a*(a + b)*Sinh[x]^2)/(a + b + a
*Sinh[x]^2) - (3*Sqrt[a]*Sqrt[a + b]*ArcSinh[(Sqrt[a]*Sinh[x])/Sqrt[a + b]]
*Sinh[x])/Sqrt[(a + b + a*Sinh[x]^2)/(a + b)]))/a^3))/(a + b + a*Sinh[x]^2)
^(3/2) - (8*(a + 2*b + a*Cosh[2*x])*(2*a + 3*b + a*Cosh[2*x])*Tanh[x])/(a +
 b)^2 + (4*(a + 2*b + a*Cosh[2*x])*(b + (3*a + 2*b)*Cosh[2*x])*Tanh[x])/(a
+ b)^2))/(384*(a + b*Sech[x]^2)^(5/2))
```

Maple [F] time = 0.093, size = 0, normalized size = 0.

$$\int (\tanh(x))^2\left(a+b(\operatorname{sech}(x))^2\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+b*sech(x)^2)^(5/2),x)
```

```
[Out] int(tanh(x)^2/(a+b*sech(x)^2)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^2}{\left(b\operatorname{sech}(x)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^2/(b*sech(x)^2 + a)^(5/2), x)
```

Fricas [B] time = 4.72217, size = 12787, normalized size = 145.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^
3 + a^2*b)*sinh(x)^8 + 4*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^6 + 4*(a^3 + 3*a
^2*b + 2*a*b^2 + 7*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^3 + a^2*b)*
cosh(x)^3 + 3*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^3 + 11*
a^2*b + 16*a*b^2 + 8*b^3)*cosh(x)^4 + 2*(35*(a^3 + a^2*b)*cosh(x)^4 + 3*a^3
 + 11*a^2*b + 16*a*b^2 + 8*b^3 + 30*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^2)*si
nh(x)^4 + 8*(7*(a^3 + a^2*b)*cosh(x)^5 + 10*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(
x)^3 + (3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3)*cosh(x))*sinh(x)^3 + a^3 + a^2
*b + 4*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^2 + 4*(7*(a^3 + a^2*b)*cosh(x)^6 +
 15*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^4 + a^3 + 3*a^2*b + 2*a*b^2 + 3*(3*a^
3 + 11*a^2*b + 16*a*b^2 + 8*b^3)*cosh(x)^2)*sinh(x)^2 + 8*((a^3 + a^2*b)*co
sh(x)^7 + 3*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^5 + (3*a^3 + 11*a^2*b + 16*a*
b^2 + 8*b^3)*cosh(x)^3 + (a^3 + 3*a^2*b + 2*a*b^2)*cosh(x))*sinh(x))*sqrt(a
)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a
*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*
(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b +
 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a
*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)
*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 +
3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a
^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(
b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 +
 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*
sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2
- a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^
2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a +
2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 -
3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*
a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*si
nh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(
x)^5 + sinh(x)^6)) + 3*((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*s
inh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 4*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^6
+ 4*(a^3 + 3*a^2*b + 2*a*b^2 + 7*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 8*(7*
(a^3 + a^2*b)*cosh(x)^3 + 3*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x))*sinh(x)^5 +
2*(3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3)*cosh(x)^4 + 2*(35*(a^3 + a^2*b)*cos
h(x)^4 + 3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3 + 30*(a^3 + 3*a^2*b + 2*a*b^2)
*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^3 + a^2*b)*cosh(x)^5 + 10*(a^3 + 3*a^2*b +
2*a*b^2)*cosh(x)^3 + (3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3)*cosh(x))*sinh(x)
^3 + a^3 + a^2*b + 4*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^2 + 4*(7*(a^3 + a^2*
b)*cosh(x)^6 + 15*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^4 + a^3 + 3*a^2*b + 2*a
*b^2 + 3*(3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3)*cosh(x)^2)*sinh(x)^2 + 8*((a
^3 + a^2*b)*cosh(x)^7 + 3*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^5 + (3*a^3 + 11
*a^2*b + 16*a*b^2 + 8*b^3)*cosh(x)^3 + (a^3 + 3*a^2*b + 2*a*b^2)*cosh(x))*s
inh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2
*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)
^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(
x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)
^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2)) - 4*sqrt(2)*((3*a^3 + 4*a^2*b)*cosh(x)^6 + 6*(3*a^3 + 4*a^2*b)*cosh(x)
*sinh(x)^5 + (3*a^3 + 4*a^2*b)*sinh(x)^6 + 3*(a^3 + 4*a^2*b + 4*a*b^2)*cosh
```

```
(x)^4 + 3*(a^3 + 4*a^2*b + 4*a*b^2 + 5*(3*a^3 + 4*a^2*b)*cosh(x)^2)*sinh(x)
^4 + 4*(5*(3*a^3 + 4*a^2*b)*cosh(x)^3 + 3*(a^3 + 4*a^2*b + 4*a*b^2)*cosh(x)
)*sinh(x)^3 - 3*a^3 - 4*a^2*b - 3*(a^3 + 4*a^2*b + 4*a*b^2)*cosh(x)^2 + 3*(
5*(3*a^3 + 4*a^2*b)*cosh(x)^4 - a^3 - 4*a^2*b - 4*a*b^2 + 6*(a^3 + 4*a^2*b
+ 4*a*b^2)*cosh(x)^2)*sinh(x)^2 + 6*((3*a^3 + 4*a^2*b)*cosh(x)^5 + 2*(a^3 +
 4*a^2*b + 4*a*b^2)*cosh(x)^3 - (a^3 + 4*a^2*b + 4*a*b^2)*cosh(x))*sinh(x))
*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
+ sinh(x)^2)))/((a^6 + a^5*b)*cosh(x)^8 + 8*(a^6 + a^5*b)*cosh(x)*sinh(x)^7
 + (a^6 + a^5*b)*sinh(x)^8 + 4*(a^6 + 3*a^5*b + 2*a^4*b^2)*cosh(x)^6 + 4*(a
^6 + 3*a^5*b + 2*a^4*b^2 + 7*(a^6 + a^5*b)*cosh(x)^2)*sinh(x)^6 + a^6 + a^5
*b + 8*(7*(a^6 + a^5*b)*cosh(x)^3 + 3*(a^6 + 3*a^5*b + 2*a^4*b^2)*cosh(x))*
sinh(x)^5 + 2*(3*a^6 + 11*a^5*b + 16*a^4*b^2 + 8*a^3*b^3)*cosh(x)^4 + 2*(3*
a^6 + 11*a^5*b + 16*a^4*b^2 + 8*a^3*b^3 + 35*(a^6 + a^5*b)*cosh(x)^4 + 30*(
a^6 + 3*a^5*b + 2*a^4*b^2)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^6 + a^5*b)*cosh(x
)^5 + 10*(a^6 + 3*a^5*b + 2*a^4*b^2)*cosh(x)^3 + (3*a^6 + 11*a^5*b + 16*a^4
*b^2 + 8*a^3*b^3)*cosh(x))*sinh(x)^3 + 4*(a^6 + 3*a^5*b + 2*a^4*b^2)*cosh(x
)^2 + 4*(7*(a^6 + a^5*b)*cosh(x)^6 + a^6 + 3*a^5*b + 2*a^4*b^2 + 15*(a^6 +
3*a^5*b + 2*a^4*b^2)*cosh(x)^4 + 3*(3*a^6 + 11*a^5*b + 16*a^4*b^2 + 8*a^3*b
^3)*cosh(x)^2)*sinh(x)^2 + 8*((a^6 + a^5*b)*cosh(x)^7 + 3*(a^6 + 3*a^5*b +
2*a^4*b^2)*cosh(x)^5 + (3*a^6 + 11*a^5*b + 16*a^4*b^2 + 8*a^3*b^3)*cosh(x)^
3 + (a^6 + 3*a^5*b + 2*a^4*b^2)*cosh(x))*sinh(x)), -1/6*(3*((a^3 + a^2*b)*c
osh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 4*
(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^6 + 4*(a^3 + 3*a^2*b + 2*a*b^2 + 7*(a^3 +
 a^2*b)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^3 + a^2*b)*cosh(x)^3 + 3*(a^3 + 3*a^
2*b + 2*a*b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3)
*cosh(x)^4 + 2*(35*(a^3 + a^2*b)*cosh(x)^4 + 3*a^3 + 11*a^2*b + 16*a*b^2 +
8*b^3 + 30*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^3 + a^2
*b)*cosh(x)^5 + 10*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^3 + (3*a^3 + 11*a^2*b
+ 16*a*b^2 + 8*b^3)*cosh(x))*sinh(x)^3 + a^3 + a^2*b + 4*(a^3 + 3*a^2*b + 2
*a*b^2)*cosh(x)^2 + 4*(7*(a^3 + a^2*b)*cosh(x)^6 + 15*(a^3 + 3*a^2*b + 2*a*
b^2)*cosh(x)^4 + a^3 + 3*a^2*b + 2*a*b^2 + 3*(3*a^3 + 11*a^2*b + 16*a*b^2 +
 8*b^3)*cosh(x)^2)*sinh(x)^2 + 8*((a^3 + a^2*b)*cosh(x)^7 + 3*(a^3 + 3*a^2*
b + 2*a*b^2)*cosh(x)^5 + (3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3)*cosh(x)^3 +
(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(b*cosh
(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 +
 a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*c
osh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^
2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 -
(a^2 + 3*a*b)*cosh(x))*sinh(x))) + 3*((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^
2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 4*(a^3 + 3*a^2*b + 2*a*b
^2)*cosh(x)^6 + 4*(a^3 + 3*a^2*b + 2*a*b^2 + 7*(a^3 + a^2*b)*cosh(x)^2)*sin
h(x)^6 + 8*(7*(a^3 + a^2*b)*cosh(x)^3 + 3*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)
)*sinh(x)^5 + 2*(3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3)*cosh(x)^4 + 2*(35*(a^
3 + a^2*b)*cosh(x)^4 + 3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3 + 30*(a^3 + 3*a^
2*b + 2*a*b^2)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^3 + a^2*b)*cosh(x)^5 + 10*(a^
3 + 3*a^2*b + 2*a*b^2)*cosh(x)^3 + (3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3)*co
sh(x))*sinh(x)^3 + a^3 + a^2*b + 4*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^2 + 4*
(7*(a^3 + a^2*b)*cosh(x)^6 + 15*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^4 + a^3 +
 3*a^2*b + 2*a*b^2 + 3*(3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3)*cosh(x)^2)*sin
h(x)^2 + 8*((a^3 + a^2*b)*cosh(x)^7 + 3*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^5
 + (3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3)*cosh(x)^3 + (a^3 + 3*a^2*b + 2*a*b
^2)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cosh(x)^2 +
a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh
(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)) + 2*sqrt(2)*((3*a^3 + 4*a^2
*b)*cosh(x)^6 + 6*(3*a^3 + 4*a^2*b)*cosh(x)*sinh(x)^5 + (3*a^3 + 4*a^2*b)*s
inh(x)^6 + 3*(a^3 + 4*a^2*b + 4*a*b^2)*cosh(x)^4 + 3*(a^3 + 4*a^2*b + 4*a*b
^2 + 5*(3*a^3 + 4*a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(5*(3*a^3 + 4*a^2*b)*cosh
(x)^3 + 3*(a^3 + 4*a^2*b + 4*a*b^2)*cosh(x))*sinh(x)^3 - 3*a^3 - 4*a^2*b -
3*(a^3 + 4*a^2*b + 4*a*b^2)*cosh(x)^2 + 3*(5*(3*a^3 + 4*a^2*b)*cosh(x)^4 -
```

```
a^3 - 4*a^2*b - 4*a*b^2 + 6*(a^3 + 4*a^2*b + 4*a*b^2)*cosh(x)^2)*sinh(x)^2
+ 6*((3*a^3 + 4*a^2*b)*cosh(x)^5 + 2*(a^3 + 4*a^2*b + 4*a*b^2)*cosh(x)^3 -
(a^3 + 4*a^2*b + 4*a*b^2)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2
 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^6 + a^5*b)*co
sh(x)^8 + 8*(a^6 + a^5*b)*cosh(x)*sinh(x)^7 + (a^6 + a^5*b)*sinh(x)^8 + 4*(
a^6 + 3*a^5*b + 2*a^4*b^2)*cosh(x)^6 + 4*(a^6 + 3*a^5*b + 2*a^4*b^2 + 7*(a^
6 + a^5*b)*cosh(x)^2)*sinh(x)^6 + a^6 + a^5*b + 8*(7*(a^6 + a^5*b)*cosh(x)^
3 + 3*(a^6 + 3*a^5*b + 2*a^4*b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^6 + 11*a^5*b
+ 16*a^4*b^2 + 8*a^3*b^3)*cosh(x)^4 + 2*(3*a^6 + 11*a^5*b + 16*a^4*b^2 + 8*
a^3*b^3 + 35*(a^6 + a^5*b)*cosh(x)^4 + 30*(a^6 + 3*a^5*b + 2*a^4*b^2)*cosh(
x)^2)*sinh(x)^4 + 8*(7*(a^6 + a^5*b)*cosh(x)^5 + 10*(a^6 + 3*a^5*b + 2*a^4*
b^2)*cosh(x)^3 + (3*a^6 + 11*a^5*b + 16*a^4*b^2 + 8*a^3*b^3)*cosh(x))*sinh(
x)^3 + 4*(a^6 + 3*a^5*b + 2*a^4*b^2)*cosh(x)^2 + 4*(7*(a^6 + a^5*b)*cosh(x)
^6 + a^6 + 3*a^5*b + 2*a^4*b^2 + 15*(a^6 + 3*a^5*b + 2*a^4*b^2)*cosh(x)^4 +
 3*(3*a^6 + 11*a^5*b + 16*a^4*b^2 + 8*a^3*b^3)*cosh(x)^2)*sinh(x)^2 + 8*((a
^6 + a^5*b)*cosh(x)^7 + 3*(a^6 + 3*a^5*b + 2*a^4*b^2)*cosh(x)^5 + (3*a^6 +
11*a^5*b + 16*a^4*b^2 + 8*a^3*b^3)*cosh(x)^3 + (a^6 + 3*a^5*b + 2*a^4*b^2)*
cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*sech(x)**2)**(5/2),x)
```

```
[Out] Integral(tanh(x)**2/(a + b*sech(x)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^2}{\left(b\operatorname{sech}(x)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(x)^2/(b*sech(x)^2 + a)^(5/2), x)
```

3.216 $\int \frac{\tanh(x)}{\left(a+b\mathbf{sech}^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{1}{a^2\sqrt{a+b\operatorname{sech}^2(x)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{1}{3a\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}$$

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/a^(5/2) - 1/(3*a*(a + b*Sech[x]^2)^(
3/2)) - 1/(a^2*Sqrt[a + b*Sech[x]^2])
```

Rubi [A] time = 0.0871114, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4139, 266, 51, 63, 208}

$$-\frac{1}{a^2\sqrt{a+b\operatorname{sech}^2(x)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{1}{3a\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/a^(5/2) - 1/(3*a*(a + b*Sech[x]^2)^(
3/2)) - 1/(a^2*Sqrt[a + b*Sech[x]^2])
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx &= -\text{Subst}\left(\int \frac{1}{x\left(a+bx^2\right)^{5/2}}\,dx,x,\text{sech}(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx,x,\text{sech}^2(x)\right)\right)\\
&= -\frac{1}{3a\left(a+b\text{sech}^2(x)\right)^{3/2}}-\frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx,x,\text{sech}^2(x)\right)}{2a}\\
&= -\frac{1}{3a\left(a+b\text{sech}^2(x)\right)^{3/2}}-\frac{1}{a^2\sqrt{a+b\text{sech}^2(x)}}-\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,\text{sech}^2(x)\right)}{2a^2}\\
&= -\frac{1}{3a\left(a+b\text{sech}^2(x)\right)^{3/2}}-\frac{1}{a^2\sqrt{a+b\text{sech}^2(x)}}-\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\text{sech}^2(x)}\right)}{a^2b}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{1}{3a\left(a+b\text{sech}^2(x)\right)^{3/2}}-\frac{1}{a^2\sqrt{a+b\text{sech}^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.773168, size = 112, normalized size = 1.81

$$-\frac{\text{sech}^5(x)(a\cosh(2x)+a+2b)\left(4a^{3/2}\cosh(3x)+12\sqrt{a}(a+b)\cosh(x)-3\sqrt{2}(a\cosh(2x)+a+2b)^{3/2}\log\left(\sqrt{a\cosh(2x)}\right.\right.}{24a^{5/2}\left(a+b\text{sech}^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] -((a + 2*b + a*Cosh[2*x])*(12*Sqrt[a]*(a + b)*Cosh[x] + 4*a^(3/2)*Cosh[3*x]
 - 3*Sqrt[2]*(a + 2*b + a*Cosh[2*x])^(3/2)*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sq
rt[a + 2*b + a*Cosh[2*x]]])*Sech[x]^5)/(24*a^(5/2)*(a + b*Sech[x]^2)^(5/2))
```

Maple [A] time = 0.014, size = 61, normalized size = 1.

$$-\frac{1}{3a}\left(a+b\left(\text{sech}\left(x\right)\right)^2\right)^{-\frac{3}{2}}-\frac{1}{a^2}\frac{1}{\sqrt{a+b\left(\text{sech}\left(x\right)\right)^2}}+\ln\left(\frac{1}{\text{sech}\left(x\right)}\left(2\,a+2\,\sqrt{a}\sqrt{a+b\left(\text{sech}\left(x\right)\right)^2}\right)\right)a^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*sech(x)^2)^(5/2),x)
```

```
[Out] -1/3/a/(a+b*sech(x)^2)^(3/2)-1/a^2/(a+b*sech(x)^2)^(1/2)+1/a^(5/2)*ln((2*a+
2*a^(1/2)*(a+b*sech(x)^2)^(1/2))/sech(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{\left(b\,\mathrm{sech}(x)^2 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)/(b*sech(x)^2 + a)^(5/2), x)
```

Fricas [B] time = 4.47824, size = 10838, normalized size = 174.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2*sinh(x)^8 + 4*(a^2
+ 2*a*b)*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 + a^2 + 2*a*b)*sinh(x)^6 + 8*(7*a^2
*cosh(x)^3 + 3*(a^2 + 2*a*b)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 8*a*b + 8*b^2)
*cosh(x)^4 + 2*(35*a^2*cosh(x)^4 + 30*(a^2 + 2*a*b)*cosh(x)^2 + 3*a^2 + 8*a
*b + 8*b^2)*sinh(x)^4 + 8*(7*a^2*cosh(x)^5 + 10*(a^2 + 2*a*b)*cosh(x)^3 + (
3*a^2 + 8*a*b + 8*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + 2*a*b)*cosh(x)^2 + 4*(
7*a^2*cosh(x)^6 + 15*(a^2 + 2*a*b)*cosh(x)^4 + 3*(3*a^2 + 8*a*b + 8*b^2)*co
sh(x)^2 + a^2 + 2*a*b)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x)^7 + 3*(a^2 + 2*a*b)
*cosh(x)^5 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x)^3 + (a^2 + 2*a*b)*cosh(x))*sin
h(x))*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a
*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*
a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*
(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2
)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a
^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4
+ 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)
^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2
*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(
x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*
cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*
b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2
*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*
a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b
+ b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*
cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh
(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^
2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6
*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*
```

```
sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
 sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b
+ 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^
3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x
)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)
*sinh(x)^5 + sinh(x)^6)) + 3*(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2
*sinh(x)^8 + 4*(a^2 + 2*a*b)*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 + a^2 + 2*a*b)*
sinh(x)^6 + 8*(7*a^2*cosh(x)^3 + 3*(a^2 + 2*a*b)*cosh(x))*sinh(x)^5 + 2*(3*
a^2 + 8*a*b + 8*b^2)*cosh(x)^4 + 2*(35*a^2*cosh(x)^4 + 30*(a^2 + 2*a*b)*cos
h(x)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(x)^4 + 8*(7*a^2*cosh(x)^5 + 10*(a^2 +
2*a*b)*cosh(x)^3 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + 2*
a*b)*cosh(x)^2 + 4*(7*a^2*cosh(x)^6 + 15*(a^2 + 2*a*b)*cosh(x)^4 + 3*(3*a^2
 + 8*a*b + 8*b^2)*cosh(x)^2 + a^2 + 2*a*b)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x)
^7 + 3*(a^2 + 2*a*b)*cosh(x)^5 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x)^3 + (a^2 +
 2*a*b)*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3
 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*
(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 +
 a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a
*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(
x)^2)) - 16*sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^
6 + 3*(a^2 + a*b)*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2 + a*b)*sinh(x)^4 + 4
*(5*a^2*cosh(x)^3 + 3*(a^2 + a*b)*cosh(x))*sinh(x)^3 + 3*(a^2 + a*b)*cosh(x
)^2 + 3*(5*a^2*cosh(x)^4 + 6*(a^2 + a*b)*cosh(x)^2 + a^2 + a*b)*sinh(x)^2 +
 a^2 + 6*(a^2*cosh(x)^5 + 2*(a^2 + a*b)*cosh(x)^3 + (a^2 + a*b)*cosh(x))*si
nh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*si
nh(x) + sinh(x)^2)))/(a^5*cosh(x)^8 + 8*a^5*cosh(x)*sinh(x)^7 + a^5*sinh(x)
^8 + 4*(a^5 + 2*a^4*b)*cosh(x)^6 + 4*(7*a^5*cosh(x)^2 + a^5 + 2*a^4*b)*sinh
(x)^6 + 8*(7*a^5*cosh(x)^3 + 3*(a^5 + 2*a^4*b)*cosh(x))*sinh(x)^5 + a^5 + 2
*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(x)^4 + 2*(35*a^5*cosh(x)^4 + 3*a^5 + 8*
a^4*b + 8*a^3*b^2 + 30*(a^5 + 2*a^4*b)*cosh(x)^2)*sinh(x)^4 + 8*(7*a^5*cosh
(x)^5 + 10*(a^5 + 2*a^4*b)*cosh(x)^3 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(x
))*sinh(x)^3 + 4*(a^5 + 2*a^4*b)*cosh(x)^2 + 4*(7*a^5*cosh(x)^6 + a^5 + 2*a
^4*b + 15*(a^5 + 2*a^4*b)*cosh(x)^4 + 3*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(
x)^2)*sinh(x)^2 + 8*(a^5*cosh(x)^7 + 3*(a^5 + 2*a^4*b)*cosh(x)^5 + (3*a^5 +
 8*a^4*b + 8*a^3*b^2)*cosh(x)^3 + (a^5 + 2*a^4*b)*cosh(x))*sinh(x)), -1/6*(
3*(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2*sinh(x)^8 + 4*(a^2 + 2*a*b
)*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 + a^2 + 2*a*b)*sinh(x)^6 + 8*(7*a^2*cosh(x
)^3 + 3*(a^2 + 2*a*b)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 8*a*b + 8*b^2)*cosh(x
)^4 + 2*(35*a^2*cosh(x)^4 + 30*(a^2 + 2*a*b)*cosh(x)^2 + 3*a^2 + 8*a*b + 8*
b^2)*sinh(x)^4 + 8*(7*a^2*cosh(x)^5 + 10*(a^2 + 2*a*b)*cosh(x)^3 + (3*a^2 +
 8*a*b + 8*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + 2*a*b)*cosh(x)^2 + 4*(7*a^2*c
osh(x)^6 + 15*(a^2 + 2*a*b)*cosh(x)^4 + 3*(3*a^2 + 8*a*b + 8*b^2)*cosh(x)^2
 + a^2 + 2*a*b)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x)^7 + 3*(a^2 + 2*a*b)*cosh(x
)^5 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x)^3 + (a^2 + 2*a*b)*cosh(x))*sinh(x))*s
qrt(-a)*arctan(sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a
+ b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(co
sh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 +
 a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2
 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 +
a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(x))*sinh(x))) + 3*(a^2*cosh(x)^8 + 8*
a^2*cosh(x)*sinh(x)^7 + a^2*sinh(x)^8 + 4*(a^2 + 2*a*b)*cosh(x)^6 + 4*(7*a^
2*cosh(x)^2 + a^2 + 2*a*b)*sinh(x)^6 + 8*(7*a^2*cosh(x)^3 + 3*(a^2 + 2*a*b)
*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 8*a*b + 8*b^2)*cosh(x)^4 + 2*(35*a^2*cosh(
x)^4 + 30*(a^2 + 2*a*b)*cosh(x)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(x)^4 + 8*(7
*a^2*cosh(x)^5 + 10*(a^2 + 2*a*b)*cosh(x)^3 + (3*a^2 + 8*a*b + 8*b^2)*cosh(
x))*sinh(x)^3 + 4*(a^2 + 2*a*b)*cosh(x)^2 + 4*(7*a^2*cosh(x)^6 + 15*(a^2 +
2*a*b)*cosh(x)^4 + 3*(3*a^2 + 8*a*b + 8*b^2)*cosh(x)^2 + a^2 + 2*a*b)*sinh(
x)^2 + a^2 + 8*(a^2*cosh(x)^7 + 3*(a^2 + 2*a*b)*cosh(x)^5 + (3*a^2 + 8*a*b
+ 8*b^2)*cosh(x)^3 + (a^2 + 2*a*b)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2
```

```
)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^
2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*
cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2
*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*
sinh(x) + a)) + 8*sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*si
nh(x)^6 + 3*(a^2 + a*b)*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2 + a*b)*sinh(x)
^4 + 4*(5*a^2*cosh(x)^3 + 3*(a^2 + a*b)*cosh(x))*sinh(x)^3 + 3*(a^2 + a*b)*
cosh(x)^2 + 3*(5*a^2*cosh(x)^4 + 6*(a^2 + a*b)*cosh(x)^2 + a^2 + a*b)*sinh(
x)^2 + a^2 + 6*(a^2*cosh(x)^5 + 2*(a^2 + a*b)*cosh(x)^3 + (a^2 + a*b)*cosh(
x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh
(x)*sinh(x) + sinh(x)^2)))/(a^5*cosh(x)^8 + 8*a^5*cosh(x)*sinh(x)^7 + a^5*s
inh(x)^8 + 4*(a^5 + 2*a^4*b)*cosh(x)^6 + 4*(7*a^5*cosh(x)^2 + a^5 + 2*a^4*b
)*sinh(x)^6 + 8*(7*a^5*cosh(x)^3 + 3*(a^5 + 2*a^4*b)*cosh(x))*sinh(x)^5 + a
^5 + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(x)^4 + 2*(35*a^5*cosh(x)^4 + 3*a^
5 + 8*a^4*b + 8*a^3*b^2 + 30*(a^5 + 2*a^4*b)*cosh(x)^2)*sinh(x)^4 + 8*(7*a^
5*cosh(x)^5 + 10*(a^5 + 2*a^4*b)*cosh(x)^3 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*
cosh(x))*sinh(x)^3 + 4*(a^5 + 2*a^4*b)*cosh(x)^2 + 4*(7*a^5*cosh(x)^6 + a^5
 + 2*a^4*b + 15*(a^5 + 2*a^4*b)*cosh(x)^4 + 3*(3*a^5 + 8*a^4*b + 8*a^3*b^2)
*cosh(x)^2)*sinh(x)^2 + 8*(a^5*cosh(x)^7 + 3*(a^5 + 2*a^4*b)*cosh(x)^5 + (3
*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(x)^3 + (a^5 + 2*a^4*b)*cosh(x))*sinh(x))]
```

Sympy [A] time = 52.2062, size = 65, normalized size = 1.05

$$-\frac{1}{3a\left(a+b\operatorname{sech}^2(x)\right)^{\frac{3}{2}}}-\frac{1}{a^2\sqrt{a+b\operatorname{sech}^2(x)}}-\frac{\operatorname{atan}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{-a}}\right)}{a^2\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)**2)**(5/2),x)
```

```
[Out] -1/(3*a*(a + b*sech(x)**2)**(3/2)) - 1/(a**2*sqrt(a + b*sech(x)**2)) - atan
(sqrt(a + b*sech(x)**2)/sqrt(-a))/(a**2*sqrt(-a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tanh(x)}{\left(b\operatorname{sech}(x)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(x)/(b*sech(x)^2 + a)^(5/2), x)
```

3.217 $\int \frac{1}{\left(a+b\,\mathbf{sech}^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=95

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}}-\frac{b(5a+3b)\tanh(x)}{3a^2(a+b)^2\sqrt{a-b\tanh^2(x)+b}}-\frac{b\tanh(x)}{3a(a+b)\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(5/2) - (b*Tanh[x])/
(3*a*(a + b)*(a + b - b*Tanh[x]^2)^(3/2)) - (b*(5*a + 3*b)*Tanh[x])/(3*a^2*
(a + b)^2*Sqrt[a + b - b*Tanh[x]^2])
```

Rubi [A] time = 0.0901705, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4128, 414, 527, 12, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}}-\frac{b(5a+3b)\tanh(x)}{3a^2(a+b)^2\sqrt{a-b\tanh^2(x)+b}}-\frac{b\tanh(x)}{3a(a+b)\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[x]^2)^(-5/2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(5/2) - (b*Tanh[x])/
(3*a*(a + b)*(a + b - b*Tanh[x]^2)^(3/2)) - (b*(5*a + 3*b)*Tanh[x])/(3*a^2*
(a + b)^2*Sqrt[a + b - b*Tanh[x]^2])
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a+b\,\text{sech}^2(x)\right)^{5/2}}\,dx &= \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\left(a+b-bx^2\right)^{5/2}}\,dx, x, \tanh(x)\right)\\
&= -\frac{b\tanh(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}}-\frac{\text{Subst}\left(\int \frac{-3a-b-2bx^2}{\left(1-x^2\right)\left(a+b-bx^2\right)^{3/2}}\,dx, x, \tanh(x)\right)}{3a(a+b)}\\
&= -\frac{b\tanh(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}}-\frac{b(5a+3b)\tanh(x)}{3a^2(a+b)^2\sqrt{a+b-b\tanh^2(x)}}+\frac{\text{Subst}\left(\int \frac{3(a+\cdots}{\left(1-x^2\right)\sqrt{\cdots}}\cdots\right)}{3a^2\cdots}\\
&= -\frac{b\tanh(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}}-\frac{b(5a+3b)\tanh(x)}{3a^2(a+b)^2\sqrt{a+b-b\tanh^2(x)}}+\frac{\text{Subst}\left(\int \frac{1\cdots}{\left(1-x^2\right)\sqrt{\cdots}}\cdots\right)}{\cdots}\\
&= -\frac{b\tanh(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}}-\frac{b(5a+3b)\tanh(x)}{3a^2(a+b)^2\sqrt{a+b-b\tanh^2(x)}}+\frac{\text{Subst}\left(\int \frac{1}{1-ax^2}\,d\cdots\right)}{\cdots}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{5/2}}-\frac{b\tanh(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}}-\frac{b(5a+3b)\tanh(x)}{3a^2(a+b)^2\sqrt{a+b-b\tan\cdots}}
\end{aligned}
$$

Mathematica [A] time = 0.422534, size = 130, normalized size = 1.37

$$
\frac{\text{sech}^5(x)\left(\frac{\sqrt{2}(a\cosh(2x)+a+2b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)}{a^{5/2}}-\frac{4b\sinh(x)(a\cosh(2x)+a+2b)\left(3a^2+a(3a+2b)\cosh(2x)+7ab+3b^2\right)}{3a^2(a+b)^2}\right)}{8\left(a+b\,\text{sech}^2(x)\right)^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[x]^2)^(-5/2),x]
```

```
[Out] (Sech[x]^5*((Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cos
h[2*x]]]*(a + 2*b + a*Cosh[2*x])^(5/2))/a^(5/2) - (4*b*(a + 2*b + a*Cosh[2*
x])*(3*a^2 + 7*a*b + 3*b^2 + a*(3*a + 2*b)*Cosh[2*x])*Sinh[x])/(3*a^2*(a +
b)^2)))/(8*(a + b*Sech[x]^2)^(5/2))
```

Maple [F] time = 0.086, size = 0, normalized size = 0.

$$\int \left(a+b\,(\operatorname{sech}(x))^{2}\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(x)^2)^(5/2),x)
```

```
[Out] int(1/(a+b*sech(x)^2)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\operatorname{sech}(x)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(x)^2 + a)^(-5/2), x)
```

Fricas [B] time = 5.24482, size = 15431, normalized size = 162.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^8 + 8*(a^4 + 2*a^3*b + a^2*b^2)
*cosh(x)*sinh(x)^7 + (a^4 + 2*a^3*b + a^2*b^2)*sinh(x)^8 + 4*(a^4 + 4*a^3*b
 + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^6 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3
+ 7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^4 + 2*a^3*b +
a^2*b^2)*cosh(x)^3 + 3*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(
x)^5 + 2*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*cosh(x)^4 + 2*(
35*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^4 + 3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24
*a*b^3 + 8*b^4 + 30*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^2)*sinh(x
)^4 + a^4 + 2*a^3*b + a^2*b^2 + 8*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^5 +
10*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^3 + (3*a^4 + 14*a^3*b + 27
*a^2*b^2 + 24*a*b^3 + 8*b^4)*cosh(x))*sinh(x)^3 + 4*(a^4 + 4*a^3*b + 5*a^2*
b^2 + 2*a*b^3)*cosh(x)^2 + 4*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^6 + 15*(a
^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^4 + a^4 + 4*a^3*b + 5*a^2*b^2 +
 2*a*b^3 + 3*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*cosh(x)^2)*
sinh(x)^2 + 8*((a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^7 + 3*(a^4 + 4*a^3*b + 5*a
^2*b^2 + 2*a*b^3)*cosh(x)^5 + (3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8
```

```
*b^4)*cosh(x)^3 + (a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(x))*s
qrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 -
 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6
 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^
2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 -
30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 -
 b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a
^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^
4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt
(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x
)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh
(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(
x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3
- (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
 a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)
^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3
 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)
^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*
sinh(x)^5 + sinh(x)^6)) + 3*((a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^8 + 8*(a^4 +
 2*a^3*b + a^2*b^2)*cosh(x)*sinh(x)^7 + (a^4 + 2*a^3*b + a^2*b^2)*sinh(x)^8
 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^6 + 4*(a^4 + 4*a^3*b + 5
*a^2*b^2 + 2*a*b^3 + 7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^2)*sinh(x)^6 + 8*(
7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^3 + 3*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*
b^3)*cosh(x))*sinh(x)^5 + 2*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b
^4)*cosh(x)^4 + 2*(35*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^4 + 3*a^4 + 14*a^3*
b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4 + 30*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3
)*cosh(x)^2)*sinh(x)^4 + a^4 + 2*a^3*b + a^2*b^2 + 8*(7*(a^4 + 2*a^3*b + a^
2*b^2)*cosh(x)^5 + 10*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^3 + (3*
a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*cosh(x))*sinh(x)^3 + 4*(a^4
 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^2 + 4*(7*(a^4 + 2*a^3*b + a^2*b^2
)*cosh(x)^6 + 15*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^4 + a^4 + 4*
a^3*b + 5*a^2*b^2 + 2*a*b^3 + 3*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 +
 8*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^7 + 3*(
a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^5 + (3*a^4 + 14*a^3*b + 27*a^2
*b^2 + 24*a*b^3 + 8*b^4)*cosh(x)^3 + (a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*
cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sin
h(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2
)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2
 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*
(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x)
 + sinh(x)^2)) - 8*sqrt(2)*((3*a^3*b + 2*a^2*b^2)*cosh(x)^6 + 6*(3*a^3*b +
2*a^2*b^2)*cosh(x)*sinh(x)^5 + (3*a^3*b + 2*a^2*b^2)*sinh(x)^6 + 3*(a^3*b +
 4*a^2*b^2 + 2*a*b^3)*cosh(x)^4 + 3*(a^3*b + 4*a^2*b^2 + 2*a*b^3 + 5*(3*a^3
*b + 2*a^2*b^2)*cosh(x)^2)*sinh(x)^4 - 3*a^3*b - 2*a^2*b^2 + 4*(5*(3*a^3*b
+ 2*a^2*b^2)*cosh(x)^3 + 3*(a^3*b + 4*a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(x)^3
 - 3*(a^3*b + 4*a^2*b^2 + 2*a*b^3)*cosh(x)^2 + 3*(5*(3*a^3*b + 2*a^2*b^2)*c
osh(x)^4 - a^3*b - 4*a^2*b^2 - 2*a*b^3 + 6*(a^3*b + 4*a^2*b^2 + 2*a*b^3)*co
sh(x)^2)*sinh(x)^2 + 6*((3*a^3*b + 2*a^2*b^2)*cosh(x)^5 + 2*(a^3*b + 4*a^2*
b^2 + 2*a*b^3)*cosh(x)^3 - (a^3*b + 4*a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(x))*
sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
 sinh(x)^2)))/((a^7 + 2*a^6*b + a^5*b^2)*cosh(x)^8 + 8*(a^7 + 2*a^6*b + a^5
*b^2)*cosh(x)*sinh(x)^7 + (a^7 + 2*a^6*b + a^5*b^2)*sinh(x)^8 + a^7 + 2*a^6
*b + a^5*b^2 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*cosh(x)^6 + 4*(a^7
 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3 + 7*(a^7 + 2*a^6*b + a^5*b^2)*cosh(x)^2)
*sinh(x)^6 + 8*(7*(a^7 + 2*a^6*b + a^5*b^2)*cosh(x)^3 + 3*(a^7 + 4*a^6*b +
5*a^5*b^2 + 2*a^4*b^3)*cosh(x))*sinh(x)^5 + 2*(3*a^7 + 14*a^6*b + 27*a^5*b^
2 + 24*a^4*b^3 + 8*a^3*b^4)*cosh(x)^4 + 2*(3*a^7 + 14*a^6*b + 27*a^5*b^2 +
24*a^4*b^3 + 8*a^3*b^4 + 35*(a^7 + 2*a^6*b + a^5*b^2)*cosh(x)^4 + 30*(a^7 +
 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^7 + 2*a^6*
```

b + a^5*b^2)*cosh(x)^5 + 10*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*cosh(x)
^3 + (3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*cosh(x))*sinh
(x)^3 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*cosh(x)^2 + 4*(a^7 + 4*a^
6*b + 5*a^5*b^2 + 2*a^4*b^3 + 7*(a^7 + 2*a^6*b + a^5*b^2)*cosh(x)^6 + 15*(a
^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*cosh(x)^4 + 3*(3*a^7 + 14*a^6*b + 27*
a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((a^7 + 2*a^6*b
+ a^5*b^2)*cosh(x)^7 + 3*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*cosh(x)^5
+ (3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*cosh(x)^3 + (a^7
 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*cosh(x))*sinh(x)), -1/6*(3*((a^4 + 2*a^
3*b + a^2*b^2)*cosh(x)^8 + 8*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)*sinh(x)^7 +
(a^4 + 2*a^3*b + a^2*b^2)*sinh(x)^8 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^
3)*cosh(x)^6 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3 + 7*(a^4 + 2*a^3*b +
a^2*b^2)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^3 +
3*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(x)^5 + 2*(3*a^4 + 14*
a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*cosh(x)^4 + 2*(35*(a^4 + 2*a^3*b + a
^2*b^2)*cosh(x)^4 + 3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4 + 30*(
a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^2)*sinh(x)^4 + a^4 + 2*a^3*b +
 a^2*b^2 + 8*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^5 + 10*(a^4 + 4*a^3*b + 5
*a^2*b^2 + 2*a*b^3)*cosh(x)^3 + (3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 +
 8*b^4)*cosh(x))*sinh(x)^3 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x
)^2 + 4*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^6 + 15*(a^4 + 4*a^3*b + 5*a^2*
b^2 + 2*a*b^3)*cosh(x)^4 + a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3 + 3*(3*a^4 +
 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((a^4 +
 2*a^3*b + a^2*b^2)*cosh(x)^7 + 3*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cos
h(x)^5 + (3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*cosh(x)^3 + (a^
4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2
)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*co
sh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2
))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)
*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cos
h(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) + 3*((a^4 + 2*a^3*b + a^2*b^2)*co
sh(x)^8 + 8*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)*sinh(x)^7 + (a^4 + 2*a^3*b +
a^2*b^2)*sinh(x)^8 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^6 + 4*
(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3 + 7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^
2)*sinh(x)^6 + 8*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^3 + 3*(a^4 + 4*a^3*b
+ 5*a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(x)^5 + 2*(3*a^4 + 14*a^3*b + 27*a^2*b^
2 + 24*a*b^3 + 8*b^4)*cosh(x)^4 + 2*(35*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^4
 + 3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4 + 30*(a^4 + 4*a^3*b + 5
*a^2*b^2 + 2*a*b^3)*cosh(x)^2)*sinh(x)^4 + a^4 + 2*a^3*b + a^2*b^2 + 8*(7*(
a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^5 + 10*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^
3)*cosh(x)^3 + (3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*cosh(x))*
sinh(x)^3 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^2 + 4*(7*(a^4 +
 2*a^3*b + a^2*b^2)*cosh(x)^6 + 15*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*co
sh(x)^4 + a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3 + 3*(3*a^4 + 14*a^3*b + 27*a^
2*b^2 + 24*a*b^3 + 8*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((a^4 + 2*a^3*b + a^2*b^
2)*cosh(x)^7 + 3*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^5 + (3*a^4 +
 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*cosh(x)^3 + (a^4 + 4*a^3*b + 5*a
^2*b^2 + 2*a*b^3)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((
a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(
x)^2))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)) + 4*sqrt(2)*(
(3*a^3*b + 2*a^2*b^2)*cosh(x)^6 + 6*(3*a^3*b + 2*a^2*b^2)*cosh(x)*sinh(x)^5
 + (3*a^3*b + 2*a^2*b^2)*sinh(x)^6 + 3*(a^3*b + 4*a^2*b^2 + 2*a*b^3)*cosh(x
)^4 + 3*(a^3*b + 4*a^2*b^2 + 2*a*b^3 + 5*(3*a^3*b + 2*a^2*b^2)*cosh(x)^2)*s
inh(x)^4 - 3*a^3*b - 2*a^2*b^2 + 4*(5*(3*a^3*b + 2*a^2*b^2)*cosh(x)^3 + 3*(
a^3*b + 4*a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(x)^3 - 3*(a^3*b + 4*a^2*b^2 + 2*
a*b^3)*cosh(x)^2 + 3*(5*(3*a^3*b + 2*a^2*b^2)*cosh(x)^4 - a^3*b - 4*a^2*b^2
 - 2*a*b^3 + 6*(a^3*b + 4*a^2*b^2 + 2*a*b^3)*cosh(x)^2)*sinh(x)^2 + 6*((3*a
^3*b + 2*a^2*b^2)*cosh(x)^5 + 2*(a^3*b + 4*a^2*b^2 + 2*a*b^3)*cosh(x)^3 - (
a^3*b + 4*a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x

```
)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^7 + 2*a^6*
b + a^5*b^2)*cosh(x)^8 + 8*(a^7 + 2*a^6*b + a^5*b^2)*cosh(x)*sinh(x)^7 + (a
^7 + 2*a^6*b + a^5*b^2)*sinh(x)^8 + a^7 + 2*a^6*b + a^5*b^2 + 4*(a^7 + 4*a^
6*b + 5*a^5*b^2 + 2*a^4*b^3)*cosh(x)^6 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a
^4*b^3 + 7*(a^7 + 2*a^6*b + a^5*b^2)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^7 + 2*a
^6*b + a^5*b^2)*cosh(x)^3 + 3*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*cosh(
x))*sinh(x)^5 + 2*(3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*
cosh(x)^4 + 2*(3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4 + 35*
(a^7 + 2*a^6*b + a^5*b^2)*cosh(x)^4 + 30*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4
*b^3)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^7 + 2*a^6*b + a^5*b^2)*cosh(x)^5 + 10*
(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*cosh(x)^3 + (3*a^7 + 14*a^6*b + 27*
a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*cosh(x))*sinh(x)^3 + 4*(a^7 + 4*a^6*b + 5
*a^5*b^2 + 2*a^4*b^3)*cosh(x)^2 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3
+ 7*(a^7 + 2*a^6*b + a^5*b^2)*cosh(x)^6 + 15*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2
*a^4*b^3)*cosh(x)^4 + 3*(3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3
*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((a^7 + 2*a^6*b + a^5*b^2)*cosh(x)^7 + 3*(a^
7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*cosh(x)^5 + (3*a^7 + 14*a^6*b + 27*a^5
*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*cosh(x)^3 + (a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a
^4*b^3)*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + b\operatorname{sech}^2(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)**2)**(5/2),x)
```

```
[Out] Integral((a + b*sech(x)**2)**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\operatorname{sech}(x)^2 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sech(x)^2 + a)^(-5/2), x)
```

3.218 $\int \frac{\coth(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=109

$$-\frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b\,\mathrm{sech}^2(x)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{b}{3a(a+b)\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}$$

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/a^(5/2) - ArcTanh[Sqrt[a + b*Sech[x]
^2]/Sqrt[a + b]]/(a + b)^(5/2) - b/(3*a*(a + b)*(a + b*Sech[x]^2)^(3/2)) -
(b*(2*a + b))/(a^2*(a + b)^2*Sqrt[a + b*Sech[x]^2])
```

Rubi [A] time = 0.199893, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {4139, 446, 85, 152, 156, 63, 208}

$$-\frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b\,\mathrm{sech}^2(x)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{b}{3a(a+b)\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/a^(5/2) - ArcTanh[Sqrt[a + b*Sech[x]
^2]/Sqrt[a + b]]/(a + b)^(5/2) - b/(3*a*(a + b)*(a + b*Sech[x]^2)^(3/2)) -
(b*(2*a + b))/(a^2*(a + b)^2*Sqrt[a + b*Sech[x]^2])
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\coth(x)}{\left(a+b\mathrm{sech}^2(x)\right)^{5/2}}\,dx = \mathrm{Subst}\left(\int \frac{1}{x\left(-1+x^2\right)\left(a+bx^2\right)^{5/2}}\,dx, x, \mathrm{sech}(x)\right)$$

$$= \frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{(-1+x)x(a+bx)^{5/2}}\,dx, x, \mathrm{sech}^2(x)\right)$$

$$= -\frac{b}{3a(a+b)\left(a+b\mathrm{sech}^2(x)\right)^{3/2}} + \frac{\mathrm{Subst}\left(\int \frac{a+b-bx}{(-1+x)x(a+bx)^{3/2}}\,dx, x, \mathrm{sech}^2(x)\right)}{2a(a+b)}$$

$$= -\frac{b}{3a(a+b)\left(a+b\mathrm{sech}^2(x)\right)^{3/2}} - \frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b\mathrm{sech}^2(x)}} - \frac{\mathrm{Subst}\left(\int \frac{-\frac{1}{2}(a+b)^2+\frac{1}{2}b(2a+b)x}{(-1+x)x\sqrt{a+bx}}\,dx, \ldots\right.}{a^2(a+b)^2}$$

$$= -\frac{b}{3a(a+b)\left(a+b\mathrm{sech}^2(x)\right)^{3/2}} - \frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b\mathrm{sech}^2(x)}} - \frac{\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \mathrm{sech}^2(x\ldots\right.}{2a^2}$$

$$= -\frac{b}{3a(a+b)\left(a+b\mathrm{sech}^2(x)\right)^{3/2}} - \frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b\mathrm{sech}^2(x)}} - \frac{\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bs\ldots}\right.}{a^2b}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\mathrm{sech}^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{b}{3a(a+b)\left(a+b\mathrm{sech}^2(x)\right)^{3/2}} - \frac{b(2\ldots}{a^2(a+b)^2\sqrt{\ldots}}$$

Mathematica [B] time = 1.04247, size = 242, normalized size = 2.22

$$\frac{\mathrm{sech}^5(x)\left(-\frac{2b\cosh(x)\left(7a^2+a(7a+4b)\cosh(2x)+16ab+6b^2\right)(a\cosh(2x)+a+2b)}{3a^2(a+b)^2} - \frac{(a\cosh(2x)+a+2b)^{5/2}\left(\sqrt{a}\left(a^2-2ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a+b}\cosh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)+(a+b)^2\ldots\right.}{\sqrt{2}\ldots}\right.}{8\left(a+b\mathrm{sech}^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] (((-2*b*Cosh[x]*(a + 2*b + a*Cosh[2*x])*(7*a^2 + 16*a*b + 6*b^2 + a*(7*a +
4*b)*Cosh[2*x]))/(3*a^2*(a + b)^2) - ((a + 2*b + a*Cosh[2*x])^(5/2)*(Sqrt[a
]*(a^2 - 2*a*b - b^2)*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Cosh[x])/Sqrt[a + 2*b +
a*Cosh[2*x]]] + (a + b)^2*(Sqrt[a]*ArcTanh[(Sqrt[2*a + 2*b]*Cosh[x])/Sqrt[a
 + 2*b + a*Cosh[2*x]]] - 2*Sqrt[a + b]*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a
 + 2*b + a*Cosh[2*x]]])))/(Sqrt[2]*a^(5/2)*(a + b)^(5/2)))*Sech[x]^5)/(8*(a
 + b*Sech[x]^2)^(5/2))
```

Maple [F] time = 0.1, size = 0, normalized size = 0.

$$\int \coth(x)\left(a+b(\mathrm{sech}(x))^2\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*sech(x)^2)^(5/2),x)
```

```
[Out] int(coth(x)/(a+b*sech(x)^2)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\left(b\operatorname{sech}(x)^2 + a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)/(b*sech(x)^2 + a)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.219 $\int \frac{\coth^2(x)}{\left(a+b\,\mathbf{sech}^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=133

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}}-\frac{(a-3b)(3a+b)\coth(x)\sqrt{a-b\tanh^2(x)+b}}{3a^2(a+b)^3}-\frac{b(7a+3b)\coth(x)}{3a^2(a+b)^2\sqrt{a-b\tanh^2(x)+b}}-\frac{\quad}{3a(a+b)\,(\cdots}$$

```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(5/2) - (b*Coth[x])/
(3*a*(a + b)*(a + b - b*Tanh[x]^2)^(3/2)) - (b*(7*a + 3*b)*Coth[x])/(3*a^2*
(a + b)^2*Sqrt[a + b - b*Tanh[x]^2]) - ((a - 3*b)*(3*a + b)*Coth[x]*Sqrt[a
+ b - b*Tanh[x]^2])/(3*a^2*(a + b)^3)
```

Rubi [A] time = 0.375975, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {4141, 1975, 472, 579, 583, 12, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}}-\frac{(a-3b)(3a+b)\coth(x)\sqrt{a-b\tanh^2(x)+b}}{3a^2(a+b)^3}-\frac{b(7a+3b)\coth(x)}{3a^2(a+b)^2\sqrt{a-b\tanh^2(x)+b}}-\frac{\quad}{3a(a+b)\,(\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(5/2) - (b*Coth[x])/
(3*a*(a + b)*(a + b - b*Tanh[x]^2)^(3/2)) - (b*(7*a + 3*b)*Coth[x])/(3*a^2*
(a + b)^2*Sqrt[a + b - b*Tanh[x]^2]) - ((a - 3*b)*(3*a + b)*Coth[x]*Sqrt[a
+ b - b*Tanh[x]^2])/(3*a^2*(a + b)^3)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
```

```
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\coth^2(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{1}{x^2\left(1-x^2\right)\left(a+b\left(1-x^2\right)\right)^{5/2}}\,dx, x, \tanh(x)\right)$$
$$= \text{Subst}\left(\int \frac{1}{x^2\left(1-x^2\right)\left(a+b-bx^2\right)^{5/2}}\,dx, x, \tanh(x)\right)$$
$$= -\frac{b\coth(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-3a+b-4bx^2}{x^2(1-x^2)\left(a+b-bx^2\right)^{3/2}}\,dx, x, \tanh(x)\right)}{3a(a+b)}$$
$$= -\frac{b\coth(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{b(7a+3b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{(a-3b)(3a+b)}{x^2(1-x^2)}\right.}{3a}$$
$$= -\frac{b\coth(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{b(7a+3b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b-b\tanh^2(x)}} - \frac{(a-3b)(3a+b)\coth}{3a^2(}$$
$$= -\frac{b\coth(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{b(7a+3b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b-b\tanh^2(x)}} - \frac{(a-3b)(3a+b)\coth}{3a^2(}$$
$$= -\frac{b\coth(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{b(7a+3b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b-b\tanh^2(x)}} - \frac{(a-3b)(3a+b)\coth}{3a^2(}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{5/2}} - \frac{b\coth(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{b(7a+3b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b-b\tanh^2(}}$$

Mathematica [A] time = 0.782152, size = 155, normalized size = 1.17

$$\frac{\text{sech}^5(x)\left(\frac{\sqrt{2}(a\cosh(2x)+a+2b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)}{a^{5/2}} - \frac{(a\cosh(2x)+a+2b)\left(3a^2\text{csch}(x)(a\cosh(2x)+a+2b)^2-4b^3(a+b)\sinh(x)+2b^2(9a+4b)\sinh(x)\right.}{3a^2(a+b)^3}\right.}{8\left(a+b\text{sech}^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] (Sech[x]^5*((Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cos
h[2*x]]]*(a + 2*b + a*Cosh[2*x])^(5/2))/a^(5/2) - ((a + 2*b + a*Cosh[2*x])*
(3*a^2*(a + 2*b + a*Cosh[2*x])^2*Csch[x] - 4*b^3*(a + b)*Sinh[x] + 2*b^2*(9
*a + 4*b)*(a + 2*b + a*Cosh[2*x])*Sinh[x]))/(3*a^2*(a + b)^3)))/(8*(a + b*S
ech[x]^2)^(5/2))
```

Maple [F] time = 0.103, size = 0, normalized size = 0.

$$\int (\coth(x))^2\left(a+b(\text{sech}(x))^2\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*sech(x)^2)^(5/2),x)
```

```
[Out] int(coth(x)^2/(a+b*sech(x)^2)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)^2}{\left(b\,\text{sech}(x)^2 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)^2/(b*sech(x)^2 + a)^(5/2), x)
```

Fricas [B] time = 10.5392, size = 26619, normalized size = 200.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^10 + 10*(a^5 + 3*a^
4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)*sinh(x)^9 + (a^5 + 3*a^4*b + 3*a^3*b^2 +
 a^2*b^3)*sinh(x)^10 + (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^
4)*cosh(x)^8 + (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4 + 45*(
a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^2)*sinh(x)^8 + 8*(15*(a^5 + 3*
a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^3 + (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 2
7*a^2*b^3 + 8*a*b^4)*cosh(x))*sinh(x)^7 + 2*(a^5 + 7*a^4*b + 23*a^3*b^2 + 3
7*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^6 + 2*(a^5 + 7*a^4*b + 23*a^3*b^2 + 3
7*a^2*b^3 + 28*a*b^4 + 8*b^5 + 105*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*co
sh(x)^4 + 14*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)
^2)*sinh(x)^6 + 4*(63*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^5 + 14*
(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^3 + 3*(a^5 +
 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x))*sinh(x)^5 -
 a^5 - 3*a^4*b - 3*a^3*b^2 - a^2*b^3 - 2*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a
^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^4 + 2*(105*(a^5 + 3*a^4*b + 3*a^3*b^2 +
a^2*b^3)*cosh(x)^6 - a^5 - 7*a^4*b - 23*a^3*b^2 - 37*a^2*b^3 - 28*a*b^4 - 8
*b^5 + 35*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^4
+ 15*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^2
)*sinh(x)^4 + 8*(15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^7 + 7*(3*
a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^5 + 5*(a^5 + 7*
a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^3 - (a^5 + 7*a^
4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x))*sinh(x)^3 - (3*a
^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^2 + (45*(a^5 + 3
*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^8 + 28*(3*a^5 + 17*a^4*b + 33*a^3*b^2
 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^6 - 3*a^5 - 17*a^4*b - 33*a^3*b^2 - 27*a^2
*b^3 - 8*a*b^4 + 30*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8
*b^5)*cosh(x)^4 - 12*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 +
8*b^5)*cosh(x)^2)*sinh(x)^2 + 2*(5*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*co
```

```
sh(x)^9 + 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^
7 + 6*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^
5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^
3 - (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x))*sinh(x)
)*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^
8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x
)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4
*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2
 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^
2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2
*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(
x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + s
qrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cos
h(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*c
osh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*co
sh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)
^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^
2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh
(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (
a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh
(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(
x)*sinh(x)^5 + sinh(x)^6)) + 3*((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(
x)^10 + 10*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)*sinh(x)^9 + (a^5 +
 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sinh(x)^10 + (3*a^5 + 17*a^4*b + 33*a^3*b^2
 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^8 + (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^
2*b^3 + 8*a*b^4 + 45*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^2)*sinh(
x)^8 + 8*(15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^3 + (3*a^5 + 17*
a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x))*sinh(x)^7 + 2*(a^5 + 7*
a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^6 + 2*(a^5 + 7*
a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5 + 105*(a^5 + 3*a^4*b + 3
*a^3*b^2 + a^2*b^3)*cosh(x)^4 + 14*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*
b^3 + 8*a*b^4)*cosh(x)^2)*sinh(x)^6 + 4*(63*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^
2*b^3)*cosh(x)^5 + 14*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4
)*cosh(x)^3 + 3*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5
)*cosh(x))*sinh(x)^5 - a^5 - 3*a^4*b - 3*a^3*b^2 - a^2*b^3 - 2*(a^5 + 7*a^4
*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^4 + 2*(105*(a^5 +
3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^6 - a^5 - 7*a^4*b - 23*a^3*b^2 - 37*
a^2*b^3 - 28*a*b^4 - 8*b^5 + 35*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3
 + 8*a*b^4)*cosh(x)^4 + 15*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*
b^4 + 8*b^5)*cosh(x)^2)*sinh(x)^4 + 8*(15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*
b^3)*cosh(x)^7 + 7*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*c
osh(x)^5 + 5*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*c
osh(x)^3 - (a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cos
h(x))*sinh(x)^3 - (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*co
sh(x)^2 + (45*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^8 + 28*(3*a^5 +
 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^6 - 3*a^5 - 17*a^4*b
 - 33*a^3*b^2 - 27*a^2*b^3 - 8*a*b^4 + 30*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*
a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^4 - 12*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37
*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^2)*sinh(x)^2 + 2*(5*(a^5 + 3*a^4*b + 3
*a^3*b^2 + a^2*b^3)*cosh(x)^9 + 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b
^3 + 8*a*b^4)*cosh(x)^7 + 6*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a
*b^4 + 8*b^5)*cosh(x)^5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a
*b^4 + 8*b^5)*cosh(x)^3 - (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a
*b^4)*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 +
 a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 +
sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cos
h(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)
) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*s
inh(x) + sinh(x)^2)) - 4*sqrt(2)*((3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)^8
```

+ 8*(3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)*sinh(x)^7 + (3*a^5 + 9*a^3*b^2
+ 4*a^2*b^3)*sinh(x)^8 + 4*(3*a^5 + 6*a^4*b + 8*a^2*b^3 + 3*a*b^4)*cosh(x)
^6 + 4*(3*a^5 + 6*a^4*b + 8*a^2*b^3 + 3*a*b^4 + 7*(3*a^5 + 9*a^3*b^2 + 4*a^
2*b^3)*cosh(x)^2)*sinh(x)^6 + 8*(7*(3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)^
3 + 3*(3*a^5 + 6*a^4*b + 8*a^2*b^3 + 3*a*b^4)*cosh(x))*sinh(x)^5 + 3*a^5 +
9*a^3*b^2 + 4*a^2*b^3 + 6*(3*a^5 + 8*a^4*b + 5*a^3*b^2 - 12*a^2*b^3 - 4*a*b
^4)*cosh(x)^4 + 2*(9*a^5 + 24*a^4*b + 15*a^3*b^2 - 36*a^2*b^3 - 12*a*b^4 +
35*(3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)^4 + 30*(3*a^5 + 6*a^4*b + 8*a^2*
b^3 + 3*a*b^4)*cosh(x)^2)*sinh(x)^4 + 8*(7*(3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*
cosh(x)^5 + 10*(3*a^5 + 6*a^4*b + 8*a^2*b^3 + 3*a*b^4)*cosh(x)^3 + 3*(3*a^5
+ 8*a^4*b + 5*a^3*b^2 - 12*a^2*b^3 - 4*a*b^4)*cosh(x))*sinh(x)^3 + 4*(3*a^
5 + 6*a^4*b + 8*a^2*b^3 + 3*a*b^4)*cosh(x)^2 + 4*(7*(3*a^5 + 9*a^3*b^2 + 4*
a^2*b^3)*cosh(x)^6 + 3*a^5 + 6*a^4*b + 8*a^2*b^3 + 3*a*b^4 + 15*(3*a^5 + 6*
a^4*b + 8*a^2*b^3 + 3*a*b^4)*cosh(x)^4 + 9*(3*a^5 + 8*a^4*b + 5*a^3*b^2 - 1
2*a^2*b^3 - 4*a*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((3*a^5 + 9*a^3*b^2 + 4*a^2*b
^3)*cosh(x)^7 + 3*(3*a^5 + 6*a^4*b + 8*a^2*b^3 + 3*a*b^4)*cosh(x)^5 + 3*(3*
a^5 + 8*a^4*b + 5*a^3*b^2 - 12*a^2*b^3 - 4*a*b^4)*cosh(x)^3 + (3*a^5 + 6*a^
4*b + 8*a^2*b^3 + 3*a*b^4)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^
2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^8 + 3*a^7*b
+ 3*a^6*b^2 + a^5*b^3)*cosh(x)^10 + 10*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3
)*cosh(x)*sinh(x)^9 + (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*sinh(x)^10 + (3
*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*cosh(x)^8 + (3*a^8 +
17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4 + 45*(a^8 + 3*a^7*b + 3*a^6
*b^2 + a^5*b^3)*cosh(x)^2)*sinh(x)^8 - a^8 - 3*a^7*b - 3*a^6*b^2 - a^5*b^3
+ 8*(15*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*cosh(x)^3 + (3*a^8 + 17*a^7*b
+ 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*cosh(x))*sinh(x)^7 + 2*(a^8 + 7*a^7
*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*cosh(x)^6 + 2*(a^8 +
7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5 + 105*(a^8 + 3*
a^7*b + 3*a^6*b^2 + a^5*b^3)*cosh(x)^4 + 14*(3*a^8 + 17*a^7*b + 33*a^6*b^2
+ 27*a^5*b^3 + 8*a^4*b^4)*cosh(x)^2)*sinh(x)^6 + 4*(63*(a^8 + 3*a^7*b + 3*a
^6*b^2 + a^5*b^3)*cosh(x)^5 + 14*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^
3 + 8*a^4*b^4)*cosh(x)^3 + 3*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*
a^4*b^4 + 8*a^3*b^5)*cosh(x))*sinh(x)^5 - 2*(a^8 + 7*a^7*b + 23*a^6*b^2 + 3
7*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*cosh(x)^4 - 2*(a^8 + 7*a^7*b + 23*a^6*b
^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5 - 105*(a^8 + 3*a^7*b + 3*a^6*b^2 +
a^5*b^3)*cosh(x)^6 - 35*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^
4*b^4)*cosh(x)^4 - 15*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4
+ 8*a^3*b^5)*cosh(x)^2)*sinh(x)^4 + 8*(15*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5
*b^3)*cosh(x)^7 + 7*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4
)*cosh(x)^5 + 5*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a
^3*b^5)*cosh(x)^3 - (a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 +
8*a^3*b^5)*cosh(x))*sinh(x)^3 - (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^
3 + 8*a^4*b^4)*cosh(x)^2 + (45*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*cosh(x
)^8 - 3*a^8 - 17*a^7*b - 33*a^6*b^2 - 27*a^5*b^3 - 8*a^4*b^4 + 28*(3*a^8 +
17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*cosh(x)^6 + 30*(a^8 + 7*a^7
*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*cosh(x)^4 - 12*(a^8
+ 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*cosh(x)^2)*si
nh(x)^2 + 2*(5*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*cosh(x)^9 + 4*(3*a^8 +
17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*cosh(x)^7 + 6*(a^8 + 7*a^7
*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*cosh(x)^5 - 4*(a^8 +
7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*cosh(x)^3 - (3
*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*cosh(x))*sinh(x)), -
1/6*(3*((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^10 + 10*(a^5 + 3*a^4*
b + 3*a^3*b^2 + a^2*b^3)*cosh(x)*sinh(x)^9 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a
^2*b^3)*sinh(x)^10 + (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)
*cosh(x)^8 + (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4 + 45*(a^
5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^2)*sinh(x)^8 + 8*(15*(a^5 + 3*a^
4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^3 + (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*
a^2*b^3 + 8*a*b^4)*cosh(x))*sinh(x)^7 + 2*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*

a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^6 + 2*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*
a^2*b^3 + 28*a*b^4 + 8*b^5 + 105*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh
(x)^4 + 14*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^2
)*sinh(x)^6 + 4*(63*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^5 + 14*(3
*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^3 + 3*(a^5 + 7
*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x))*sinh(x)^5 - a
^5 - 3*a^4*b - 3*a^3*b^2 - a^2*b^3 - 2*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2
*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^4 + 2*(105*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^
2*b^3)*cosh(x)^6 - a^5 - 7*a^4*b - 23*a^3*b^2 - 37*a^2*b^3 - 28*a*b^4 - 8*b
^5 + 35*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^4 +
15*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^2)*
sinh(x)^4 + 8*(15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^7 + 7*(3*a^
5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^5 + 5*(a^5 + 7*a^
4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^3 - (a^5 + 7*a^4*
b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x))*sinh(x)^3 - (3*a^5
 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^2 + (45*(a^5 + 3*a
^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^8 + 28*(3*a^5 + 17*a^4*b + 33*a^3*b^2 +
 27*a^2*b^3 + 8*a*b^4)*cosh(x)^6 - 3*a^5 - 17*a^4*b - 33*a^3*b^2 - 27*a^2*b
^3 - 8*a*b^4 + 30*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b
^5)*cosh(x)^4 - 12*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*
b^5)*cosh(x)^2)*sinh(x)^2 + 2*(5*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh
(x)^9 + 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^7
+ 6*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^5
- 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^3
- (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x))*sinh(x))*
sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 +
a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(
x)*sinh(x) + sinh(x)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sin
h(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^
2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) + 3*((a^5 +
 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^10 + 10*(a^5 + 3*a^4*b + 3*a^3*b^2
+ a^2*b^3)*cosh(x)*sinh(x)^9 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sinh(x
)^10 + (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^8 + (
3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4 + 45*(a^5 + 3*a^4*b +
3*a^3*b^2 + a^2*b^3)*cosh(x)^2)*sinh(x)^8 + 8*(15*(a^5 + 3*a^4*b + 3*a^3*b^
2 + a^2*b^3)*cosh(x)^3 + (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*
b^4)*cosh(x))*sinh(x)^7 + 2*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a
*b^4 + 8*b^5)*cosh(x)^6 + 2*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a
*b^4 + 8*b^5 + 105*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^4 + 14*(3*
a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^2)*sinh(x)^6 +
4*(63*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^5 + 14*(3*a^5 + 17*a^4*
b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^3 + 3*(a^5 + 7*a^4*b + 23*a^
3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x))*sinh(x)^5 - a^5 - 3*a^4*b -
 3*a^3*b^2 - a^2*b^3 - 2*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^
4 + 8*b^5)*cosh(x)^4 + 2*(105*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)
^6 - a^5 - 7*a^4*b - 23*a^3*b^2 - 37*a^2*b^3 - 28*a*b^4 - 8*b^5 + 35*(3*a^5
 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^4 + 15*(a^5 + 7*a^
4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^2)*sinh(x)^4 + 8*
(15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^7 + 7*(3*a^5 + 17*a^4*b +
 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^5 + 5*(a^5 + 7*a^4*b + 23*a^3*b
^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^3 - (a^5 + 7*a^4*b + 23*a^3*b^2
 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x))*sinh(x)^3 - (3*a^5 + 17*a^4*b +
33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^2 + (45*(a^5 + 3*a^4*b + 3*a^3*b
^2 + a^2*b^3)*cosh(x)^8 + 28*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 +
8*a*b^4)*cosh(x)^6 - 3*a^5 - 17*a^4*b - 33*a^3*b^2 - 27*a^2*b^3 - 8*a*b^4 +
 30*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^4
- 12*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^2
)*sinh(x)^2 + 2*(5*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^9 + 4*(3*a
^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^7 + 6*(a^5 + 7*a

```
^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^5 - 4*(a^5 + 7*a
^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^3 - (3*a^5 + 17*
a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x))*sinh(x))*sqrt(-a)*arcta
n(sqrt(2)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh
(x)^2 + a)) + 2*sqrt(2)*((3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)^8 + 8*(3*a
^5 + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)*sinh(x)^7 + (3*a^5 + 9*a^3*b^2 + 4*a^2*
b^3)*sinh(x)^8 + 4*(3*a^5 + 6*a^4*b + 8*a^2*b^3 + 3*a*b^4)*cosh(x)^6 + 4*(3
*a^5 + 6*a^4*b + 8*a^2*b^3 + 3*a*b^4 + 7*(3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*co
sh(x)^2)*sinh(x)^6 + 8*(7*(3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)^3 + 3*(3*
a^5 + 6*a^4*b + 8*a^2*b^3 + 3*a*b^4)*cosh(x))*sinh(x)^5 + 3*a^5 + 9*a^3*b^2
 + 4*a^2*b^3 + 6*(3*a^5 + 8*a^4*b + 5*a^3*b^2 - 12*a^2*b^3 - 4*a*b^4)*cosh(
x)^4 + 2*(9*a^5 + 24*a^4*b + 15*a^3*b^2 - 36*a^2*b^3 - 12*a*b^4 + 35*(3*a^5
 + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)^4 + 30*(3*a^5 + 6*a^4*b + 8*a^2*b^3 + 3*a
*b^4)*cosh(x)^2)*sinh(x)^4 + 8*(7*(3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)^5
 + 10*(3*a^5 + 6*a^4*b + 8*a^2*b^3 + 3*a*b^4)*cosh(x)^3 + 3*(3*a^5 + 8*a^4*
b + 5*a^3*b^2 - 12*a^2*b^3 - 4*a*b^4)*cosh(x))*sinh(x)^3 + 4*(3*a^5 + 6*a^4
*b + 8*a^2*b^3 + 3*a*b^4)*cosh(x)^2 + 4*(7*(3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*
cosh(x)^6 + 3*a^5 + 6*a^4*b + 8*a^2*b^3 + 3*a*b^4 + 15*(3*a^5 + 6*a^4*b + 8
*a^2*b^3 + 3*a*b^4)*cosh(x)^4 + 9*(3*a^5 + 8*a^4*b + 5*a^3*b^2 - 12*a^2*b^3
 - 4*a*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*cosh(
x)^7 + 3*(3*a^5 + 6*a^4*b + 8*a^2*b^3 + 3*a*b^4)*cosh(x)^5 + 3*(3*a^5 + 8*a
^4*b + 5*a^3*b^2 - 12*a^2*b^3 - 4*a*b^4)*cosh(x)^3 + (3*a^5 + 6*a^4*b + 8*a
^2*b^3 + 3*a*b^4)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2
*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^8 + 3*a^7*b + 3*a^6*b
^2 + a^5*b^3)*cosh(x)^10 + 10*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*cosh(x)
*sinh(x)^9 + (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*sinh(x)^10 + (3*a^8 + 17
*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*cosh(x)^8 + (3*a^8 + 17*a^7*b
 + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4 + 45*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^
5*b^3)*cosh(x)^2)*sinh(x)^8 - a^8 - 3*a^7*b - 3*a^6*b^2 - a^5*b^3 + 8*(15*(
a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*cosh(x)^3 + (3*a^8 + 17*a^7*b + 33*a^6
*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*cosh(x))*sinh(x)^7 + 2*(a^8 + 7*a^7*b + 23*a
^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*cosh(x)^6 + 2*(a^8 + 7*a^7*b
+ 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5 + 105*(a^8 + 3*a^7*b + 3
*a^6*b^2 + a^5*b^3)*cosh(x)^4 + 14*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*
b^3 + 8*a^4*b^4)*cosh(x)^2)*sinh(x)^6 + 4*(63*(a^8 + 3*a^7*b + 3*a^6*b^2 +
a^5*b^3)*cosh(x)^5 + 14*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4
*b^4)*cosh(x)^3 + 3*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 +
 8*a^3*b^5)*cosh(x))*sinh(x)^5 - 2*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3
 + 28*a^4*b^4 + 8*a^3*b^5)*cosh(x)^4 - 2*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a
^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5 - 105*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)
*cosh(x)^6 - 35*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*co
sh(x)^4 - 15*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*
b^5)*cosh(x)^2)*sinh(x)^4 + 8*(15*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*cos
h(x)^7 + 7*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*cosh(x)
^5 + 5*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*c
osh(x)^3 - (a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^
5)*cosh(x))*sinh(x)^3 - (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4
*b^4)*cosh(x)^2 + (45*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*cosh(x)^8 - 3*a
^8 - 17*a^7*b - 33*a^6*b^2 - 27*a^5*b^3 - 8*a^4*b^4 + 28*(3*a^8 + 17*a^7*b
+ 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*cosh(x)^6 + 30*(a^8 + 7*a^7*b + 23*a
^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*cosh(x)^4 - 12*(a^8 + 7*a^7*b
 + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*cosh(x)^2)*sinh(x)^2 +
 2*(5*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*cosh(x)^9 + 4*(3*a^8 + 17*a^7*b
 + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*cosh(x)^7 + 6*(a^8 + 7*a^7*b + 23*a
^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*cosh(x)^5 - 4*(a^8 + 7*a^7*b
+ 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*cosh(x)^3 - (3*a^8 + 17
*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*cosh(x))*sinh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+b*sech(x)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)^2}{\left(b\,\mathrm{sech}(x)^2 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(coth(x)^2/(b*sech(x)^2 + a)^(5/2), x)
```

3.220 $\int \frac{1}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=183

$$-\frac{b\left(33a^2+40ab+15b^2\right)\tanh(c+dx)}{15a^3d(a+b)^3\sqrt{a-b\tanh^2(c+dx)+b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a-b\tanh^2(c+dx)+b}}\right)}{a^{7/2}d}-\frac{b(9a+5b)\tanh(c+dx)}{15a^2d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)^{3/2}}\cdot$$

```
[Out] ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + b - b*Tanh[c + d*x]^2]]/(a^(7/2)*d
) - (b*Tanh[c + d*x])/(5*a*(a + b)*d*(a + b - b*Tanh[c + d*x]^2)^(5/2)) - (
b*(9*a + 5*b)*Tanh[c + d*x])/(15*a^2*(a + b)^2*d*(a + b - b*Tanh[c + d*x]^2
)^(3/2)) - (b*(33*a^2 + 40*a*b + 15*b^2)*Tanh[c + d*x])/(15*a^3*(a + b)^3*d
*Sqrt[a + b - b*Tanh[c + d*x]^2])
```

Rubi [A] time = 0.191089, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4128, 414, 527, 12, 377, 206}

$$-\frac{b\left(33a^2+40ab+15b^2\right)\tanh(c+dx)}{15a^3d(a+b)^3\sqrt{a-b\tanh^2(c+dx)+b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a-b\tanh^2(c+dx)+b}}\right)}{a^{7/2}d}-\frac{b(9a+5b)\tanh(c+dx)}{15a^2d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)^{3/2}}\cdot$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^(-7/2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + b - b*Tanh[c + d*x]^2]]/(a^(7/2)*d
) - (b*Tanh[c + d*x])/(5*a*(a + b)*d*(a + b - b*Tanh[c + d*x]^2)^(5/2)) - (
b*(9*a + 5*b)*Tanh[c + d*x])/(15*a^2*(a + b)^2*d*(a + b - b*Tanh[c + d*x]^2
)^(3/2)) - (b*(33*a^2 + 40*a*b + 15*b^2)*Tanh[c + d*x])/(15*a^3*(a + b)^3*d
*Sqrt[a + b - b*Tanh[c + d*x]^2])
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+b\operatorname{sech}^2(c+dx)\right)^{7/2}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\left(a+b-bx^2\right)^{7/2}}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\tanh(c+dx)}{5a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^{5/2}} - \frac{\operatorname{Subst}\left(\int \frac{-5a-b-4bx^2}{(1-x^2)\left(a+b-bx^2\right)^{5/2}}\,dx, x, \tanh(c+dx\right.}{5a(a+b)d}$$

$$= -\frac{b\tanh(c+dx)}{5a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^{5/2}} - \frac{b(9a+5b)\tanh(c+dx)}{15a^2(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)^{3/2}} +$$

$$= -\frac{b\tanh(c+dx)}{5a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^{5/2}} - \frac{b(9a+5b)\tanh(c+dx)}{15a^2(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)^{3/2}} -$$

$$= -\frac{b\tanh(c+dx)}{5a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^{5/2}} - \frac{b(9a+5b)\tanh(c+dx)}{15a^2(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)^{3/2}} -$$

$$= -\frac{b\tanh(c+dx)}{5a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^{5/2}} - \frac{b(9a+5b)\tanh(c+dx)}{15a^2(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)^{3/2}} -$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a+b-b\tanh^2(c+dx)}}\right)}{a^{7/2}d} - \frac{b\tanh(c+dx)}{5a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^{5/2}} - \frac{b(9a+}{15a^2(a+b)^2d\,(a}$$

Mathematica [A] time = 7.45835, size = 330, normalized size = 1.8

$$\frac{\operatorname{sech}^7(c+dx)\left(\frac{15}{4}e^{-7(c+dx)}\left(a\left(e^{2(c+dx)}+1\right)^2+4be^{2(c+dx)}\right)^{7/2}\left(\tanh^{-1}\left(\frac{ae^{2(c+dx)}+a+2b}{\sqrt{a}\sqrt{a\left(e^{2(c+dx)}+1\right)^2+4be^{2(c+dx)}}}\right)-\tanh^{-1}\left(\frac{ae^{2(c+dx)}+a}{\sqrt{a}\sqrt{a\left(e^{2(c+dx)}\right.}}\right.\right.\right.}{96}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^(-7/2),x]
```

```
[Out] (Sech[c + d*x]^7*((15*(4*b*E^(2*(c + d*x)) + a*(1 + E^(2*(c + d*x)))^2)^(7/
2)*(ArcTanh[(a + 2*b + a*E^(2*(c + d*x)))/(Sqrt[a]*Sqrt[4*b*E^(2*(c + d*x))
 + a*(1 + E^(2*(c + d*x)))^2])] - ArcTanh[(a + a*E^(2*(c + d*x)) + 2*b*E^(2
*(c + d*x)))/(Sqrt[a]*Sqrt[4*b*E^(2*(c + d*x)) + a*(1 + E^(2*(c + d*x)))^2]
)]))/(4*E^(7*(c + d*x))) - (4*Sqrt[a]*b*(a + 2*b + a*Cosh[2*(c + d*x)])*(13
5*a^4 + 480*a^3*b + 709*a^2*b^2 + 460*a*b^3 + 120*b^4 + 4*a*(45*a^3 + 135*a
^2*b + 117*a*b^2 + 35*b^3)*Cosh[2*(c + d*x)] + a^2*(45*a^2 + 60*a*b + 23*b^
2)*Cosh[4*(c + d*x)])*Sinh[c + d*x])/(a + b)^3))/(960*a^(7/2)*d*(a + b*Sech
[c + d*x]^2)^(7/2))
```

Maple [F] time = 0.152, size = 0, normalized size = 0.

$$\int \left(a+b\left(\operatorname{sech}\left(dx+c\right)\right)^{2}\right)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(d*x+c)^2)^(7/2),x)
```

```
[Out] int(1/(a+b*sech(d*x+c)^2)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\operatorname{sech}\left(dx+c\right)^{2}+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(d*x + c)^2 + a)^(-7/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\operatorname{sech}(dx+c)^2+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*x + c)^2 + a)^(-7/2), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
    (flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                         #since this estimate of leaf count is bit lower than
```

```
                    #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```